Computing the Central Charge of the 3D Ising CFT Using Quantum Finite Elements

ANNA-MARIA E. GLÜCK – YALE UNIVERSITY & UNIVERSITÄT HEIDELBERG LATTICE 2022 – THEORETICAL DEVELOPMENTS II

Introduction

• 3D Ising model: interacting spins $\sigma = \pm 1$ in 3D









 $T \gg T_c$

• 3D Ising model: interacting spins σ^{\perp}







[Tong 2017]

• 3D Ising model: interacting spins σ^{\perp} '



POV Condensed Matter Physics

• Physical quantities: Power laws

Determination of critical exponents using MC & finite size scaling

[Swendsen, Wang 1987], [Wolff 1989], [Pelissetto, Vicari, 2000]

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$$egin{aligned} C \propto t^{-lpha} & m \propto t^{eta} & \chi \propto t^{-\gamma} & \xi \propto t^{-
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angle \propto r^{-d+2-\eta} & & \ t = rac{|T-T_c|}{T_c} \end{aligned}$$

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POV Continuum Field Theory



Conformal bootstrap (crossing symmetry constraints on n-point functions)+ new quantities beyond FSS results[El-Showk et al.,- unproven assumptions & truncations2012, 20141

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This work: Lattice Method for calculating the same quantities

as the conformal bootstrap and checking their values.

POV Continuum Field Theory



Conformal bootstrap (crossing symmetry constraints on n-

point functions)

[El-Showk et al., 2012, 2014]

- + new quantities beyond FSS results
- unproven assumptions & truncations

Antipodal 4-point function on $\mathbb{R} \times \mathbb{S}^2 \ni (t, \vec{x})$

• OPE for the 4-point function of σ -operators in the 3D Ising CFT

Conformal blocks

$$\frac{\langle \sigma(x_1)\sigma(x_2)\sigma(x_3)\sigma(x_4)\rangle}{\langle \sigma(x_1)\sigma(x_2)\rangle\langle \sigma(x_3)\sigma(x_4)\rangle} = 1 + \sum_{\mathcal{O}} \int_{\sigma\sigma\mathcal{O}}^2 G_{\mathcal{O}}(\Delta_{\mathcal{O}}; x_1, x_2, x_3, x_4)$$

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• In special antipodal frame on $\mathbb{R} \times \mathbb{S}^2$, $G_{\mathcal{O}}$ are particularly simple [Costa et al., 2016]:

$$G_{\mathcal{O},l} = \sum_{\substack{n=0,2,4,\dots \ j \\ j \in \{\max(0,l-n),\dots,l+n-2,l+n\}}} \sum_{\substack{p=0,2,4,\dots \ j \\ expansion}} e^{-(\Delta_{\mathcal{O}}+n)t} B_{n,j}(\Delta_{\mathcal{O}}) P_j(\cos(\theta)) \xrightarrow{\text{Partial wave expansion}} expansion$$



Antipodal 4-point function on $\mathbb{R} \times \mathbb{S}^2 \ni (t, \vec{x})$

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Boltain $f_{\sigma\sigma O}$ and Δ_O by fits to partial wave expansion coefficients for 4-pt function on $\mathbb{R} imes \mathbb{S}^2$

Quantum Finite Elements [Brower et al., 2018, 2021]

- LFT on curved manifolds (here: $\mathbb{R}\times\mathbb{S}^2$)
- Discretize φ^4 -theory on $\mathbb{R} \times \mathbb{S}^2$ using Regge Calculus, DEC & FEM



⇒ convergence to spherically symmetric continuum theory





 $c_{1}>$ Lattice simulations of critical $arphi^{4}$ -theory on $\ \mathbb{R} imes \mathbb{S}^{2}$ possible

Numerical Results

MONTE CARLO SIMULATIONS OF CRITICIAL φ^4 -THEORY,

PERIODIC BOUNDARY CONDITIONS



Numerical results

$$\frac{\langle \sigma(x_1)\sigma(x_2)\sigma(x_3)\sigma(x_4)\rangle}{\langle \sigma(x_1)\sigma(x_2)\rangle\langle \sigma(x_3)\sigma(x_4)\rangle} = \sum_{\text{even } j} c_j(\Delta t)P_j(\cos(\theta)) = 1 + \sum_{\mathcal{O}} f_{\sigma\sigma\mathcal{O}}^2 \sum_{n=0,2,4,\dots} \sum_j e^{-(\Delta_{\mathcal{O}}+n)c_Rgt} B_{n,j}(\Delta_{\mathcal{O}})P_j(\cos(\theta))$$



Simultaneous fits of c_0(t) and c_2(t) using primaries $\,\epsilon,\ T,\ \epsilon',\ T'\,$ up to n=20



Model averaging results for c_0 and c_2

Simultaneous fits of $c_0(t)$ and $c_2(t)$ using 4 primaries ϵ , T, ϵ' , T' & different t-ranges



Model averaging – leading terms





Model averaging – central charge



$$c/c_{free} = \frac{\Delta_{\sigma}^2 \Delta_T^2}{3f_{\sigma\sigma T}^2}$$

 $c^{bootstrap}/c_{free} = 0.946534(11)$

$$c^{fit}/c_{free} = 0.9041(37)$$

Model averaging - ratios

Simultaneous fits of $c_0(t)$ and $c_2(t)$ using 4 primaries ϵ , T, ϵ' , T' & different t-ranges



$$c/c_{free} = \frac{\Delta_{\sigma}^2 \Delta_T^2}{3f_{\sigma\sigma T}^2}$$

Conclusion and outlook

- Good agreeance for results already established in Monte Carlo
- Central charge result differs from bootstrap values

Improve estimates & elimate possible systematic errors by

- 1. Increasing statistics
- 2. Including more primaries -> less excited state contamination
- 3. Adding higher partial waves c_4 , c_6 , ... to the joined fits
- 4. Investigate other possible sources for systematic errors
- Soon: use slightly different approaches to investigate the CFT

Acknowledgements

I would like to thank my collaborators and co-authors

- George T. Fleming, Yale University
- Richard C. Brower, Boston University
- Venkitesh Ayyar, Boston University
- Evan Owen, Boston University
- Timothy G. Raben, Michigan State University
- Chung-I Tan, Brown University

Thank you for your attention!

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First recursive fits for c₄

