# The emergence of expanding space-time in a novel large-N limit of the Lorentzian type IIB matrix model

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39th International Symposium on Lattice Field Theory 11 August 2022, Bonn, Germany

#### superstring theory and the matrix model

superstring theory

- promising candidate for quantum gravity
- consistent only in 10D space-time
- how to describe our 4D space-time in superstring theory
   → compactification
  - (3+1)D space-time + 6 extra dimension(compact, small)
  - Number of perturbatively stable vacua is extremely large.

Type IIB matrix model is a promising candidate for non-perturbative formulation of superstring theory.

## definition of the Lorentzian type IIB matrix model

• partition function and the action

 $Z = \int dA d\Psi d\bar{\Psi} \ e^{iS}$   $S = -N\beta \text{Tr} \left\{ -\frac{1}{2} [A_0, A_i]^2 + \frac{1}{4} [A_i, A_j]^2 + \frac{1}{2} \bar{\Psi}_{\alpha} (C\Gamma^{\mu})_{\alpha\beta} [A_{\mu}, \Psi_{\beta}] \right\}$   $A_{\mu}, \Psi_{\alpha} : N \times N \text{ Hermitian matrices}$   $(\mu = 0, \dots, 9, \quad \alpha = 1, 2, \dots, 16)$ 

- This model has  $\mathcal{N}=2$  SUSY. evidence for the fact that this model includes gravity
- emergence of space-time space-time coordinates : eigenvalues of  $A_{\mu}$
- This model has SO(9,1) Lorentz symmetry.

• Wick rotation of the matrices

Lorentzian 
$$\rightarrow$$
 Euclidean  
 $A_0 \rightarrow \tilde{A}_0 = \frac{ie^{-i\frac{1}{8}\pi}A_0}{A_i} = e^{i\frac{3}{8}\pi}A_0$   
 $A_i \rightarrow \tilde{A}_i = e^{-i\frac{1}{8}\pi}A_i$ 

i : the target space Wick rotation  $e^{-i\frac{1}{8}\pi}$  : the worldsheet Wick rotation

• equivalence between the Euclidean and Lorentzian model  $\left\langle \frac{1}{N} \operatorname{Tr}(A_0)^2 \right\rangle_{\mathrm{L}} = e^{-i\frac{3}{4}\pi} \left\langle \frac{1}{N} \operatorname{Tr}(\tilde{A}_0)^2 \right\rangle_{\mathrm{E}}, \qquad \left\langle \frac{1}{N} \operatorname{Tr}(A_i)^2 \right\rangle_{\mathrm{L}} = e^{i\frac{1}{4}\pi} \left\langle \frac{1}{N} \operatorname{Tr}(\tilde{A}_i)^2 \right\rangle_{\mathrm{E}}$ 



## previous works on type IIB matrix model

#### Euclidean version of type IIB matrix model

[J. Nishimura, T. Okubo, and F. Sugino (2011)]

[K. Anagnostopoulos, T. Azuma, Y. Ito, J. Nishimura, T. Okubo, S. Papadoudis (2020)]

- Complex Langevin method works in this model.
- SSB: SO(10)  $\rightarrow$  SO(3) occurs dynamically.
  - SO(4) does not appear.
- Relation between the emergent space and our universe is not clear.

# previous works on type IIB matrix model

classical solutions of the Lorentzian type IIB matrix model

[K. Hatakeyama, A. Matsumoto, J. Nishimura, A. Tsuchiya, A. Yosprakob (2019)]

• solving the equation of motion.

$$[A^{\nu}, [A_{\nu}, A_{\mu}]] = 0.$$

- The solution to this EOM is exhausted by diagonal matrices.
- Non-trivial vacua do not appear.
- introducing an additional term

$$[A^{\nu}, [A_{\nu}, A_{\mu}]] - \gamma A_{\mu} = 0.$$
  
(\gamma > 0)

 Typical solutions have expanding space although its dimensionality is not fixed.

# In this talk, we focus on the non-perturbative aspects of the Lorentzian type IIB matrix model.

## novel large-N limit

• In order to obtain a large-N limit inequivalent to the Euclidean model, we add a Lorentz invariant "mass" term to the action.

$$S_{\gamma} = -\frac{1}{2}N\gamma \operatorname{Tr}(A_{\mu})^{2} = \frac{1}{2}N\gamma \left\{ \operatorname{Tr}(A_{0})^{2} - \operatorname{Tr}(A_{i})^{2} \right\}$$

Motivation for this extra mass term comes from the previous work on classical solutions.

$$[A^{\nu}, [A_{\nu}, A_{\mu}]] - \gamma A_{\mu} = 0$$

[K. Hatakeyama, A. Matsumoto, J. Nishimura, A. Tsuchiya, A. Yosprakob (2019)] [H. Steinacker (2017)]

We consider taking the  $\gamma \to 0^+$  limit after taking the large-N limit.

#### complex Langevin method

• We choose an SU(N) basis :

 $A_0 = \operatorname{diag}(\alpha_1, \alpha_2, \dots, \alpha_N)$ 

• a way to realize the ordering :  $\alpha_1 < \alpha_2 < \cdots < \alpha_N$ 

$$\alpha_1 = 0, \ \alpha_2 = e^{\tau_1}, \ \alpha_3 = e^{\tau_1} + e^{\tau_2}, \ \dots, \ \alpha_N = \sum_{a=1}^{N-1} e^{\tau_a}$$

• complexify the variables

[J. Nishimura, A. Tsuchiya (2019)]

 $A_i:$  Hermitian matrices  $\rightarrow\,$  general matrices  $\tau_a:\, {\rm real}\,\,\rightarrow\, {\rm complex}$ 

complex Langevin equation

 $\frac{d\tau_a}{dt_{\rm L}} = -\frac{\partial S}{\partial \tau_a} + \eta_a(t_{\rm L}), \quad \frac{d(A_i)_{ab}}{dt_{\rm L}} = -\frac{\partial S}{\partial (A_i)_{ba}} + (\eta_i)_{ab}(t_{\rm L})$ 

#### criterion for the correct convergence

The drift histogram falls off exponentially or faster with the magnitude of the drift term.

[K. Nagata, J. Nishimura, S. Shimasaki (2016)]

• singular drift problem - a cause of wrong convergence -

If the Dirac operator has eigenvalues that are almost 0, then the criterion is not satisfied.

adding fermionic mass term

 $S_{m_{\rm f}} = iNm_{\rm f} {\rm Tr}[\bar{\Psi}_{\alpha}(\Gamma_7\Gamma_8^{\dagger}\Gamma_9)_{\alpha\beta}\Psi_{\beta}]$ 

[K. Anagnostopoulos, T. Azuma, Y. Ito, J. Nishimura, T. Okubo, S. Papadoudis (2020)]  $m_{\rm f}=\infty$  corresponds to the fermion quenched model.

We need to make the  $m_{\rm f} \rightarrow 0$  extrapolation eventually.

• We perform the following procedure at each Langevin step for stabilization. (c.f. [F. Attanasio, B. Jäger (2018]])

$$A_i \to \frac{1}{1+\epsilon} \left( A_i + \epsilon A_i^{\dagger} \right)$$

#### phase structure for various $\gamma$

Lorentz invariant mass term:  $\frac{1}{2}N\gamma \{ \operatorname{Tr}(A_0)^2 - \operatorname{Tr}(A_i)^2 \}$ 



#### how to extract time-evolution

• band diagonal structure (dynamical property)





how to extract time-evolution



• definition of time  $t_a = \sum_{i=1}^{a} |\bar{\alpha}_i - \bar{\alpha}_{i-1}|, \quad \bar{\alpha}_i = \frac{1}{n} \sum_{j=0}^{n-1} \alpha_{i+j}$ (n: band width) •  $\bar{A}_i(t_a)$  ( $n \times n$  matrix) represents the state of the universe at  $t_a$ .

# SSB of SO(9) symmetry

- order parameter for SSB of SO(9) the eigenvalues of "moment of inertia tensor"  $T_{ij}(t) = \frac{1}{n} \operatorname{tr} (X_i(t)X_j(t)), \quad X_i(t) \equiv \frac{1}{2} \left( \bar{A}_i(t) + \bar{A}_i^{\dagger}(t) \right)$ 
  - SO(9) symmetric: 9 eigenvalues are almost degenerate.
  - SO(9) broken: 9 eigenvalues are NOT degenerate.

$$N = 64, m_{\rm f} = 10, \gamma = 2.6, n = 12$$



## $\gamma$ dependence



- 1d expansion occurs.
- Time range becomes longer at smaller  $\gamma$ .
- $\bullet\,$  In the real time phase, space expansion increases as  $\gamma$  decreases.

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### fermionic effects

 $N = 64, \ \gamma = 2.6, \ n = 12$ fermionic mass term:  $iNm_{\rm f}{\rm Tr}[\bar{\Psi}_{\alpha}(\Gamma_{7}\Gamma_{8}^{\dagger}\Gamma_{9})_{\alpha\beta}\Psi_{\beta}]$  $m_{\rm f} = \infty$  corresponds to the fermion quenched model.



- In the real time phase, space expansion increases as m<sub>f</sub> decreases.
  - The attractive force between space-time eigenvalues is weakened by the SUSY effects.

#### summary and discussion

- We successfully applied the complex Langevin method to the Lorentzian type IIB matrix model.
- $\bullet\,$  equivalence between the Euclidean and Lorentzian model in the conventional large- $N\,$  limit
  - Euclidean model exhibits SSB:  $SO(10) \rightarrow SO(3)$ . The space-time becomes complex and it has Euclidean signature.
- introducing the Lorentz invariant mass term
  - Real space-time appears at late times.
  - expansion of space (1d)
  - Fermionic effects are important for the emergence of 3d space. Pfaffian becomes zero if there are only two large matrices:

 $A_1, A_2 \neq 0, A_3, \dots, A_9 = 0. \quad \mbox{[W. Krauth, H. Nicolai, M. Staudacher (1998)]} \\ \mbox{[J. Nishimura, G. Vernizzi (2000)]}$ 

(Due to the exponential expansion of space,  ${\it A}_0$  cannot play any role here.)

 $\rightarrow$  3d expanding space may be favored by the Pfaffian.

### summary and discussion

Novel large-N limit

our prediction for a phase diagram (  $\gamma,\ 1/N$  )



- We expect that 3d expanding space appears in the novel large-N limit.
- We are now trying to see whether  $SO(9) \rightarrow SO(3)$  occurs by decreasing  $m_f$  further. (*c.f.* Studies of the Euclidean model)

#### Thank you for listening!

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# $\gamma$ dependence



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#### fermionic effects

$$N = 64, \ \gamma = 2.6, \ n = 12$$

fermionic mass term:  $iNm_{\rm f} {\rm Tr}[\bar{\Psi}_{\alpha}(\Gamma_7 \Gamma_8^{\dagger} \Gamma_9)_{\alpha\beta} \Psi_{\beta}]$  $m_{\rm f} = \infty$  corresponds to the fermion quenched model.

