The emergence of expanding space-time in a novel large-$N$ limit of the Lorentzian type IIB matrix model

Mitsuaki Hirasawa

Based on the collaboration with Konstantinos Anagnostopoulos, Takehiro Azuma, Kohta Hatakeyama, Jun Nishimura, Stratos Papadoudis, Asato Tsuchiya

$^1$INFN Milano-Bicocca, $^2$NTUA, $^3$Setsunan Univ., $^4$KEK, $^5$SOKENDAI, $^6$Shizuoka Univ.

39th International Symposium on Lattice Field Theory
11 August 2022, Bonn, Germany
superstring theory and the matrix model

superstring theory

- promising candidate for quantum gravity
- consistent only in 10D space-time

how to describe our 4D space-time in superstring theory
→ compactification

- $(3+1)D$ space-time + 6 extra dimension (compact, small)
- Number of perturbatively stable vacua is extremely large.

Type IIB matrix model is a promising candidate for non-perturbative formulation of superstring theory.
partition function and the action

\[ Z = \int dA d\Psi d\bar{\Psi} \ e^{iS} \]

\[ S = -N\beta \text{Tr} \left\{ -\frac{1}{2} [A_0, A_i]^2 + \frac{1}{4} [A_i, A_j]^2 + \frac{1}{2} \bar{\Psi}_\alpha (CT^\mu)_{\alpha\beta} [A_\mu, \Psi_\beta] \right\} \]

\[ A_\mu, \Psi_\alpha : N \times N \text{ Hermitian matrices} \]

\[ (\mu = 0, \ldots, 9, \quad \alpha = 1, 2, \ldots, 16) \]

This model has \( \mathcal{N} = 2 \) SUSY.

evidence for the fact that this model includes gravity

emergence of space-time

space-time coordinates : eigenvalues of \( A_\mu \)

This model has SO(9,1) Lorentz symmetry.
equivalence between the Euclidean and Lorentzian model

- Wick rotation of the matrices

  Lorentzian $\rightarrow$ Euclidean

  \[
  A_0 \rightarrow \tilde{A}_0 = ie^{-i\frac{1}{8}\pi}A_0 = e^{i\frac{3}{8}\pi}A_0
  \]

  \[
  A_i \rightarrow \tilde{A}_i = e^{-i\frac{1}{8}\pi}A_i
  \]

  $i$: the target space Wick rotation
  $e^{-i\frac{1}{8}\pi}$: the worldsheet Wick rotation

- equivalence between the Euclidean and Lorentzian model

  \[
  \langle \frac{1}{N} \text{Tr}(A_0)^2 \rangle_L = e^{-i\frac{3}{4}\pi} \langle \frac{1}{N} \text{Tr}(\tilde{A}_0)^2 \rangle_E ,
  \]

  \[
  \langle \frac{1}{N} \text{Tr}(A_i)^2 \rangle_L = e^{i\frac{1}{4}\pi} \langle \frac{1}{N} \text{Tr}(\tilde{A}_i)^2 \rangle_E
  \]
previous works on type IIB matrix model

Euclidean version of type IIB matrix model

[J. Nishimura, T. Okubo, and F. Sugino (2011)]

[K. Anagnostopoulos, T. Azuma, Y. Ito, J. Nishimura, T. Okubo, S. Papadoudis (2020)]

- Complex Langevin method works in this model.

- SSB: $\text{SO}(10) \rightarrow \text{SO}(3)$ occurs dynamically.
  - $\text{SO}(4)$ does not appear.

- Relation between the emergent space and our universe is not clear.
previous works on type IIB matrix model

classical solutions of the Lorentzian type IIB matrix model

[K. Hatakeyama, A. Matsumoto, J. Nishimura, A. Tsuchiya, A. Yosprakob (2019)]

- solving the equation of motion.

\[ [A^\nu, [A_\nu, A_\mu]] = 0. \]

- The solution to this EOM is exhausted by diagonal matrices.

- Non-trivial vacua do not appear.

- introducing an additional term

\[ [A^\nu, [A_\nu, A_\mu]] - \gamma A_\mu = 0. \]

\( (\gamma > 0) \)

- Typical solutions have expanding space although its dimensionality is not fixed.

In this talk, we focus on the non-perturbative aspects of the Lorentzian type IIB matrix model.
In order to obtain a large-$N$ limit inequivalent to the Euclidean model, we add a Lorentz invariant “mass” term to the action.

\[ S_\gamma = -\frac{1}{2} N \gamma \text{Tr}(A_\mu)^2 = \frac{1}{2} N \gamma \left\{ \text{Tr}(A_0)^2 - \text{Tr}(A_i)^2 \right\} \]

Motivation for this extra mass term comes from the previous work on classical solutions.

\[ [A^\nu, [A_\nu, A_\mu]] - \gamma A_\mu = 0 \]


We consider taking the $\gamma \rightarrow 0^+$ limit after taking the large-$N$ limit.
complex Langevin method

- We choose an SU(\(N\)) basis:
  \[ A_0 = \text{diag}(\alpha_1, \alpha_2, \ldots, \alpha_N) \]

- a way to realize the ordering: \(\alpha_1 < \alpha_2 < \cdots < \alpha_N\)
  \[ \alpha_1 = 0, \alpha_2 = e^{\tau_1}, \alpha_3 = e^{\tau_1 + \tau_2}, \ldots, \alpha_N = \sum_{a=1}^{N-1} e^{\tau_a} \]

- complexify the variables
  \[ A_i : \text{Hermitian matrices} \rightarrow \text{general matrices} \]
  \[ \tau_a : \text{real} \rightarrow \text{complex} \]

- complex Langevin equation
  \[ \frac{d\tau_a}{dt_L} = -\frac{\partial S}{\partial \tau_a} + \eta_a(t_L), \quad \frac{d(A_i)_{ab}}{dt_L} = -\frac{\partial S}{\partial (A_i)_{ba}} + (\eta_i)_{ab}(t_L) \]

**criterion for the correct convergence**

The drift histogram falls off exponentially or faster with the magnitude of the drift term.

- [K. Nagata, J. Nishimura, S. Shimasaki (2016)]
- [J. Nishimura, A. Tsuchiya (2019)]
complex Langevin method

- singular drift problem - a cause of wrong convergence -
  If the Dirac operator has eigenvalues that are almost 0, then the criterion is not satisfied.

- adding fermionic mass term

\[ S_{m_f} = iN m_f \text{Tr}[\bar{\Psi}_\alpha (\Gamma_7 \Gamma_8^\dagger \Gamma_9)_{\alpha\beta} \Psi_\beta] \]

[K. Anagnostopoulos, T. Azuma, Y. Ito, J. Nishimura, T. Okubo, S. Papadoudis (2020)]

\[ m_f = \infty \] corresponds to the fermion quenched model.

We need to make the \( m_f \to 0 \) extrapolation eventually.

- We perform the following procedure at each Langevin step for stabilization.

\[ A_i \to \frac{1}{1+\epsilon} \left( A_i + \epsilon A_i^\dagger \right) \]

(c.f. [F. Attanasio, B. Jäger (2018)])
phase structure for various $\gamma$

Lorentz invariant mass term: $\frac{1}{2} N \gamma \left\{ \text{Tr}(A_0)^2 - \text{Tr}(A_i)^2 \right\}$

- $\gamma \leq 1.8$: qualitatively the same as the Euclidean model ($\gamma = 0$)
- $\gamma \geq 2.6$: Time is almost real.

We focus on the real time phase.
how to extract time-evolution

- band diagonal structure (dynamical property)

\[ A_{pq} \equiv \frac{1}{9} \sum_i |(A_i)_{pq}|^2 \]

- how to extract time-evolution

\[
A_0 = \begin{pmatrix}
\alpha_1 & \alpha_2 \\
\bar{\alpha}_1 & \bar{\alpha}_2 \\
\alpha_3 & \alpha_4 \\
\bar{\alpha}_3 & \bar{\alpha}_4 \\
\alpha_N & \alpha_N \\
\end{pmatrix}
\]

\[
A_i = \begin{pmatrix}
1 & 2 & \cdots & \cdots & a \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
2 & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
-0 & \cdots & \cdots & \cdots & \cdots \\
\end{pmatrix}
\]

- definition of time

\[
t_a = \sum_{i=1}^{a} |\bar{\alpha}_i - \bar{\alpha}_{i-1}|, \quad \bar{\alpha}_i = \frac{1}{n} \sum_{j=0}^{n-1} \alpha_{i+j}
\]

(n: band width)

- \( \bar{A}_i(t_a) \) (n \times n matrix) represents the state of the universe at \( t_a \).
SSB of SO(9) symmetry

- order parameter for SSB of SO(9)
- the eigenvalues of “moment of inertia tensor”
  \[ T_{ij}(t) = \frac{1}{n} \text{tr} \left( X_i(t)X_j(t) \right), \quad X_i(t) \equiv \frac{1}{2} \left( \bar{A}_i(t) + \bar{A}_i^\dagger(t) \right) \]
- SO(9) symmetric: 9 eigenvalues are almost degenerate.
- SO(9) broken: 9 eigenvalues are NOT degenerate.

\[ N = 64, \; m_f = 10, \; \gamma = 2.6, \; n = 12 \]

SSB of SO(9) occurs.
1d space expands exponentially.

\[ \text{tr}(\bar{A}_i(t))^2 = e^{2i\theta_s(t)} \text{tr}(\bar{A}_i(t))^2 \]

Space becomes real at late times.

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γ dependence

\[ N = 64, \ m_f = 10, \ \gamma = 2.6, \ n = 12 \]

\[ \gamma = 4.0 \quad \gamma = 2.6 \]

- 1d expansion occurs.
- Time range becomes longer at smaller \( \gamma \).
- In the real time phase, space expansion increases as \( \gamma \) decreases.


fermionic effects

\[ N = 64, \ \gamma = 2.6, \ \eta = 12 \]

fermionic mass term: \( iN m_f \text{Tr} \left[ \bar{\Psi}_\alpha (\Gamma_7 \Gamma_8 \Gamma_9)_{\alpha \beta} \Psi_\beta \right] \)

\( m_f = \infty \) corresponds to the fermion quenched model.

\[ m_f = 10 \]

\[ m_f = 5 \]

- In the real time phase, space expansion increases as \( m_f \) decreases.
  - The attractive force between space-time eigenvalues is weakened by the SUSY effects.
summary and discussion

- We successfully applied the complex Langevin method to the Lorentzian type IIB matrix model.

- Equivalence between the Euclidean and Lorentzian model in the conventional large-$N$ limit
  - Euclidean model exhibits SSB: $\text{SO}(10) \rightarrow \text{SO}(3)$.
  - The space-time becomes complex and it has Euclidean signature.

- Introducing the Lorentz invariant mass term
  - Real space-time appears at late times.
  - Expansion of space (1d)
  - Fermionic effects are important for the emergence of 3d space.
    Pfaffian becomes zero if there are only two large matrices:
    $$A_1, A_2 \neq 0, A_3, \ldots, A_9 = 0.$$  
    [W. Krauth, H. Nicolai, M. Staudacher (1998)]  
    [J. Nishimura, G. Vernizzi (2000)]
    (Due to the exponential expansion of space, $A_0$ cannot play any role here.)

- $\rightarrow$ 3d expanding space may be favored by the Pfaffian.
Novel large-$N$ limit

our prediction for a phase diagram $(\gamma, 1/N)$

We expect that 3d expanding space appears in the novel large-$N$ limit.

We are now trying to see whether $SO(9) \rightarrow SO(3)$ occurs by decreasing $m_f$ further.  

(c.f. Studies of the Euclidean model)

Thank you for listening!
\[ N = 64, \quad m_f = 10, \quad \gamma = 2.6, \quad n = 12 \]

\[ \gamma = 4.0 \]

\[ a = 0.44(2) \]
\[ b = 0.94(2) \]
\[ c = -0.47(4) \]

\[ \gamma = 2.6 \]

\[ a = 0.82(9) \]
\[ b = 0.78(3) \]
\[ c = -0.98(17) \]
fermionic effects

$N = 64, \ \gamma = 2.6, \ n = 12$

fermionic mass term: $iN m_f \text{Tr}[\bar{\Psi}_\alpha (\Gamma_7 \Gamma_8 \Gamma_9)_{\alpha \beta} \Psi_\beta]$

$m_f = \infty$ corresponds to the fermion quenched model.

$m_f = 10$

$a = 0.82(9)$

$b = 0.78(3)$

$c = -0.98(17)$

$m_f = 5$

$a = 2.54(15)$

$b = 0.54(1)$

$c = -3.1(2)$