The emergence of expanding space-time in a novel large-N limit of the Lorentzian type IIB matrix model

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superstring theory and the matrix model

superstring theory

- promising candidate for quantum gravity
- consistent only in 10D space-time
- how to describe our 4D space-time in superstring theory

  ! **compactification**

  - (3+1)D space-time + 6 extra dimension(compact, small)
  - Number of perturbatively stable vacua is extremely large.

Type IIB matrix model is a promising candidate
for non-perturbative formulation of superstring theory.

de nition of the Lorentzian type IIB matrix model

- partition function and the action

$$Z = \int dA d \;\; d \;\; e^{iS}$$

$$S = \;\; N\beta \mathrm{Tr} \;\; \frac{1}{2}[A_0, A_i]^2 + \frac{1}{4}[A_i, A_j]^2 + \frac{1}{2} \;\; _\alpha(C \;\; ^\mu)_{\alpha\beta}[A_\mu, \;\; _\beta]$$

$$A_\mu, \;\; _\alpha : N \;\; N \text{ Hermitian matrices}$$

$$(\mu = 0, \ldots, 9, \quad \alpha = 1, 2, \ldots, 16)$$

- This model has $N = 2$ SUSY.
  evidence for the fact that this model includes gravity
- emergence of space-time
  space-time coordinates : eigenvalues of A
- This model has SO(9,1) Lorentz symmetry.

- Wick rotation of the matrices

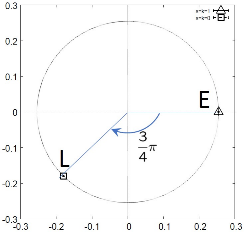
Lorentzian	!	Euclidean
A_0	!	$\tilde{A}_0 = i e^{i\frac{1}{8}\pi} A_0 = e^{i\frac{3}{8}\pi} A_0$
A_i	!	$\tilde{A}_i = e^{i\frac{1}{8}\pi} A_i$

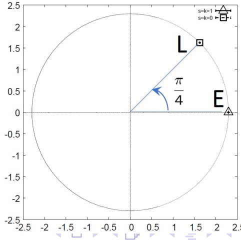
i : the target space Wick rotation

$e^{i\frac{1}{8}\pi}$: the worldsheet Wick rotation

- equivalence between the Euclidean and Lorentzian model

$$\left\langle \frac{1}{N}\mathrm{Tr}(A_0)^2 \right\rangle_{\mathrm{L}} = e^{i\frac{3}{4}\pi} \left\langle \frac{1}{N}\mathrm{Tr}(\tilde{A}_0)^2 \right\rangle_{\mathrm{E}} ; \qquad \left\langle \frac{1}{N}\mathrm{Tr}(A_i)^2 \right\rangle_{\mathrm{L}} = e^{i\frac{1}{4}\pi} \left\langle \frac{1}{N}\mathrm{Tr}(\tilde{A}_i)^2 \right\rangle_{\mathrm{E}}$$





previous works on type IIB matrix model

Euclidean version of type IIB matrix model

[J. Nishimura, T. Okubo, and F. Sugino (2011)]

[K. Anagnostopoulos, T. Azuma, Y. Ito, J. Nishimura, T. Okubo, S. Papadoudis (2020)]

- Complex Langevin method works in this model.
- SSB: SO(10) ! SO(3) occurs dynamically.
  - SO(4) does not appear.
- Relation between the emergent space and our universe is not clear.

previous works on type IIB matrix model

classical solutions of the Lorentzian type IIB matrix model

[K. Hatakeyama, A. Matsumoto, J. Nishimura, A. Tsuchiya, A. Yosprakob (2019)]

- solving the equation of motion.

$$[A \;;[A \;;A \;]] = 0:$$

  - The solution to this EOM is exhausted by diagonal matrices.
  - Non-trivial vacua do not appear.
- introducing an additional term

$$[A \;;[A \;;A \;]] \qquad A \quad = 0:$$
$$(\quad > 0)$$

  - Typical solutions have expanding space although its dimensionality is not fixed.

In this talk, we focus on the non-perturbative aspects of the Lorentzian type IIB matrix model.

novel large-N limit

- In order to obtain a large-N limit inequivalent to the Euclidean model, we add a Lorentz invariant "mass" term to the action.

$$S \;=\; \frac{1}{2}N \;\; \mathrm{Tr}(A\quad)^2 = \frac{1}{2}N \quad \mathrm{Tr}(A_0)^2 \quad \mathrm{Tr}(A_i)^2$$

Motivation for this extra mass term comes from the previous work on classical solutions.

$$[A\ ;[A\ ;A\ ]] \qquad A\ = 0$$

[K. Hatakeyama, A. Matsumoto, J. Nishimura, A. Tsuchiya, A. Yosprakob (2019)] [H. Steinacker (2017)]

We consider taking the $\;!\; 0^+$ limit after taking the large-N limit.

complex Langevin method

[G. Parisi (1983)] [J. Klauder (1984)]

- We choose an SU(N) basis :

$$A_0 = \mathrm{diag}(\alpha_1, \alpha_2, \ldots, \alpha_N)$$

- a way to realize the ordering : $\alpha_1 < \alpha_2 < \cdots < \alpha_N$

$$\alpha_1 = 0, \quad \alpha_2 = e^{\tau_1}, \quad \alpha_3 = e^{\tau_1} + e^{\tau_2}, \ \ldots, \quad \alpha_N = \sum_{a=1}^{N-1} e^{\tau_a}$$

[J. Nishimura, A. Tsuchiya (2019)]

- complexify the variables

$$A_i : \text{ Hermitian matrices } \to \text{ general matrices}$$
$$\tau_a : \text{ real } \to \text{ complex}$$

- complex Langevin equation

$$\frac{d\tau_a}{dt_{\mathrm{L}}} = -\frac{\partial S}{\partial \tau_a} + \eta_a(t_{\mathrm{L}}), \quad \frac{d(A_i)_{ab}}{dt_{\mathrm{L}}} = -\frac{\partial S}{\partial (A_i)_{ba}} + (\eta_i)_{ab}(t_{\mathrm{L}})$$

criterion for the correct convergence

The drift histogram falls off exponentially or faster
with the magnitude of the drift term.

[K. Nagata, J. Nishimura, S. Shimasaki (2016)]

complex Langevin method

[G. Parisi (1983)] [J. Klauder (1984)]

- singular drift problem - a cause of wrong convergence -

  If the Dirac operator has eigenvalues that are almost 0,
  then the criterion is not satisfied.

  - adding fermionic mass term

$$S_{m_\mathrm{f}} = iNm_\mathrm{f}\mathrm{Tr}[\quad (\ _7 \ {}^{y}_{8} \ _9) \qquad]$$

[K. Anagnostopoulos, T. Azuma, Y. Ito, J. Nishimura, T. Okubo, S. Papadoudis (2020)]

m_f = *1* corresponds to the fermion quenched model.

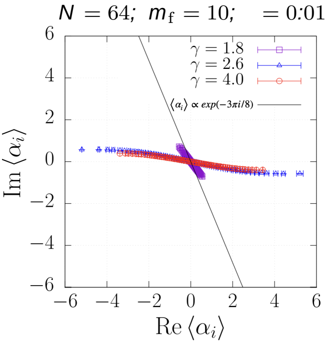
We need to make the m_f ! 0 extrapolation eventually.

- We perform the following procedure at each Langevin step for stabilization. (*c.f.* [F. Attanasio, B. Jäger (2018)])

$$A_i \ ! \ \frac{1}{1+} \ A_i + A_i^{y}$$

phase structure for various

Lorentz invariant mass term: $\frac{1}{2}N \quad f\,\mathrm{Tr}(A_0)^2 \quad \mathrm{Tr}(A_i)^2 g$



- 1:8 :
  qualitatively the same as the Euclidean model (= 0)

  1st order phase transition (?)

- 2:6 :
  Time is almost real.

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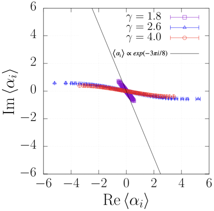
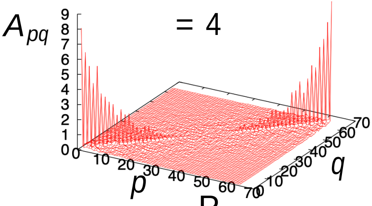
We focus on the real time phase.

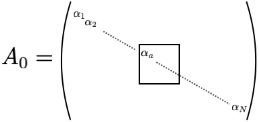
- band diagonal structure (dynamical property)

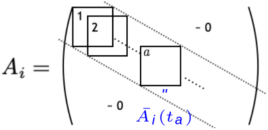




$$A_{pq} \quad \frac{1}{9} \sum_i |(A_i)_{pq}|^2$$

- how to extract time-evolution





- definition of time

$$t_a = \sum_{i=1}^{a} |\bar{\alpha}_i - \bar{\alpha}_{i-1}|, \quad \bar{\alpha}_i = \frac{1}{n} \sum_{j=0}^{n-1} \alpha_{i+j}$$

(n: band width)

- $A_i(t_a)$ ($n \times n$ matrix) represents the state of the universe at t_a.

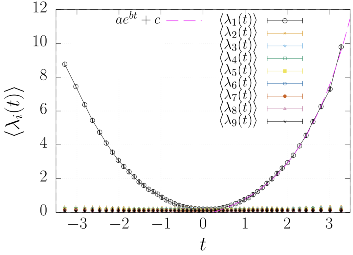
- order parameter for SSB of SO(9)

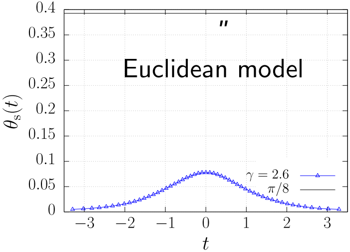
  the eigenvalues of "moment of inertia tensor"

$$T_{ij}(t) = \frac{1}{n}\mathrm{tr}\,(X_i(t)X_j(t))\,, \quad X_i(t) \quad \frac{1}{2} \quad A_i(t) + A_i^{y}(t)$$

  - SO(9) symmetric: 9 eigenvalues are almost degenerate.
  - SO(9) broken: 9 eigenvalues are NOT degenerate.

$N = 64,\ m_{\mathrm{f}} = 10,\ \gamma = 2.6,\ n = 12$





SSB of SO(9) occurs.

1d space expands exponentially.

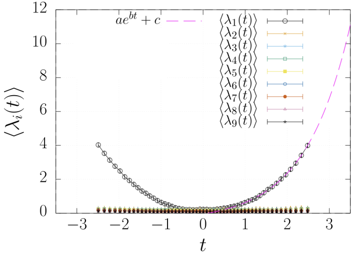
$$\mathrm{tr}(A_i(t))^2 = e^{2i\theta_s(t)} j\mathrm{tr}(A_i(t))^2 j$$

Space becomes real at late times.

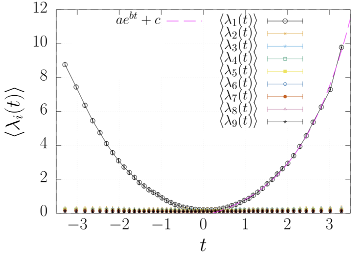
dependence

$N = 64$; $m_{\mathrm{f}} = 10$; $= 2{:}6$; $n = 12$

$= 4{:}0$



$= 2{:}6$



- 1d expansion occurs.
- Time range becomes longer at smaller .
- In the real time phase, space expansion increases as decreases.

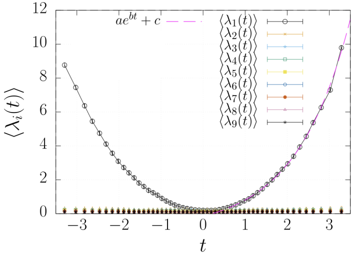
fermionic e ects

$$N = 64; \quad = 2{:}6; \; n = 12$$

fermionic mass term: $iNm_{\mathrm{f}}\mathrm{Tr}[\quad (\; _7 \; ^y_8 \; _9) \quad]$

$m_{\mathrm{f}} = 1$ corresponds to the fermion quenched model.

$m_{\mathrm{f}} = 10$ $\qquad\qquad$ $m_{\mathrm{f}} = 5$



- In the real time phase, space expansion increases as m_{f} decreases.
  - The attractive force between space-time eigenvalues is weakened by the SUSY effects.

summary and discussion

- We successfully applied the complex Langevin method to the Lorentzian type IIB matrix model.

- equivalence between the Euclidean and Lorentzian model in the conventional large-N limit
  - Euclidean model exhibits SSB: SO(10) $\to$ SO(3).
    The space-time becomes complex and it has Euclidean signature.

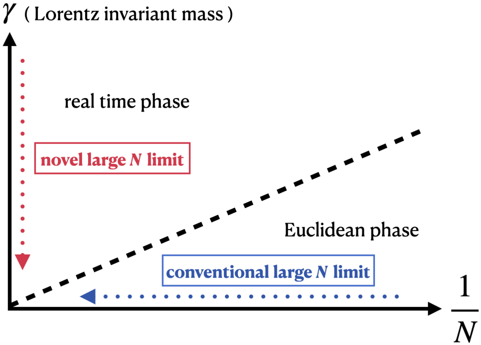
- introducing the Lorentz invariant mass term
  - Real space-time appears at late times.
  - expansion of space (1d)
  - Fermionic effects are important for the emergence of 3d space.
    Pfaffian becomes zero if there are only two large matrices:
    $$A_1, A_2 \neq 0, A_3, \ldots, A_9 = 0.$$
    [W. Krauth, H. Nicolai, M. Staudacher (1998)]
    [J. Nishimura, G. Vernizzi (2000)]

    (Due to the exponential expansion of space, A_0 cannot play any role here.)

$\to$ 3d expanding space may be favored by the Pfaffian.

Novel large-N limit

our prediction for a phase diagram (; 1=N)



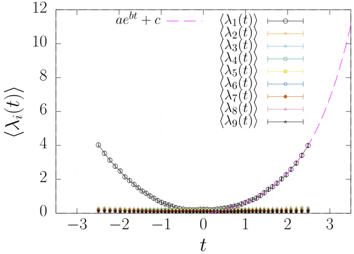
- We expect that 3d expanding space appears in the novel large-N limit.
- We are now trying to see whether SO(9) ! SO(3) occurs by decreasing m_f further. (*c.f.* Studies of the Euclidean model)

Thank you for listening!

dependence

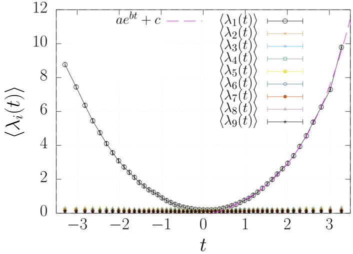
$$N = 64;\ m_{\mathrm{f}} = 10;\quad = 2{:}6;\ n = 12$$

$= 4{:}0$



$$a = 0{:}44(2)$$
$$b = 0{:}94(2)$$
$$c = \quad 0{:}47(4)$$

$= 2{:}6$



$$a = 0{:}82(9)$$
$$b = 0{:}78(3)$$
$$c = \quad 0{:}98(17)$$

fermionic e ects

$N = 64; \quad = 2{:}6; \; n = 12$

fermionic mass term: $iNm_{\mathrm{f}}\mathrm{Tr}[\quad (\; _7 \; {}^{y}_{8} \; _9) \qquad]$

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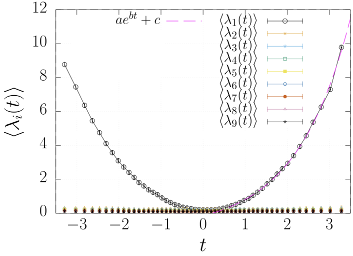

$a = 0{:}82(9)$

$b = 0{:}78(3)$

$c = \quad 0{:}98(17)$

$m_{\mathrm{f}} = 5$

$a = 2{:}54(15)$

$b = 0{:}54(1)$

$c = \quad 3{:}1(2)$