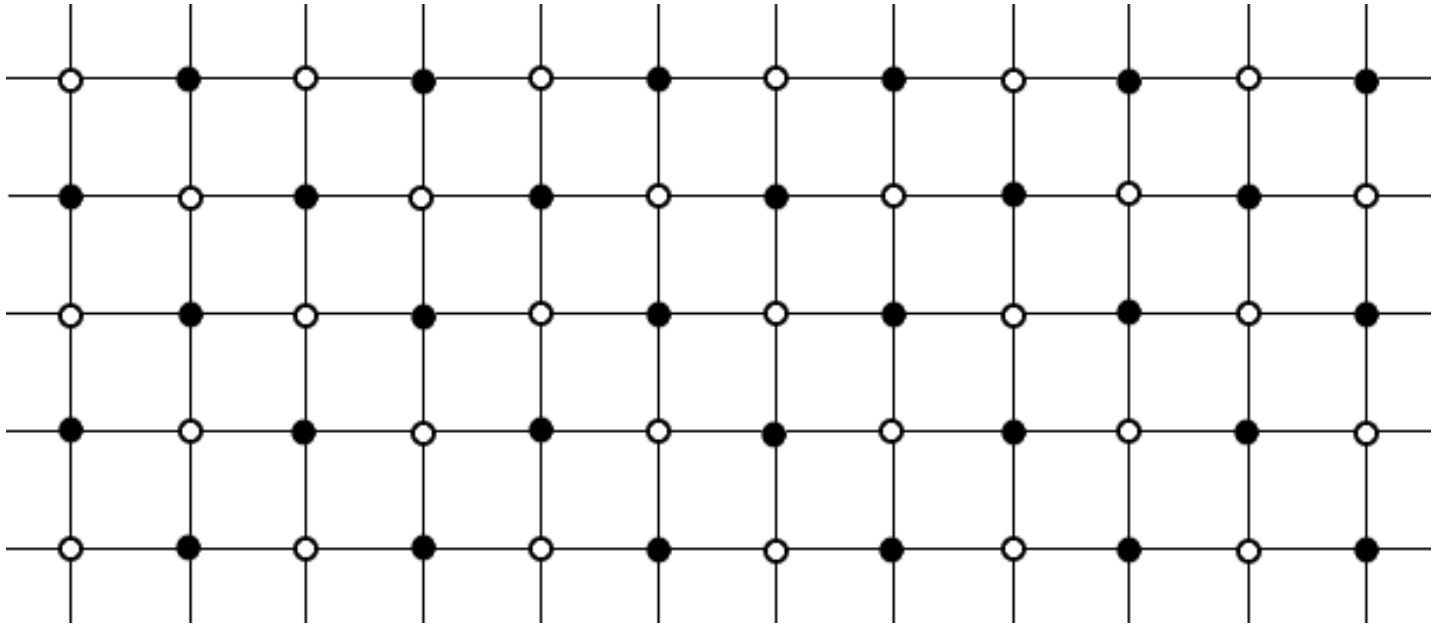
# A New Type of Lattice Gauge Theory through Self-adjoint Extensions



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## Hamiltonian U(1) Lattice Gauge Theory

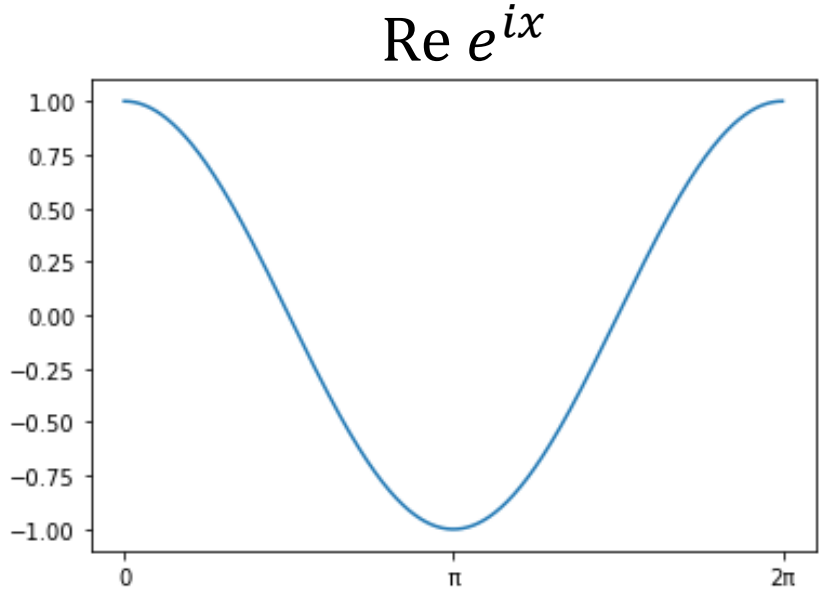
For $\mathrm{U}(1)$ gauge theory, assign $e^{i\varphi_l} \in \mathrm{U}(1)$ to each link

$$H = \frac{e^2}{2}\sum_l E_l^2 - \frac{1}{e^2}\sum_{\square} B_{\square}^2$$

Single-link wavefunctions $\psi(\varphi)$ are periodic

$$\psi(2\pi) = \psi(0)$$

The electric field $E = -i\partial_\varphi$ is self-adjoint.




*But this is not the most general thing one can do!*

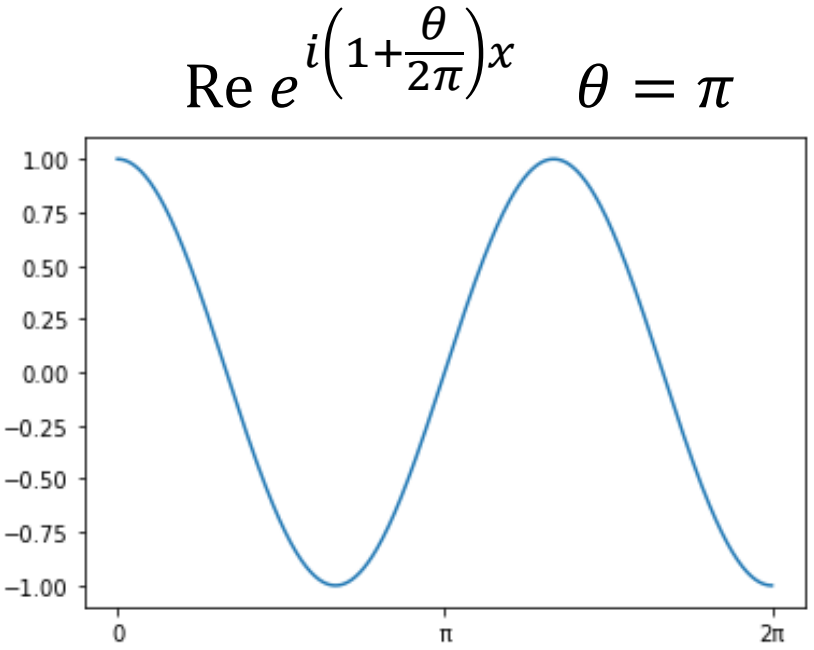
## A twisted Hilbert space

Introduce a twist (*self-adjoint extension*[1])

$$\psi(2\pi) = e^{i\theta}\psi(0)$$

The electric field $E = -i\partial_\varphi$ is still self-adjoint.

- Different Hilbert space *but*
- Still a $\mathrm{U}(1)$ gauge theory!
- Change magnetic term $B_\square^2 \to (B'_\square)^2$ to restore lattice cubic symmetry




[1]see e.g. quant-ph/9907069

## A new type of lattice gauge theory

Natural extension in the Hamiltonian picture,
but not in the path-integral approach

This construction provides an **extension to the Wilsonian framework** of gauge theories

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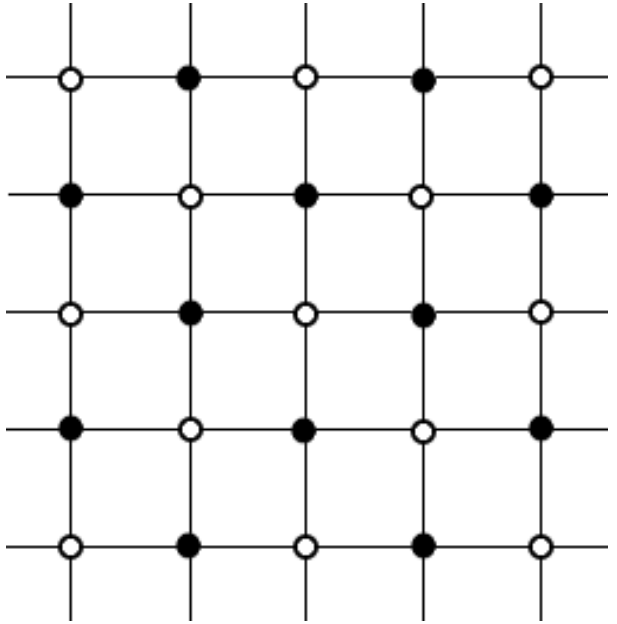
Now focus on extended $\mathrm{U}(1)$ in **2+1 dimensions**

*Does this extension give something new in the continuum?*

## Dualization in 3D



Go to the path-integral approach and dualize:

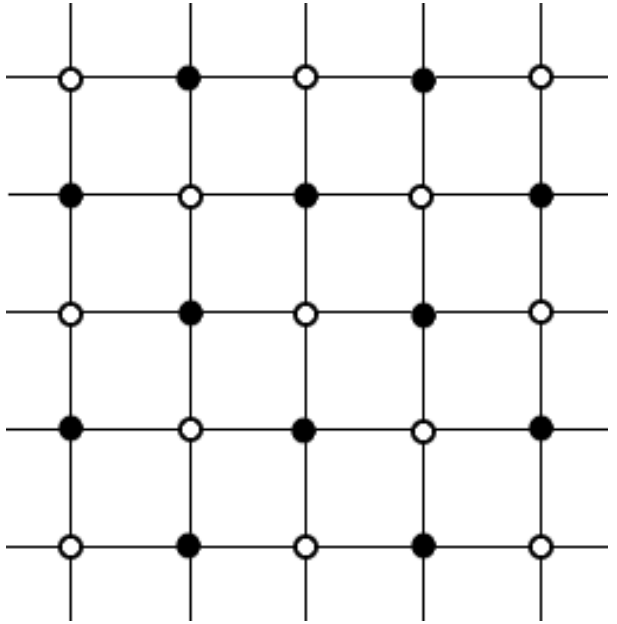
$$Z = \sum_{\{h\}} \exp\left[-\frac{e^2}{2}\sum_{\langle xy\rangle}\left(h_x - h_y\right)^2\right]$$

The $h_x$ are **scalars** defined on the sites of the **dual lattice**.

In 3D a site on the dual lattice corresponds to a **cube** in the original lattice.

Why dualization? Original partition function has a **sign problem**

## Dualization in 3D



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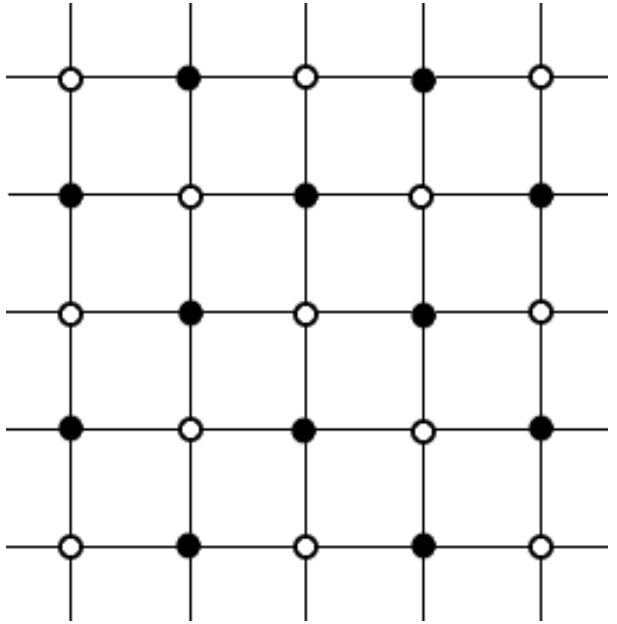
For $\theta = 0$ (usual $\mathrm{U}(1)$) all the $h_x$ are integer-valued.

For $\theta = \pi$ the $h_x$ are staggered integer and half-integer.

For $\theta \neq 0, \pi$ charge conjugation is explicitly broken.

## Dualization in 3D



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For $\theta = 0$ (usual $\mathrm{U}(1)$) all the $h_x$ are integer-valued. $\longrightarrow$ well-understood[1]

For $\theta = \pi$ the $h_x$ are staggered integer and half-integer.

For $\theta \neq 0, \pi$ charge conjugation is explicitly broken. $\longrightarrow$ future work

[1]Göpfert and Mack 1981

## Symmetries of the dual $\theta = \pi$ theory

The $h_x$ are staggered integer and half-integer on even and odd lattice sites:

$$Z = \sum_{\{h\}} \exp\left[-\frac{e^2}{2}\sum_{\langle xy\rangle}\left(h_x - h_y\right)^2\right]$$

Global $\mathbb{Z}$ invariance: $h_x \to h_x + n$ for $n$ integer.

Charge conjugation $C$: $h_x \to -h_x$

Shift by one lattice spacing $S$: $h_x \to h_{x+\hat{\mu}} + \frac{1}{2}$

## Order parameters for *C* and *S* breaking

$$O_{CS} = \sum_x (-1)^x h_x$$

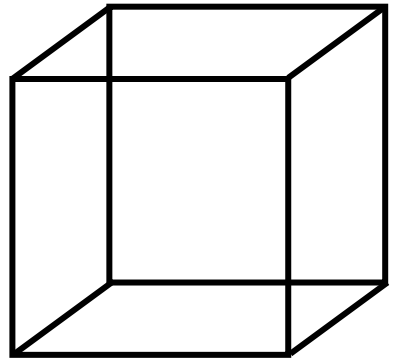
Under $C$ and $S$ separately,

$$O_{CS} \to -O_{CS}$$

Both $O_{CS}$ and $O_S$ are *local* and $\mathbb{Z}$ *-invariant*.

$$O_S = \sum_{cubes} \sum_{x \in cube} (-1)^x \left(h_x - \bar{h}_{cube}\right)^2$$

$O_S$ is $C$ invariant but changes sign under $S$.



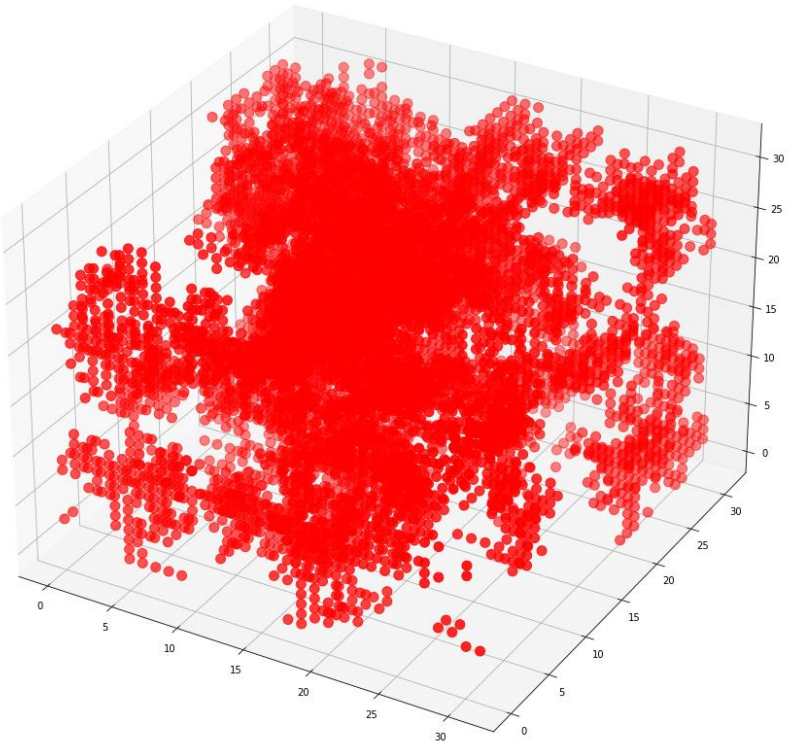
## Numerical simulation of the dual $\theta = \pi$ theory

We simulate the staggered height model with an efficient cluster algorithm.

Range of couplings $e^2 = 0.3$ up to $e^2 = 2.0$

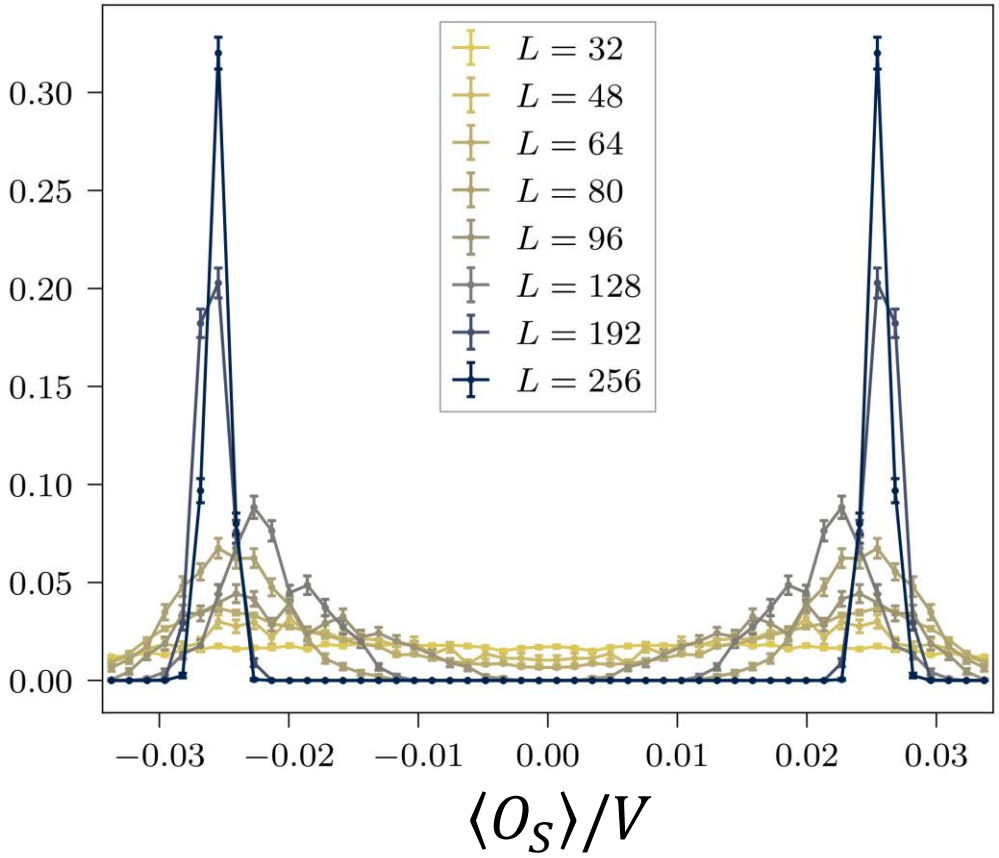
Lattices from $L = 32$ up to $L = 256$

Continuum limit expected as $e^2 \to 0$
(same as usual $\theta = 0$ theory)

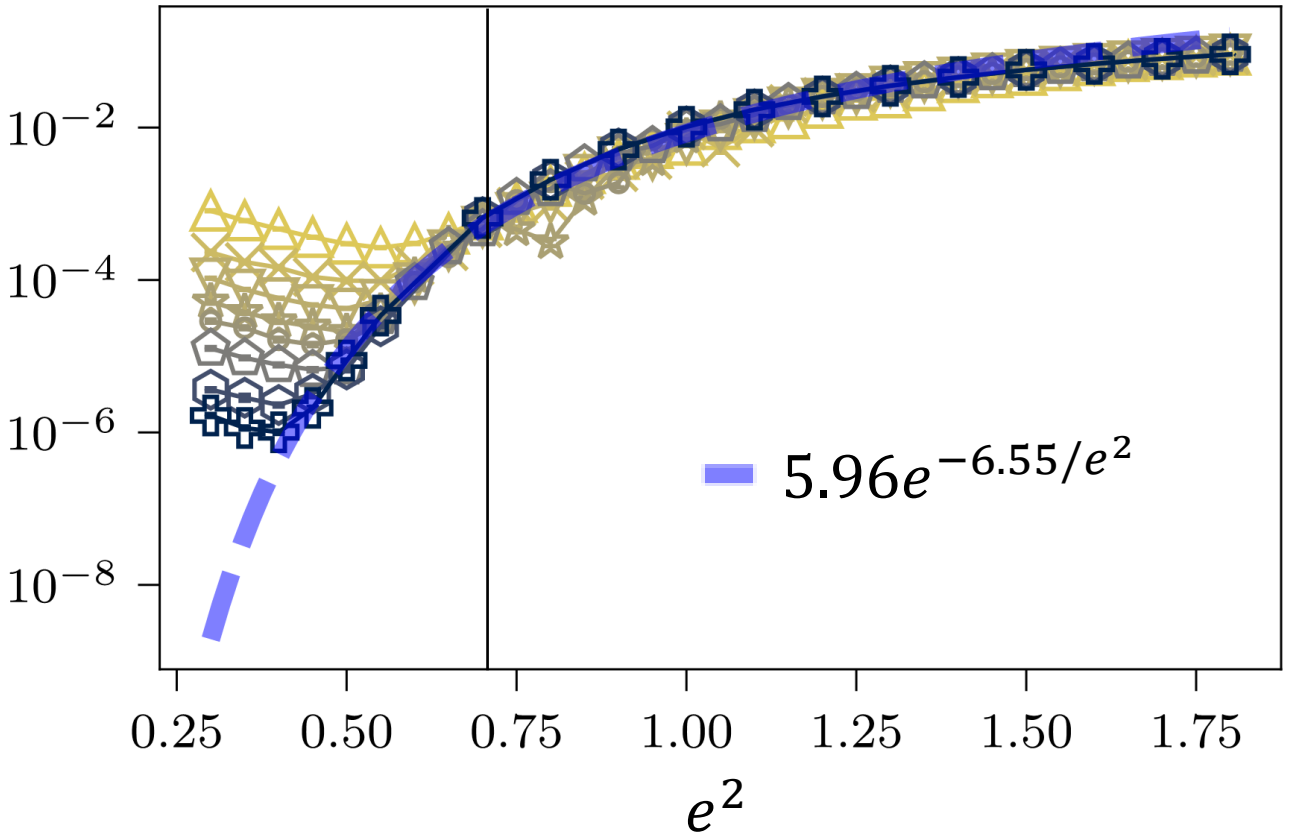


# Shift by one lattice spacing S is broken

$e^2 = 0.70$




$\langle (O_S)^2 \rangle / V^2$



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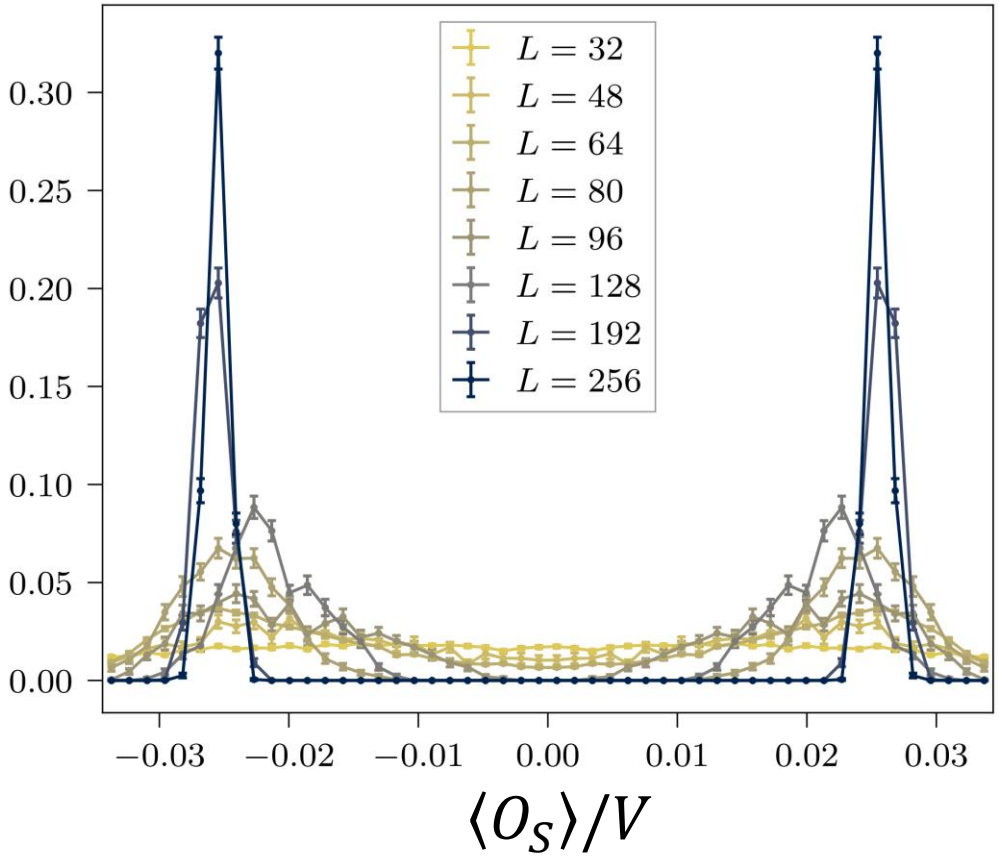


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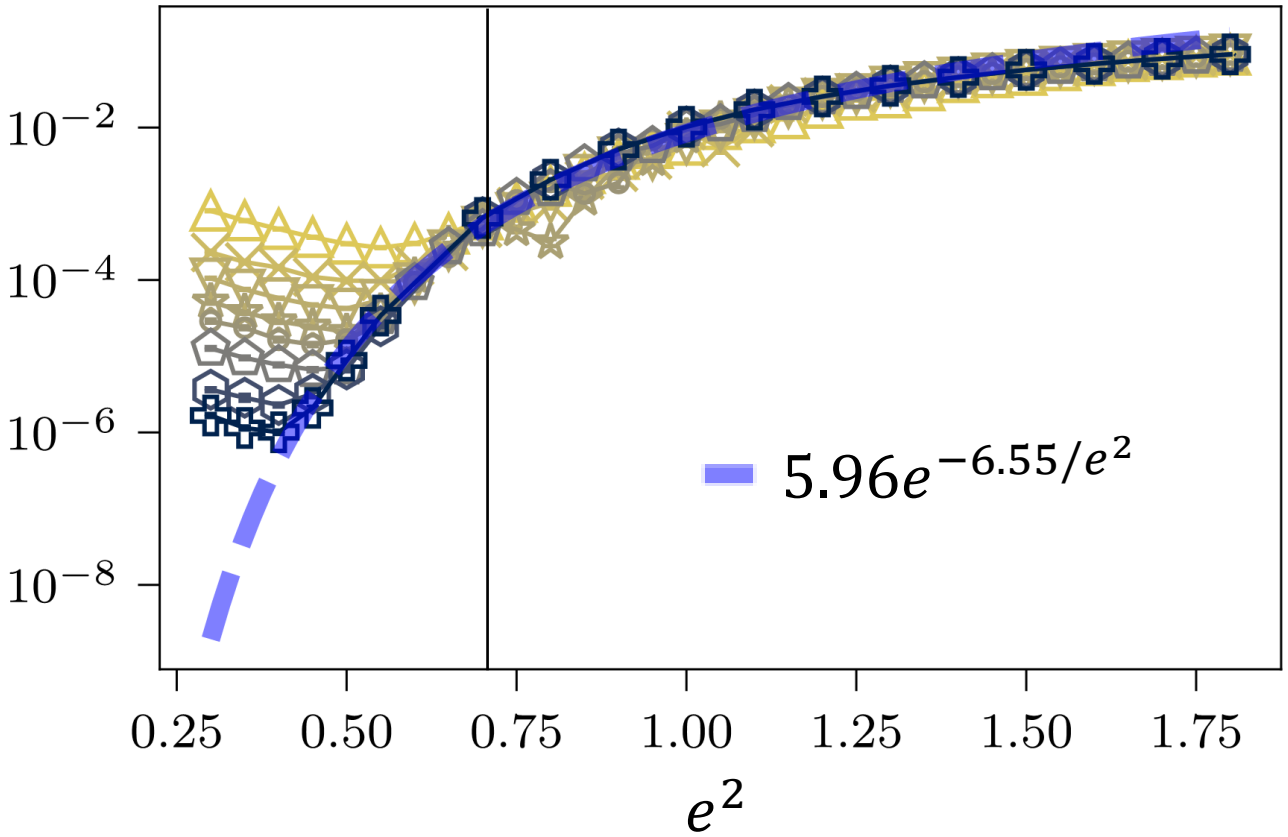
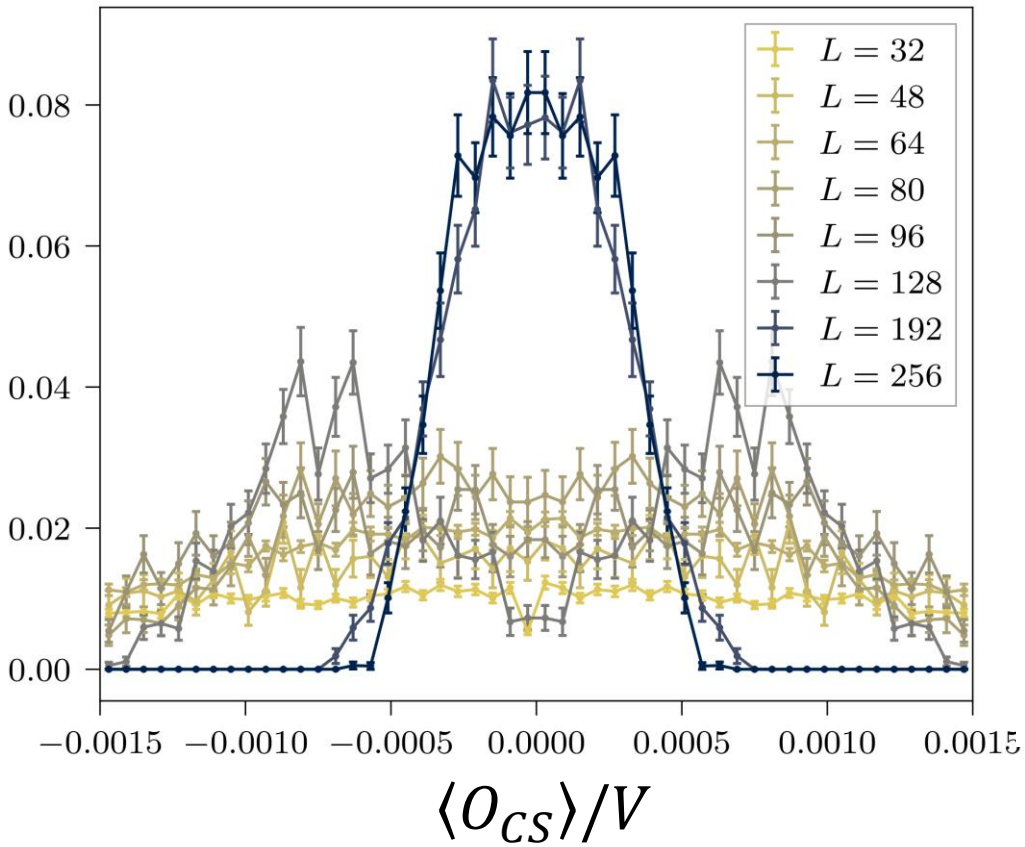


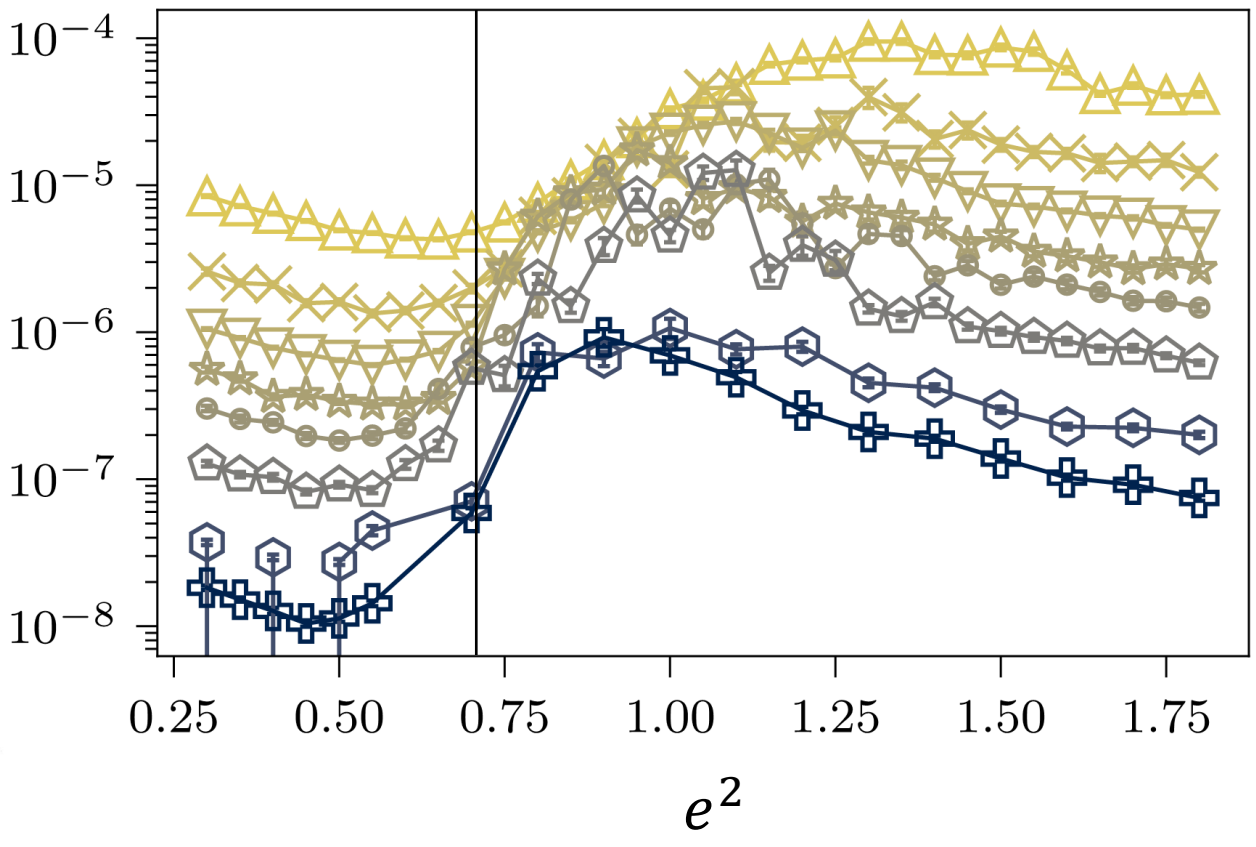

Evidence for *S* breaking down to the continuum limit $e^2 \to 0$

# Charge conjugation is not broken

$e^2 = 0.70$




$\langle (O_{cs})^2 \rangle / V^2$



## Phase diagram

Charge conjugation $C$ is **preserved** for all values of $e^2$

Shift by one lattice spacing $S$ appears to be **broken** for all values of $e^2$

$U(1)$ gauge theory with $\theta = \pi$ has a **broken** $\mathbb{Z}_2$ **symmetry** absent in the usual $\theta = 0$ case.

In the continuum, the broken $S$ symmetry may manifest in **internal degrees of freedom**

## Conclusion

We described a **non-perturbative extension** of the Hilbert space of $U(1)$ gauge theory.

We simulated the extended 3D $U(1)$ theory with $\theta = \pi$ and shown that it has a **broken** $\mathbb{Z}_2$ **symmetry** absent in the usual case.

*Further questions:*
What is the **universality class** of the $\theta = \pi$ $U(1)$ theory?

What happens for $\theta \neq 0, \pi$? What happens in 4D?

What happens in the non-Abelian case?

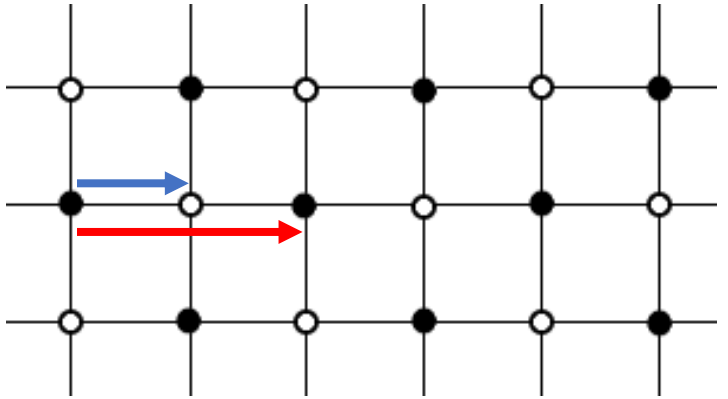
# Backup slides

## Phase diagram

Charge conjugation $C$ is **preserved** for all values of $e^2$

Shift by one lattice spacing $S$ appears to be **broken** for all values of $e^2$

But translation by an even number of lattice spacings remains unbroken



## Intuition for *C* and *S* breaking

$$O_{CS} = \sum_x (-1)^x h_x$$

Under $C$ and $S$ separately,

$$O_{CS} \to -O_{CS}$$

$\langle O_{CS} \rangle \neq 0$ if both the even and odd sublattices order simultaneously.

$$O_S = \sum_{cubes} \sum_{x \in cube} (-1)^x \left(h_x - \bar{h}_{cube}\right)^2$$

$O_S$ is $C$ invariant but changes sign under $S$.

$\langle O_S \rangle \neq 0$ if one sublattice orders and the other fluctuates.

## Cluster algorithm

We simulated the staggered height model using the Swendsen-Wang multicluster algorithm

1. At each step, pick a “reflection plane” $h_0$ equal to one of the $h_x$ chosen at random

2. Activate links between $h_x$ and $h_y$ **only** if they are on the same side of the reflection plane with probability

$$p = 1 - e^{-2e^2(h_x - h_0)(h_y - h_0)}$$

3. This partitions the $h_x$ into clusters. Then reflect each cluster $h_x \to 2h_0 - h_x$ with probability $1/2$