Extracting Non-Abelian Coulomb Potential from $SU(3)$ $\mathcal{N} = 4$ Lattice SYM

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Based on: Work done in colloboration with S. Catterall and J. Giedt,

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Overview

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   - Brief Review of Lattice $\mathcal{N} = 4$ super Yang-Mills Theory
   - Details of the $\mathcal{Q}$ Invariant Construction
2. $SU(3)$ Lattice SYM
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   - Supersymmetric Wilson Loops
   - Polyakov Line Correlators
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Brief Review of Lattice $\mathcal{N} = 4$ super Yang-Mills Theory

- We start with a twisted $4d$ $\mathcal{N} = 4$ supersymmetric theory has 16 supercharges. On lattice we can only preserve one of them exactly.
- The other 15 are broken by lattice artifacts and recovered only in the continuum limit.
- Bosons and fermions treated symmetrically meaning that they both live on links as required by the exact susy and lattice gauge invariance.

Brief Review of Lattice $\mathcal{N} = 4$ super Yang-Mills Theory

- A_4^* lattice is used as the underlying lattice structure which packs the 4 gauge fields and 6 scalars into 5 complex bosons each associated with one of the basis vectors of the lattice. They are also valued in the adjoint representation of the algebra not in the group.
- All fields transform under the twisted rotation group

$$\mathrm{diag}(SO(4)_L \times SO(4)_R) \tag{1}$$

Where L denotes the Lorentz Symmetry and R the R-Symmetry

$\mathcal{Q}$ Invariant Construction

Let's start with the supersymmetric lattice action

$$S = \frac{N}{4\lambda} \mathcal{Q} \sum_x \mathrm{Tr}\ \left(\chi_{ab} \mathcal{F}_{ab} + \eta \overline{\mathcal{D}}_a \mathcal{U}_a + \frac{1}{2} \eta d \right) + S_{\mathrm{closed}} \quad (2)$$

- The second term in the action S_{closed} is given as.

$$S_{\mathrm{closed}} = -\frac{N}{16\lambda} \sum_x \mathrm{Tr}\ \epsilon_{abcde} \chi_{ab} \overline{\mathcal{D}}_c \chi_{de} \quad (3)$$

$\mathcal{Q}$ Invariant Construction

Carrying out the $\mathcal{Q}$ variation and integrating out the auxiliary field d we obtain the supersymmetric lattice action $S = S_b + S_f$ where

$$S_b = \frac{N}{4\lambda} \sum_x \mathrm{Tr} \left(\mathcal{F}_{ab} \overline{\mathcal{F}}_{ab} + \frac{1}{2} \mathrm{Tr} \left(\overline{\mathcal{D}}_a \mathcal{U}_a \right)^2 \right) \tag{4}$$

and

$$S_f = -\frac{N}{4\lambda} \sum_x \left(\mathrm{Tr}\ \chi_{ab} \mathcal{D}_{[a} \psi_{b]} + \mathrm{Tr}\ \eta \overline{\mathcal{D}}_a \psi_a \right) \tag{5}$$

Fermionic part of this action is also known as Kähler-Dirac action.

- The continuum limit of this action corresponds to the Marcus or GL twist of $\mathcal{N} = 4$ Yang-Mills.

Regulating the flat directions

- There are flat directions that corresponds to the classical vacuum solutions of the bosonic action.
- To regulate these flat directions we add the term

$$S_{\rm mass} = \mu^2 \sum_x {\rm Tr}\ \left(\overline{\mathcal{U}}_a(x)\mathcal{U}_a(x) - I\right)^2 \tag{6}$$

- This term gives masses to the scalars.
- Lifts the degeneracy and provides a unique ground state.

Controlling the $U(1)$ modes

- Since each link is an element of the algebra $\mathfrak{gl}(N, C)$, this formulations naturally describes the gauge group $U(N) = SU(N) \times U(1)$.
- Even though the $U(1)$ gauge degrees of freedom decouple in the continuum limit they introduce lattice artifacts at strong coupling and need to be suppressed to access strong coupling regimes.

Previous attempts to control this $U(1)$ mode include adding

- A plaquette determinant term can go up to $\lambda \sim 6$
  arXiv: 1505.03135 by S. Catterall and D. Schaich

New Action

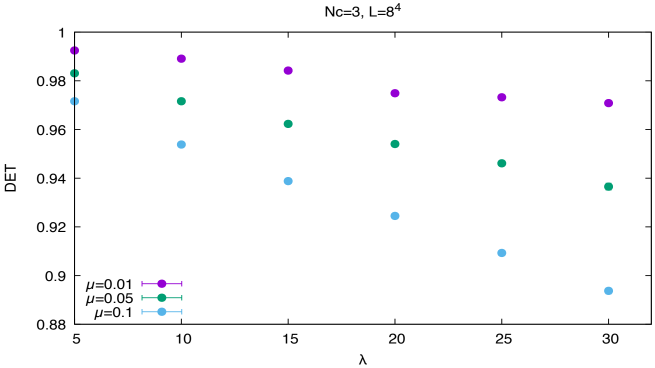
Instead, we include a new term to the action which drives the determinant of each individual gauge link to unity.

$$\frac{N}{4\lambda}\kappa\mathcal{Q}\sum_{x,a}\mathrm{Tr}\,(\eta)\,(\mathrm{Re}\det(U_a(x)-1)) \tag{7}$$

- This term breaks the $U(1)$ symmetry explicitly. But since $U(1)$ is a decoupled free theory in the continuum limit we are still preserving the $SU(N)$ gauge invariance.
- Most important result of this new term is that it allows simulation with arbitrarily large coupling.
- Results for the $SU(2)$ SYM using this new term can be seen from our paper arXiv:2009.07334

Phase structure of the $SU(3)$ Lattice SYM

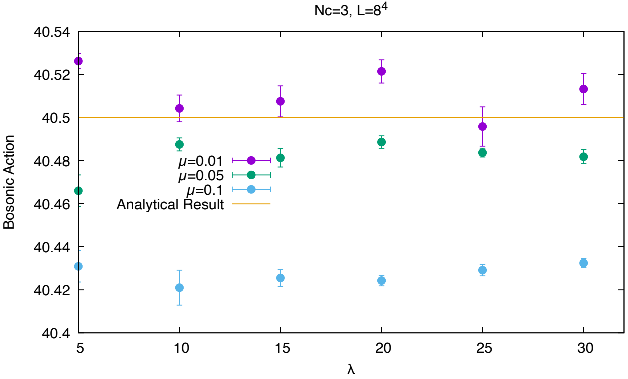
As a first test, We plot the Expectation value of the link determinant vs λ for 8^4 lattices at $\mu = 0.1, 0.05, 0.01$.



The expectation value is close to unity out to very large λ provided μ^2 is small enough confirming that we have effectively reduced the gauge fields to $SU(3)$.

Phase structure of the $SU(3)$ Lattice SYM

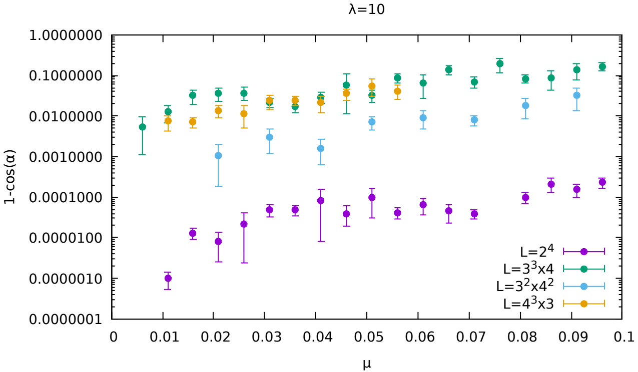
Next we look at the expectation value of the bosonic action $\frac{1}{V} < S_b >= \frac{9N^2}{2}$ for an N color theory on a system with (lattice) volume V *independent* of coupling λ.



- From these two plots we see that there is no phase transition as we vary λ as expected for a $\mathcal{N} = 4$ SYM.

Absence of a sign problem

- Writing the Pfaffian phase as $e^{i\alpha(\lambda,U)}$ we plot the quantity $1 - \cos\alpha$ as a function of μ for $\lambda = 10$.



- Pfaffian phase saturates as $L \to \infty$ and decreases as $\mu \to 0$.

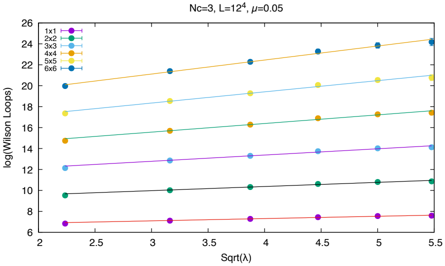
Supersymmetric Wilson Loops & Polyakov Line Correlators

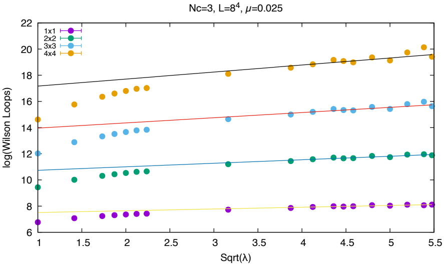
We showed strong evidence that the lattice theory

- Exists in a single phase with unbroken supersymmetry out to very large coupling
- And it can be simulated with a Monte Carlo algorithm without encountering a sign problem.

We can turn on to confirming known results for $\mathcal{N} = 4$ Yang-Mills at strong coupling for Supersymmetric Wilson loops and static potential.

- Supersymmetric Wilson loops are generalization of regular Wilson loops by including contibutions from the scalars and are realized in the twisted construction by forming path ordered products of complexified lattice gauge fields $\mathcal{U}_a$
- The holographic prediction for the supersymmetric Wilson loops is that at strong coupling they depend on $W(R,T) = e^{(c\sqrt{\lambda}T/R)}$ not on λ as expected from perturbation theory.





Polyakov Line Correlators

Another interesting question is whether we can see evidence for a non-abelian Coulomb potential.

- To probe for this is we calculated the correlators of (smeared) Polyakov lines defined as.

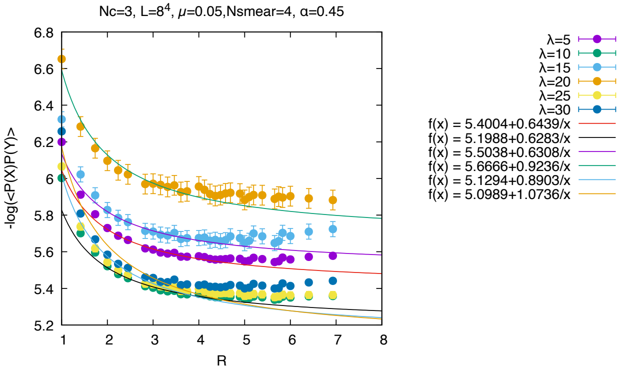
$$P(R) = \sum_{x,y} \left[< P(x)P^{\dagger}(y) > - < P(x) >< P^{\dagger}(y) > \delta(R, |x-y|) \right] \quad (8)$$

where $|x - y|$ is the distance in the A_4^* lattice.

- This is expected to vary like $P(R) \sim e^{-V(R)T}$ with $V(R)$ the static potential.

Polyakov Line Correlators

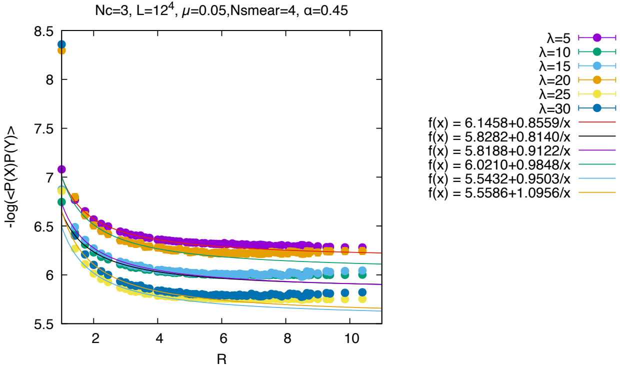
Here we show the logarithm of the Polyakov line correlators with fits of form $a + b/R$ for $L = 8^4$ lattices with smearing paramaters set at $N_{smear} = 4, \alpha = 0.45$



- As can be seen from the fits our data fits nicely to a function of form $1/R$ as expected from the holographic predictions.

Polyakov Line Correlators

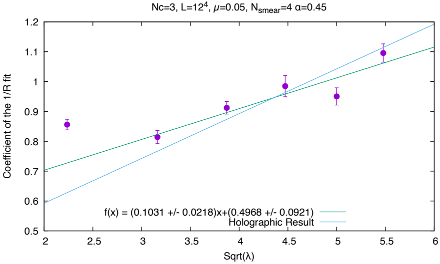
We can do the same calculation for a $L = 12^4$ lattice as well

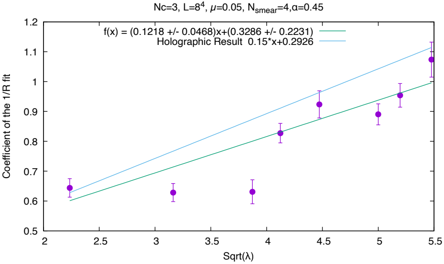


- Again we see that we have a clear $1/R$ dependence for the correlators.

Extracting the Non-Abelian Coulomb Potential

We can take the coefficient of these $1/R$ fits and plot them as a function of the coupling to compare our results with the holographic predictions for the Static potential





- We see that our results show a $\sqrt{\lambda}$ dependence as expected and are close to the holograhic predicitions which is shown as the blue line above.

Conclusions

- The square root behavior at large λ is consistent with the result for circular Wilson loops in $\mathcal{N} = 4$ SYM derived by N. Drukker and D. Gross. arXiv:hep-th/0010274 This strange $\sqrt{\lambda}$ dependence *cannot* be seen in perturbation theory.
- Furthermore we showed that the static potential shows the expected $1/R$ dependence.
- Comparing the functional dependence on the $\sqrt{\lambda}$ with the holographic results derived by J.K. Erickson, G.W. Semenoff, R.J. Szabo,K. Zarembo in arXiv:hep-th/9911088v1 and further we see that our results are close to the holographic predictions.
- For future work we need to figure out the dependence on the smearing parameters, bosonic mass and try to get closer to the holographic prediction.

Thanks for listening.

Fitting results for the $1/R$ fits for $L = 8^4, \mu = 0.05, N_{smear} = 4, \alpha = 0.45$

λ	$a + b/R$	Reduced-χ^2
5	$5.40(1) + 0.64(3)/R$	0.13
10	$5.19(1) + 0.62(3)/R$	0.06
15	$5.50(1) + 0.63(4)/R$	0.99
20	$5.66(2) + 0.92(4)/R$	0.96
23	$5.27(1) + 0.82(3)/R$	0.08
25	$5.12(1) + 0.89(3)/R$	0.04
27	$5.69(1) + 0.71(3)/R$	0.99
30	$5.09(2) + 1.07(6)/R$	0

Table: Fitting results for the $1/R$

To obtain these fits we fitted the data between $1 < R < 4$ for $N_{smear} = 4, \alpha = 0.45$ and $\mu = 0.05$

Fitting results for the $1/R$ fits for $L = 12^4, \mu = 0.05, N_{smear} = 4, \alpha = 0.45$

λ	$a + b/R$	Reduced-χ^2
5	$6.15(1) + 0.85(3)/R$	0.99
10	$5.82(1) + 0.81(2)/R$	0.95
15	$5.82(7) + 0.91(4)/R$	0.88
20	$6.02(1) + 0.98(4)/R$	0.99
25	$5.54(1) + 0.95(3)/R$	0.76
30	$5.55(2) + 1.09(3)/R$	0.51

Table: Fitting results for the $1/R$

To obtain these fits we fitted the data between $1.5 < R < 4$ for $N_{smear} = 4, \alpha = 0.45$ and $\mu = 0.05$

L	$a\sqrt{\lambda}+b$	Reduced-χ^2
8	$0.12(4)\sqrt{\lambda} + 0.3(2)$	0.18
12	$0.10(2)\sqrt{\lambda} + 0.49(9)$	0.07

Table: Fitting parameters for $\sqrt{\lambda}$ dependence of $1/R$ terms

Ward Identities for $SU(3)$

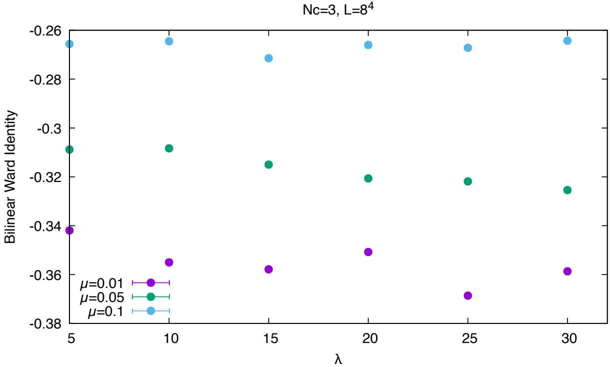


Figure: Ward Identity vs λ for $\mu = 0.01, 0.05, 0.1$