Thermal QCD for non-perturbative renormalization of composite operators

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Overview

1. **Renormalization strategy**
2. **Warm-up: flavor-singlet vector current**
3. **QCD energy-momentum tensor**
4. **Conclusions & outlook**

Renormalization strategy

- Bare lattice operator $O \Rightarrow$ Renormalized operator O^R

$$O^R = Z_O \left[O + \sum_k \frac{c_k}{a^{d-d_k}} O_k \right] , \quad d = \dim(O)$$

- In our case Z_O, c_k fixed by imposing continuum Ward Identities on O^R

$\star$ Thermal QCD with shifted and twisted boundary conditions

$$U_\mu(x_0 + L_0, \boldsymbol{x}) = U_\mu(x_0, \boldsymbol{x} - L_0\boldsymbol{\xi})$$
$$\psi(x_0 + L_0, \boldsymbol{x}) = -e^{i\theta_0}\, \psi(x_0, \boldsymbol{x} - L_0\boldsymbol{\xi})$$
$$\bar{\psi}(x_0 + L_0, \boldsymbol{x}) = -e^{-i\theta_0}\, \bar{\psi}(x_0, \boldsymbol{x} - L_0\boldsymbol{\xi})$$

Flavor-singlet vector current

- Lattice local current

$$V_\mu^l(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$$

- Lattice conserved current (Wilson fermions)

$$V_\mu^c(x) = \frac{1}{2}\left[\bar{\psi}(x+a\hat{\mu})U_\mu^\dagger(x)\left(\gamma_\mu+1\right)\psi(x) + \bar{\psi}(x)U_\mu(x)\left(\gamma_\mu-1\right)\psi(x+a\hat{\mu})\right]$$

- Definition of the renormalization constant ($L/a \to \infty$)

$$Z_V(g_0^2) = \lim_{a/L_0\to 0} \left.\frac{\langle V_\mu^c\rangle_{\boldsymbol{\xi},\theta_0}}{\langle V_\mu^l\rangle_{\boldsymbol{\xi},\theta_0}}\right|_{g_0^2,L_0/a}$$

- Shift and twist $\boldsymbol{\xi} = (1,0,0)$, $\theta_0 = \pi/6$

Flavor-singlet vector current

- $\mathcal{O}(a)$-improvement [Bhattacharya, Gupta, Lee, Sharpe, Wu, Phys. Rev. D **73** (2006), 034504]

$$\hat{V}_\mu^{c,l}(x) = V_\mu^{c,l}(x) - \frac{a}{4}\, c_V^{c,l}\, (\partial_\nu + \partial_\nu^*)\, \left(\bar{\psi}(x)\, [\gamma_\mu, \gamma_\nu]\, \psi(x)\right)$$

$$\Rightarrow \langle V_\mu^{c,l} \rangle \quad \text{automatically improved}$$

- 1-loop improvement [Skouroupathis, Panagopoulos, Phys. Rev. D **79** (2009), 094508]

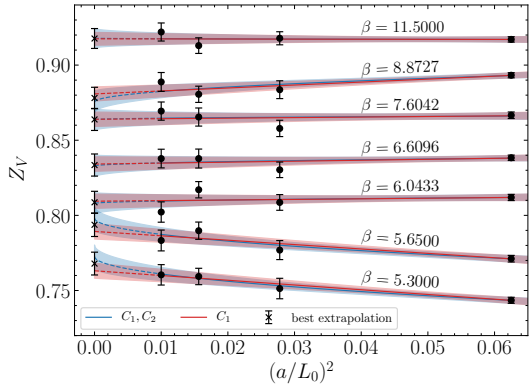
$$Z_V(g_0^2) = \lim_{a/L_0 \to 0} \left\{ \frac{\langle V_\mu^c \rangle_{\boldsymbol{\xi},\theta_0}}{\langle V_\mu^l \rangle_{\boldsymbol{\xi},\theta_0}}\bigg|_{g_0^2, L_0/a} - \left[1 + c_1 g_0^2 - Z_V^{(0)} \left(\frac{a}{L_0} \right) - g_0^2 Z_V^{(1)} \left(\frac{a}{L_0} \right) \right] \right\}$$

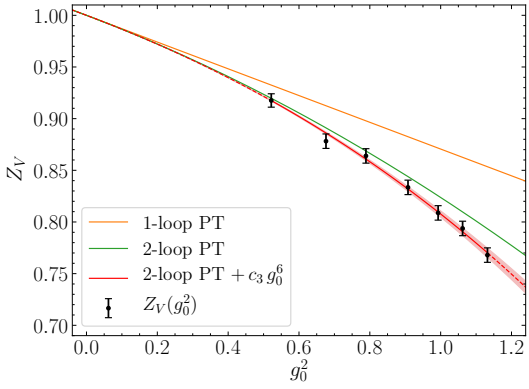
- Expected scaling for $a/L_0 \to 0$

$$Z_V \left(g_0^2, \frac{a}{L_0} \right) = Z_V(g_0^2) + C_1 \cdot \left(\frac{a}{L_0} \right)^2 + C_2 \cdot (a\Lambda_{QCD}) \left(\frac{a}{L_0} \right) + C_3 \cdot (a\Lambda_{QCD})^2 + \ldots$$

Flavor-singlet vector current

Wilson gauge action with $N_f = 3$ $\mathcal{O}(a)$-improved Wilson fermions





$$Z_V^{\text{fit}}(g_0^2) = 1 - 0.129 g_0^2 - 0.047 g_0^4 - 0.016(3) g_0^6$$

[Bresciani, Dalla Brida, Giusti, Pepe, Rapuano, arXiv:2203.14754]

Energy-momentum tensor

- Lattice regularization

$$\begin{aligned} T^{\text{lat}}_{\mu\nu} &= T^{\{1\}}_{\mu\nu} + T^{\{3\}}_{\mu\nu} + T^{\{6\}}_{\mu\nu} \\ &= T^{G,\{1\}}_{\mu\nu} + T^{F,\{1\}}_{\mu\nu} + T^{G,\{3\}}_{\mu\nu} + T^{F,\{3\}}_{\mu\nu} + T^{G,\{6\}}_{\mu\nu} + T^{F,\{6\}}_{\mu\nu} \end{aligned}$$

- $T^{\{3\}}_{\mu\nu}$, $T^{\{6\}}_{\mu\nu}$ relevant for our approach to QCD thermodynamics
- No other operator mixing in renormalization:

$$T^{R,\{3\}}_{\mu\nu} = Z^{\{3\}}_G T^{G,\{3\}}_{\mu\nu} + Z^{\{3\}}_F T^{F,\{3\}}_{\mu\nu}$$

$$T^{R,\{6\}}_{\mu\nu} = Z^{\{6\}}_G T^{G,\{6\}}_{\mu\nu} + Z^{\{6\}}_F T^{F,\{6\}}_{\mu\nu}$$

- 4 conditions needed to determine the Zs

Energy-momentum tensor

- Continuum Ward Identity imposed on the sextet component at two chemical potentials

$$\begin{cases} Z_G^{\{6\}} \langle T_{0k}^{G,\{6\}} \rangle_{\boldsymbol{\xi},\theta^A} + Z_F^{\{6\}} \langle T_{0k}^{F,\{6\}} \rangle_{\boldsymbol{\xi},\theta^A} = -\dfrac{\Delta f(L_0, \boldsymbol{\xi}, \theta^A)}{\Delta \xi_k} + \mathcal{O}(a^2) \\ \\ Z_G^{\{6\}} \langle T_{0k}^{G,\{6\}} \rangle_{\boldsymbol{\xi},\theta^B} + Z_F^{\{6\}} \langle T_{0k}^{F,\{6\}} \rangle_{\boldsymbol{\xi},\theta^B} = -\dfrac{\Delta f(L_0, \boldsymbol{\xi}, \theta^B)}{\Delta \xi_k} + \mathcal{O}(a^2) \end{cases}$$

- The same for the triplet

$$\langle T_{0k}^{R,\{6\}} \rangle_{\boldsymbol{\xi},\theta} = \xi_k \langle T_{0j}^{R,\{3\}} \rangle_{\boldsymbol{\xi},\theta} \quad (j \neq k, \xi_j = 0)$$

[Dalla Brida, Giusti, Pepe, JHEP 04 (2020) 043]

Energy-momentum tensor

- $N_f = 3,\ L/a = 288,\ \boldsymbol{\xi} = (1, 0, 0)$
- $\theta_0 = 0$

β	L_0/a	N_{traj}	$\langle T_{\mu\nu}^{G,\{6\}} \rangle / T^4$	$\langle T_{\mu\nu}^{F,\{6\}} \rangle / T^4$	$\langle T_{\mu\nu}^{G,\{3\}} \rangle / T^4$	$\langle T_{\mu\nu}^{F,\{3\}} \rangle / T^4$
6.0433	4	50	-2.325(8)	-6.335(3)	-2.760(11)	-7.081(5)
	6	100	-2.314(19)	-5.772(8)	-2.63(3)	-6.303(15)
8.8727	4	50	-2.842(8)	-6.982(3)	-3.223(12)	-7.692(4)
	6	100	-2.822(24)	-6.343(9)	-3.13(4)	-6.846(11)

- $\theta_0 = \pi/6$

β	L_0/a	N_{traj}	$\langle T_{\mu\nu}^{G,\{6\}} \rangle / T^4$	$\langle T_{\mu\nu}^{F,\{6\}} \rangle / T^4$	$\langle T_{\mu\nu}^{G,\{3\}} \rangle / T^4$	$\langle T_{\mu\nu}^{F,\{3\}} \rangle / T^4$
6.0433	4	50	-2.247(5)	-5.5939(21)	-2.642(7)	-6.2617(29)
	6	100	-2.198(19)	-5.115(9)	-2.59(4)	-5.587(15)
8.8727	4	50	-2.784(7)	-6.1605(29)	-3.164(12)	-6.783(4)
	6	100	-2.753(26)	-5.615(7)	-3.00(4)	-6.077(13)

Conclusions & outlook

- Thermal QCD as a non-perturbative renormalization framework
- Lower discretization errors with $\boldsymbol{\xi} = (1, 0, 0)$
- Non-trivial Ward Identities at $\boldsymbol{\xi} \neq 0$

⋆ Non-perturbative renormalization of $T_{\mu\nu}$

⋆ Properties of Quark-Gluon Plasma

⋆ Non-perturbative determination of the QCD equation of state from 1 GeV to 100 GeV with target precision of 1%

$$\frac{s}{T^3} = -L_0^4 \frac{(1+\boldsymbol{\xi}^2)^3}{\xi_k} \langle T_{0k}^R \rangle \,, \quad T = \frac{1}{L_0\sqrt{1+\boldsymbol{\xi}^2}}$$