General kinematics three-gluon vertex in Landau-gauge from quenched-lattice QCD

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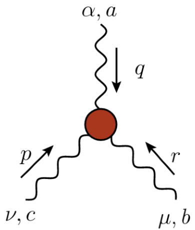
The three-gluon Vertex

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a \qquad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\, f^{abc} A_\mu^b A_\nu^c$$

- Responsible for the main differences between gluon and photon dynamics.
- Non-perturbative object which can be computed from the lattice and DSE.
- Key ingredient in others DSE.
- Intimately linked to asymptotic freedom.



Computing three-gluon vertex in Landau gauge

Landau gauge

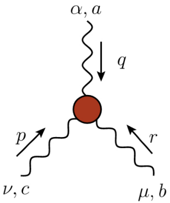
Landau gauge $\partial_\mu A^a_\mu = 0$ fixed numerically, allowing to compute gauge dependent quantities.

- Gluon propagator:

$$\Delta^{ab}_{\mu\nu}(q^2) = \langle A^a_\mu(q) A^b_\nu(-q) \rangle = \delta^{ab} \Delta(q^2) P_{\mu\nu}(q)$$

- Three-gluon vertex:

$$f^{abc} \mathcal{G}_{\alpha\mu\nu}(q, r, p) = \langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p) \rangle$$



The transversely projected vertex

$$\mathcal{G}^{\alpha\mu\nu}(q,r,p) = g\overline{\Gamma}^{\alpha\mu\nu}(q,r,p)\ \Delta(q^2)\ \Delta(r^2)\ \Delta(p^2)$$ which corresponds to the transverse projection of the 1PI vertex:

$$\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) = \Gamma^{\alpha'\mu'\nu'}(q,r,p) P^{\alpha}_{\alpha'}(q) P^{\mu}_{\mu'}(r) P^{\nu}_{\nu'}(p)$$

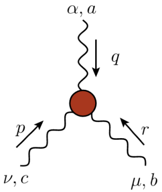
No access to longitudinal part

If the 1PI vertex, $\Gamma^{\alpha\mu\nu}(q,r,p)$ has a longitudinal and a transverse part:

$$\Gamma^{\alpha\mu\nu}(q,r,p) = \Gamma_L{}^{\alpha\mu\nu}(q,r,p) + \Gamma_T{}^{\alpha\mu\nu}(q,r,p)$$

we will only access the transverse part,

Tensorial structure of the three-gluon vertex



Momentum conservation

$$q + r + p = 0$$

Ball-Chiu decomposition [Phys. Rev. D22 (1980) 2550]

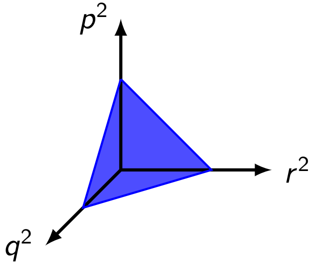
- 14 independent tensors $(\ell_1, \ell_2, \cdots, \ell_{10}, t_1, \cdots, t_4)$
- Transverse projection $\rightarrow$ 4 independent tensors

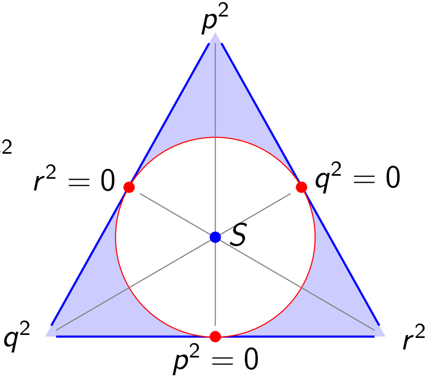
Transversely projected vertex

$$\bar{\Gamma}^{\alpha\mu\nu}(q, r, p) = \bar{\Gamma}_A \lambda_A^{\alpha\mu\nu} + \bar{\Gamma}_B \lambda_B^{\alpha\mu\nu} + \bar{\Gamma}_C \lambda_C^{\alpha\mu\nu} + \bar{\Gamma}_D \lambda_D^{\alpha\mu\nu}$$

Kinematics of the three-gluon vertex

$\bar{\Gamma}^{\alpha\mu\nu}(q, r, p)$ depends on three momenta, with $q + r + p = 0$.
The scalar form factors can be cast in terms of q^2, r^2 and p^2.





Kinematics of the three-gluon vertex

Particular cases:

Case	Def.	$\hat{qr}$	Tensors
Sym.	$q^2 = r^2 = p^2$	$\frac{2\pi}{3}$	$\lambda_{1,2}^{sym}$
Soft gluon	$p = 0$	π	λ_3^{sg}
Collinear	$q = r = -p/2$	0	(none)
Bisectoral	$q^2 = r^2$	$(0, \pi)$	3
General		–	4

Symmetric and soft-gluon cases already studied in [Phys.Lett.B 818 (2021) 136352]

Tensorial structure of the three-gluon vertex

$$\bar{\Gamma}^{\alpha\mu\nu} = \underbrace{\bar{\Gamma}_A \lambda_A^{\alpha\mu\nu}}_{Tree-level} + \bar{\Gamma}_B \lambda_B^{\alpha\mu\nu} + \bar{\Gamma}_C \lambda_C^{\alpha\mu\nu} + \bar{\Gamma}_D \lambda_D^{\alpha\mu\nu}$$

Basis for the transevsely projected vertex

$$\lambda_A^{\alpha\mu\nu}(q,r,p) = \left(\ell_1^{\alpha'\mu'\nu'} + \ell_4^{\alpha'\mu'\nu'} + \ell_7^{\alpha'\mu'\nu'}\right) P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p)$$

$$\lambda_B^{\alpha\mu\nu}(q,r,p) = 3\frac{(r-p)^{\alpha'}(p-q)^{\mu'}(q-r)^{\nu'}}{q^2+r^2+p^2} P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p)$$

$$\lambda_C^{\alpha\mu\nu}(q,r,p) = \frac{3(\ell_3^{\alpha'\mu'\nu'} + \ell_6^{\alpha'\mu'\nu'} + \ell_9^{\alpha'\mu'\nu'})}{q^2+r^2+p^2} P_{\alpha'}^{\alpha}(q) P_{\mu'}^{\mu}(r) P_{\nu'}^{\nu}(p)$$

$$\lambda_D^{\alpha\mu\nu}(q,r,p) = \left(\frac{3}{q^2+r^2+p^2}\right)^2 \left(t_1^{\alpha\mu\nu} + t_2^{\alpha\mu\nu} + t_3^{\alpha\mu\nu}\right)$$

Lattice setups

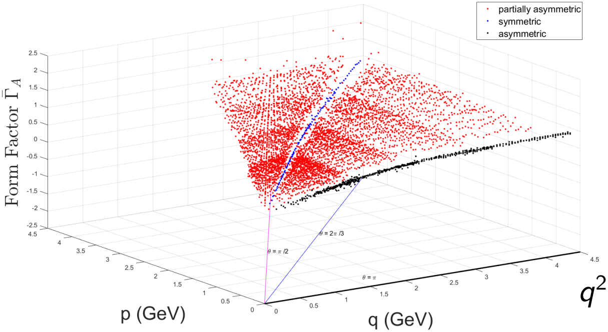
Exploited quenched gauge field configurations with:

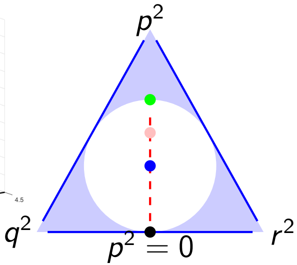
β	L^4/a^4	a (fm)	confs
5.6	32^4	0.236	2000
5.8	32^4	0.144	2000
6.0	32^4	0.096	2000
6.2	32^4	0.070	2000

- Absolute calibration for $\beta = 5.8$ taken from [S. Necco and R. Sommer, Nucl. Phys. B622, 328 (2002)].
- Relative calibrations based in gluon propagator scaling [Phys. Rev. D 98, 114515 (2018)],

Results for the biscetoral case $q^2 = r^2$.

The form factors $\bar{\Gamma}(q^2, q^2, p^2)$ depend on two momenta.





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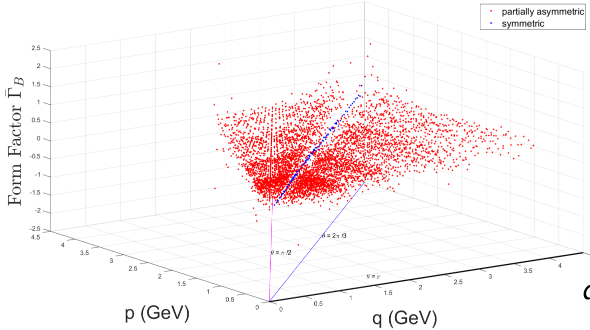

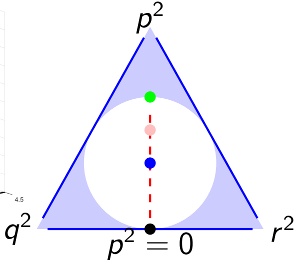

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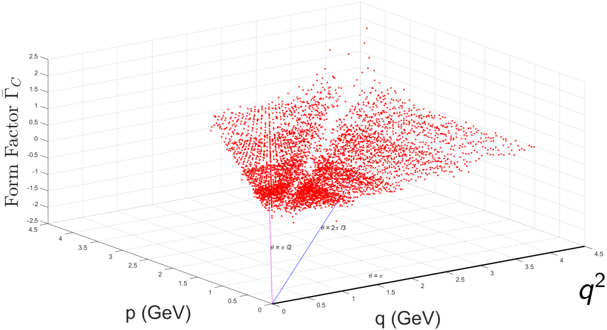

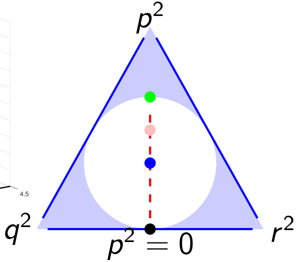

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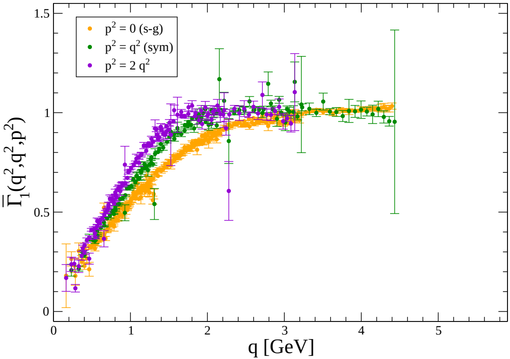
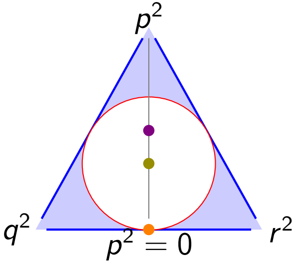


- Different running for each angle $\hat{qr}$ ($\pi/2$, $2\pi/3$, and π. in the plot)



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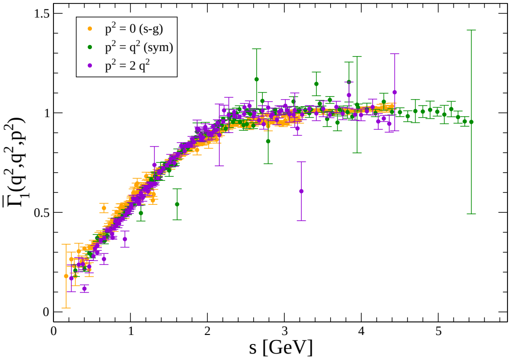
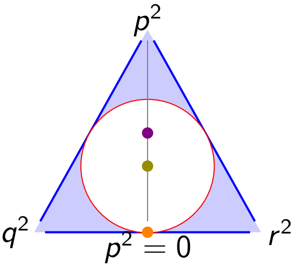
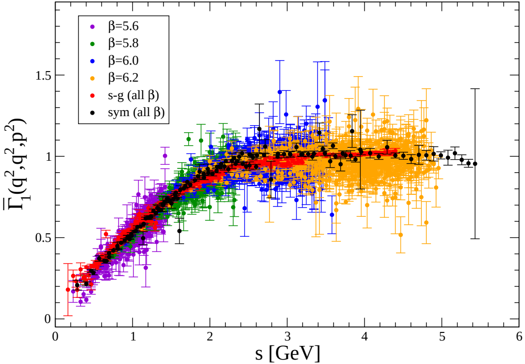


- The different angles scale in terms of $\sqrt{(q^2 + r^2 + p^2)/2}$.

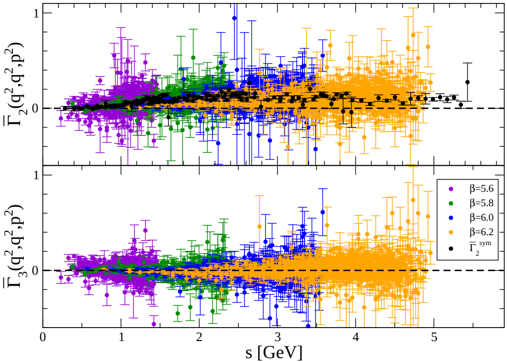


Results for the biscetoral case $q^2 = r^2$.



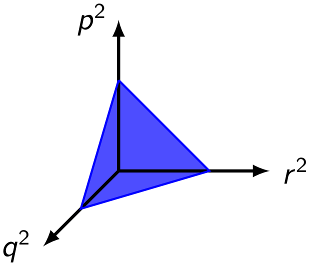
- Scales in terms of $(q^2 + r^2 + p^2)/2$.
- Renormalized $\bar{\Gamma}_3^{sg}(q^2)\Big|_{q^2=\mu^2} = 1$ at $\mu = 4.3 GeV$.

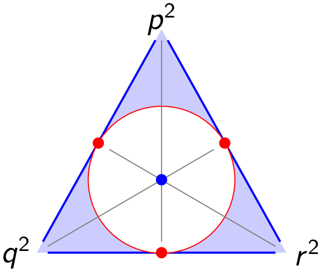
Results for the biscetoral case $q^2 = r^2$.



- Scales in terms of $(q^2 + r^2 + p^2)/2$.
- Excluded momenta with angles $\hat{q}r \approx 0, \frac{2\pi}{3}, \pi$.
- Smaller than $\overline{\Gamma}_A$.

Results for the biscetoral case $q^2 = r^2$.



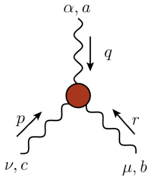


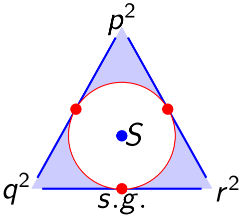
Indeed,the scalar form factors could only depend on symmetric momentum variables [Phys. Rev. D 89, 105014 (2014)]:

- $s^2 = \frac{q^2+r^2+p^2}{2}$ (plane)
- $(q^2 - r^2)^2 + (r^2 - p^2)^2 + (p^2 - q^2)^2$ (radius)
- $(q^2 + r^2 - 2p^2)(r^2 + p^2 - 2q^2)(p^2 + q^2 - 2r^2)$ (phase)

Results for the biscetoral case $q^2 = r^2$.

- Scale with $s^2 = (q^2 + r^2 + p^2)/2$.
- $\bar{\Gamma}_A$ dominates.
- $\bar{\Gamma}_B \ll \bar{\Gamma}_A$ (and $\bar{\Gamma}_C, \bar{\Gamma}_D \approx 0$).
- Qualitative agreement between $\bar{\Gamma}_1^{sym}$ and $\bar{\Gamma}_3^{sg}$.
- General case $q^2 \neq r^2 \neq p^2$ studied later.





The full vertex seems to be well described by:

$$\bar{\Gamma}^{\alpha\mu\nu}(q,r,p) \approx \bar{\Gamma}_3^{sg}(s^2)\Big|_{s^2} \bar{\Gamma}_0^{\alpha\mu\nu}(q,r,p)$$

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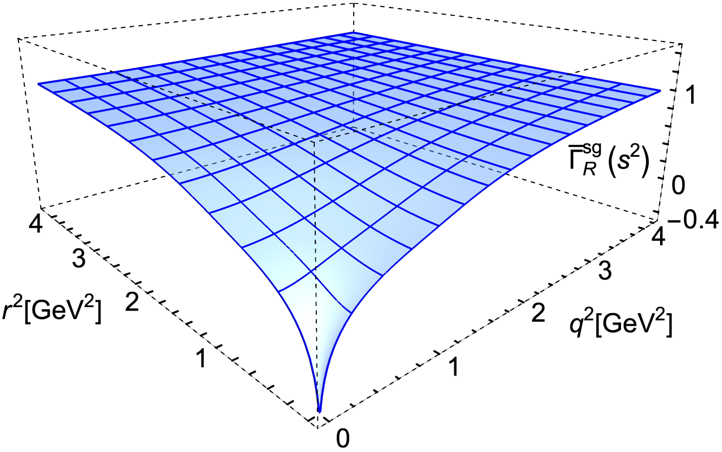

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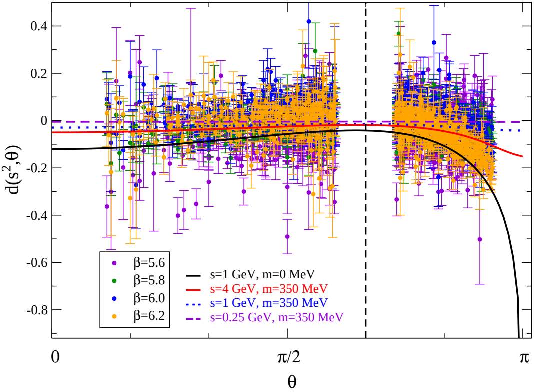
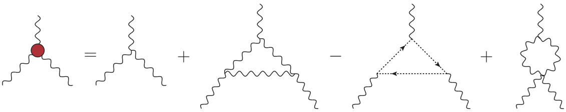




Applications

Let us study the contraction:

$$K_{\mathcal{W}}(q^2, r^2, p^2) = \overline{\Gamma}^{\alpha\mu\nu}(q, r, p)\ g_{\alpha\mu}(q - r)_\nu$$

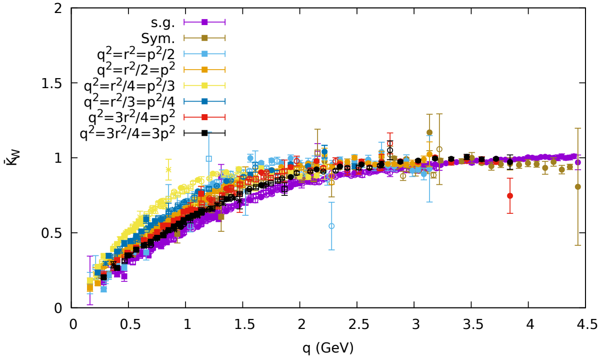
and compute the ratio to its tree-level counterpart:

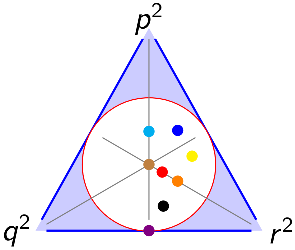
$$\bar{K}_{\mathcal{W}}(q^2, r^2, p^2) = \frac{K_{\mathcal{W}}(q^2, r^2, p^2)}{K^0_{\mathcal{W}}(q^2, r^2, p^2)}\ .$$

Contraction $K_{\mathcal{W}}$

- This contraction leads to Γ_3^{sg} for the soft-gluon case.
- Appears in the context of the Schwinger mechanism for gluon mass generation [Phys.Rev.D 105 (2022) 014030].
- The lattice extraction of $K_{\mathcal{W}}$ does not rely on the tensor decomposition of $\overline{\Gamma}^{\alpha\mu\nu}$.

Applications





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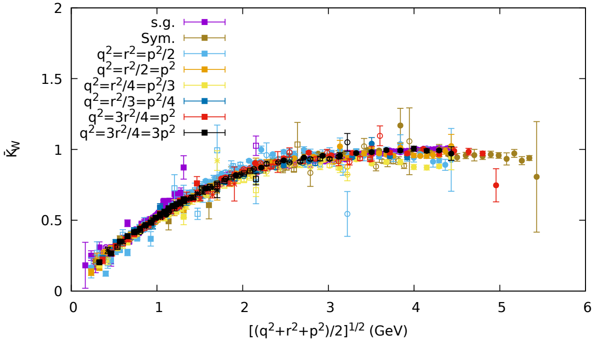

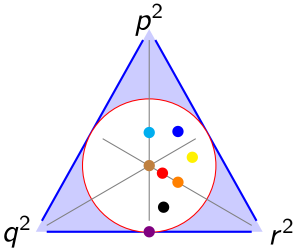

Scale in terms of
$q^2 + r^2 + q^2$

Conclusions

Conclusions

- Computed Landau-gauge 3g vertex for general kinematic configurations from quenched lattice-QCD.
- Nice scaling in terms of a single scale $s^2 = \frac{q^2+r^2+p^2}{2}$.
- Dominance of the tree-level tensor $\lambda_A^{\alpha\mu\nu}(q, r, p)$.
- The full vertex can be described from the soft-gluon vertex dressing $\overline{\Gamma}_3^{sg}(s^2)$.

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