Thermodynamics with Möbius domain wall fermions near the physical point (I)

JLQCD collaboration:

Y. Aoki[1](presenter), S. Aoki[2,1], H. Fukaya[3], S. Hashimoto[4,5,1], I. Kanamori[1], T. Kaneko[4,5,6], Y. Nakamura[1] , Y. Zhang[1]

1: R-CCS, 2: YITP, 3: Osaka, 4: KEK, 5: SOKENDAI, 6: KMI

Lattice 2022 @ Bonn

August 12, 2022

Two talks for N_f=2+1 thermo from JLQCD

- Aoki (I)
  - Set up: LCP, m_{res}
  - Discussion of DWF fermionic measurements and renormalization
- Kanamori (II)
  - Simulations
  - Physical Results

acknowledgements

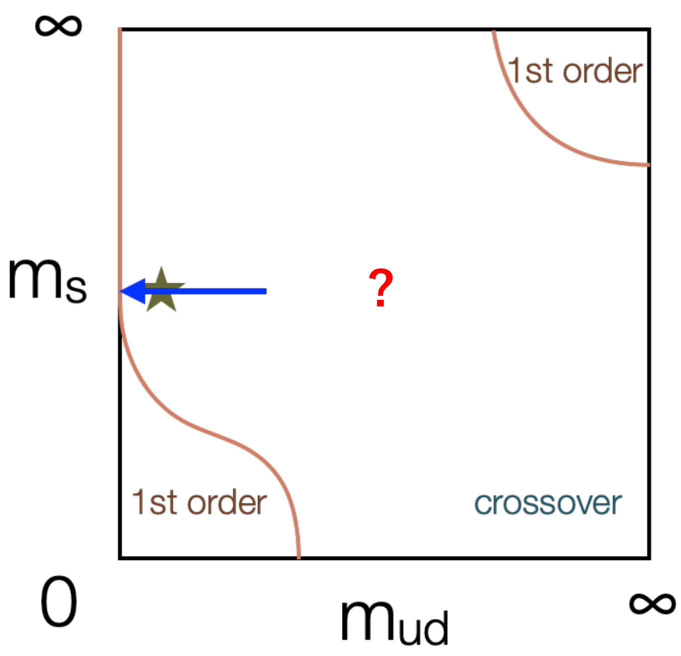
- Codes used:
    - HMC
        - Grid / Regensburg
    - Measurements:
        - BQCD
        - Bridge++
        - Hadrons / Grid

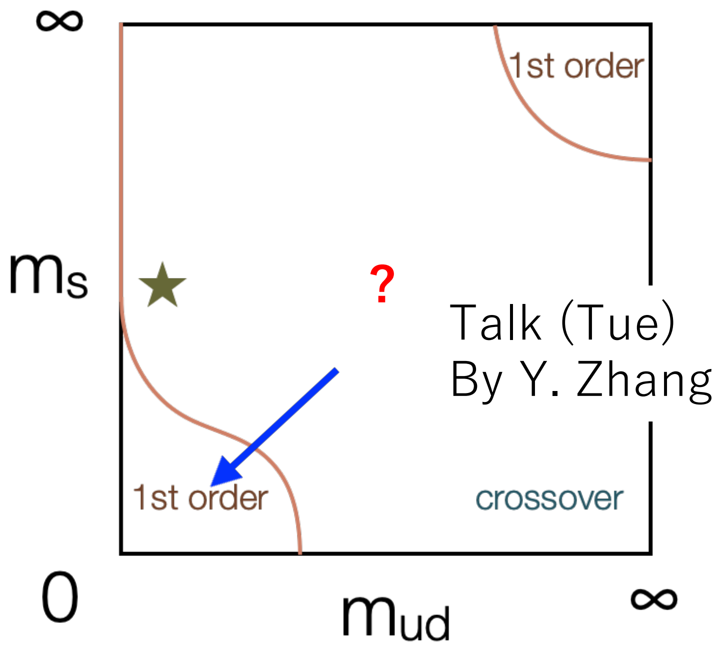
- MEXT program
  「富岳」成果創出加速プログラム
  Program for Promoting Researches on the Supercomputer Fugaku
  - Simulation for basic science: from fundamental laws of particles to creation of nuclei

- Computers
    - supercomputer Fugaku provided by the RIKEN Center for Computational Science
    - Oakforest-PACS
    - Polaire and Grand Chariot at Hokkaido University

Intro

- N_f=2+1 thermodynamic property
  - through chiral symmetric formulation
  - Order of the transition
  - (pseudo) critical temperature
  - Location of the phase boundary
  - Near the physical point
- Chiral symmetric formulation
  - Ideal to treat flavor SU(2) and $U(1)_A$ properly
  - Domain wall fermion (DWF) : practical choice
- DWF and chirality
  - Fine lattice needed
  - Aiming for $a < 0.08$ fm (eventually)
  - Current search domain: $0.07 \leq a \leq 0.14$ fm
  - Current criticality range: $0.08 \leq a \leq 0.13$ fm





N_f=2+1 Möbius DWF LCP

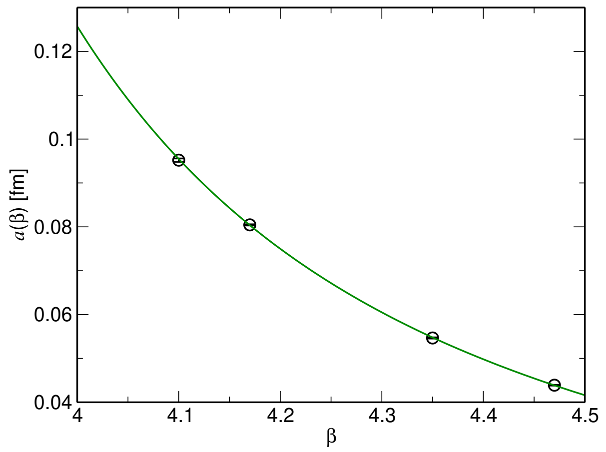
For the Line of constant physics: $am_s(\beta)$ with $a(\beta)$

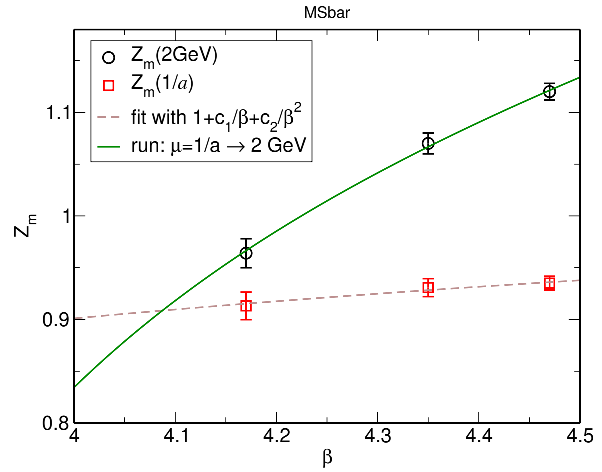
- Step 1: determine $a(\beta)$ [fm] with t_0 (BMW) input
  - at $\beta = 4.1^*, 4.17, 4.35, 4.47$

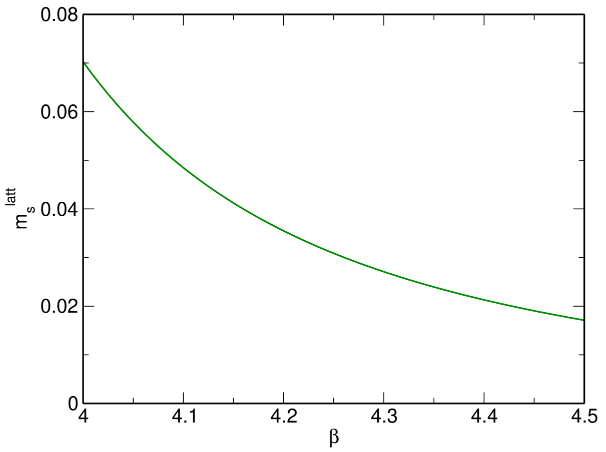
    * β=4.1 from unpublished pilot data, to add support at small β
- Step 2: determine $Z_m(\beta)$ using NPR results
  - at $\beta =$ $4.17, 4.35, 4.47$
  - And use $Z_m(\beta)$ so obtained for $\beta \geq 4.0$: $\beta < 4.17$ region is extrapolation
  - $1/Z_m(\beta)$ will be used to renormalize scalar operator
- Step 3: solve $am_s(\beta)$ with input:
  - $m_s^R = Z_m \cdot am_s^{latt} \cdot a^{-1} = 92$ MeV
  - $\frac{m_s}{m_{ud}} = 27.4$ (See for example FLAG 2019)
- See for details in Lattice 2021 proc by S.Aoki et al.

Do simulation

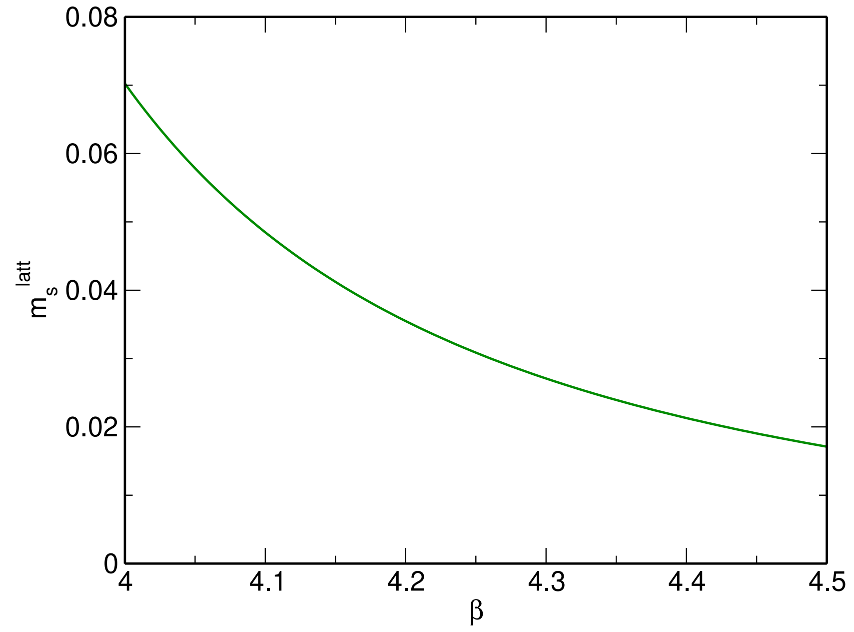
- Step 4: use $a(\beta)$ including new data at $\beta = 4.0$ (preliminary)
  - For dimension-full quantities







LCP remarks

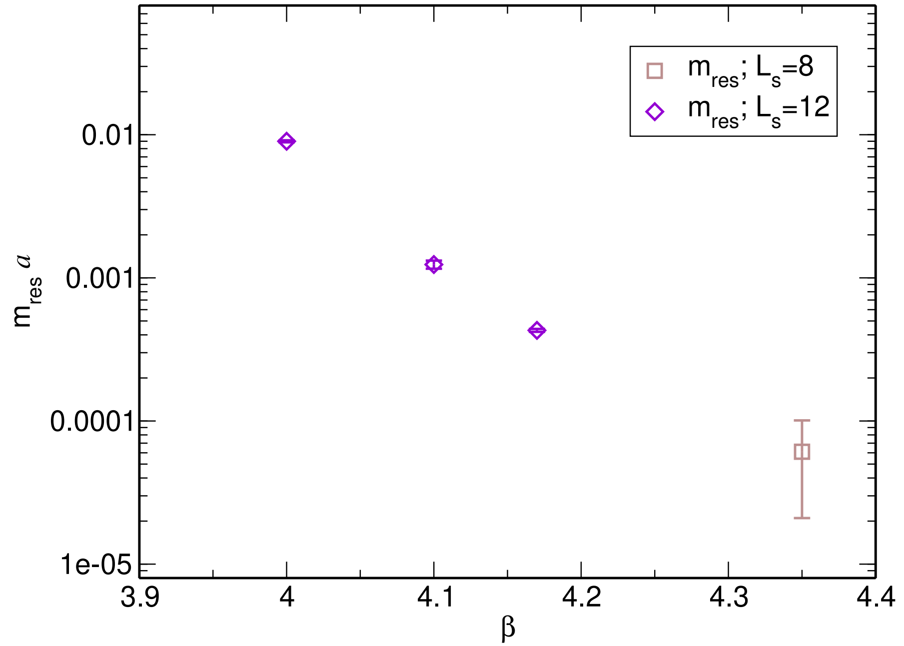


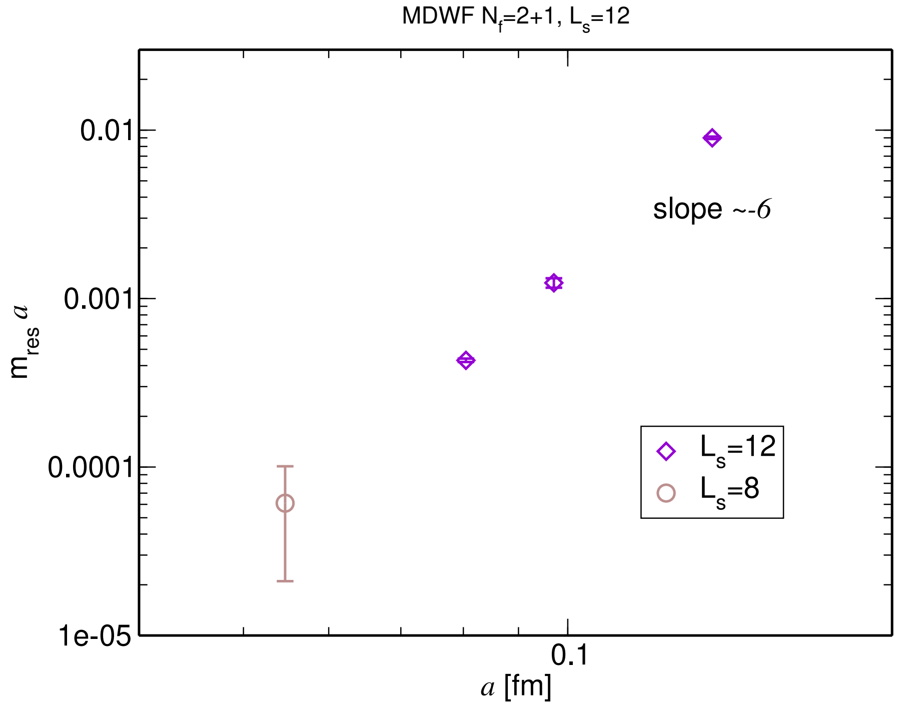
Features

- Fine lattice: use of existing results ($0.04 \le a \le 0.08$ fm)
  - Granted preciseness towards continuum limit
- Coarse lattice parametrization is an extrapolation
  - Preciseness might be deteriorated
  - Newly computing Z_m e.g. at $\beta = 4.0$ (lower edge) might improve, but not done so far
    - NPR of Z_m at a^{-1} ≃1.4 GeV may have sizable error (window problem) anyway
- Smooth connection from fine to coarse should not alter leading $O(a^2)$
  - Difference should be higher order
- Error estimated from Kaon mass
  - Δm_K ~ 10 % at $\beta = 4.0$ ($a \simeq 0.14$ fm)
  - Δm_K ~ a few % at $\beta = 4.17$ ($a \simeq 0.08$ fm)

Domain wall fermion !

- Möbius DWF → OVF by reweighting
  - Successful (w/ error growth) at $\beta = 4.17$ ($a \simeq 0.08$ fm)
    - See Lattice 2021 JLQCD (presenter: K.Suzuki)
  - Questionable for
    - Coarser lattice: rough gauge, DWF chiral symmetry breaking
    - Finer lattice: larger V (# sites)
- Chiral fermion with continuum limit
  - A practical choice is to stick on DWF
- Controlling chiral symmetry breaking with DWF
  - WTI residual mass m_{res}: $m_\pi^2 \propto (m_f + m_{res})(1 + h.o.)$
  - Understanding $m_{res}(\beta)$ with fixed L_s (5-th dim size)
- $m_{res}[MeV] \sim a^X$, where $X \sim 5$
  - Vanishes quickly as $a \to 0$
  - 1st (dumb) approximation: forget about m_{res}
  - Better : $m_f^{cont} \leftrightarrow (m_f + m_{res})$ but, this is not always enough

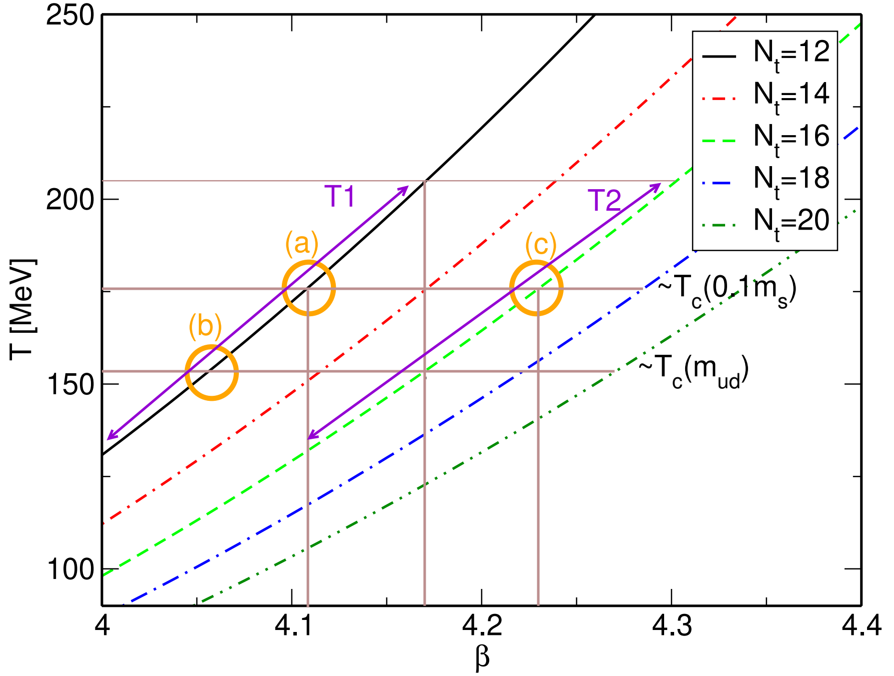


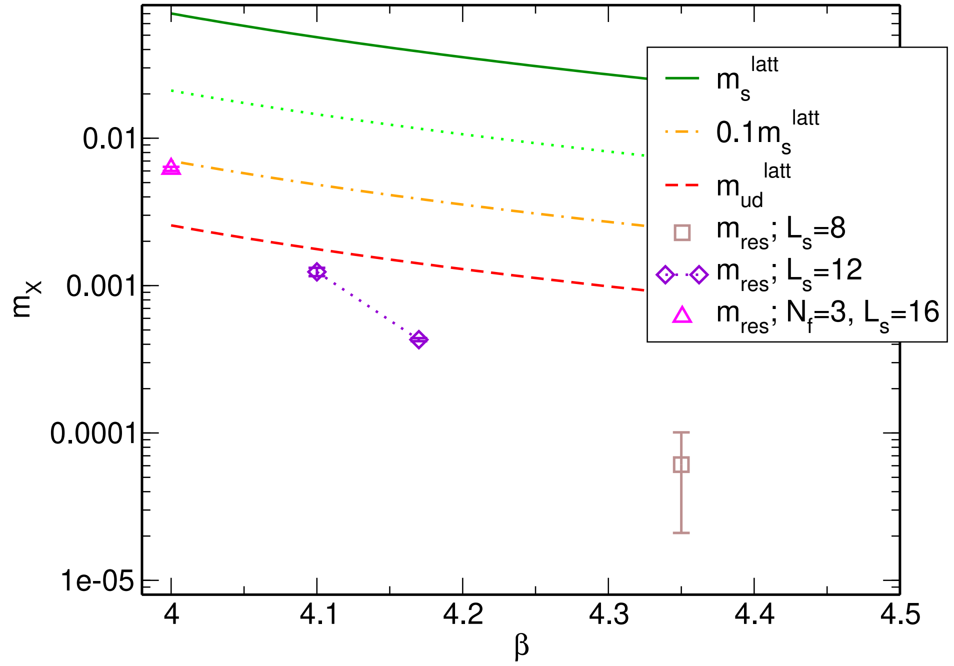


Simulation plan: 1st round

$L_s = 12$ fixed throughout this study

- T1-(a)
  - $N_t = 12$
  - $m_l = 0.1 m_s$
  - $V_s = 24^3$
- T2-(c)
  - $N_t = 16$
  - $m_l = 0.1 m_s$
  - $V_s = 32^3$
- T1-(b)
  - $N_t = 12$
  - $m_l \simeq m_{ud} \rightarrow m_l^{input} = 0$
  - $V_s = 24^3$





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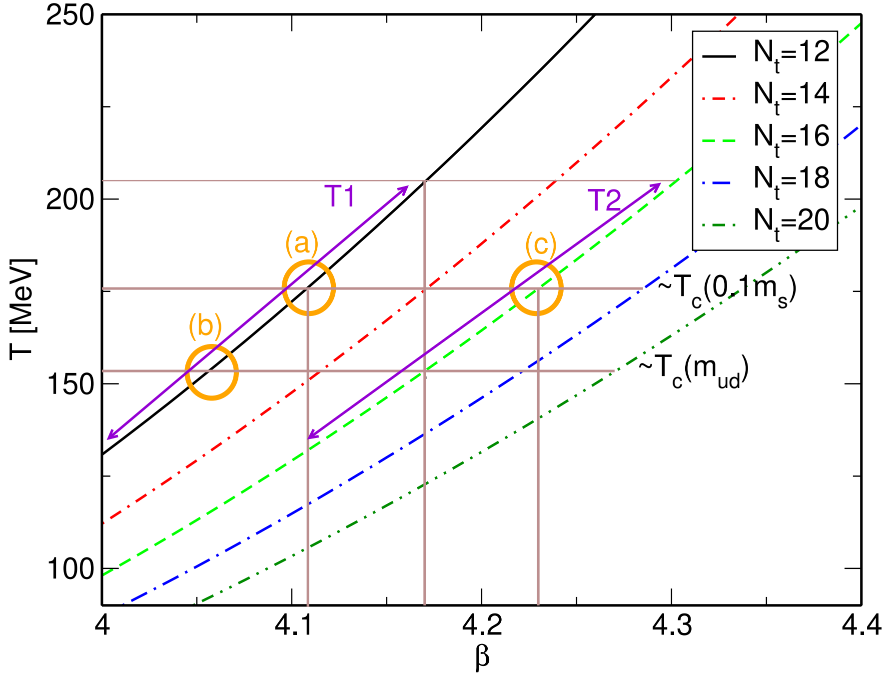
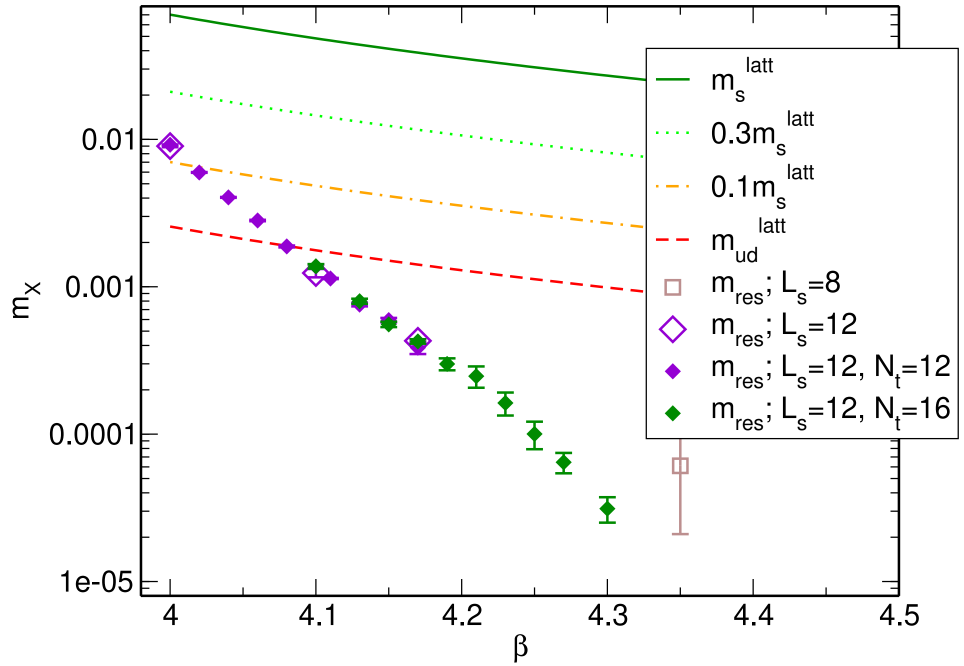
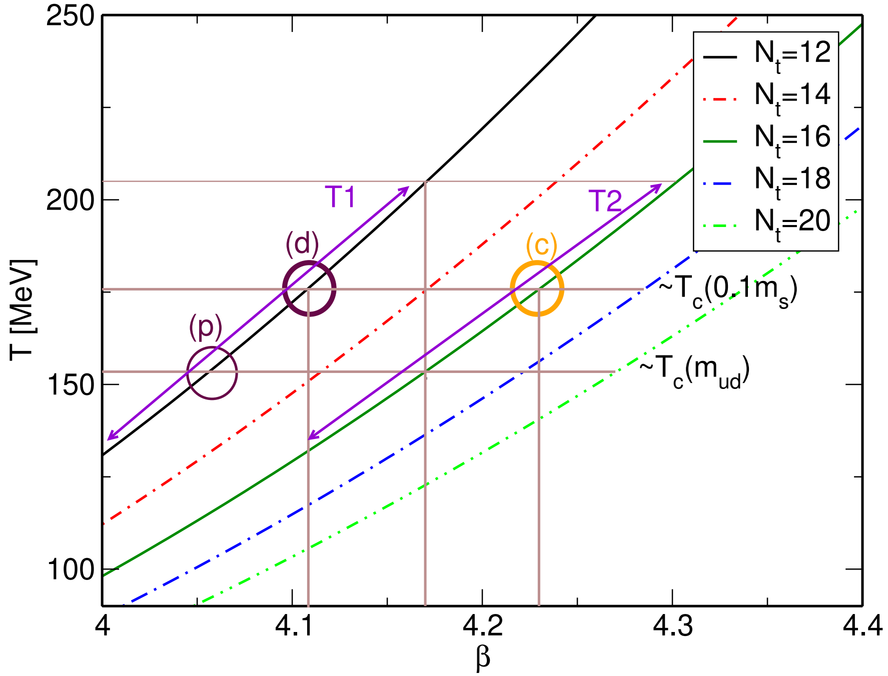


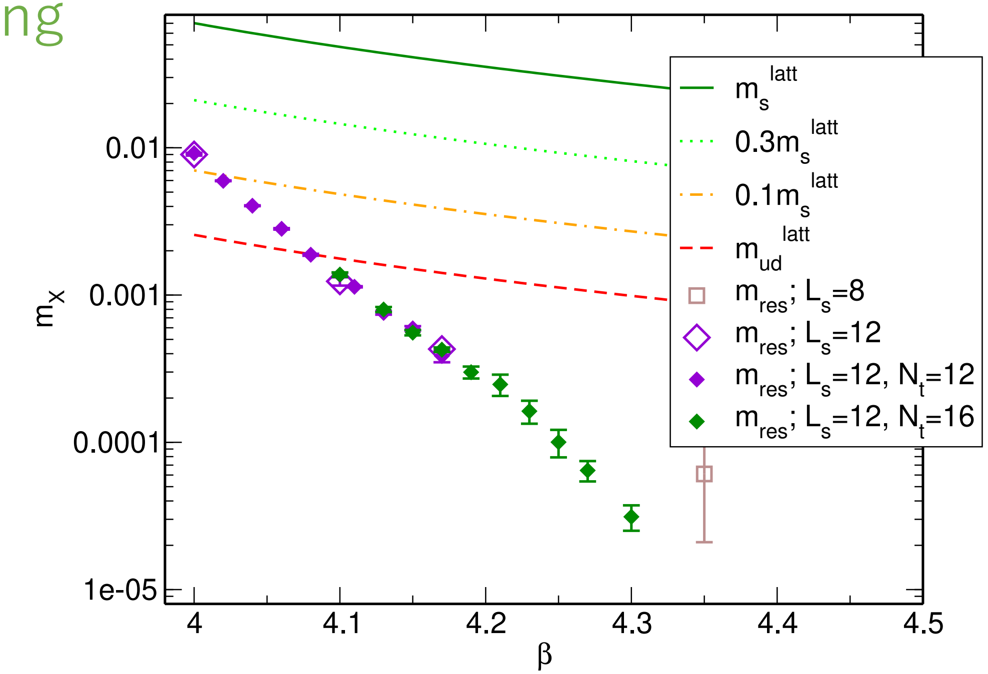


Simulation plan: 2nd round w/ treatment of $\boldsymbol{m_{res}}$ effect

$L_s = 12$ fixed throughout this study

- T1-(d)
  - $N_t = 12$
  - $m_l = 0.1 m_s$
  - $m_q^{input} = m_q^{LCP} - m_{res}$
  - $V_s = 24^3$, $\mathbf{32^3}$
- T2-(c)
  - $N_t = 16$
  - $m_l = 0.1 m_s$
  - m_{res} shift by reweighting
  - $V_s = 32^3$
- T1-(p)
  - $N_t = 12$
  - $m_l = m_{ud}$
  - $m_q^{input} = m_q^{LCP} - m_{res}$
  - $V_s = 24^3$

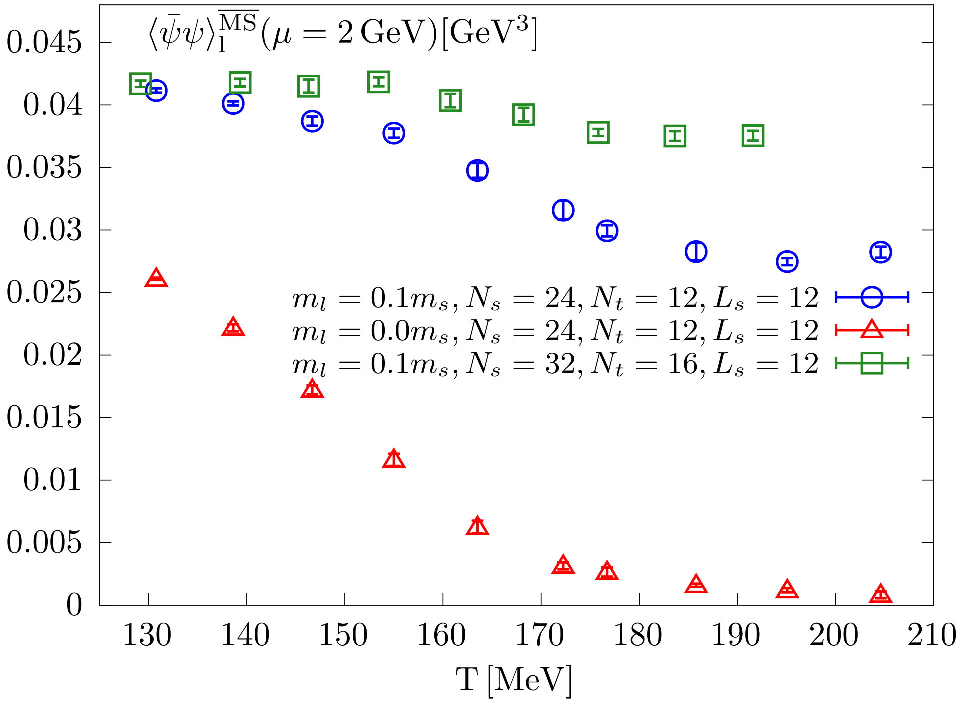


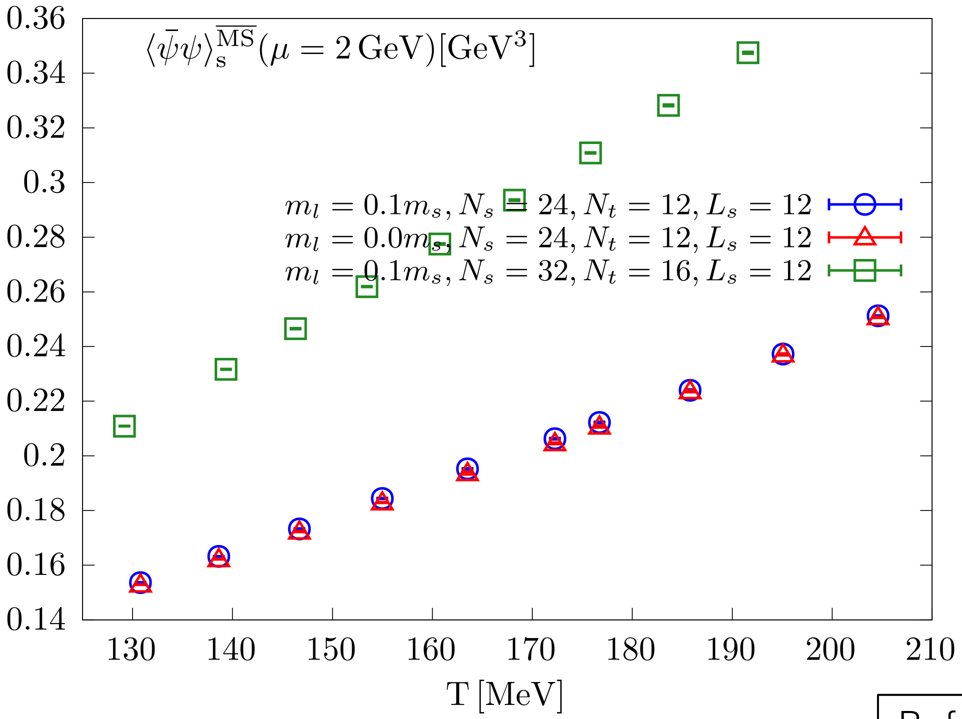


Results and discussion on round 1

Light quark $\Sigma = -\langle\overline{\psi}\psi\rangle$

- Two step UV renormalization necessary (naively)
  - Logarithmic divergence (multiplicative): $Z_S(\overline{MS}, 2\text{ GeV})$
  - Power divergence (additive): $\propto m_f\, a^{-2}$
    - Subtracted using $\langle\bar{s}s\rangle$





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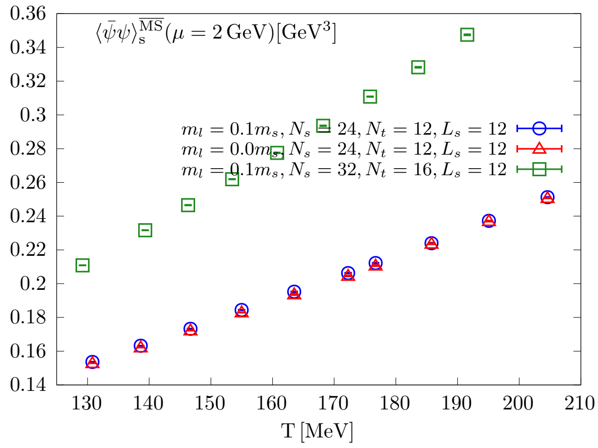

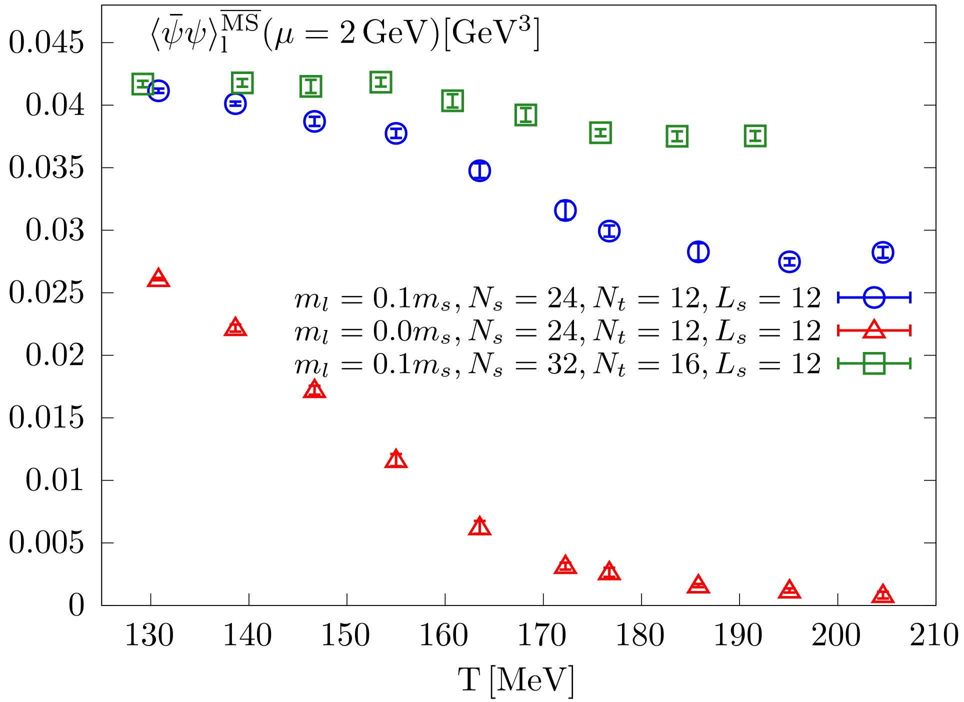

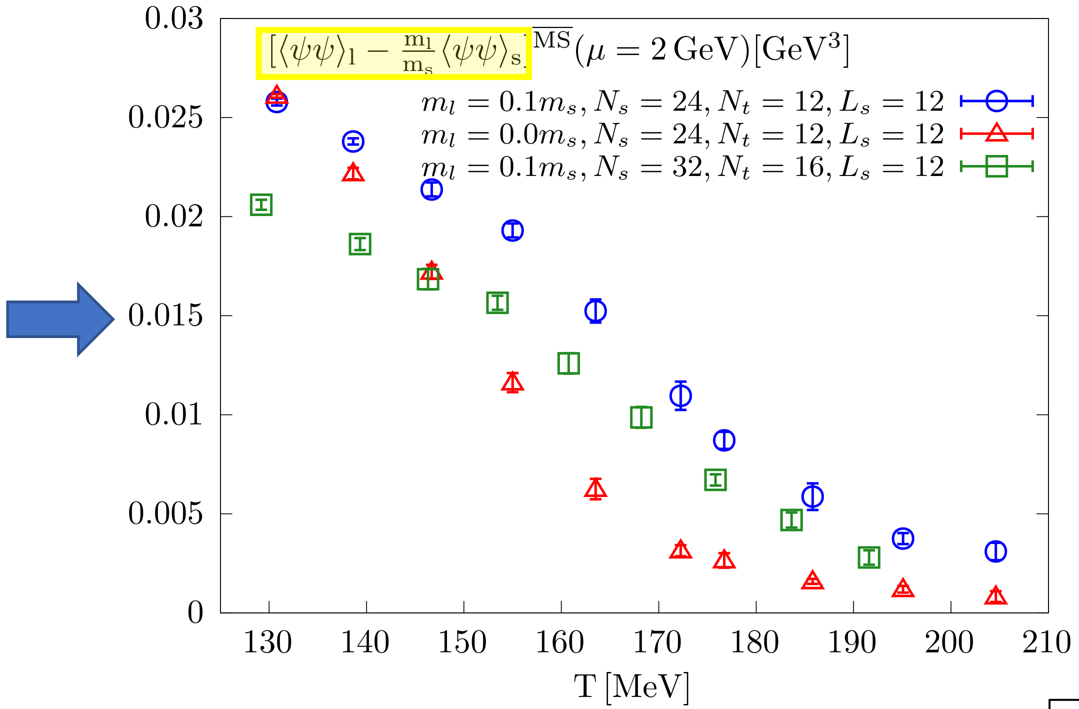

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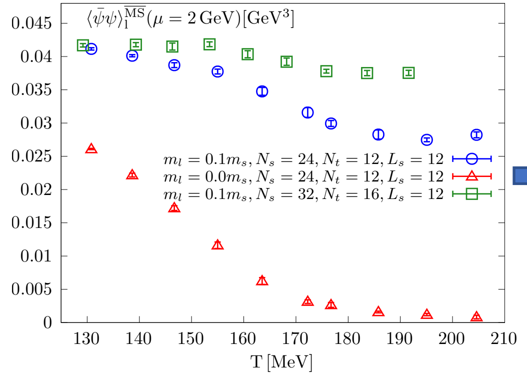
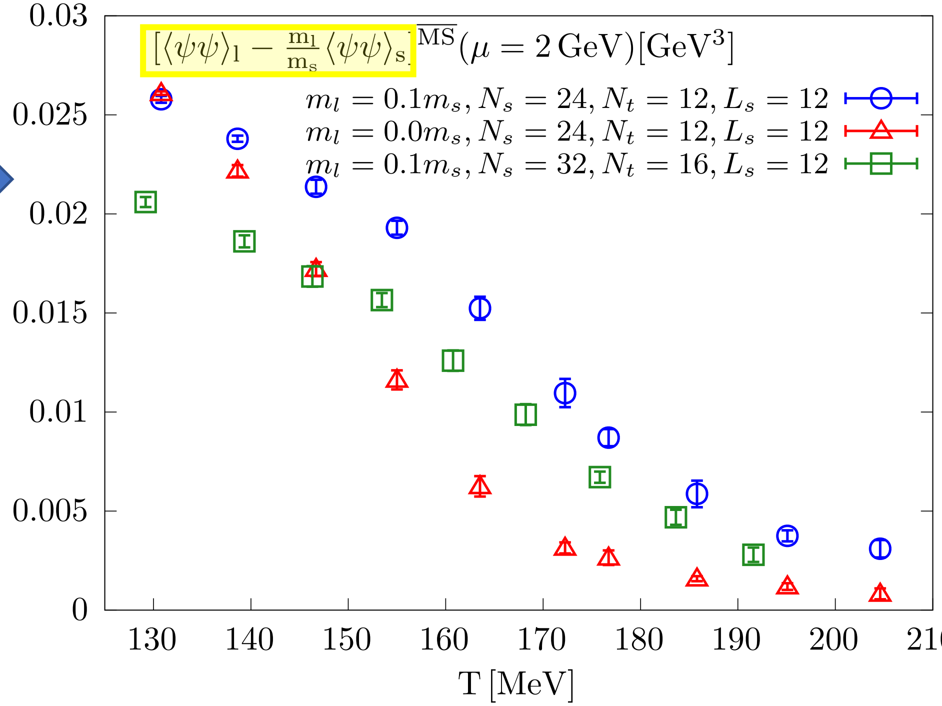
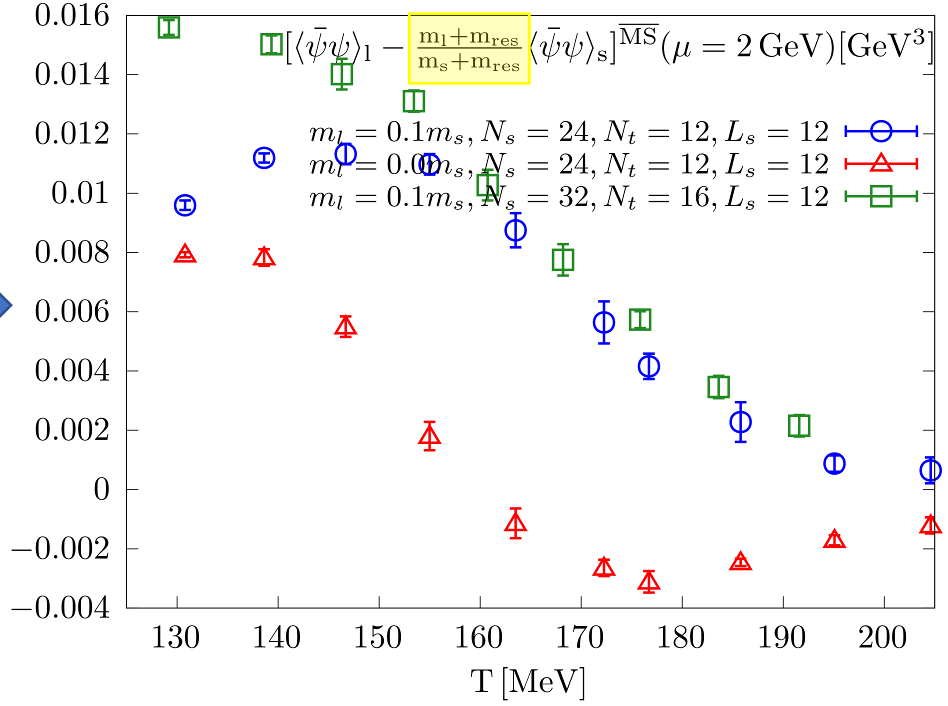
$$m_{res} = \frac{\langle 0|J_{5q}|\pi\rangle}{\langle 0|P|\pi\rangle}$$

$$m_\pi^2 \propto (m_f + m_{res})$$



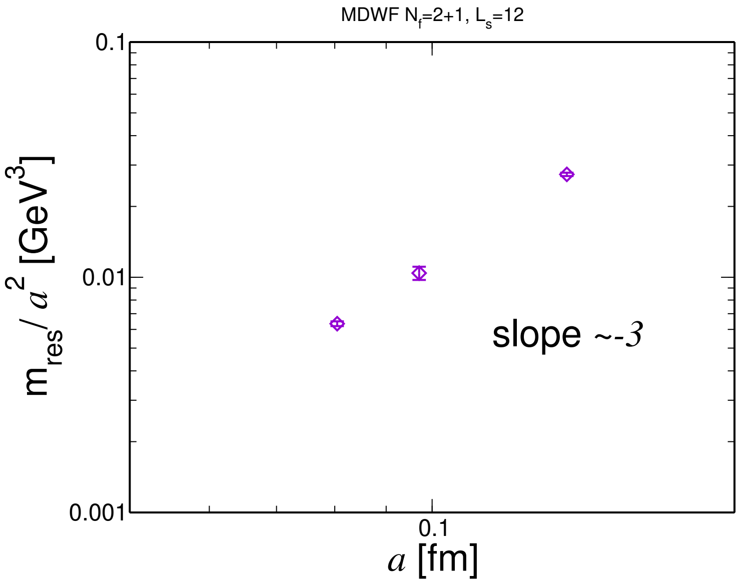




Before "step 4"

Light quark $\Sigma = -\langle\overline{\psi}\psi\rangle$: residual power divergence

- $\Sigma|_{DWF} \sim \frac{m_f + x m_{res}}{a^2} + \Sigma|_{cont.} + \cdots$ S. Sharpe (arXiv: 0706.0218)
  - $m_{res} \neq x m_{res}$; $x = O(1) \neq 1$
    - "Since x is not known, this term gives an uncontrolled error in the condensate. It can be studied and reduced only by increasing L_s - a very expensive proposition." – S. Sharpe.



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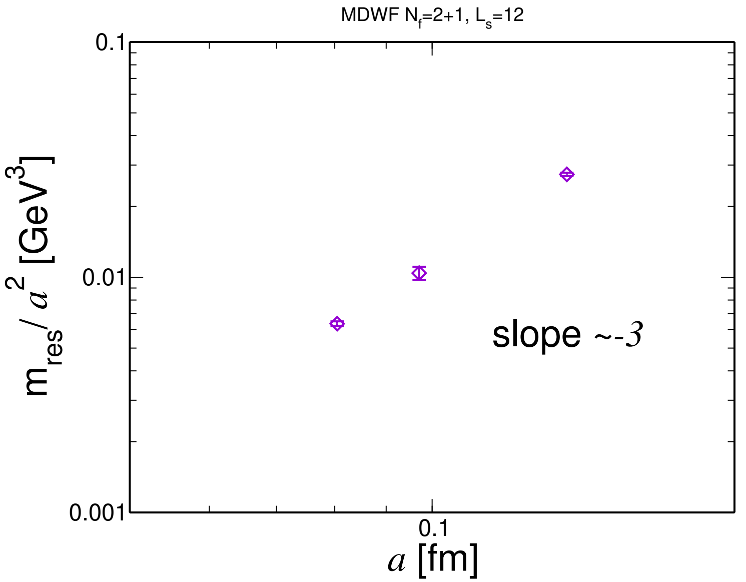
- There is a way to estimate $x m_{res}$ using m'_{res}
  - If chiral symmetry is restored $\rightarrow \Sigma|_{cont.} = 0$
  - $\boldsymbol{-x m_{res}}$ is a **zero** of $\Sigma|_{DWF}$ which is **related** with
  - $m'_{res} = \frac{\sum_x \langle J_{5q}(x) P(0)\rangle}{\sum_x \langle P(x)\, P(0)\rangle}$ $\left(\leftrightarrow\ m_{res} = \frac{\sum_{\vec{x}} \langle J_{5q}(\vec{x},t) P(0)\rangle}{\sum_{\vec{x}} \langle P(\vec{x},t)\, P(0)\rangle}\right.$ at large t)
    - Axial WT identity: $(m_f + m'_{res}) \sum_x \langle P(x)\, P(0)\rangle = \Sigma$



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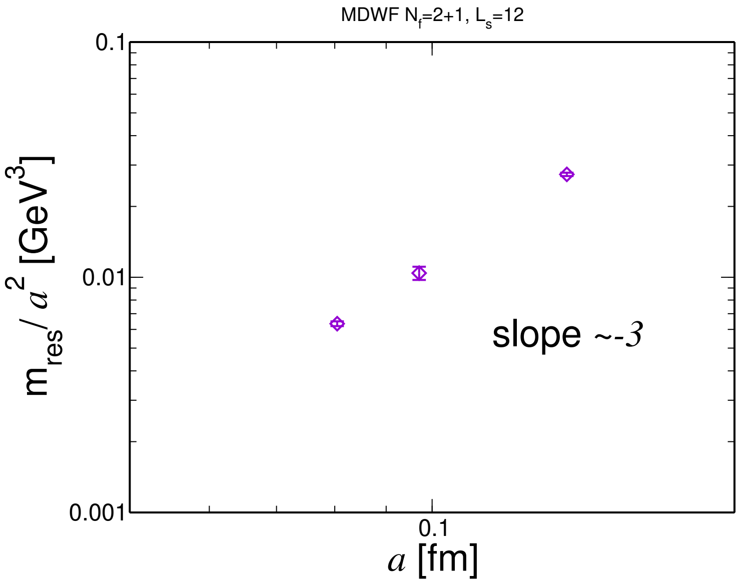
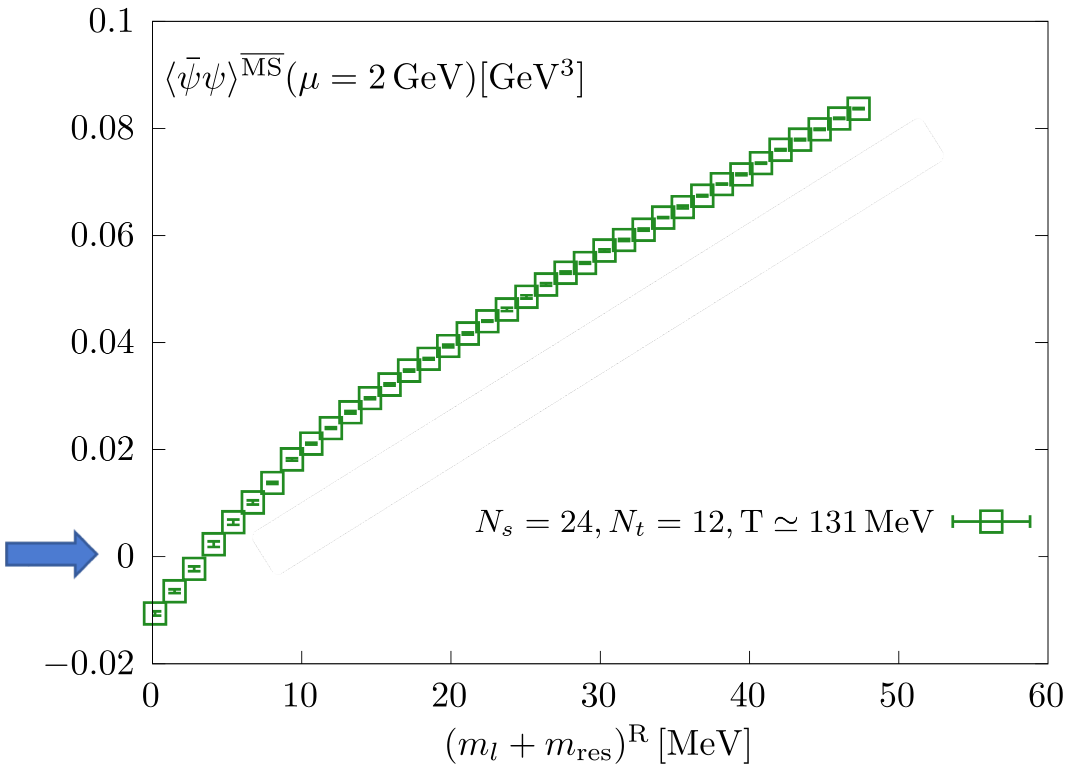


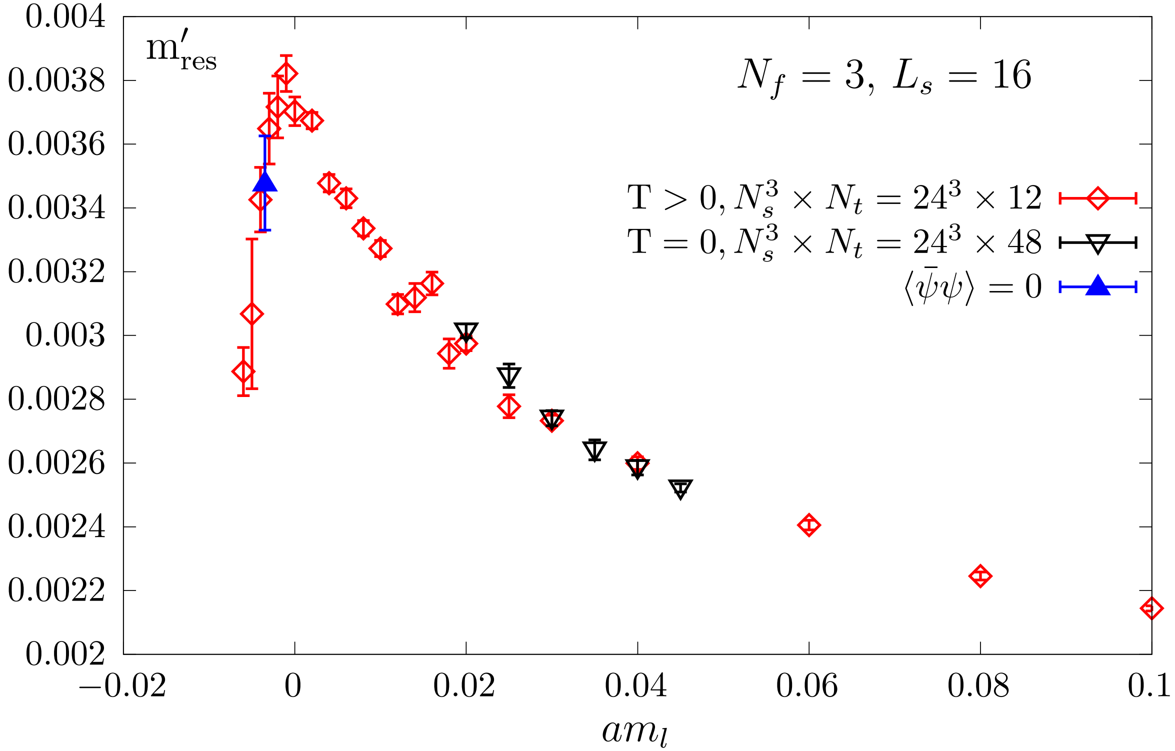
"Forget about m_{res}" is dumber for Σ, but…

m'_{res}: example in N_f=3 case

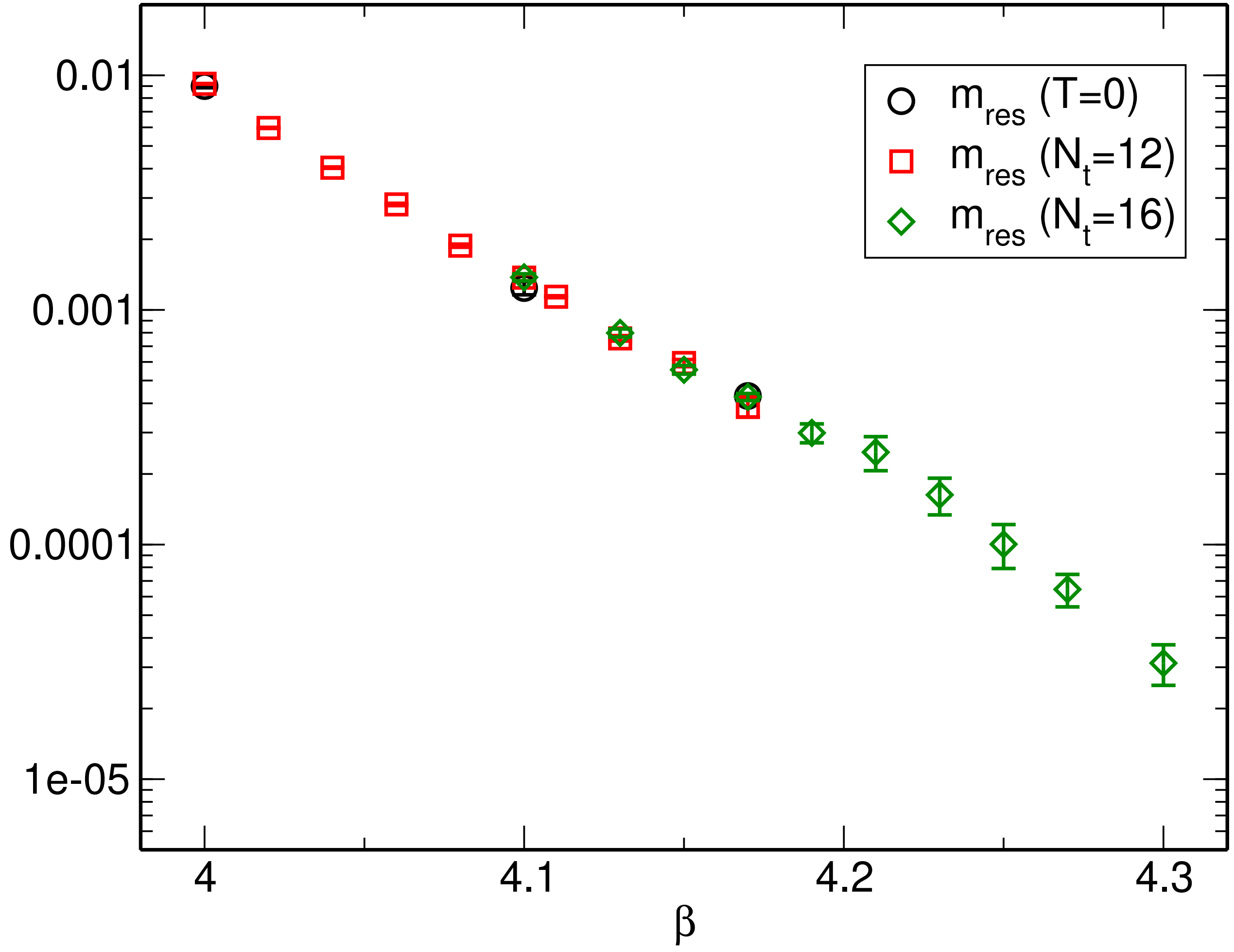
$$m'_{res} = \frac{\sum_x \langle J_{5q}(x) P(0)\rangle}{\sum_x \langle P(x)\, P(0)\rangle}$$

WTI: $(m_f + m'_{res}) \sum_x \langle P(x)\, P(0)\rangle = \Sigma$





m_{res} and m'_{res} for N_f=2+1



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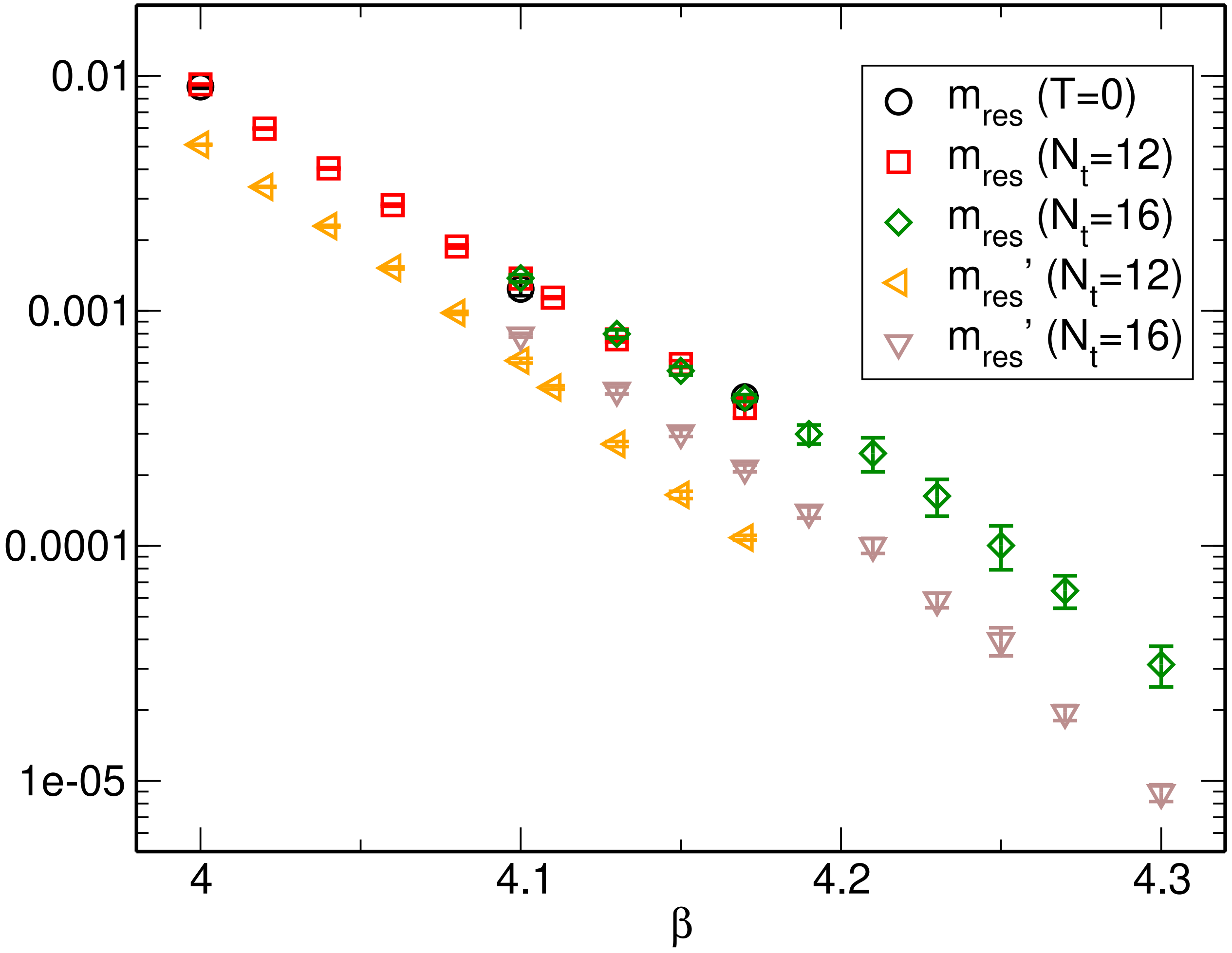

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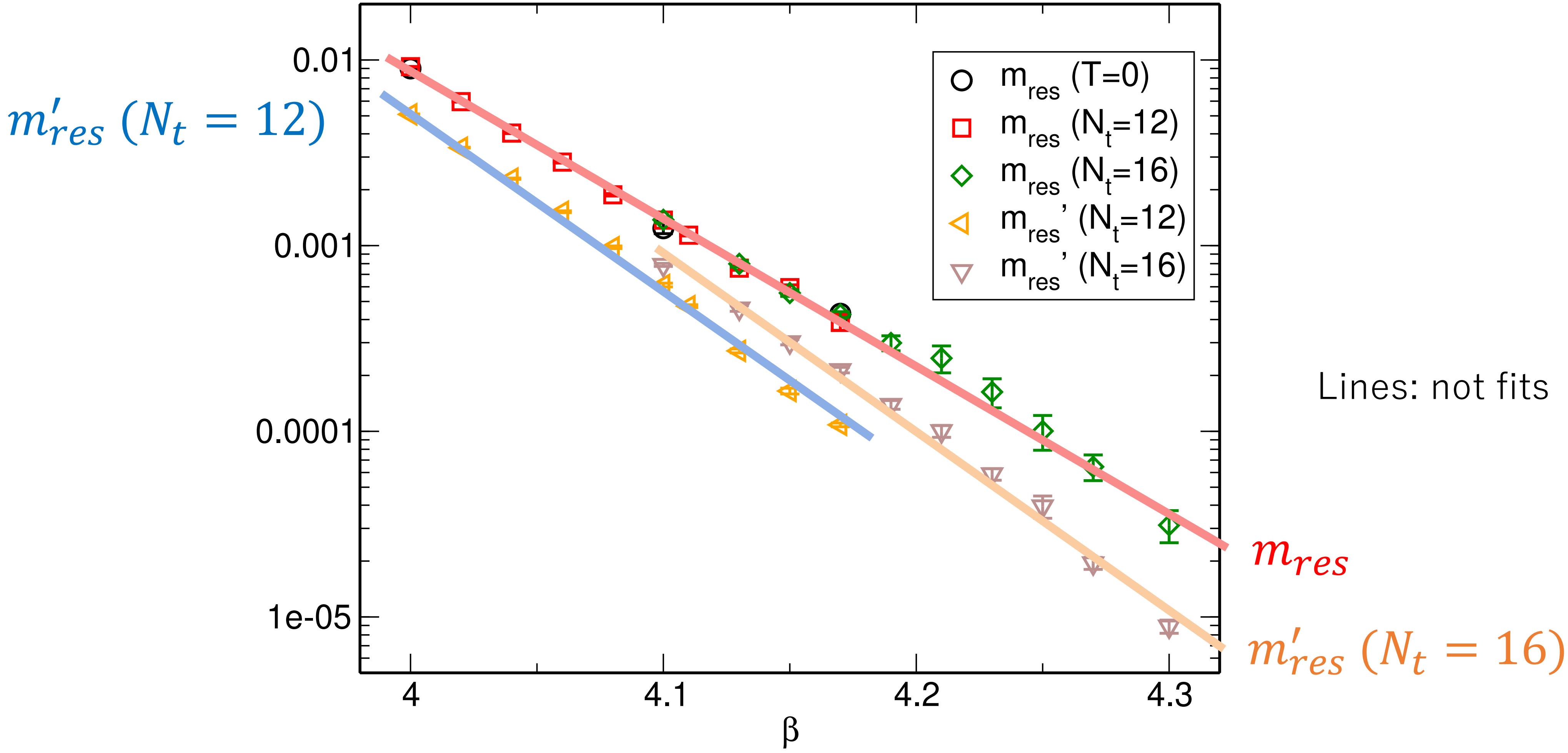
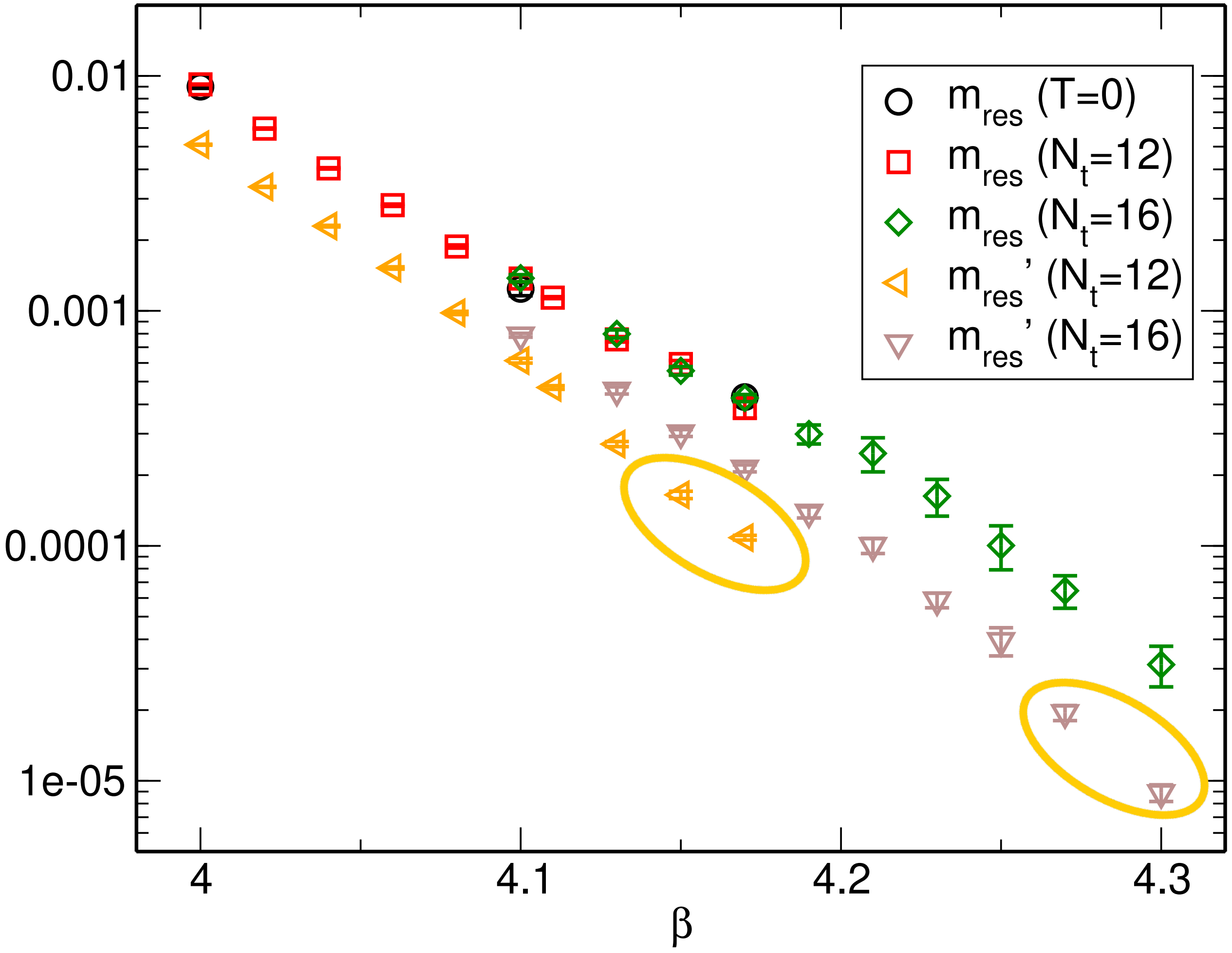


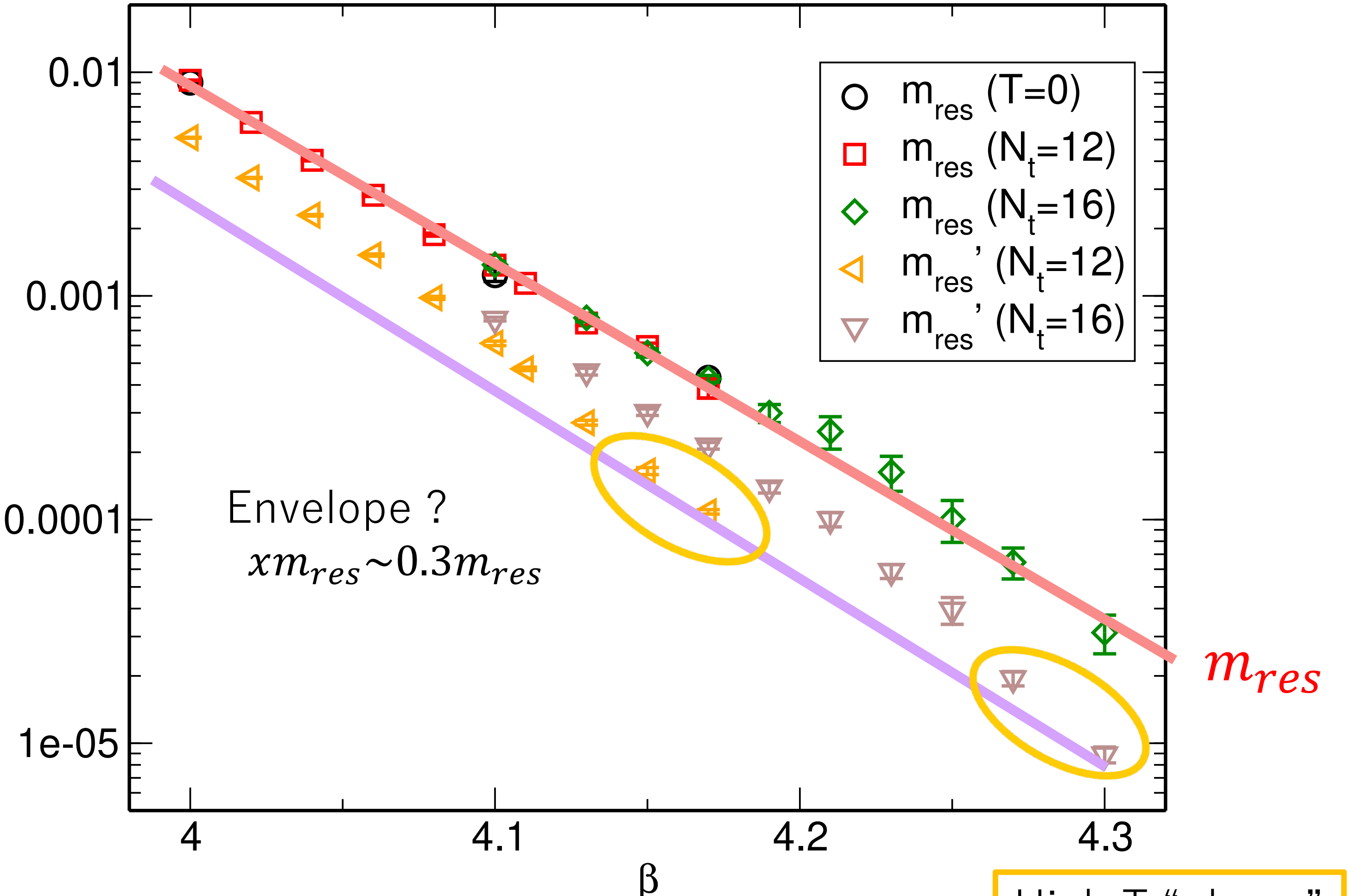
Lines: not fits

m_{res} and m'_{res} for N_f=2+1



High T “phase”

m_{res} and m'_{res} for N_f=2+1



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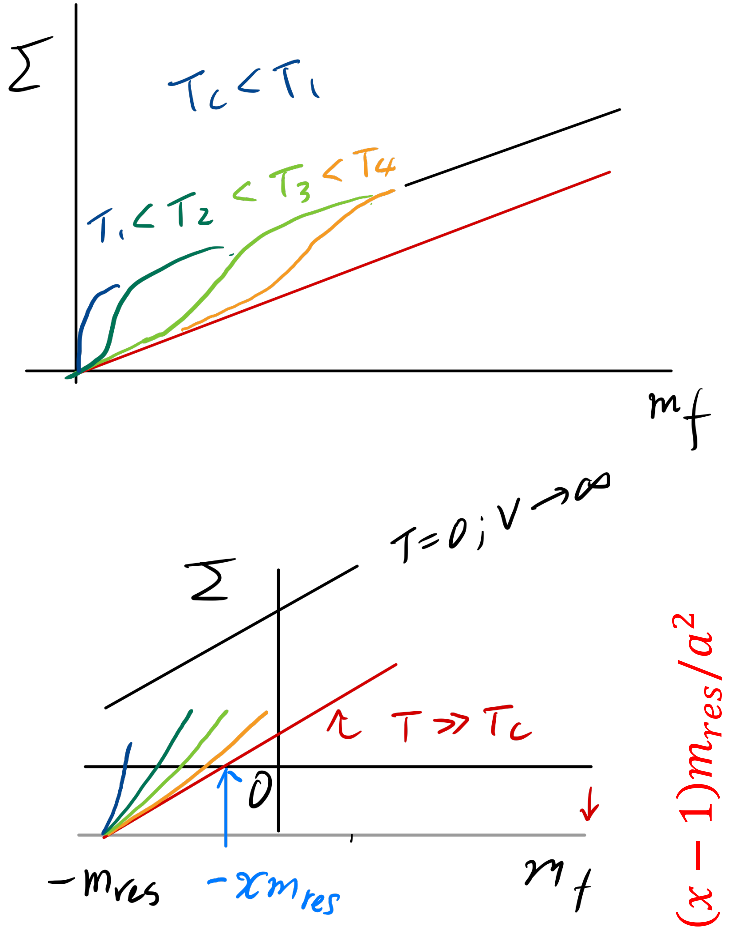
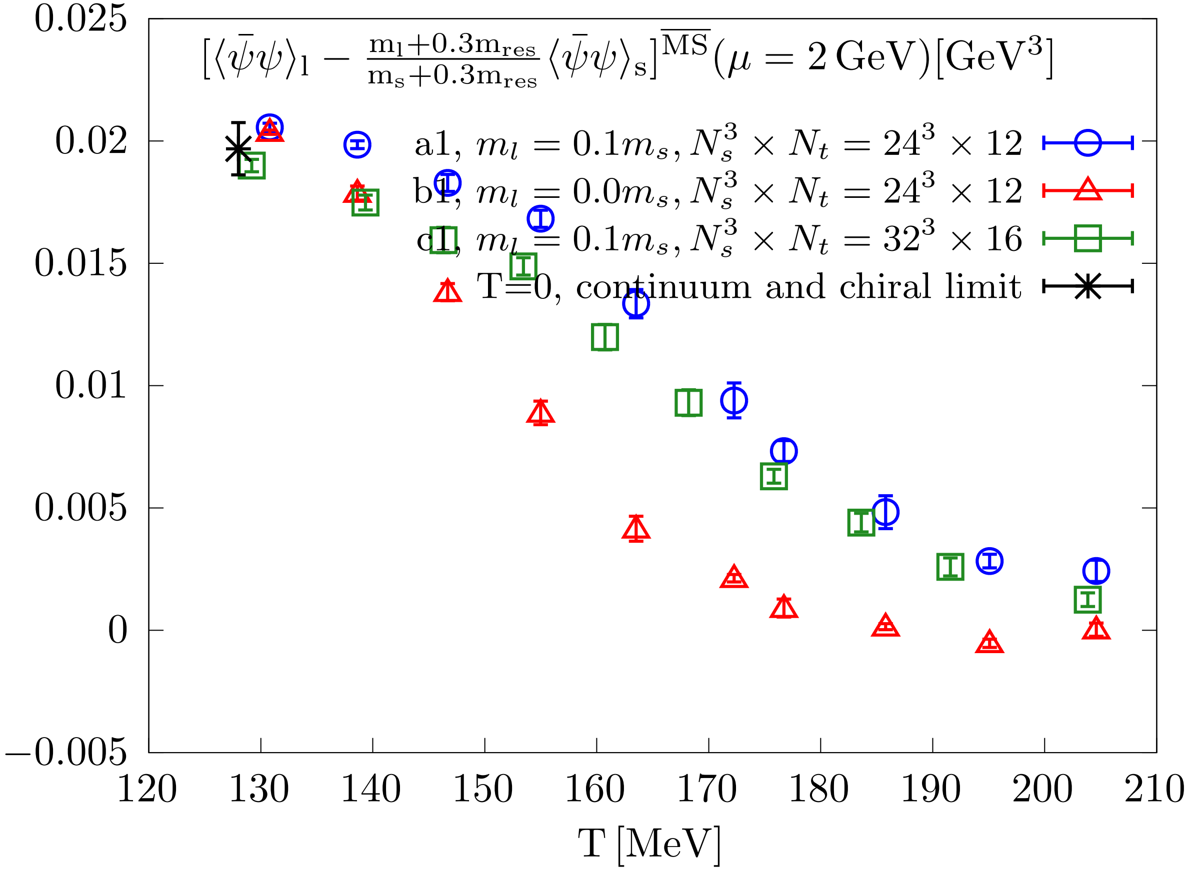


$$xm_{res} = \lim_{T\to\infty} m'_{res}$$

Subtraction with $x = 0.3$



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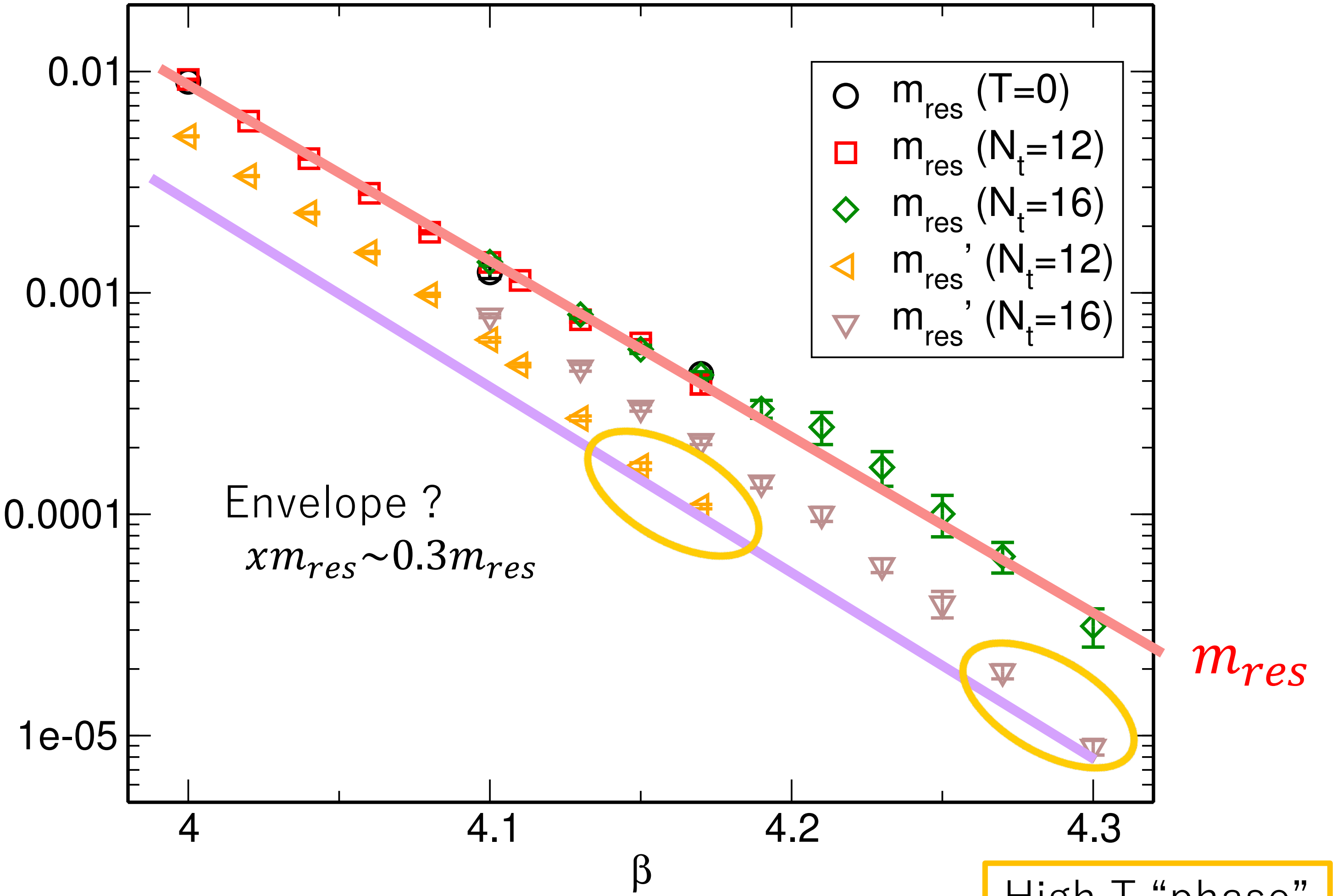

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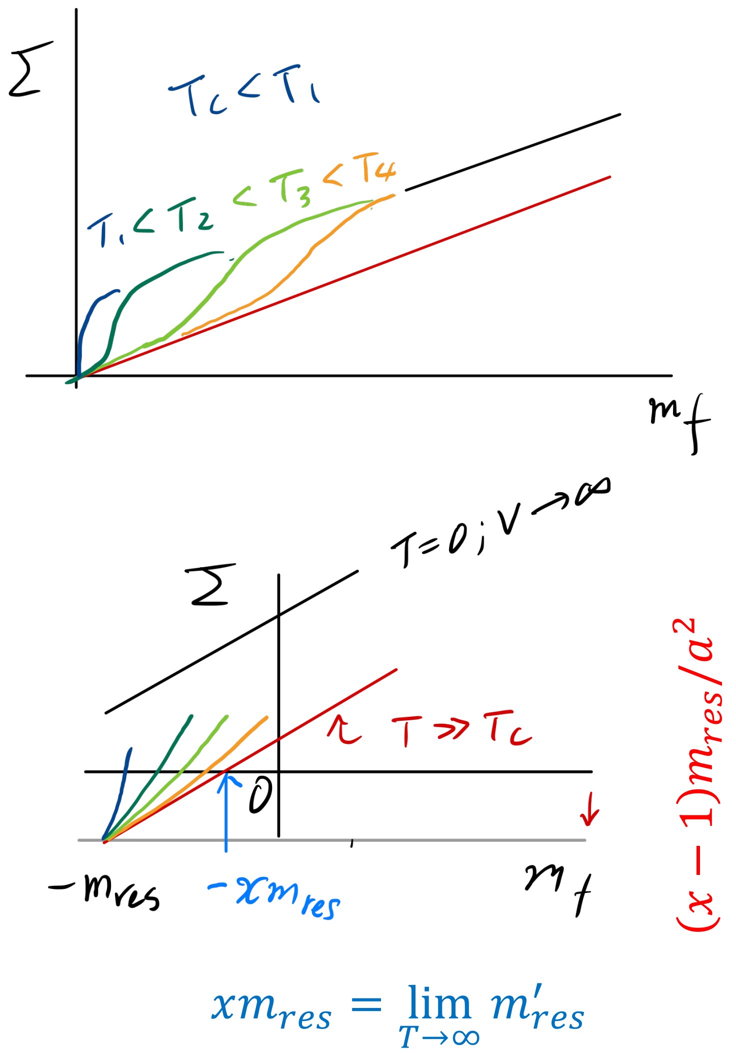

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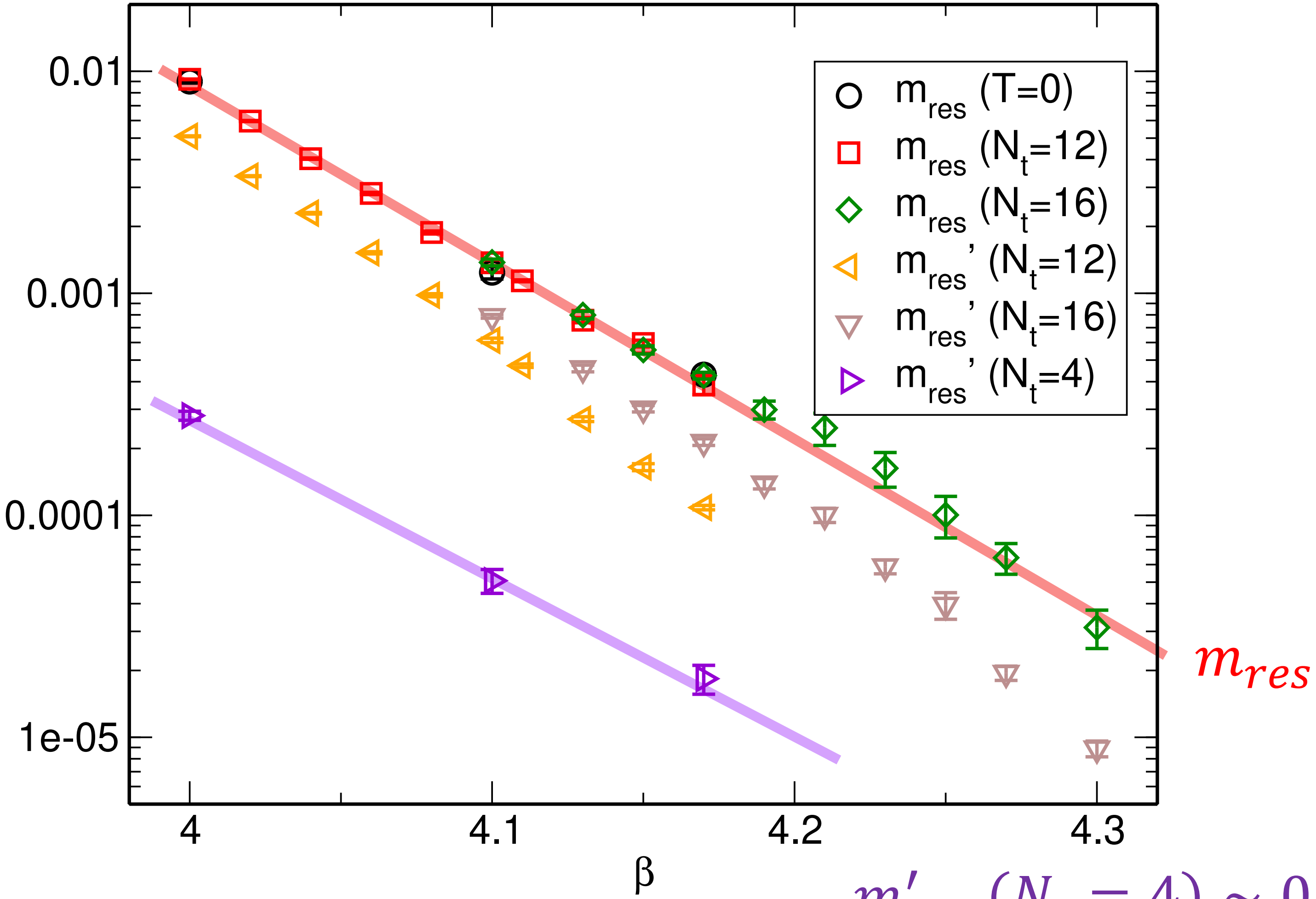
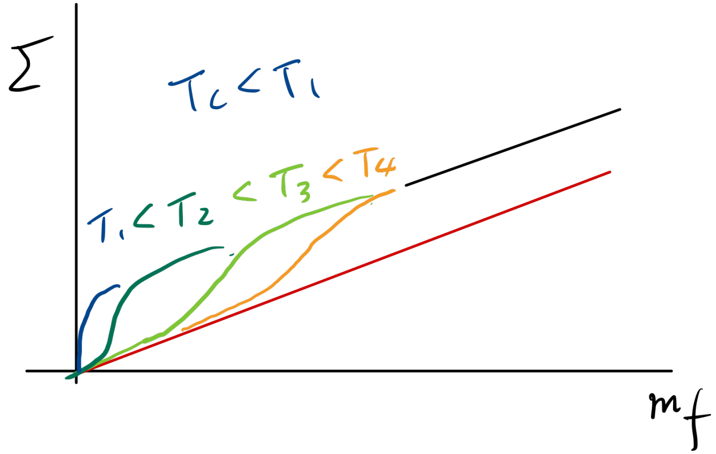
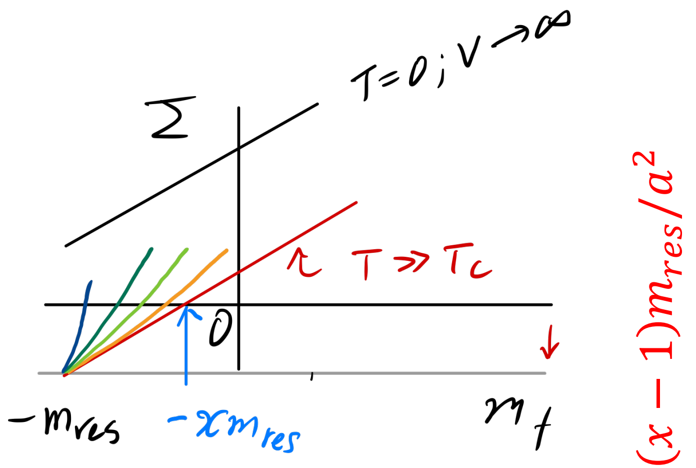


Lines: not fits

$m'_{res}\,(N_t = 4) \sim 0.03\, m_{res}$





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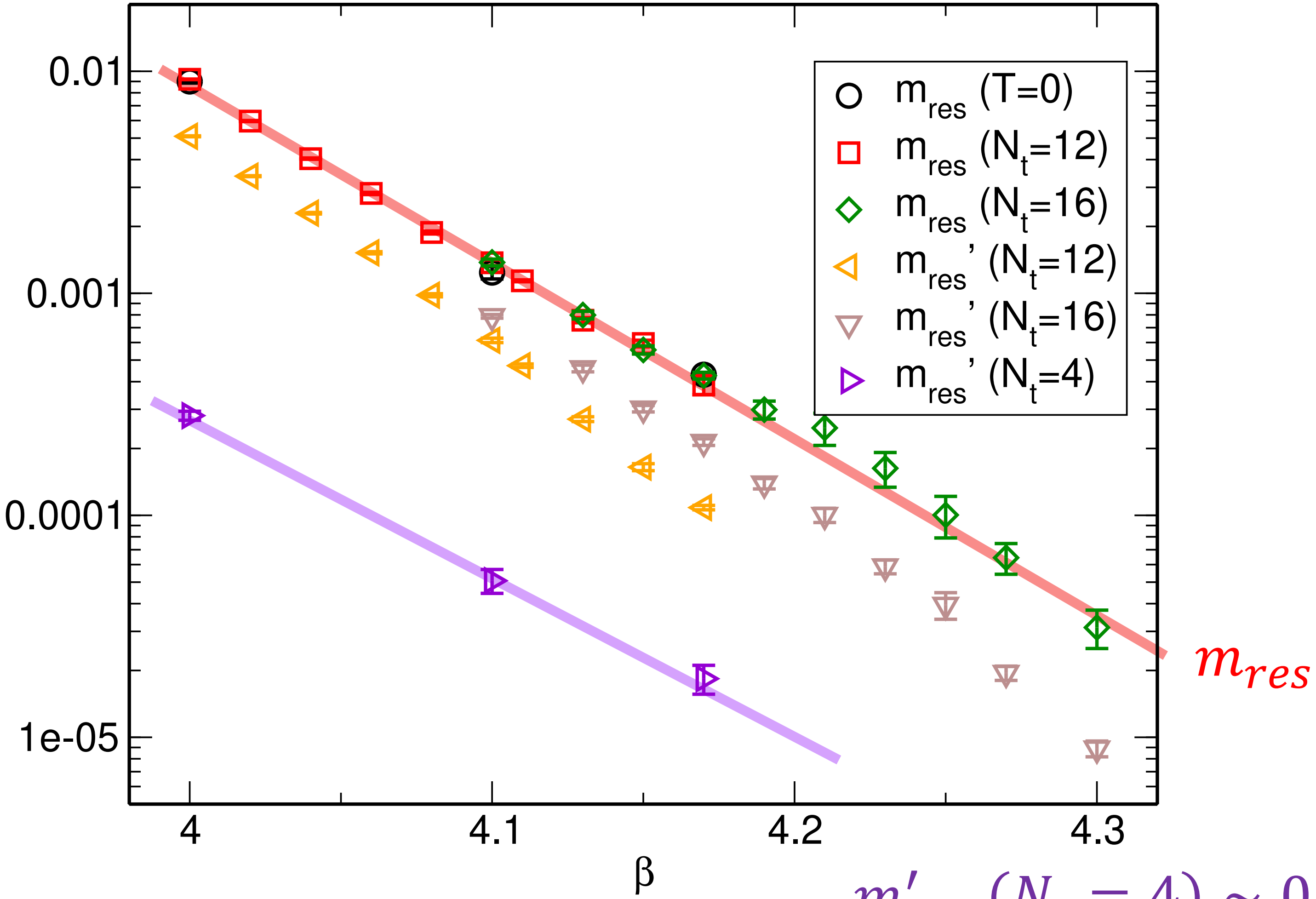
m_{res} and m'_{res} for N_f=2+1



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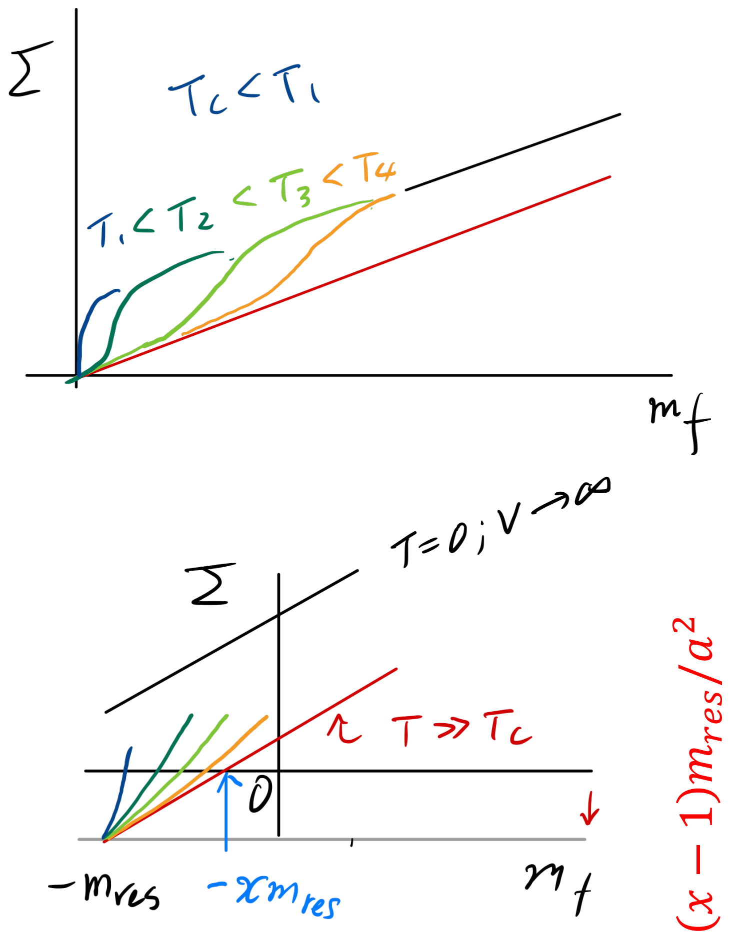


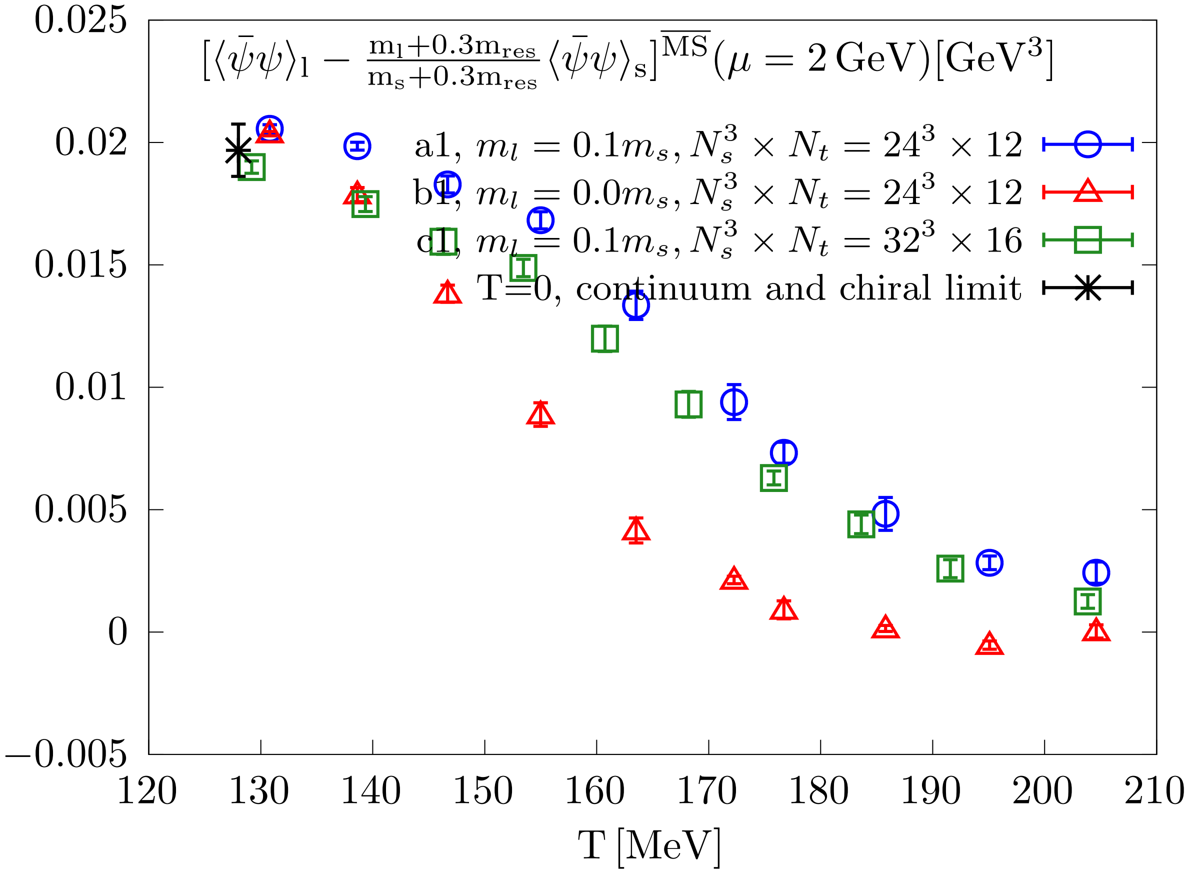
$$x m_{res} = \lim_{T\to\infty} m'_{res}$$

$$m'_{res}\,(N_t = 4) \sim 0.03\, m_{res}$$

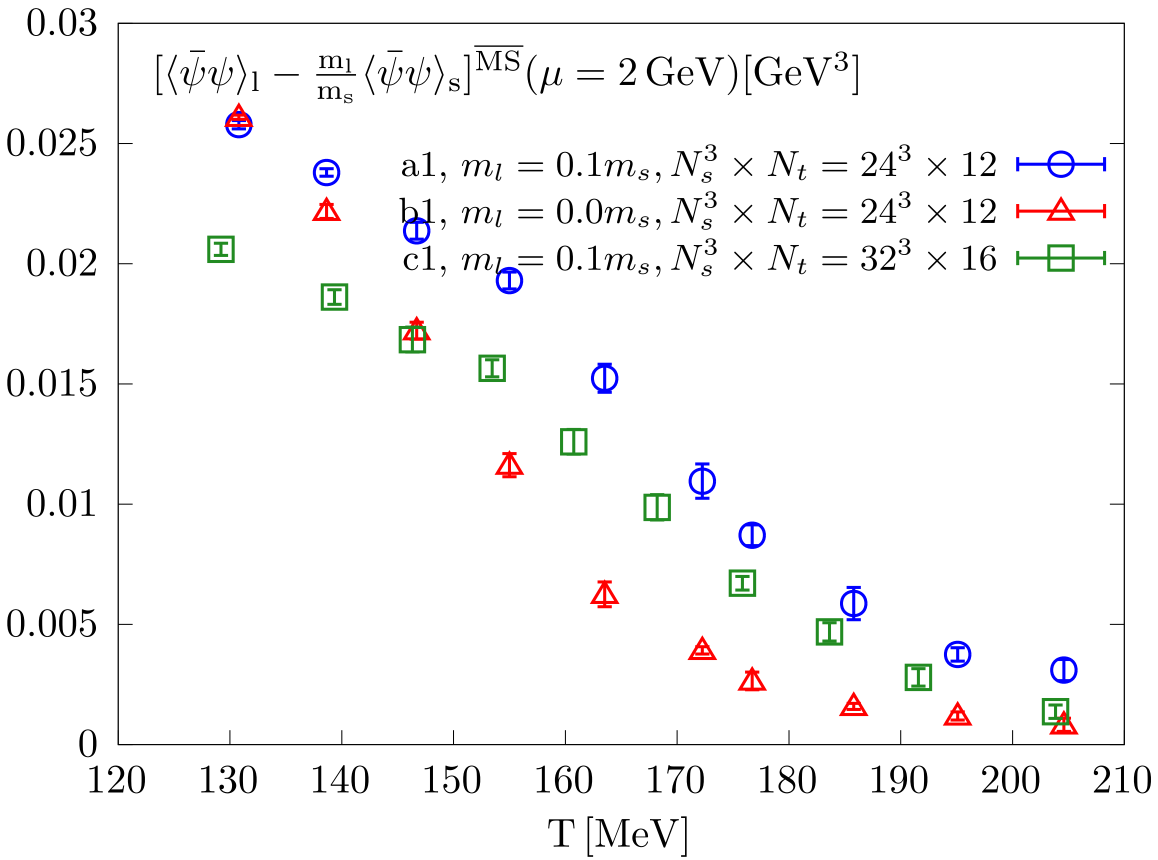
$x < 0.03$!

Subtraction with $x = 0.3$ and $x = 0$

$x = 0.3$



$x = 0$: this should be closer to the truth



- Note: quark mass tuning w/o caring m_{res}

Round 2 → see next talk

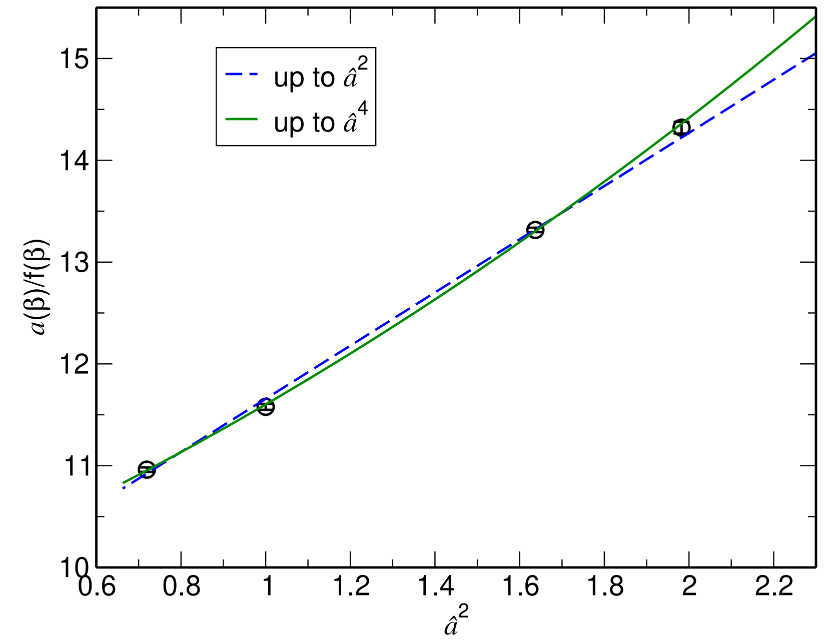
Summary

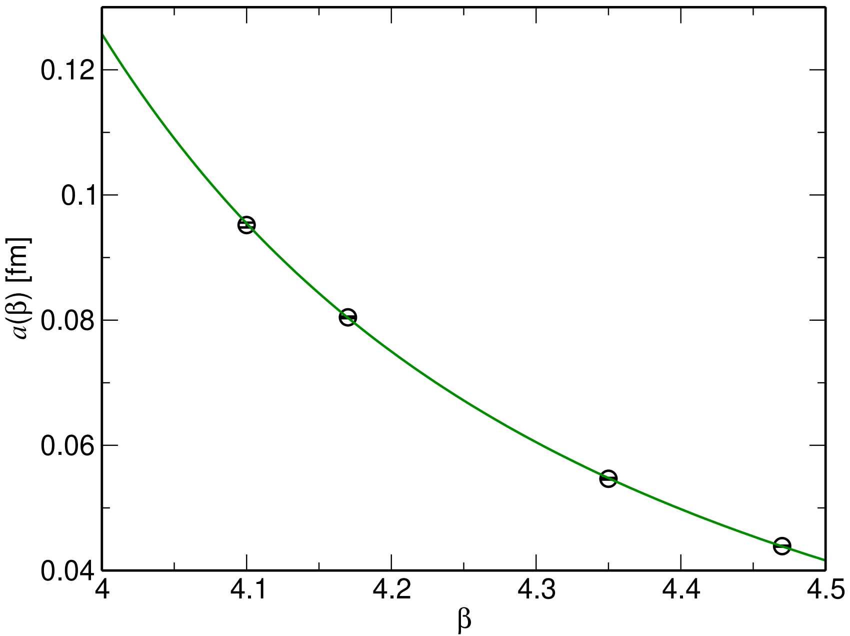
- Möbius DWF simulation for T>0 with N_t=12, 16
  - ↔ N_t=8 by HotQCD (2012)
- Along the Line of Constant Physics
  - Using quark mass input
- Fixed L_s computation : good chiral symmetry ($a > 0$) → exact symmetry ($a \to 0$)
- But, requires a delicate treatment depending on quantity of interest
  - One of the most difficult quantity may be the chiral condensate
  - method to subtract residual power divergence under development
    - Using m'_{res}
    - S. Sharpe's x is not $O(1)$ but seemingly very small (for MDWF)
    - Residual power "divergence" term ($\propto (1-x)$) is larger than that for $x = O(1)$
- First round simulations with $m_l^{input} = 0.1\, m_s$, (and 0): N_s/N_t=2
  - using Supercomputer Fugaku
  - All results here are still preliminary
- 2nd round and further discussion is given by I. Kanamori

backup

N_f=2+1 Möbius DWF

- $a(\beta)$
- Using
  - JLQCD T=0 lattices with t_0 meas.
    - a=0.080, 0.055, 0.044 fm (published)
    - a=0.095 fm (pilot study) to guide LCP
    - a=0.136 fm added later for precision scale
  - Parameterization of Edwards et al (1998)
    - $a = c_0 f(g^2)(1 + c_2 \hat{a}(g)^2 + c_4 \hat{a}(g)^4).$
    - $$\hat{a}(g)^2 \equiv [f(g^2)/f(g_0^2)]^2,$$
      $$f(g^2) \equiv (b_0 g^2)^{-b_1/2b_0^2} \exp\left(-\frac{1}{2b_0 g^2}\right),$$
      $$b_0 = \frac{1}{(4\pi)^2}\left(11 - \frac{2}{3}N_f\right), \quad b_1 = \frac{1}{(4\pi)^4}\left(102 - \frac{38N_f}{3}\right),$$
    - Fit to $\hat{a}^4$ works well





N_f=2+1 Möbius DWF LCP

- Quark mass as function of β [fixed physics]
- We use quark mass input
  - $m_s = 92\ MeV$ (MSb 2GeV)
  - $\frac{m_s}{m_{ud}} = 27.4$ (See for example FLAG 2019)
  - $m_q^R = Z_m \cdot (am_q^{latt}) \cdot a^{-1}(\beta)$

- Parameterizing $Z_m(\beta)$
  - Take $Z_m(2GeV)$ w/ NPR Tomii et al 2016
  - $Z_m(2GeV) \rightarrow Z_m(a^{-1})$ NNNLO pert.
    - No (large) $\log(a\mu)$
    - Should behave like $1 + d_1 g^2 + d_2 g^4 + \cdots$
  - Fit $Z_m(a^{-1})$ with $1 + c_1\beta^{-1} + c_2\beta^{-2}$
  - $Z_m(a^{-1}) \rightarrow Z_m(2GeV)$ NNNLO pert.