

Photon production rate from Transverse-Longitudinal (T-L) mesonic correlator on the lattice

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HotQCD Colloboration

Outline

- 1 Motivation and Observables
- 2 Lattice Details and Correlator
- 3 Spectral function reconstruction
 - Polynomial ansatz
 - Hydro motivated spectral function
 - Backus-Gilbert estimate
- 4 Comparison of spectral functions
- 5 Summary

- Photons and dileptons produced from QGP are important probes to study Quark-Gluon-Plasma.
- Photons and dileptons will directly come out of the plasma without further interaction with the plasma.
- The photon production rate (R_γ) and di-lepton production rate (R_{l+l-}) from a thermalized QGP can be calculated in-terms of spectral function
L.D. McLerran and T. Toimela, Phys. Rev. D 31 (1985) 545.

$$\frac{d\Gamma_\gamma}{d^3\vec{k}} = \frac{\alpha_{em} n_b(\omega)}{2\pi^2 k} g^{\mu\nu} \rho_{\mu\nu}(\omega = |\vec{k}|, \vec{k})$$

$$\frac{d\Gamma_{l+l-}}{d\omega d^3\vec{k}} = \frac{\alpha_{em}^2 n_b(\omega)}{3\pi^2(\omega^2 - k^2)} g^{\mu\nu} \rho_{\mu\nu}(\omega, \vec{k})$$

- Need to estimate $\rho_{\mu\nu}$ from lattice.
- We need correlation function of the current operator $J_\mu(\vec{x}, \tau) = \bar{\psi}(\vec{x}, \tau)\gamma_\mu\psi(\vec{x}, \tau)$ on the lattice.

$$G_{\mu\nu}^E(\tau, \vec{k}) = \int d^3\vec{x} \exp(i\vec{k}\cdot\vec{x}) \langle J_\mu(\vec{x}, \tau) J_\nu(\vec{0}, 0) \rangle$$

- Relation with spectral function,

$$G_{\mu\nu}^E(\tau, \vec{k}) = \int_0^\infty \frac{d\omega}{\pi} \rho_{\mu\nu}(\omega, \vec{k}) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

- Numerically unstable problem.
 - 1) Difference in the number of degrees of freedom.
 - 2) Small error in G^E become very large error in ρ .

- $\rho_{\mu\nu}$ can be decomposed,

$$\rho_{\mu\nu}(\omega, \vec{k}) = P_{\mu\nu}^T \rho_T(\omega, \vec{k}) + P_{\mu\nu}^L \rho_L(\omega, \vec{k})$$

$$\rho_V(\omega, \vec{k}) = \rho_\mu^\mu(\omega, \vec{k}) = 2\rho_T(\omega, \vec{k}) + \rho_L(\omega, \vec{k})$$

- At the photon point $\rho_L(|\vec{k}|, \vec{k}) = 0$.

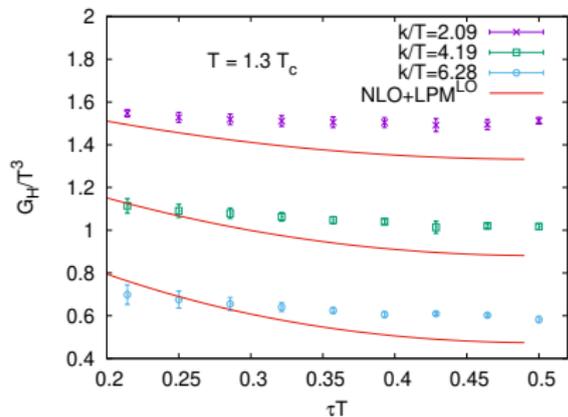
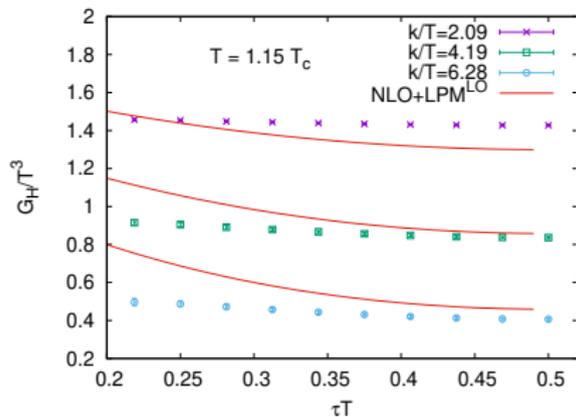
$$\frac{d\Gamma_\gamma}{d^3k} \propto 2\rho_T(|\vec{k}|, \vec{k})$$

- $\rho_H = 2(\rho_T - \rho_L)$ has small UV part. G_H^E has less UV contribution.
- Sum rule $\int_0^\infty d\omega \omega \rho_H(\omega) = 0$
- Photon production rate can be estimated from ρ_H .

M. Ce, T. Harris, H. B. Meyer, A. Steinberg, and A. Toniato, Phys. Rev. D 102, 091501(R)

- We use $N_f = 2 + 1$ flavor HISQ configurations with $m_l = \frac{m_s}{5}$.
- Lattice spacing $a \sim 0.028$ fm and pion mass $m_\pi \sim 313$ MeV.
- Lattice size: $96^3 \times 32$ and $96^3 \times 28$ (spatial extent ~ 2.67 fm).
- Temperatures: $1.15 T_c$ (221 MeV) and $1.3 T_c$ (250 MeV).
- On these configurations we calculate correlation function of clover Wilson fermion at finite momentum $\frac{k}{T} = \frac{2\pi n}{3}$.
- $\kappa = 0.13515$ and $c_{sw} = 1.34108$ (Tadpole improvement)
- This correspond to a quark mass $\ll T$.

$$G_H^E(\tau, \vec{k}) = \int_0^\infty \frac{d\omega}{\pi} 2(\rho_T(\omega, \vec{k}) - \rho_L(\omega, \vec{k})) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$



Non-perturbative modeling of the spectral function is required.

- Polynomial fit ansatz for spectral function,

$$G_H^E(\tau, \vec{k}) = \int_0^\infty \frac{d\omega}{\pi} \rho_H(\omega, \vec{k}) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

- For $\omega \leq \omega_0$

$$\rho_H(\omega) = P_5(\omega, \beta, \gamma, \delta_0, \delta_1, \delta_2)$$

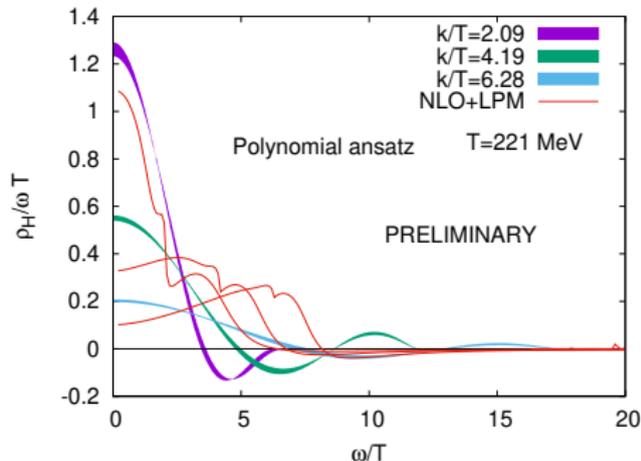
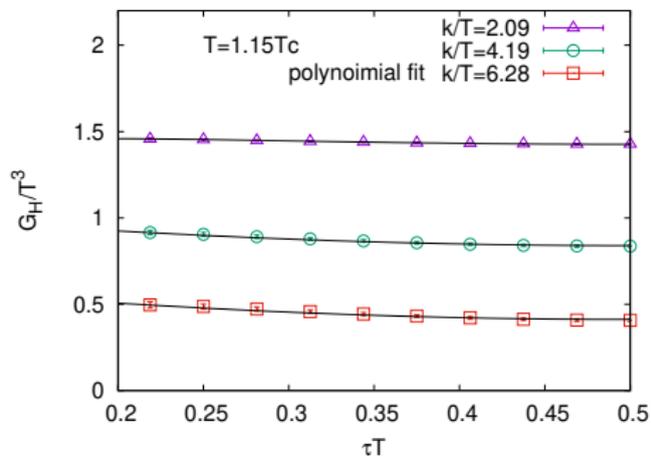
and $\omega \geq \omega_0$

$$\rho_H(\omega) = \rho_H^{\text{pert}}(\omega)$$

J. Ghiglieri, O. Kaczmarek, M. Laine, and F. Meyer, Phys. Rev. D 94, 016005.

- $\beta = \rho_V^{\text{pert}}(\omega_0, \vec{k})$ and $\gamma = \left. \frac{\partial \rho_V^{\text{pert}}(\omega, \vec{k})}{\partial \omega} \right|_{\omega=\omega_0}$
- For ρ_{pert} NLO-LPM re-summed spectral function has been used.
G. Jackson & M. Laine, J. High Energy. Phys. 2019, 144
- The parameter δ_0, δ_1 and δ_2 has been chosen to satisfy the sum rule
- $\omega_0 \gg k$.

- $\omega_0 = \sqrt{k^2 + 8(\pi T)^2}$



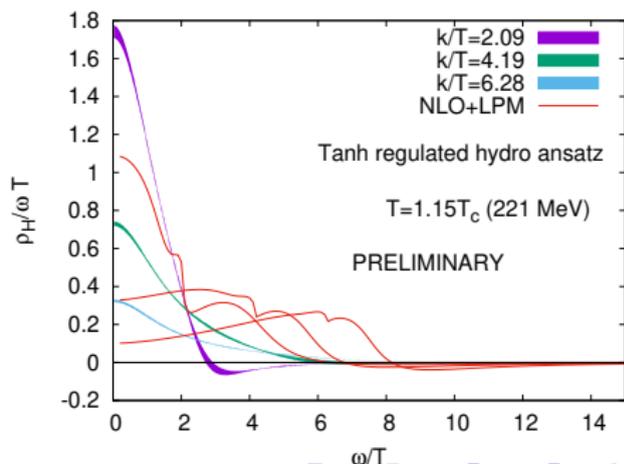
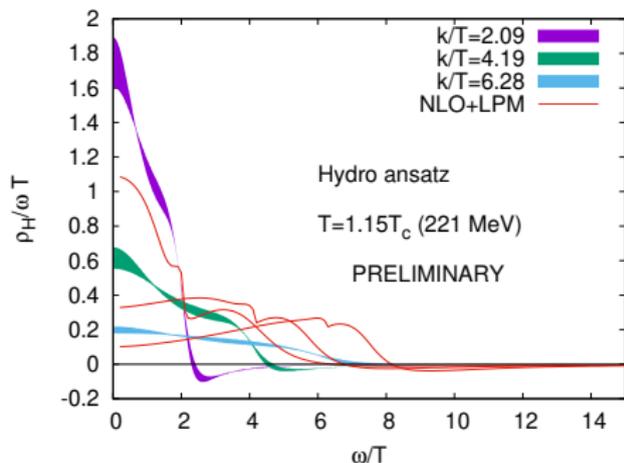
- At the photon point the non-perturbative estimate is smaller than the perturbative estimate.

- Hydro motivated spectral function, [arxiv.2112.12497](https://arxiv.org/abs/2112.12497)

$$\rho_H^{\text{hydro}}(\omega, \vec{k}) = A \frac{\omega k^2 (1 + a^2 k^2 - 2 a b \omega^2 + b^2 \omega^2)}{(1 + b^2 \omega^2)(a^2 k^4 + \omega^2 - 2 a k^2 b \omega^2 + b^2 \omega^4)}$$

- ρ_H^{hydro} will satisfy the sum rule exactly when $a = b$
- At large ω this spectral function goes like $\frac{1}{\omega^3}$.
- OPE prediction $\frac{1}{\omega^4}$.

M. Ce, T. Harris, H. B. Meyer, A. Steinberg, and A. Toniato, Phys. Rev. D 102, 091501(R)

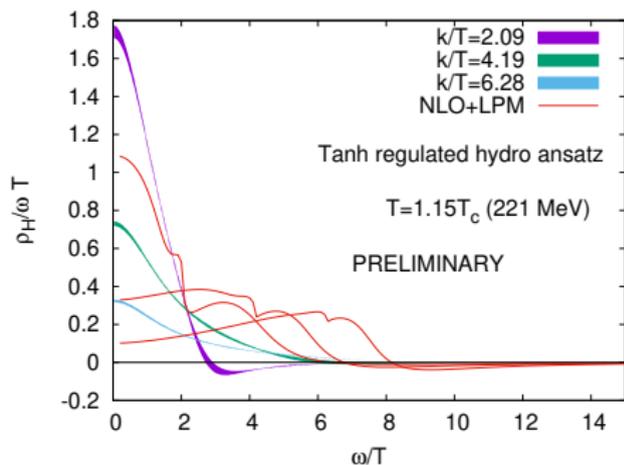
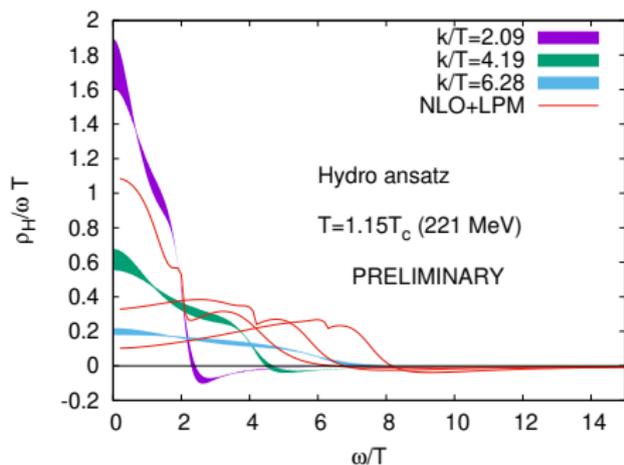


- One can get $1/\omega^4$ behaviour by the following ansatz,

$$\rho_H^{hydro}(\omega, \vec{k}) = A \frac{\tanh(\omega/T) k^2 (1 + a^2 k^2 - 2 a b \omega^2 + b^2 \omega^2)}{(1 + b^2 \omega^2)(a^2 k^4 + \omega^2 - 2 a k^2 b \omega^2 + b^2 \omega^4)}$$

- Very similar ansatz

M. Ce, T. Harris, H. B. Meyer, A. Steinberg, and A. Toniato, Phys. Rev. D 102, 091501(R)



- Backus Gilbert estimate of the spectral function,

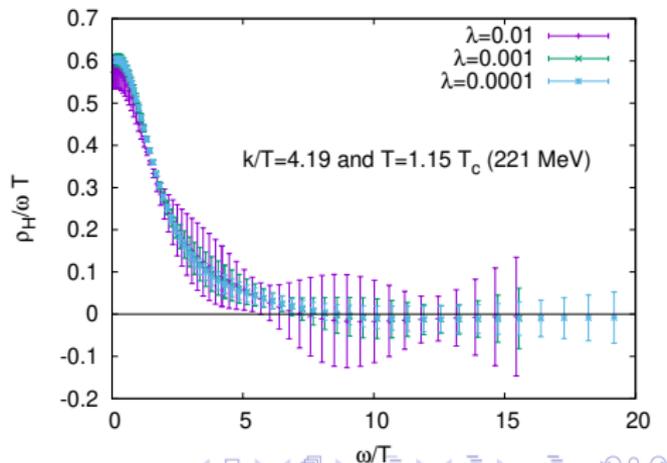
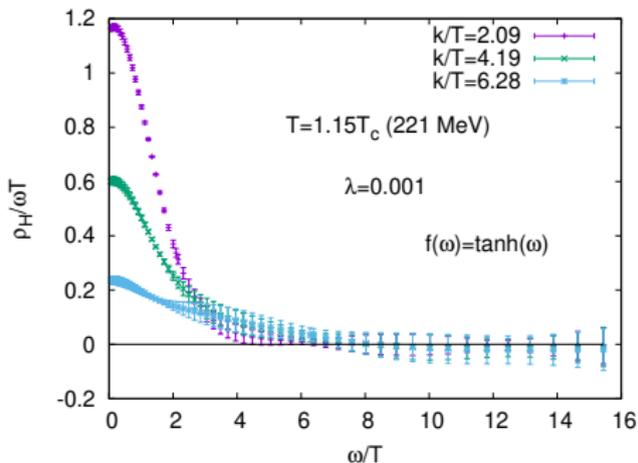
G. Backus, F. Gilbert, *Geophysical Journal of the Royal Astronomical Society* 16, 169 (1968)

- $$G_H(\tau) = \int_0^\infty \frac{d\omega}{\pi} \frac{\rho_H(\omega)}{f(\omega)} f(\omega) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

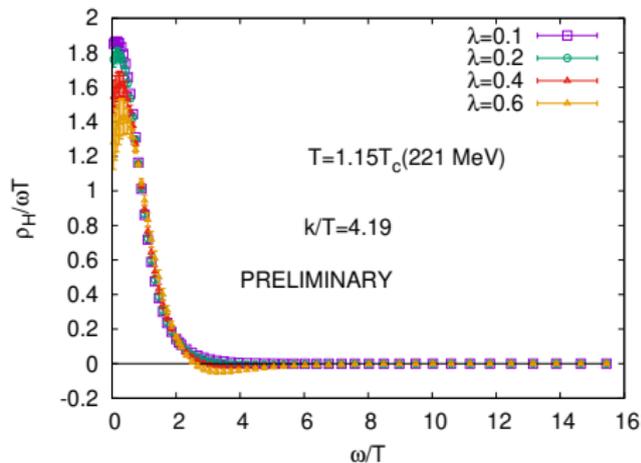
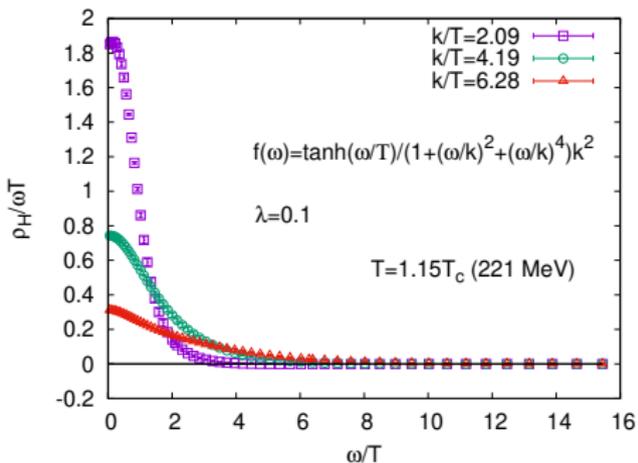
- $$\frac{\rho^{BG}(\omega)}{f(\omega)} = \sum_i q_i(\omega) G(\tau_i) = \int_0^\infty d\bar{\omega} \delta(\omega, \bar{\omega}) \frac{\rho(\bar{\omega})}{f(\bar{\omega})}.$$

$$\delta(\omega, \bar{\omega}) = \sum_i q_i(\omega) K(\bar{\omega}, \tau_i) f(\bar{\omega}).$$

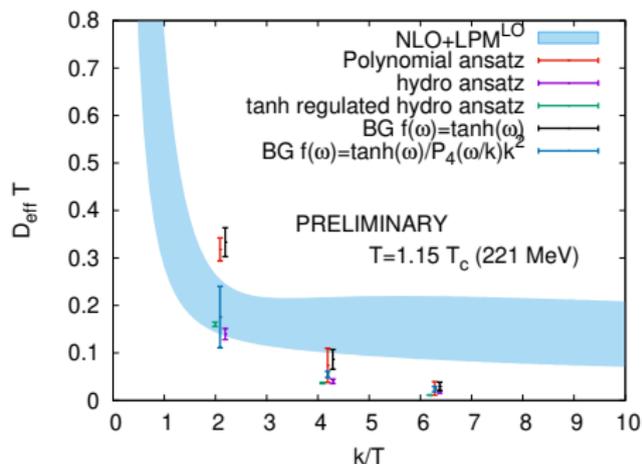
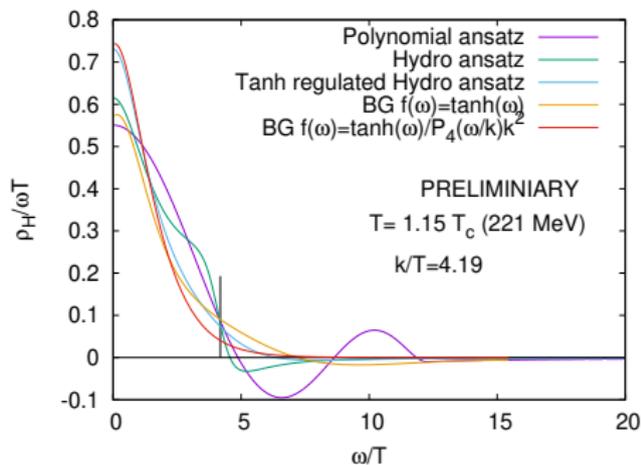
- Minimize $F(\omega) = \lambda \text{Width}[\delta(\omega, \bar{\omega})] + (1 - \lambda) \text{var}[\rho_{BG}(\omega)]$



$$f(\omega) = \frac{\tanh(\omega/T)}{k^2(1 + (\omega/k)^2 + (\omega/k)^4)}$$

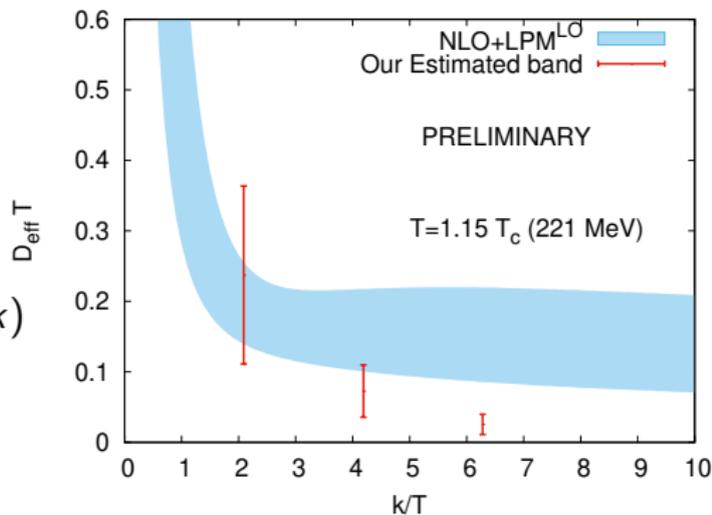


- Effective diffusion coefficient, $D_{\text{eff}}(k) = \frac{\rho_H(|\vec{k}|, \vec{k})}{2\chi_q |\vec{k}|} \lim_{k \rightarrow 0} D_{\text{eff}}(k) = D$



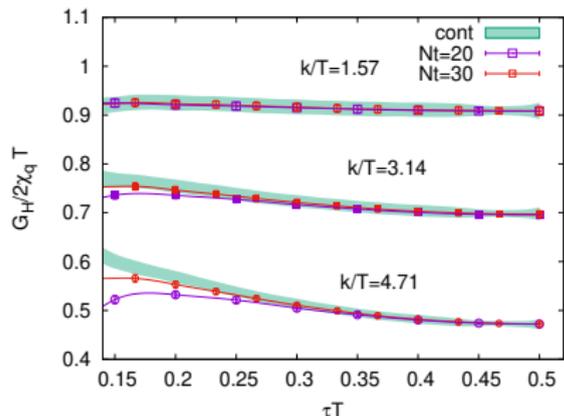
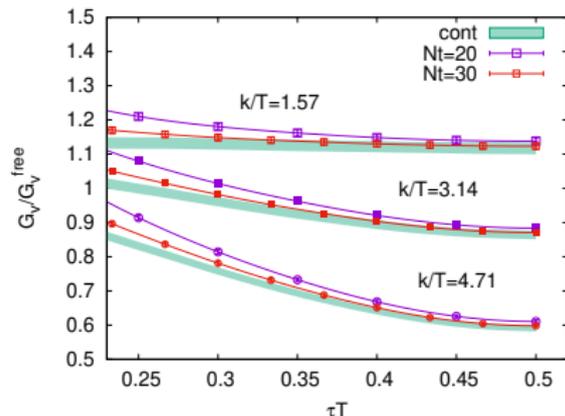
- Photon production rate,

$$\frac{d\Gamma_\gamma}{d^3k} = \frac{\alpha_{em} n_b(\omega) \chi_q}{\pi^2} Q_i^2 D_{eff}(k)$$



- We calculated T-L correlator in Full QCD.
- Polynomial spectral function with smoothly connected UV NLO-LPM-resummed spectral function has been used to fit the lattice correlator.
- We have used spectral function motivated from Hydro ansatz.
- Backus-Gilbert method has also been used.
- Photon production rate estimated from all these method has been compared.

- Continuum extrapolation of Clover Wilson fermion correlation function in Quenched QCD.



- Motivated from the quenched data, we assume the cut-off effects on T-L correlator is also small in 2+1 flavor QCD and show the result from a single lattice spacing.

- Backus Gilbert estimate of the spectral function,

$$G(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) K(\omega, \tau).$$

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$$\rho^{BG}(\omega) = q_i(\omega) G(\tau_i) = \int_0^\infty d\bar{\omega} \delta(\omega, \bar{\omega}) \rho(\bar{\omega}).$$

$$\delta(\omega, \bar{\omega}) = \sum_i q^i(\omega) K(\bar{\omega}, \tau_i) = \mathbf{q}^t(\omega) \cdot \mathbf{K}(\bar{\omega}).$$

- $A(\omega) = \int d\bar{\omega} (\omega - \bar{\omega})^2 \delta(\omega, \bar{\omega})^2 = \mathbf{q}^t(\omega) \cdot \mathbf{W}(\omega) \cdot \mathbf{q}(\omega).$

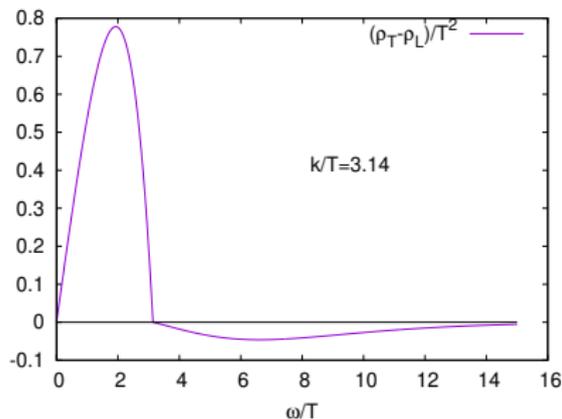
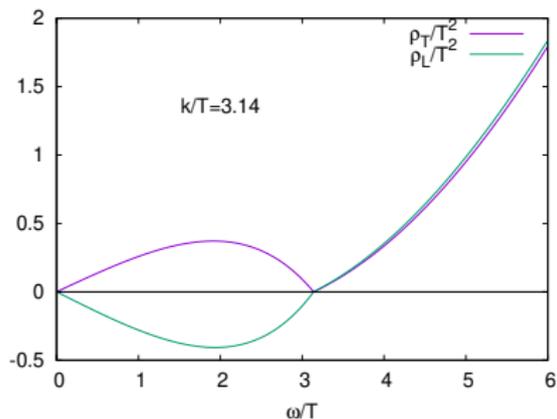
$$W_{ij}(\omega) = \int_0^\infty (\omega - \bar{\omega})^2 K(\omega, \tau_i) K(\bar{\omega}, \tau_j)$$

\mathbf{W} has very small eigenvalue.

- Regulate the problem with, $B(\omega) = \text{var}(\rho^{BG}(\omega)) = \mathbf{q}^t(\omega) \cdot \mathbf{S} \cdot \mathbf{q}(\omega).$
 $S_{ij} = \text{COV}(\tau_i, \tau_j).$
- Minimize, $F(\omega) = \lambda A(\omega) + (1 - \lambda) B(\omega).$

$$\rho_H(\omega) = \frac{\beta\omega^3}{2\omega_0^3} \left(5 - 3\frac{\omega^2}{\omega_0^2} \right) - \frac{\gamma\omega^3}{2\omega_0^2} \left(1 - \frac{\omega^2}{\omega_0^2} \right) + \sum_{i=0}^n \delta_i \left(\frac{\omega}{\omega_0} \right)^i \left(1 - \frac{\omega^2}{\omega_0^2} \right)^2$$

- Free result for ρ_T and ρ_L , G. Aarts and J.M. Resco, Nucl. Phys. B 726 (2005) 93



- $\rho_V = 2\rho_T + \rho_L$ has large UV part. G_V^E has large UV contribution.
- $\rho_H = 2(\rho_T - \rho_L)$ has small UV part. G_H^E has less UV contribution.
- M. Ce, T. Harris, H. B. Meyer, A. Steinberg, and A. Toniato, Phys. Rev. D 102, 091501(R)
- Sum rule $\int_0^\infty d\omega \omega \rho_H(\omega) = 0$
- Photon production rate can be estimated from ρ_H .

