Anomalous transport phenomena on the lattice

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Anomalous transport phenomena

Introduction

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Anomalous transport phenomena

I Examples:

- Chiral Magnetic Effect (CME)
- Chiral Separation Effect (CSE)
- Chiral Vortical Effect (CVE)
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For a review see Kharzeev, Liao, Voloshin, Wang '16

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  - ...

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- Event-by-event CP-violation ! non-trivial topology of QCD vacuum

CRC-TR 211

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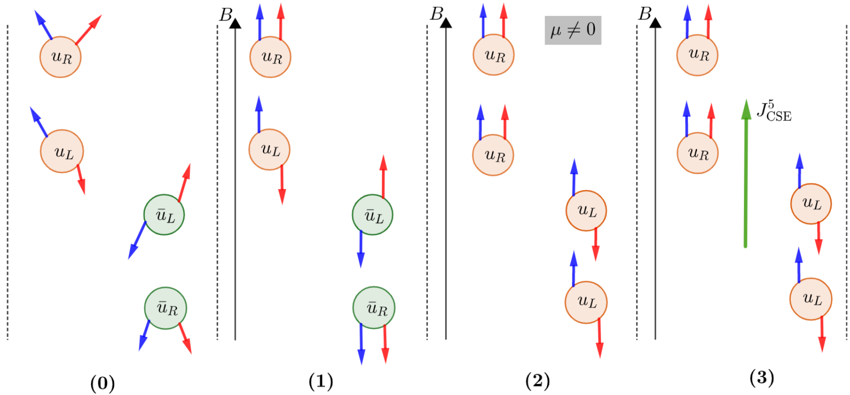
    Latest signals suggest suppression of CME:
    Can we understand this from theory?

Example: CSE

1. Magnetic field induces polarization
2. Finite density: N_q N_q /
3. **Chiral Separation E ect** (CSE):

Magnetic field + Finite density ! Axial current



spin, momentum

- Quark chemical potential induces imbalance between n_q and n_q:

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Introduction to CSE

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- Corrections in the interacting case? $C_{\mathrm{CSE}}(m; T)$?

CSE on the lattice

- Some previous results:
  - Overlap: Quenched QCD Puhr, Buividovich '17
    No significant corrections found
  - Wilson/Domain Wall: SU(2) Buividovich, Smith, von Smekal '21
    CSE suppressed at low T

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- Simulations at finite real suffer from sign problem

- Measure derivatives of the current:

$$\left.\frac{\mathrm{d}\langle j_z^A\rangle}{\mathrm{d}\mu}\right|_{\mu=0} = \frac{T}{V}\Big[\langle \mathrm{Tr}(\gamma_4 M^{-1})\mathrm{Tr}(\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0} - \langle \mathrm{Tr}(\gamma_4 M^{-1}\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0}\Big] = C_{\mathrm{cse}} eB_z$$

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- Simulations at $\mu = 0 \rightarrow$ no sign problem
- Numerical derivative (linear fit) w.r.t. B to obtain C_{cse}:

CSE on the lattice

- Measure derivatives of the current:

$$\frac{\mathrm{d} h j_z^A i}{\mathrm{d}} \Big|_{=0} = \frac{T}{V} [h \mathrm{Tr}({}_4 M \;\; {}^1) \mathrm{Tr}({}_3 \; {}_5 M \;\; {}^1) i_{=0}$$

$$h \mathrm{Tr}({}_4 M \;\; {}^1 \; {}_3 \; {}_5 M \;\; {}^1) i_{=0}]$$

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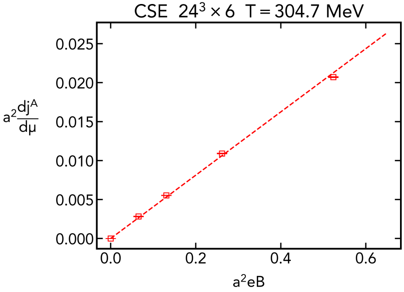

Results for CSE: free case

- Consistency check in the free case

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- Importance of continuum limit
- C_{cse} approaches $1/2\pi^2$ when $LT \to \infty$

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- 2+1 flavours, physical masses

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- 2+1 flavours, physical masses

- High T ($T > T_c$): approaches free case value
- Low T ($T < T_c$): CSE suppressed Buividovich, Smith, von Smekal '21
  Chiral effective theories Avdoshkin, Sadofyev, Zakharov '18

Summary & Outlook

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- Study of CSE with staggered fermions, 2+1 flavours, physical masses
- Free case consistent with analytical prediction
- Full QCD: suppression at low T, approach free case value at high T

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Outlook

- Check mass dependence of CSE in full QCD
- Perform continuum limit for C_{CSE}
- Apply same method to study CME

Backup slides

- Transport effects:

$$\begin{pmatrix} \vec{J} \\ \vec{J}_5 \end{pmatrix} = \begin{pmatrix} \text{Ohm} & \text{CME} \\ \text{CESE} & \text{CSE} \end{pmatrix} \begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix}$$

- Chiral Vortical Effect: vector/axial current generated by rotation $+$ $+$ $_5$:

$$\vec{J} = \frac{1}{\ 2} \ _5 \ \vec{}$$

$$\vec{J}_5 = \left(\frac{1}{6} T^2 + \frac{1}{2 \ ^2} (\ _5^2 + \ ^2) \right) \vec{}$$

Free case continuum limit

- Continuum limit: $a \to 0$, $V = L^3 = a^3 N_s^3 =$ fixed (L fixed)
- Also keep $T = 1/(N_t a)$, $m = \hat{m}/a$ and B fixed
- Then $LT =$ fixed so

$$\frac{N_s}{N_t} = \text{fixed}$$

- Also $mL =$ fixed, $m/T =$ fixed so

$$\hat{m}N_s = \text{fixed}, \quad \hat{m}N_t = \text{fixed}$$

- And

$$B = \frac{2\pi N_b}{L_y L_x} \Rightarrow N_b = \text{fixed}$$

Staggered gammas

- Staggered gammas Taste singlets ($\mathbb{1}$), ($_5$ $\mathbb{1}$):

$$(x;y) = \frac{1}{2} (x)[U(x)u(x)e^{a \; q} {}_{x+\hat{};y} + U^y(y)u^y(y)e \; {}^{a \; q} {}_{x \; \hat{};y}]$$

$$_5(x;y) = \frac{1}{4!} \sum_{\text{perm}} {}_{\text{perm}} \; {}_1 \; {}_2 \; {}_3 \; {}_4$$

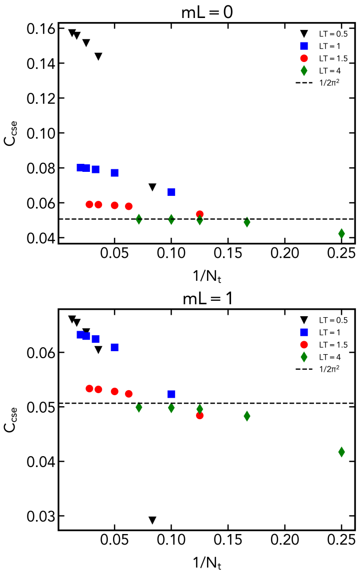
$$(\; _5 \; _4)(x;y) = \frac{1}{3!} \sum_{\text{perm}} {}_{\text{perm}} \; {}_1 \; {}_2 \; {}_3$$

with

$$(x) = (\; 1)^{\sum_{\nu<\mu} x_\nu}; \qquad {}_1(x) = 1$$

and U 2 SU(3); u 2 U(1)

Free case



mL = 0
LT = 0.5
LT = 1
LT = 1.5
LT = 4
1/2π²
C_{cse}
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.05
0.10
0.15
0.20
0.25
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