A $(2+1)$-flavor lattice study of the pion quasiparticle in the thermal hadronic phase at physical quark masses

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Overview

- Introduction/Preliminaries
- Results
  - Modified dispersion relation
  - Dey-Eletsky-Ioffe mixing theorem at finite quark mass
  - Quark number susceptibility (QNS)
- Conclusion

Introduction

- in the early universe weakly coupled quarks and gluons have been in a hot and dense phase (QGP) $\rightarrow$ no individual color charges
- heavy-ion collisions offer possibility to study QCD matter under these conditions
- expansion $\rightarrow$ universe cooled down $\rightarrow$ phase transition
- how do zero temperature excitations get modified with increasing T? $\rightarrow$ HRG Model in hadronic phase
- extend studies [Brandt et al. PRD '14, Brandt et al. PRD '15] to the $(2+1)-$flavor case on an ensemble with quasi-physical quark masses

Numerical Setup

Table 1: Parameters and lattice spacing of the ensemble analyzed in this work. The lattice spacing determination is from Ref. [Bruno et al. PRD '17].

β/a	L/a	$6/g_0^2$	κ_l	κ_s	$a\,[\mathrm{fm}]$
24	96	3.55	0.137232867	0.136536633	0.06426(76)

- $\mathcal{O}(a)$ improved Wilson Fermions
- single gauge ensemble at quasi-physical quark masses
- $T = \frac{1}{\beta} = \frac{1}{24a} = (127.9 \pm 1.5)\,\mathrm{MeV}$
- $T = 0$: $m_\pi = (128.1 \pm 1.3 \pm 1.5)\,\mathrm{MeV}$,
  $\mathrm{m}_K = (488.98 \pm 0.3 \pm 5.8)\,\mathrm{MeV}$ [Ce et al. '22, 2206.06582]

Preliminaries

- $P^a(x) = \bar{\psi}(x)\frac{\tau^a}{2}\psi(x)\,,\ \ V^a_\mu(x) = \bar{\psi}(x)\gamma_\mu\frac{\tau^a}{2}\psi(x)\,,\ \ A^a_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x)$

- PCAC relation: $\partial_\mu A^a_\mu(x) = 2m_{\mathrm{PCAC}}P^a(x)$

  $\hookrightarrow$ implies $G^s_P(x_3, T, \mathbf{p}=0) = -\frac{1}{4m^2_{\mathrm{PCAC}}}\frac{\partial^2}{\partial x_3^2}G^s_A(x_3, T, \mathbf{p}=0)$ $\ (*)$

static screening correlator

$$G^s_A(x_3, T, \mathbf{p}=0) = \int \mathrm{d}x_0\mathrm{d}^2x_\perp \langle A^{a,\mathrm{imp}}_3(x)A^{a,\mathrm{imp}}_3(0)\rangle \overset{|x_3|\to\infty}{=} \frac{f^2_\pi m_\pi}{2}e^{-m_\pi|x_3|}$$

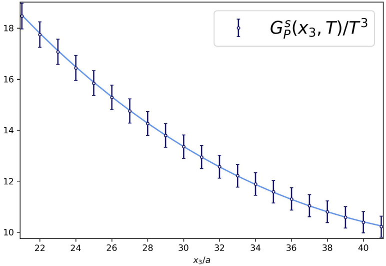
Results: Extracting m_π and f_π

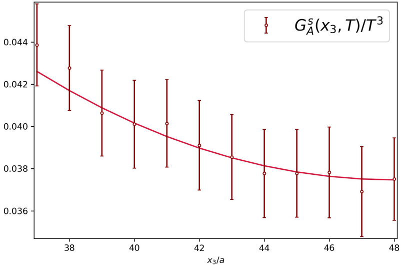
Fit ansatz for screening correlators

making use of the PCAC-based relation $(*)$ one formulates:

$$G_A^s(x_3, T, \mathbf{p}=0) = \frac{A_1^2 m_1}{2} \cosh[(m_1(x_3 - L/2)] \,,$$

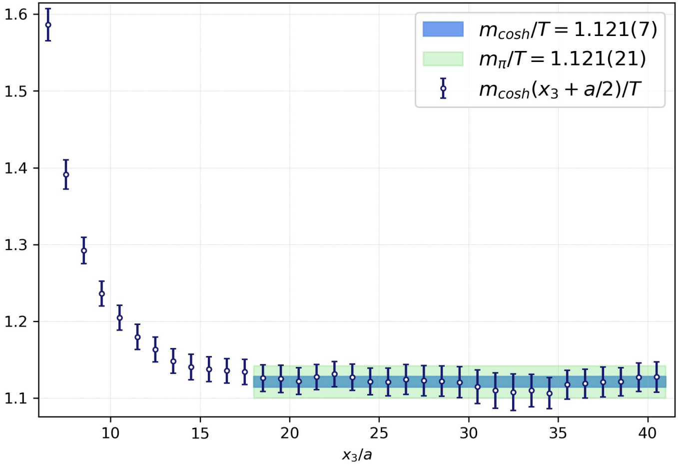
$$G_P^s(x_3, T, \mathbf{p}=0) = -\frac{A_1^2 m_1^3}{8 m_{\mathrm{PCAC}}^2} \cosh[(m_1(x_3 - L/2)]$$





Definition: cosh-mass

$$\frac{G_P(x_3, T, \mathbf{p} = 0)}{G_P(x_3 + a, T, \mathbf{p} = 0)} = \frac{\cosh[m_{\cosh}(x_3 + a/2) * (x_3 - L/2)]}{\cosh[m_{\cosh}(x_3 + a/2) * (x_3 + a - L/2)]}$$



modified dispersion relation [Son et al., PRL, 2001, Son et al., PRD, 2002]

$$\omega_{\mathbf{p}} = u(T)\sqrt{m_\pi^2 + \mathbf{p}^2}\,, \quad \text{for any } T \lesssim T_C$$

- Son and Stephanov showed that u is the ratio of static quant.:

$$u^2 = \frac{f_\pi^2}{\int_0^\beta \mathrm{d}x_0\, G_A(x_0,\mathbf{p}{=}0)} \qquad (m_q = 0)$$

- assuming that the pion dominates the Eucl. 2-pt. func. of A_0 and P:

$$\omega_{\mathbf{0}}^2 = \left.\frac{\partial_0^2 G_A(x_0, T, \mathbf{p}=0)}{G_A(x_0, T, \mathbf{p}=0)}\right|_{x_0=\beta/2} = -4m_{\mathrm{PCAC}}^2 \left.\frac{G_P(x_0, T, \mathbf{p}=0)}{G_A(x_0, T, \mathbf{p}=0)}\right|_{x_0=\beta/2} \Rightarrow$$

Estimators for the pion velocity [Brandt et al., PRD, '14, Brandt et al., PRD, '15]

$$u_m = \left[-\frac{4m_q^2}{m_\pi^2} \left.\frac{G_P(x_0, T, \mathbf{p}=0)}{G_A(x_0, T, \mathbf{p}=0)}\right|_{x_0=\beta/2}\right]^{1/2},$$

$$u_f = \frac{f_\pi^2 m_\pi}{2G_A(\beta/2, T, \mathbf{p}=0)\sinh(u_f m_\pi \beta/2)}$$

Results: Screening quantities

Table 2: Summary of the results of the E250 thermal ensemble with $N_\tau = 24$. The pion quasiparticle mass $\omega_{\mathbf{0}}$ is calculated using $\omega_{\mathbf{0}} = u_m m_\pi$. Analogously: $f_\pi^t = f_\pi / u_m$.

$m_\pi\big\vert_{T=128\,\mathrm{MeV}}$	144(3) MeV
$\omega_{\mathbf{0}}\big\vert_{T=128\,\mathrm{MeV}}$	113(3) MeV
$f_\pi\big\vert_{T=128\,\mathrm{MeV}}$	72(2) MeV
$f_\pi^t\big\vert_{T=128\,\mathrm{MeV}}$	91(2) MeV
u_f	0.787(15)
u_m	0.786(18)
u_f / u_m	1.001(27)
$m_\pi\big\vert_{T=0\,\mathrm{MeV}}$	128(1) MeV
$f_\pi\big\vert_{T=0\,\mathrm{MeV}}$	87(1) MeV

Dey-Eletsky-Ioffe mixing theorem at finite quark mass

- in chir. limit in hadr. phase: heat bath dominated by massless pions
- taking only the contr. of the two lowest states into acc., to $O(T^2)$ [Dey et al., Phys. Lett. B, '90, Eletsky et al., PRD, '93]:

$$\rho_V(\omega, \mathbf{p}, T) = (1-\epsilon)\rho_V(\omega, \mathbf{p}, T=0) + \epsilon\rho_A(\omega, \mathbf{p}, T=0)\,, \ \epsilon \equiv \frac{T^2}{6F_\pi^2}$$

$$\rho_A(\omega, \mathbf{p}, T) = (1-\epsilon)\rho_A(\omega, \mathbf{p}, T=0) + \epsilon\rho_V(\omega, \mathbf{p}, T=0)$$

$$\Rightarrow \rho_V(\omega, \mathbf{p}, T) - \rho_A(\omega, \mathbf{p}, T) = (1-2\epsilon)\left[\rho_V(\omega, \mathbf{p}, 0) - \rho_A(\omega, \mathbf{p}, 0)\right]$$

$\Rightarrow$ order parameter for chiral symmetry restoration

Dey-Eletsky-Ioffe mixing theorem at finite quark mass

$\Rightarrow$ analyze

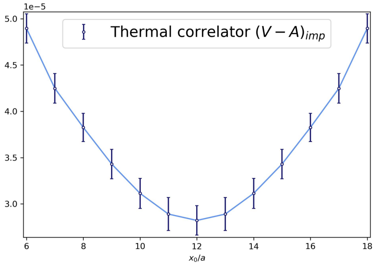
$$G_V(x_0, T, \mathbf{p}=0) - G_A(x_0, T, \mathbf{p}=0) \equiv \\ -\frac{1}{3}\int \mathrm{d}^3x \sum_{i=1}^{3} \left[\langle V_i^a(x) V_i^a(0)\rangle - \langle A_i^a(x) A_i^a(0)\rangle\right]$$

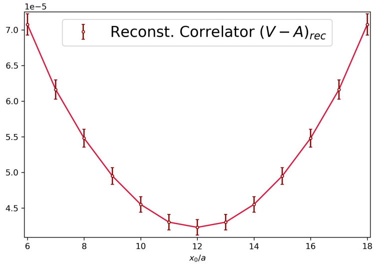
even at non-vanishing quark mass

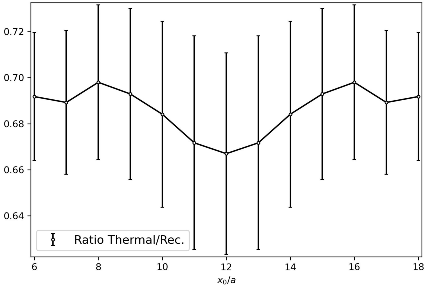
- reduction by a factor of $(1-2\epsilon)$ compared to $G_V^{\mathrm{rec}} - G_A^{\mathrm{rec}}$

$$G_J^{\mathrm{rec}}(x_0, T, \mathbf{p}) = \textstyle\sum_{m\in \mathrm{Z}} G_J(|x_0 + m\beta|, 0, \mathbf{p}) \qquad (J \in \{V, A\})$$

Dey-Eletsky-Ioffe mixing theorem at finite quark mass







Quark number susceptibility (QNS)

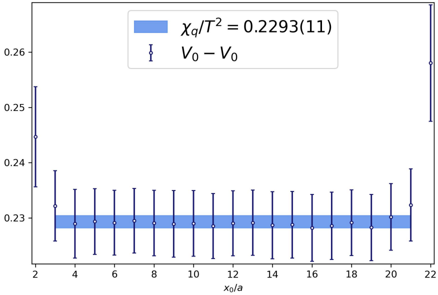
- $\chi_q(T) = \frac{\partial \rho_q}{\partial \mu_q}\Big|_{\mu_q=0}$
- on the lattice: $\chi_q(T) = \beta \int \mathrm{d}^3 x \, \langle V_0(0,\mathbf{x})\, V_0(0,\mathbf{0})\rangle$
- comparison with HRG model: $\chi_q(T) = (\chi_q)_{\text{mesons}} + (\chi_q)_{\text{baryons}}$

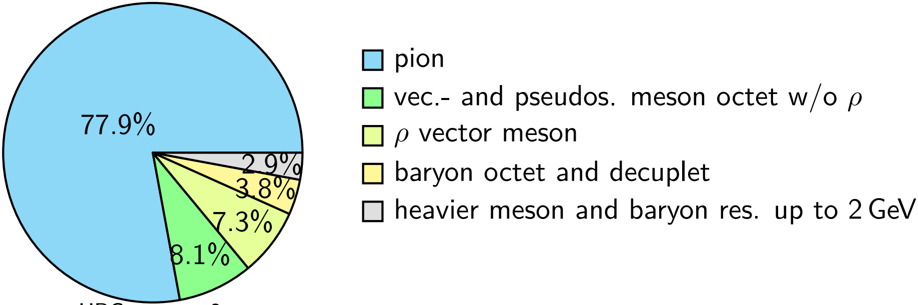
$$\frac{(\chi_q)_{\text{mesons}}}{T^2} = \frac{2\beta^3}{3} \sum_{\text{multiplets}} (2J+1)I(I+1)(2I+1) \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} f^B_{\mathbf{p}}(1+f^B_{\mathbf{p}}),$$

$$\frac{(\chi_q)_{\text{baryons}}}{T^2} = \frac{2\beta^3}{3} \sum_{\text{multiplets}} (2J+1)I(I+1)(2I+1) \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} f^F_{\mathbf{p}}(1-f^F_{\mathbf{p}}),$$

- Alternative: include only pions, but with mod. disp. rel. up to $\Lambda_p = 400\,\mathrm{MeV}$

$$\frac{\chi_q}{T^2} = 4\beta^3 \int_{|\mathbf{p}|<\Lambda_p} \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} f^B_{\mathbf{p}}(\omega_{\mathbf{p}})(1+f^B_{\mathbf{p}}(\omega_{\mathbf{p}}))$$





- $\chi_q^{\mathrm{HRG}}(T)/T^2 = 0.2428$ (5.8% above lattice estimate)
- $\chi_q^{\mathrm{mod.}}(T)/T^2 = 0.2163$ (5.3% below lattice estimate)

Conclusion

- pion velocity $u \approx 0.79$

$$T = 0: \qquad \text{pion mass} = 128(1)\,\text{MeV}$$

$$T = 128\,\text{MeV}: \qquad \underbrace{\omega_{\mathbf{0}}}_{quasip.mass} = 113(3)\,\text{MeV} \qquad \underbrace{m_\pi}_{scr.mass} = 144(3)\,\text{MeV}$$

- chiral symmetry restoration already at an advanced stage: V-A correlator difference consistent with an overall 0.68(3) damping of the vacuum V-A spectral function

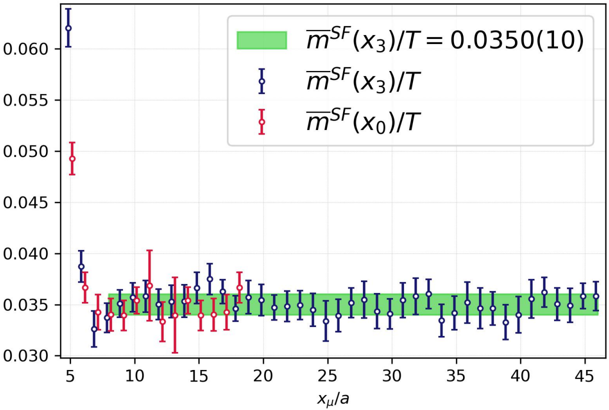
- pion contribution dominates quark number susceptibility

Thank You for Your Attention!

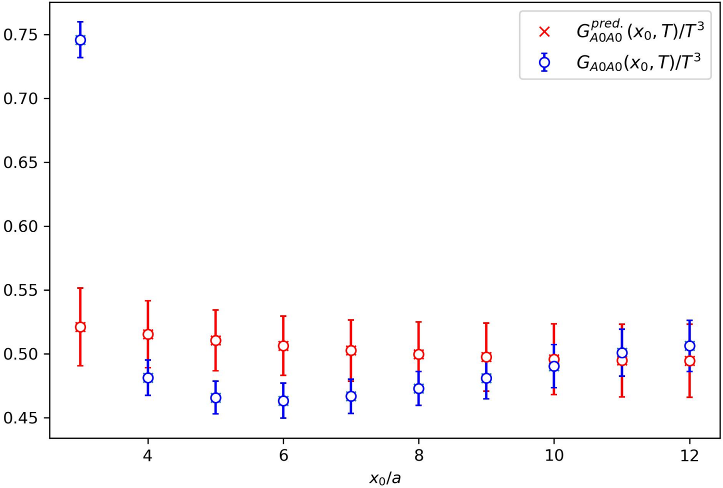
Definition: PCAC mass [Luscher et al., Nucl. Phys. B, '97]

$$m_{\mathrm{PCAC}}(x_3) = \frac{1}{2}\frac{Z_A(g_0^2)}{Z_P(g_0^2)}\frac{\int \mathrm{d}x_0 \mathrm{d}^2 x_\perp \, \langle \partial_3^{\mathrm{imp}} A_3^{a,\mathrm{imp}}(x) P^a(0)\rangle}{\int \mathrm{d}x_0 \mathrm{d}^2 x_\perp \, \langle P^b(x) P^b(0)\rangle} \quad x_\perp = (x_1, x_2)\,,$$

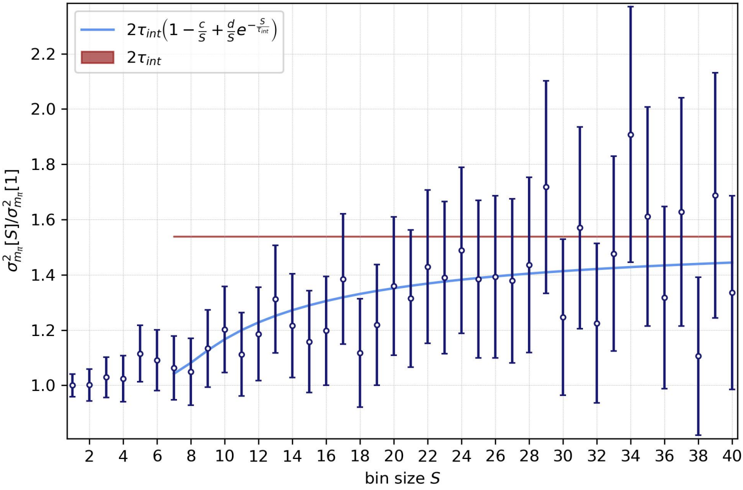
$$A_\mu^a(x) \longrightarrow A_\mu^{a,\mathrm{imp}}(x) = A_\mu^a(x) + a c_A(g_0^2)\partial_\mu^{\mathrm{imp}} P^a(x)$$



Back Up Slides: Predicted $A0_t$ vs lattice estimate



Back Up Slides: Error estimation pion screening mass [Bouma et al., PoS, LATTICE2021]



Back Up Slides: Chiral effective theory Lagrangian of Son and Stephanov

$$\mathcal{L}_{\text{eff}} = \frac{f_t^2}{4}\langle \nabla_0 \Sigma \nabla_0 \Sigma^\dagger \rangle - \frac{f_\pi^2}{4}\langle \partial_i \Sigma \partial_i \Sigma^\dagger \rangle + \frac{m_\pi^2 f_\pi^2}{2} \text{Re}\langle \Sigma \rangle$$

- Σ denotes an $SU(2)$ matrix
- $\nabla_0 \Sigma = \partial_0 \Sigma - \frac{i}{2}\mu_{I5}(\tau_3 \Sigma + \Sigma \tau_3)$: covariant derivative
- μ_{I5}: axial isospin chemical potential
- Lorentz invar. broken $\Rightarrow$ two indep. decay const. related through u