Progress on the QCD Deconfinement Critical Point for $N_f=2$ Staggered Fermions

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in collaboration with

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Outline

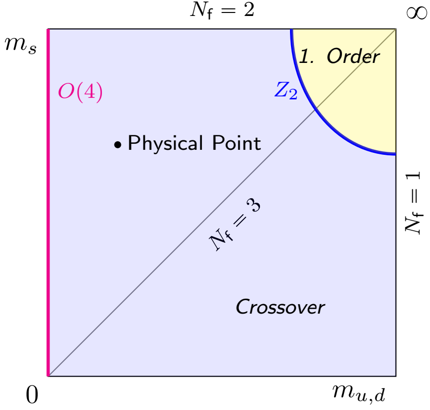
1. The Columbia Plot of QCD
2. Simulation Details
3. Analysis of the Deconfinement Transition
4. Results for the Critical Mass Value
5. Development of an Effective Ginzburg-Landau Theory

The Columbia Plot of QCD

- type of thermal transition as function of $m_{u,d}, m_s$
- pure gauge theory: deconfinement due to the spontaneous breaking of the Z_3 center symmetry
- dynamical quarks break center symmetry explicitly

Columbia Plot

(Cuteri, Philipsen, and Sciarra 2021)



The Columbia Plot of QCD

- type of thermal transition as function of $m_{u,d}, m_s$
- pure gauge theory: deconfinement due to the spontaneous breaking of the Z_3 center symmetry
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Goal

Localize the Z_2 critical point for $N_\mathrm{f} = 2$ with lattice QCD approaching the continuum limit.

- motivation: first principles benchmark for effective theories without sign problem

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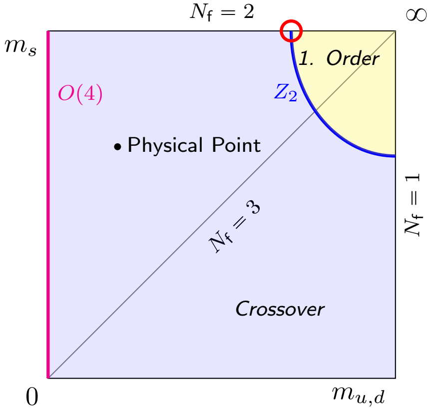

Simulation Details

discretization of QCD action:

- Wilson gauge action
- Staggered fermion action

parameter	label	values
am	bare quark mass	5-6 values around am_c
N_τ	temporal lattice points	$8,\ 10$
N_σ	spatial lattice points	$N_\sigma/N_\tau \in \{4,5,6,7,8\}$
$\beta(a)$	inverse gauge coupling	2-4 values around β_c

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observable: Polyakov loop $\langle L \rangle$:

$$L = \frac{1}{N_\sigma^3} \sum_{\boldsymbol{n}} \frac{1}{3} \mathrm{Tr} \left[\prod_{n_4=0}^{N_\tau - 1} U_4(\boldsymbol{n}, n_4) \right]$$

→ approximate order parameter for deconfinement transition

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Numerical Tools:

- CL^{2}QCD: LQCD code[1] (`rhmc` executable)
- BaHaMAS: run and monitor LQCD simulations[2]
- PLASMA: analyze LQCD data

[1] Pinke et al. 2018
[2] Sciarra 2021

Analysis of the Deconfinement Transition

- distribution of order parameter $|L|$ is continuous for finite systems
- analyze skewness B_3 and kurtosis B_4 of $|L|$
- $B_3(\beta_c) = 0$ determines β_c
- $B_4(\beta_c) \rightarrow$ information on type of transition

infinite volume kurtosis values

Type	1. Order	Z_2 (Ising 3D)	Crossover
$\boldsymbol{B_4(T_c)}$	1	$1.604(1)$[3]	3

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- analyze quantities using jackknife resampling $\rightarrow$ determine errors
- reweight with respect to β using multiple histogram method[4]
- extract $B_4(\beta_c)$ from reweighted kurtosis data

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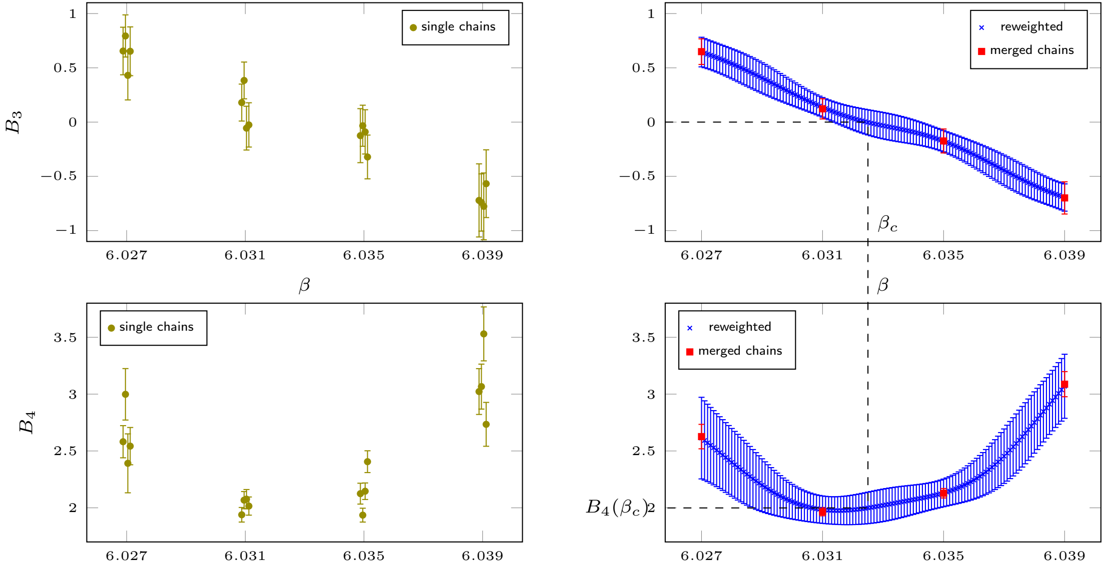
Exemplary Analysis of $B_{3,4}$ for am=0.25, N_τ=10, N_σ=50



Kurtosis Finite Size Scaling Formula

- finite size scaling (FSS) formula for kurtosis of observable[5] $O = c_M \cdot M$

$$B_4(N_\sigma, \beta_c, m) = A + B \cdot x + \mathcal{O}(x^2)$$

scaling variable

$$x = \left(\frac{1}{m} - \frac{1}{m_c}\right) N_\sigma^{1/\nu}$$

critical exponents from Ising 3D universality class[6]

$y_t = 1/\nu$	y_t
1.5870(10)	2.4818(3)

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Kurtosis Finite Size Scaling Formula

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  $O = c_M \cdot M + c_E \cdot E$

$$B_4(N_\sigma, \beta_c, m) = \left(A + B \cdot x + \mathcal{O}(x^2)\right) \times \left(1 + C N_\sigma^{y_t - y_h} + \mathcal{O}\left(N_\sigma^{2(y_t - y_h)}\right)\right)$$

- correction term becomes irrelevant for sufficiently large volumes
- fit kurtosis data to FSS formula to determine m_c

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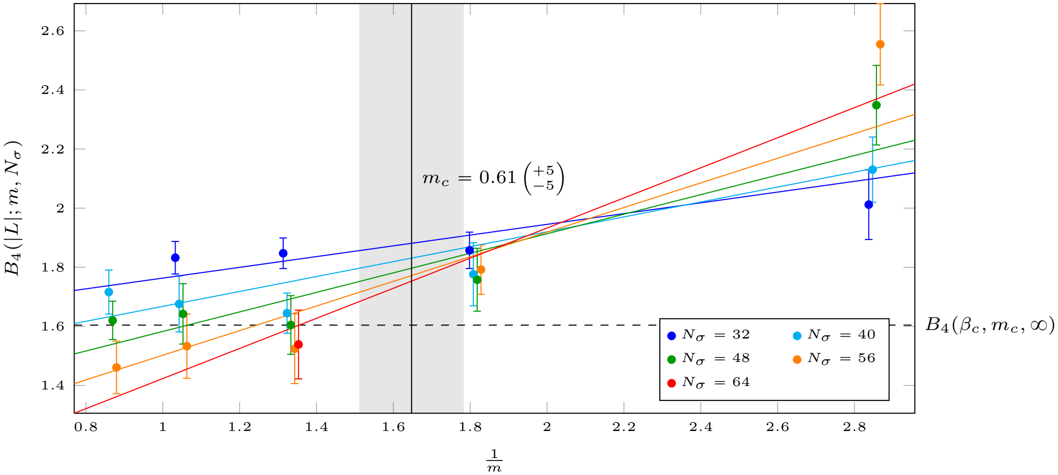
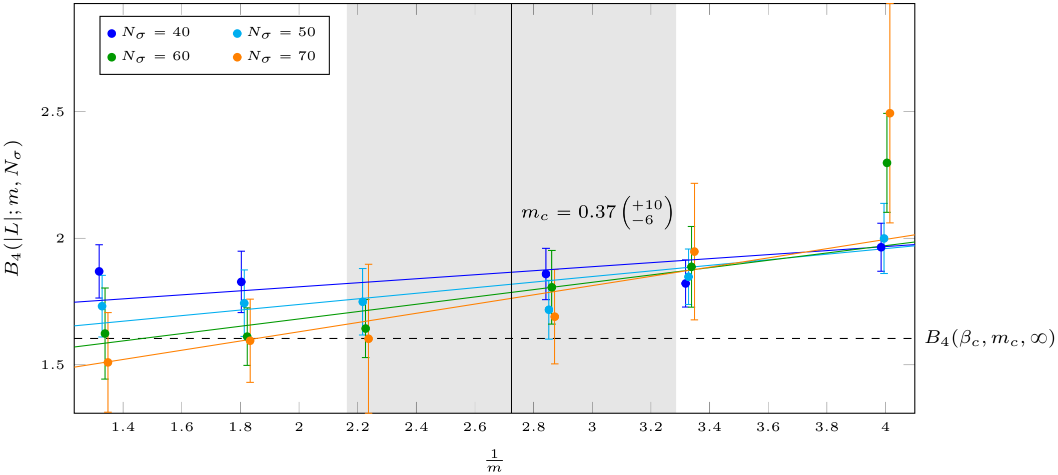
Results for the Critical Mass for N_τ=8



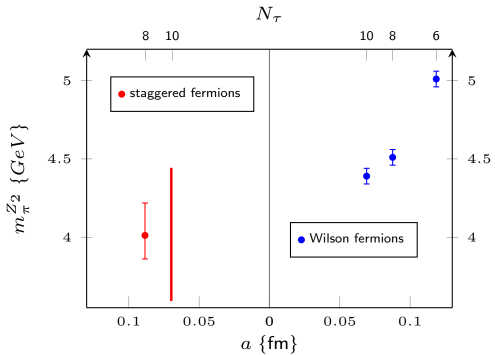
$1/m_c$	a_1	c	ndf	χ^2_{ndf}	Q
1.65(14)	$6.4(7) \cdot 10^{-4}$	3.8(6)	17	1.03	42.0%

Preliminary Results for the Critical Mass for $N_\tau = 10$



$1/m_c$	a_1	c	ndf	χ^2_{ndf}	Q
2.7(6)	$2.0(6) \cdot 10^{-4}$	4.4(1.3)	20	0.4611	98.02%

Results for the Critical Point



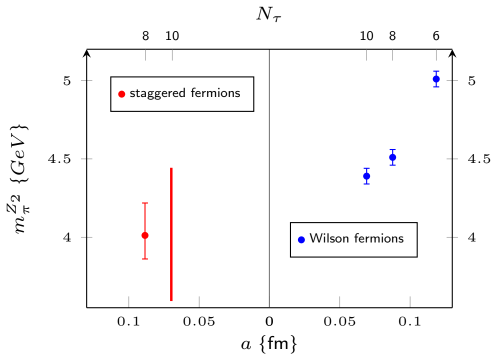
- set the scale using w_0 scale[7] based on Wilson flow[8]
- pion mass is not resolved by the lattice
- comparison with Wilson fermions[9]

$N_\tau=8$:

am	β_{pc}	am_π	a {fm}	m_π {GeV}	T_c {MeV}	m_π/T_c
0.5610	5.9859	1.73568(13)	0.0887(9)	3.86(4)	278(3)	13.89
0.6070	5.9940	1.79918(9)	0.0885(9)	4.01(4)	278(3)	14.39
0.6613	6.0022	1.87063(7)	0.0875(9)	4.22(4)	282(3)	14.97

[7] Borsányi et al. 2010
[8] Lüscher 2010
[9] Cuteri, Philipsen, Schön, et al. 2021

Results for the Critical Point



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$N_\tau = 10$:

am	β_{pc}	am_π	a {fm}	m_π {GeV}	T_c {MeV}	m_π/T_c
0.30	6.0600	1.28023(15)	0.0703(8)	3.60(4)	281(3)	12.80
0.45	6.1139	1.55837(15)	0.0693(8)	4.44(5)	284(3)	15.58

[7] Borsányi et al. 2010
[8] Lüscher 2010
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Development of an Effective Ginzburg-Landau Theory

- approaching continuum limit via simulations is very cumbersome
- an effective Ginzburg-Landau theory might offer a short-cut

Idea

Find the effective Ginzburg-Landau theory that describes the staggered LQCD data in the neighborhood of the deconfinement critical point.

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general Landau functional

$$\mathcal{L}(\eta) = C_2(\beta, m)\eta^2 + C_3(\beta, m)\eta^3 + C_4(\beta, m)\eta^4 - h\eta$$

with the order parameter $\eta = 2\text{Re}(L)$
and the symmmetry breaking field $h \propto 1/m$

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- first step: determine Landau coefficients C_i from data for a single N_τ
- expand C_i around the critical values of the parameters (similarly as in Hatta and Ikeda 2003)

$$C_i(\tilde{\beta}, \tilde{m}) = c_{i,0} + c_{i,\beta}\tilde{\beta} + c_{i,m}\tilde{m}$$

with $\tilde{\beta} = \frac{\beta - \beta_c}{\beta_c}$ and $\tilde{m} = \frac{m_c - m}{m}$

Development of an Effective Ginzburg-Landau Theory

constrain coefficients at the critical point (saddle point condition):

$$\begin{aligned}\mathcal{L}'(\eta_c) &= 0 \\ \mathcal{L}''(\eta_c) &= 0 \\ \mathcal{L}'''(\eta_c) &= 0\end{aligned} \quad \longrightarrow \quad \begin{aligned}\eta_c &= 3/2 \cdot h_c/c_{2,0} \\ c_{3,0} &= -4/9 \cdot c_{2,0}^2/h_c \\ c_{4,0} &= 2/27 \cdot c_{2,0}^3/h_c^2\end{aligned}$$

pseudo-critical line $\beta_{pc}(m)$ for reduced Landau functional ($c_{2,m} = c_{3,\beta} = c_{4,\beta} = c_{4,m} = 0$):

$$\begin{aligned}\mathcal{L}'(\eta_{pc}) &= 0 \\ \mathcal{L}'''(\eta_{pc}) &= 0\end{aligned}$$

$$\tilde{\beta}_{pc}(\tilde{m}) = \frac{32c_{2,0}^6(h_c - h) + 27c_{3,m}h_c^2\tilde{m}\left(8c_{2,0}^4 - 36c_{2,0}^2c_{3,m}h_c\tilde{m} + 27c_{3,m}^2h_c^2\tilde{m}^2\right)}{24c_{2,0}^3c_{2,\beta}h_c\left(9c_{3,m}h_c\tilde{m} - 4c_{2,0}^2\right)}$$

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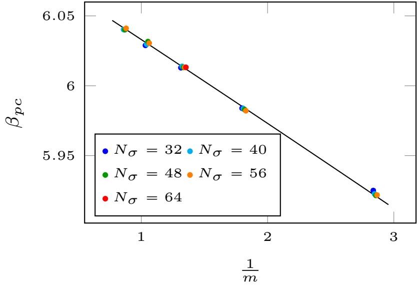
Fit of $\beta_{pc}(m)$ (m_c, β_c fixed):



- linear nature of $\beta_{pc}(1/m)$ does not constrain the fit sufficiently
- need more data points outside the linear region

Conclusion and Outlook

- critical quark mass has been found for $N_\tau = 8$ and preliminary for $N_\tau = 10$
- fit results for $N_\tau = 10$ will improve (currently running simulations)
- too early to perform continuum limit based on simulation data
- at least one larger N_τ must be added in future for continuum limit
- increasing computational effort for larger N_τ motivates search for alternative approaches

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- finding the effective Ginzburg-Landau theory in the neighborhood of the deconfinement critical point has been presented
- more data is needed for conclusive fit results
- in future: extension of Ginzburg-Landau theory to several N_τ

Thank you for your attention!

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