Charm Fluctuations in (2+1)-flavor QCD at High Temperature

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Outline

- Motivation
- Charm mass tuning and parametrization on the lattice
- Hadron resonance gas (HRG) model
- Generalized susceptibilities of conserved charges
- Simulation details
- Results and comparison with older HotQCD analysis
- Conclusions and outlook.

Motivation

- Strong interaction matter undergoes a chiral crossover at $\mathrm{T_{pc}} = 156.5 \pm 1.5$ MeV. [HotQCD Collaboration, 2019]
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- Probing relevant charm degrees of freedom in both temperature regimes will also help in understanding the hadronisation mechanisms inside the deconfined QGP medium.
- Signals of exotic charm states such as tetraquarks can shed light on how quarks arrange inside the bound states.

Charm Mass Tuning on Lattice

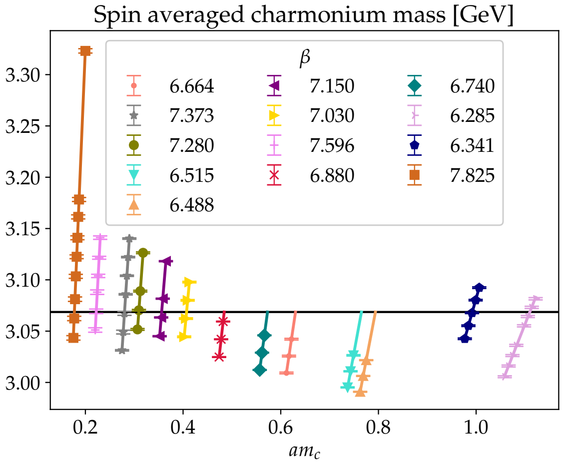
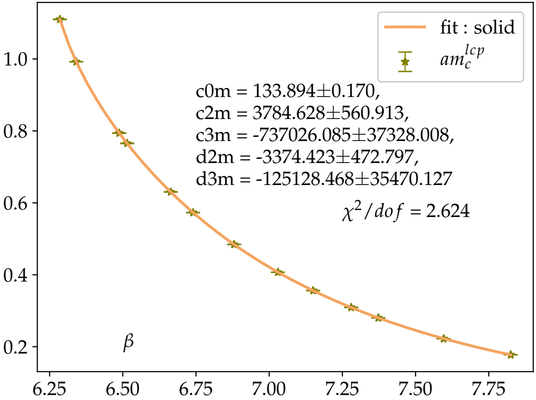


Figure: f_K scale setting from [D.Bollweg et al., 2021] has been used. Black horizontal line corresponds to mass $[(3m_{J/\psi} + m_{\eta_{c\bar{c}}})/4] = 3.06865$ GeV.

Charm Mass Parametrization

$$am_c^{lcp} = \left(\frac{20b_0}{\beta}\right)^{\frac{4}{9}} c_{0m} f(\beta) \left[\frac{1 + c_{2m}(\frac{10}{\beta}) f^2(\beta) + c_{3m}(\frac{10}{\beta}) f^3(\beta)}{1 + d_{2m}(\frac{10}{\beta}) f^2(\beta) + d_{3m}(\frac{10}{\beta}) f^3(\beta)} \right].$$

$f(\beta)$ is the 2-loop QCD beta function.



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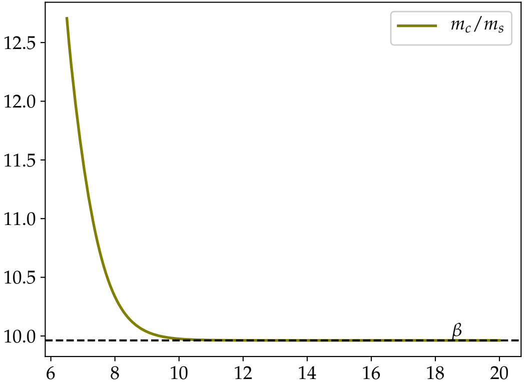

Figure: $\mathrm{m_c/m_s}$ quoted by PDG is $11.76^{+0.05}_{-0.10}$. In the continuum we obtain, $\mathrm{m_c/m_s} = 9.96$ (error analysis is in the progress). $\mathrm{am_s}$ parametrization from [HotQCD Collaboration, 2014] is used for the comparision.

Hadron Resonance Gas (HRG) model

- HRG describes a non-interacting gas of hadron resonances. This model has been successful in describing particle abundance ratios in heavy ion experiment results. $\mathrm{T} < \mathrm{T}_{\mathrm{pc}}$ is the validity regime of HRG. [C. R. Allton et al., 2005]

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- Charmed baryons and mesons separately contribute to the partition function of HRG, which in turn reflects in contributions to the pressure. $\mathrm{P_C}(\mathrm{T}, \overrightarrow{\mu})/\mathrm{T}^4 = \mathrm{M_C}(\mathrm{T}, \overrightarrow{\mu}) + \mathrm{B_C}(\mathrm{T}, \overrightarrow{\mu})$.

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- $$\mathrm{M_C}(\mathrm{T}, \overrightarrow{\mu}) = \frac{1}{2\pi^2} \sum_{\mathrm{i}} \mathrm{g_i} \left(\frac{\mathrm{m_i}}{\mathrm{T}}\right)^2 \mathrm{K_2}(\mathrm{m_i}/\mathrm{T}) \cosh(\mathrm{Q_i}\hat{\mu}_{\mathrm{Q}} + \mathrm{S_i}\hat{\mu}_{\mathrm{S}} + \mathrm{C_i}\hat{\mu}_{\mathrm{C}}).$$
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- $\hat{\mu}_\mathrm{X} = \mu/\mathrm{T}$, $\mathrm{X} \in \{\mathrm{B}, \mathrm{Q}, \mathrm{S}, \mathrm{C}\}$.

Generalized susceptibilities of conserved charges

$$\mathrm{M_C(T, \vec{\mu}) = \frac{1}{2\pi^2} \sum_i g_i \left(\frac{m_i}{T}\right)^2 K_2(m_i/T) \cosh(Q_i\hat{\mu}_Q + S_i\hat{\mu}_S + C_i\hat{\mu}_C)}\ .$$

- $\mathrm{K_2(x)} \sim \sqrt{\pi/2x}\ e^{-x}\ [1 + \mathbb{O}(x^{-1})]$. If $\mathrm{m_i} \gg \mathrm{T}$, then contribution to $\mathrm{P_C}$ will be exponentially suppressed.
- Λ_c^+ mass ~ 2286 MeV, Ξ_{cc}^{++} mass ~ 3621 MeV. At $\mathrm{T_{pc}}$, contribution to $\mathrm{B_C}$ from Ξ_{cc}^{++} will be suppressed by a factor of 10^{-3} in relation to Λ_c^+.

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$$\chi_{klmn}^{BQSC} = \left. \frac{\partial^{(k+l+m+n)} \ [P\ (\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)\ /T^4]}{\partial\hat{\mu}_B^k\ \partial\hat{\mu}_Q^l\ \partial\hat{\mu}_S^m\ \partial\hat{\mu}_C^n} \right|_{\overrightarrow{\mu}=0} \ .$$

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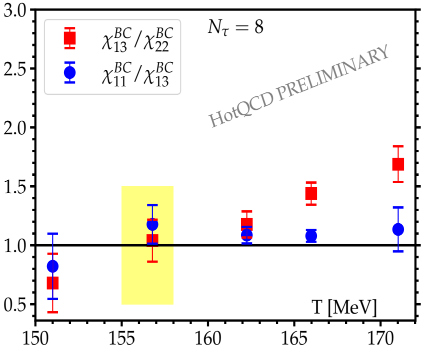
- Ratios like $\chi_{1n}^{BC}/\chi_{1l}^{BC}$ will be unity for all temperatures, for odd n, l.
- At present, we have gone upto fourth order in calculating various cumulants.

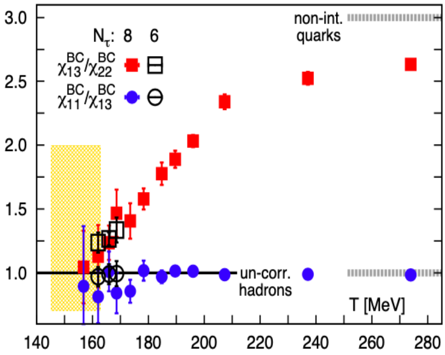
Simulation details

- Partition function of QCD with 2 light, 1 strange and 1 charm quark flavors is :
$$\mathcal{Z} = \int \mathcal{D}[\mathrm{U}]\{\det \mathrm{D}(\mathrm{m_l})\}^{2/4}\{\det \mathrm{D}(\mathrm{m_s})\}^{1/4}\{\det \mathrm{D}(\mathrm{m_c})\}^{1/4}\mathrm{e}^{-\mathrm{S_g}}.$$
This can used to calculate susceptibilities is the BQSC basis.
- We used (2+1)-flavor HISQ configurations generated by HotQCD collaboration for $\mathrm{m_s}/\mathrm{m_l} = 27$ and $\mathrm{N}_\tau = 8$.
- We treated charm sector quark in the quenched approximation.
- We made use of 500 random vectors to calculate various traces per configuration, expect for Tr $\left(\mathrm{D}^{-1}\frac{\partial \mathrm{D}}{\partial \mu}\right)$, for which we used 2000 random vectors.

Results

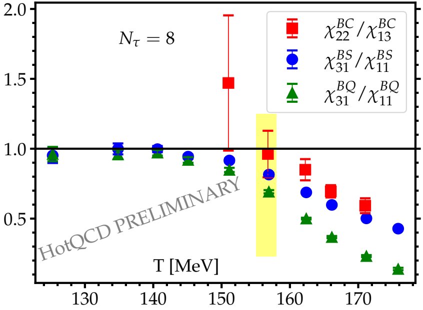
Near T $=$ 156 MeV, left figure has on order of magnitude higher statistics. Right figure is the older HotQCD analysis [A. Bazavov et al., 2014].

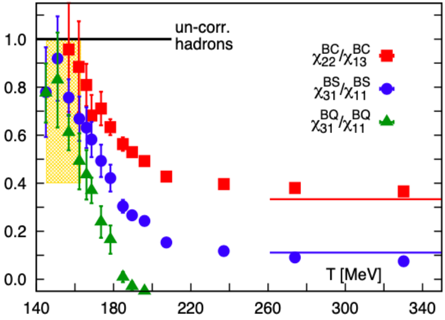




Crossover region has shrunk and we have better control over the errors there. With further addition of statistics, results will look more precise.

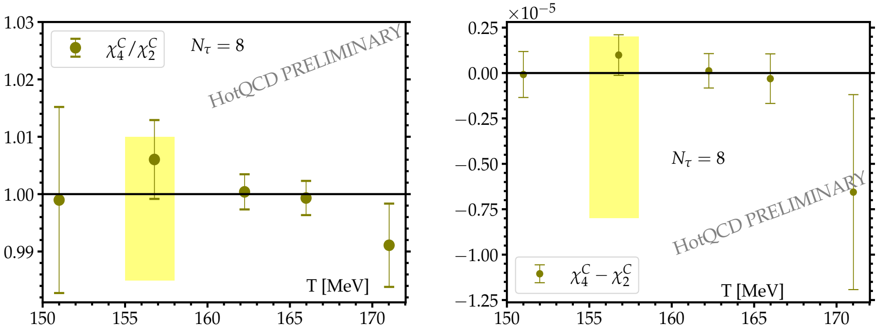
Right figure is the older HotQCD analysis. BQ and BS ratios in the left figure were calculated by making use of all the available statistics. This is clearly reflected in the errors.





BQ and BS correlations separated by even $\hat{\mu}_{\mathrm{B}}$ derivatives should be unity in the HRG phase.

$|\mathrm{C}| > 1$ states



We also want to explore the multiple charm sector. Ratio on left indicates that the contribution to partial pressure from $|\mathrm{C}| > 1$ sector is indeed very small and difference on right quantifies it. For HRG,

$$\chi_4^{\mathrm{C}} - \chi_2^{\mathrm{C}} = 12\mathrm{B}_{\mathrm{C},2} + 72\mathrm{B}_{\mathrm{C},3}.$$

Conclusions & Outlook

- Until now only one-third of the available statistics was used. Our aim is to make use of the available high statistics.

$N_\tau = 8$				$N_\tau = 12$			
β	m_l	T[MeV]	#conf.	β	m_l	T[MeV]	#conf.
6.175	0.003307	125.28	1,471,861				
6.245	0.00307	134.84	1,275,380	6.640	0.00196	135.24	330,447
6.285	0.00293	140.62	1,598,555	6.680	0.00187	140.80	441,115
6.315	0.00281	145.11	1,559,003	6.712	0.00181	145.40	416,703
6.354	0.00270	151.14	1,286,603	6.754	0.00173	151.62	323,738
6.390	0.00257	156.92	1,602,684	6.794	0.00167	157.75	299,029
6.423	0.00248	162.39	1,437,436	6.825	0.00161	162.65	214,671
6.445	0.00241	166.14	1,186,523	6.850	0.00157	166.69	156,111
6.474	0.00234	171.19	373,644	6.880	0.00153	171.65	144,633
6.500	0.00228	175.84	294,311	6.910	0.00148	176.73	131,248

Conclusions & Outlook

- We see deviations from HRG model in the open charm sector near $\mathrm{T_{pc}} = 156.5 \pm 1.5$ MeV. This is the consistent with the older HotQCD analysis.
- Unlike the ratios presented in this talk, ratios of contributions to partial pressure coming from charmed baryons and open charm mesons are sensitive to the mass spectrum in the charm sector. We are going to investigate this in detail.
- We want to go to $\mathrm{N}_\tau = 12$ to quantify the cut-off effects.