Thermal phase transition in rotating QCD with dynamical quarks

A. A. Roenko,

in collaboration with

V. V. Braguta, A. Yu. Kotov, D. A. Sychev

Joint Institute for Nuclear Research, Bogoliubov Laboratory of Theoretical Physics

roenko@theor.jinr.ru

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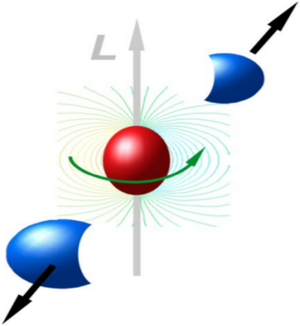
- In non-central heavy ion collisions creation of QGP with angular momentum is expected.



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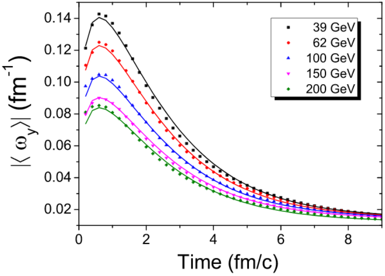
- The rotation occurs with relativistic velocities.



Au+Au, b = 7 fm

[Y. Jiang et al., Phys. Rev. C 94, 044910 (2016), arXiv:1602.06580 [hep-ph]]

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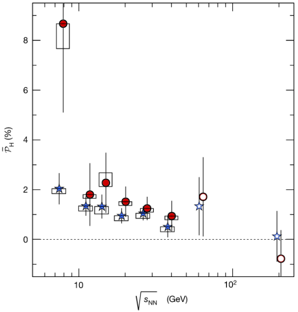

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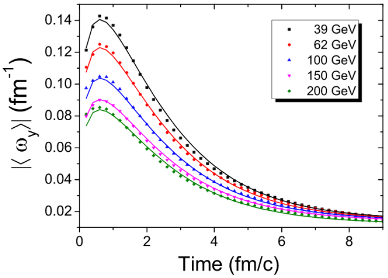

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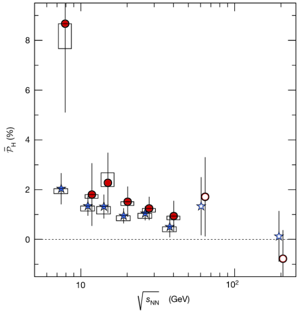

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- How does the rotation affect to phase transitions in QCD?

Related papers

Rotation on the lattice (phase transitions were not considered):

- A. Yamamoto and Y. Hirono, Phys. Rev. Lett. 111, 081601 (2013), arXiv:1303.6292 [hep-lat]

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- X. Wang et al., Phys. Rev. D 99, 016018 (2019), arXiv:1808.01931 [hep-ph]
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Rotation suppress the chiral condensate ($S = 0$), states with $S \neq 0$ are preferable.
$\Rightarrow$ Critical temperature decreases due to the rotation.

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- Holography: N. R. F. Braga et al., Phys. Rev. D 105, 106003 (2022), arXiv:2201.05581 [hep-th], A. A. Golubtsova et al., Nucl. Phys. B 979, 115786 (2022), arXiv:2107.11672 [hep-th], X. Chen et al., JHEP 07, 132 (2021), arXiv:2010.14478 [hep-ph], ...
- Compact QED in 2+1-D M. N. Chernodub, Phys. Rev. D 103, 054027 (2021), arXiv:2012.04924 [hep-ph]
- HRG model: Y. Fujimoto et al., Phys. Lett. B 816, 136184 (2021), arXiv:2101.09173 [hep-ph]
- Instantons in rotating YM: M. N. Chernodub, (2022), arXiv:2208.04808 [hep-th]
- Polyakov loop potential in YM with , (perturbatively, finite T): S. Chen et al., (2022), arXiv:2207.12665 [hep-ph]
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The rotation affect both gluon and quark degrees of freedom!

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The running e ective coupling G(!) is introduced.

) Critical temperature increases due to the rotation.

Rotating reference frame

QCD (at thermal equilibrium) is investigated in the reference frame which rotates with the system with angular velocity .

In this reference frame there appears an external gravitational eld

$$
g = \begin{pmatrix} 1 \quad r^2 \quad {}^2 & y & x & 0 \\ y & 1 & 0 & 0 \\ x & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} :
$$

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Rotating reference frame

QCD (at thermal equilibrium) is investigated in the reference frame which rotates with the system with angular velocity Ω.

In this reference frame there appears an external gravitational field

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Tolman-Ehrenfest effect: In gravitational field the temperature isn't a constant in space at thermal equilibrium:

$$T(r)\sqrt{1 - r^2\Omega^2} = \text{const} \equiv T;$$

One could expect, that the rotation effectively warm up the periphery and as a result, from kinematics, the critical temperature should decreases.

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The partition function is [1]

$$Z = \int D \; D \quad DA \quad \exp \quad S_G[A; \;] \quad S_F[\;;\;;A;m; \quad] \quad : \qquad (1)$$

[1] A. Yamamoto and Y. Hirono, Phys. Rev. Lett. 111 , 081601 (2013), arXiv: 1303.6292 [hep-lat].

The Euclidean gluon action can be written as

$$S_G = \frac{1}{4g^2}\int d^4x \sqrt{g_E}\; g_E \; g_E \; F^a \; F^a \; : \qquad (2)$$

And the quark action reads as follows[2]

$$S_F = \int d^4x \sqrt{g_E} \qquad (D \qquad) + m \quad ; \qquad (3)$$

The covariant derivative D and spinor a ne connection is

$$D = @ \quad iA \; ; \qquad (4)$$

$$= \frac{i}{4} \; {}^{ij} \; !\,_{ij} \; ; \qquad (5)$$

$${}^{ij} = \frac{i}{2}(\; {}^{i} \; {}^{j} \qquad {}^{j} \; {}^{i}) \qquad (6)$$

$$!\,_{ij} = g^E \; e_i \; (@ e_j + \qquad e_j \;) \qquad (7)$$

where e_i is the vierbein and is the Christo el symbol.

[2] A. Yamamoto and Y. Hirono, Phys. Rev. Lett. 111, 081601 (2013), arXiv: 1303.6292 [hep-lat].

The Euclidean metric tensor can be obtained from $g_{\mu\nu}$ by Wick rotation $t \to i\tau$

$$g^E_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & y\Omega_I \\ 0 & 1 & 0 & -x\Omega_I \\ 0 & 0 & 1 & 0 \\ y\Omega_I & -x\Omega_I & 0 & 1 + r^2\Omega_I^2 \end{pmatrix},$$

where imaginary angular velocity $\Omega_I = -i\Omega$ is introduced.

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where imaginary angular velocity $\Omega_I = i\Omega$ is introduced.

Sign problem

- The Euclidean action is complex-valued function with real rotation!
- The Monte–Carlo simulations are conducted with imaginary angular velocity $\Omega_I = i\Omega$.
- The results are analytically continued to the region of the real angular velocity.

The resulting partition function is

$$Z = \int D \quad D \quad DU \exp \quad S_G[U; \quad {}_I] \quad S_F[\quad ; \quad ; m; U; \quad {}_I] \quad = \int DU \det M[m; U; \quad {}_I]\, e^{(\quad S_G[U; \quad {}_I])} \tag{8}$$

- The rotation affect both gluon and quark degrees of freedom!

Lattice setup

The resulting partition function is

$$Z = \int D\ D\ DU \exp\ \ S_G[U;\ _I]\ \ S_F[\ ;\ ; m; U;\ _I] = $$

$$= \int DU \det M[m; U;\ _I]\, e^{(\ S_G[U;\ _I])} \quad (8)$$

- The rotation a ect both gluon and quark degrees of freedom!
- $N_f = 2$ clover-improved Wilson fermions (c_{SW} from one-loop) + RG-improved (Iwasaki) gauge action are used.
- We reanalyze data for $m_{PS}a$ and $m_V a$ at zero temperature from CP-PACS and WHOT-QCD collaborations to restore LCP's more frequently in and set the scale.
- Simulation is performed on the lattice $N_t \quad N_z \quad N_s^2$ ($N_s = N_x = N_y$), which rotates around z-axis.
  Up to now, only results with $N_t = 4$ are available, work in progress...

To set the temperature along the given LCP we use the zero-temperature mass of vector meson (m_V -input)

$$\frac{T}{m_V}(m_{PS}=m_V;\ \ ) = \frac{1}{N_t\ \ \ m_V\, a(m_{PS}=m_V;\ \ )}: \tag{9}$$

and nd

$$\frac{T}{T_{pc}}(\ \ ) = \frac{m_V\, a(\ _{pc;\ =0}\ )}{m_V\, a(\ \ )}$$

The system should be limited in the directions, which are orthogonal to the rotation axis: $\Omega_I (N_s - 1)a/\sqrt{2} < 1$

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The boundary conditions in directions $x; y$ have to be treated carefully! The results depend on BC for any approach.

The use of periodic/open/Dirichlet BC gives qualitatively the same results for rotating gluodynamics. PBC in directions $x; y$ have been imposed.

Lattice setup: rotation and boundary conditions

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The use of periodic/open/Dirichlet BC gives qualitatively the same results for rotating gluodynamics. PBC in directions $x; y$ have been imposed.

The critical temperature in gluodynamics depends mainly on the linear velocity on the boundary $v_I = {}_I(N_s \quad 1)a$. Thus, v_I is xed in simulations instead of angular velocity $_I$ in physical units (e.g., MeV).

The Polyakov loop is

$$L(\mathbf{x}) = \mathrm{Tr}\left[\prod_{\tau=0}^{N_\tau-1} U_4(\mathbf{x};\tau)\right]; \qquad L = \frac{1}{N_s^2 N_z}\sum_{\mathbf{x}} L(\mathbf{x}). \tag{10}$$

The pseudo-critical temperature T_{pc} of the confinement/deconfinement phase transition is determined using the Polyakov loop susceptibility

$$\chi_L = N_s^2 N_z \left(\langle |L|^2\rangle - \langle |L|\rangle^2\right); \tag{11}$$

by means of the Gaussian fit and as inflection point of Polyakov loop.

The (bare) chiral condensate is

$$\langle\bar\psi\psi\rangle^{bare} = \frac{N_f T}{V}\,\mathrm{Tr}\,(M^{-1}) \tag{12}$$

For the chiral transition, pseudo-critical temperature T_{pc} is determined using peak of the (disconnected) chiral susceptibility:

$$\chi^{bare}_{\langle\bar\psi\psi\rangle} = \frac{N_f T}{V}\left[\langle \mathrm{Tr}\,(M^{-1})^2\rangle - \langle\mathrm{Tr}\,(M^{-1})\rangle^2\right] \tag{13}$$

Rotating QCD: Periodic boundary conditions

Figure: The Polyakov loop as a function of $T = T_{pc}(= 0)$ for di erent values of imaginary linear velocity on the boundary v_I. Lattice 4 16 17^2, LCP $m_{PS} = m_V = 0:80$.

- Pseudo-critical temperature decreases due to imaginary rotation (same as in gluodynamics).

Rotating QCD: Periodic boundary conditions

Figure: The Polyakov loop susceptibility and chiral susceptibility as a function of $T/T_{pc}(= 0)$ for di erent values of imaginary linear velocity on the boundary v_I. Lattice 4 16 17^2, LCP $m_{PS}/m_V = 0.80$.

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In order to disentangle the e ect of the rotation on fermions and gluons, the separate angular velocities are introduced: $S_G(\ _G) + S_F(\ _F)$.

Figure: The Polyakov loop as a function of $T = T_{pc}$ for various rotation regimes. Lattice $4 \quad 16 \quad 17^2$, $m_{PS} = m_V = 0:80$.

Rotating QCD: various rotation regimes

Figure: The Polyakov loop susceptibility and chiral susceptibility as a function of $T{=}T_{pc}$ for various rotation regimes. Lattice $4 \quad 16 \quad 17^2$, $m_{PS}{=}m_V = 0{:}80$.

Rotation of fermions and gluons separately has the opposite in uence on the critical temperature.

Figure: The pseudo-critical temperature as a function of imaginary linear velocity on the boundary for various rotation regimes (full, only gluons, only fermions).

$$\frac{T_{pc}(v_I)}{T_{pc}(0)} = 1 \quad B_2 \frac{v_I^2}{c^2} \tag{14}$$

${}_G = {}_F \neq 0$	${}_G \neq 0$	${}_F \neq 0$
$B_2 > 0$	$B_2^{(G)} > B_2$	$B_2^{(F)} < 0$

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How do the results depend on $m_{PS} = m_V$?

Rotating QCD: critical temperature

LCP's with $m_{PS} = m_V = 0{:}65; 0{:}70; 0{:}75; 0{:}80; 0{:}85$ were considered; $v_I = c < 0{:}3$.

$$\frac{T_{pc}(v_I)}{T_{pc}(0)} = 1 \quad B_2 \frac{v_I^2}{c^2}$$

LCP's with $m_{PS}=m_V = 0:65; 0:70; 0:75; 0:80; 0:85$ were considered; $v_I=c < 0:3$.

$$\frac{T_{pc}(v_I)}{T_{pc}(0)} = 1 \quad B_2 \frac{v_I^2}{c^2} \qquad =) \qquad \frac{T_{pc}(v)}{T_{pc}(0)} = 1 + \; B_2 \frac{v^2}{c^2}$$

The pseudo-critical temperature increases with the angular velocity (v /).

The coe cient B_2 slightly grows with decreasing pion mass in considered range.

The chiral transition shifts to the same direction as con nement-decon nement transition.

Conclusions

The separate rotation of quarks and gluons in QCD has the opposite in uence on the critical temperature.

The critical temperature in $N_f = 2$ QCD increases with angular velocity (v /)

$$\frac{T_{pc}(v)}{T_{pc}(0)} = 1 + B_2 \frac{v^2}{c^2} .$$

It's not Tolman-Ehrenfest e ect!

The coe cient B_2 slightly grows with decreasing pion mass in considered range ($m_{PS}/m_V = 0.65 \ldots 0.85$).

The (preliminary) results are similar to gluodynamics, where the critical temperature also increases with angular velocity.

It should be noted, that NJL (and other phenomenological models) predicts that critical temperature decreases due to the rotation. But taking into account the contribution of rotating gluons leads to an increase in T_c.

Future plans: increase statistics; simulations with smaller pion mass, on ner lattices ($N_t = 6, 8$), with an open BC.

Thank you for your attention!

The Euclidean metric tensor can be obtained from g by Wick rotation $t \to \; i$

$$g^E = \begin{pmatrix} 1 & 0 & 0 & y\,_I \\ 0 & 1 & 0 & x\,_I \\ 0 & 0 & 1 & 0 \\ y\,_I & x\,_I & 0 & 1 + r^2\,{}^2_I \end{pmatrix};$$

where imaginary angular velocity $_I = \; i$ is introduced. Substituting the (g_E) to formula (15) one gets

$$S_G = \frac{1}{2g^2}\int d^4x \Big[(1 + r^2\,{}^2_I)F^a_{xy}F^a_{xy} + (1 + y^2\,{}^2_I)F^a_{xz}F^a_{xz} + (1 + x^2\,{}^2_I)F^a_{yz}F^a_{yz} +$$
$$+ F^a_{x}F^a_{x} + F^a_{y}F^a_{y} + F^a_{z}F^a_{z}$$
$$+ 2y\,_I(F^a_{xy}F^a_{y} + F^a_{xz}F^a_{z}) \quad 2x\,_I(F^a_{yx}F^a_{x} + F^a_{yz}F^a_{z}) + 2xy\,{}^2_I F^a_{xz}F^a_{zy}\Big].$$

The covariant Dirac operator depends on the choice of the vierbein. We choose the vierbein in the form [3]

$$e_1^x = e_2^y = e_3^z = e_4 = 1\,; \quad e_4^x = \quad y \;_{I}\,; \quad e_4^y = x \;_{I}\,; \quad \text{and other } e_i \;= 0$$

As the result, the Euclidean quark action is

$$S_F = \int d^4x \quad {}^x D_x + {}^y D_y + {}^z D_z + \quad D \;+ i \;_{I} \frac{12}{2} \; + m \quad ; \qquad (15)$$

where the gamma matrices are given by $\; = \; {}^i e_i$

$$^x = \;^1 \;\; y \;_{I} \;^4\,; \quad ^y = \;^2 + x \;_{I} \;^4\,; \quad ^z = \;^3\,; \quad = \;^4\,: \qquad (16)$$

The quark action contains orbit-rotation coupling term $\;_{I}(xD_y \;\; yD_x)$ and spin-rotation coupling term $i \;\;_{I} \;^{12}=2$.

[3] A. Yamamoto and Y. Hirono, Phys. Rev. Lett. 111 , 081601 (2013), arXiv: 1303.6292 [hep-lat] .

Rotating QCD: gluon lattice action

We use RG-improved (Iwasaki) lattice gauge action (for non-rotating part):

$$S_G = \sum_x \Big((c_0 + r^2 \; {}^2_{\text{I}}) W^{1\;\;1}_{xy} + (c_0 + y^2 \; {}^2_{\text{I}}) W^{1\;\;1}_{xz} + (c_0 + x^2 \; {}^2_{\text{I}}) W^{1\;\;1}_{yz} +$$

$$+ c_0 \, W^{1\;\;1}_{x} + W^{1\;\;1}_{y} + W^{1\;\;1}_{z} + y \;{}_{\text{I}}\; W^{1\;\;1\;\;1}_{xy} + W^{1\;\;1\;\;1}_{xz}$$

$$x \;{}_{\text{I}}\; W^{1\;\;1\;\;1}_{yx} + W^{1\;\;1\;\;1}_{yz} + xy \; {}^2_{\text{I}} W^{1\;\;1\;\;1}_{xzy} + \sum_{\ne} c_1 W^{1\;\;2} \Big) ; \qquad (17)$$

with $= 6 = g^2$, and $c_0 = 1 \quad 8c_1$, and $c_1 = \quad 0{:}331$, where

$$W^{1\;\;1}(x) = 1 \quad \frac{1}{3}\text{Re Tr } U \quad (x) ; \qquad (18)$$

$$W^{1\;\;2}(x) = 1 \quad \frac{1}{3}\text{Re Tr } R \quad (x) ; \qquad (19)$$

$$W^{1\;\;1\;\;1}(x) = \quad \frac{1}{3}\text{Re Tr } V \quad (x) ; \qquad (20)$$

U denotes clover-type average of 4 plaquettes,

R is a rectangular loop,

V is asymmetric chair-type average of 8 chairs.

The lattice quark action has the following form ($N_f = 2$ clover-improved Wilson fermions are used)

$$S_F = \mathrm{X}_{f} \mathrm{X}_{x_1;x_2} {}^{f}(x_1) \quad {}_{x_1;x_2} \Big((1 \quad {}^{x})T_{x+} + (1 + {}^{x})T_{x} + (1 \quad {}^{y})T_{y+} +$$

$$(1 + {}^{y})T_{y} + (1 \quad {}^{z})T_{z+} + (1 + {}^{z})T_{z} + (1 \quad) \exp\Big(ia \,{}_{l} \frac{12}{2}\Big) T_{+}$$

$$+ (1 + \quad) \exp\Big(ia \,{}_{l} \frac{12}{2}\Big) T \quad {}_{x_1;x_2} c_{SW} \mathrm{X}_{<} F \quad {}^{f}(x_2) ; \quad (21)$$

where $= 1=(8 + 2\,am)$, $T_{+} = U(x_1)\, {}_{x_1+;x_2}$, $T = U^{y}(x_1)\, {}_{x_1;x_2}$ and

$${}^{x} = {}^{1} \quad y\, {}_{l} \, {}^{4}; \quad {}^{y} = {}^{2} + x\, {}_{l} \, {}^{4}; \quad {}^{z} = {}^{3}; \quad = {}^{4}.$$

The clover coe cient is taken as $c_{SW} = (1 \quad W^{1\;1})^{3=4} = (1 \quad 0{:}8412=)^{3=4}$ (one-loop result for the plaquette are used).

The spin-rotation coupling term is exponentiated like chemical potential.

The lattice quark action has the following form ($N_f = 2$ clover-improved Wilson fermions are used)

$$S_F = \sum_f \sum_{x_1;x_2} {}^f(x_1) \Big({}_{x_1;x_2} \quad (1 \quad {}^x)T_{x+} + (1 + \quad {}^x)T_x \quad + (1 \quad {}^y)T_{y+} +$$

$$(1 + \quad {}^y)T_y \quad + (1 \quad {}^z)T_{z+} + (1 + \quad {}^z)T_z \quad + (1 \quad) \exp \Big(ia \; {}_l \frac{12}{2} \Big) T \, {}_+$$

$$+ (1 + \quad) \exp \quad ia \; {}_l \frac{12}{2} \; T \qquad {}_{x_1;x_2} c_{SW} \sum_{<} \quad F \quad {}^f(x_2) ; \qquad (21)$$

where $= 1=(8 + 2\,am)$, $T \, {}_+ = U \; (x_1) \; {}_{x_1+ \; ;x \; 2}$, $T \quad = U^y(x_1) \; {}_{x_1 \quad ;x \; 2}$ and

$${}^x = \quad {}^1 \quad y \; {}_l \quad {}^4 ; \qquad {}^y = \quad {}^2 + x \; {}_l \quad {}^4 ; \qquad {}^z = \quad {}^3 ; \qquad = \quad {}^4 :$$

The clover coe cient is taken as $c_{SW} = (1 \quad W^{1 \quad 1}) \;{}^{3=4} = (1 \quad 0{:}8412= \;) \;{}^{3=4}$ (one-loop result for the plaquette are used).

The spin-rotation coupling term is exponentiated like chemical potential.

Figure: The local Polyakov loop $|\langle L(x,y)\rangle|$ as a function of coordinate for OBC and $\Omega_I = 0$ MeV (left), $\Omega_I = 24$ MeV (right). Points with $x \neq 0, y = 0$ from the lattice $8 \times 24 \times 49^2$ are shown.

- The local Polyakov loop $|\langle L(x,y)\rangle|$ is zero for all spatial points in the confinement phase, both with and without rotation $\Rightarrow$ Polyakov loop still acts as the order parameter.
- In deconfinement phase the boundary is screened.

Rotating gluodynamics: PBC, Polyakov loop distribution

Figure: The local Polyakov loop $|\langle L(x,y)\rangle|$ as a function of coordinate for OBC and $\Omega_I = 0$ MeV (left), $\Omega_I = 24$ MeV (right). Points with $x \neq 0, y = 0$ from the lattice $8 \times 24 \times 49^2$ are shown.

- The local Polyakov loop $|\langle L(x,y)\rangle|$ is zero for all spatial points in the confinement phase, both without rotation and with nonzero angular velocity.
- The local Polyakov loop demonstrates weak dependence on the coordinate in the deconfinement phase.

Rotating gluodynamics: DBC, Polyakov loop distribution

Figure: The local Polyakov loop $|\langle L(x;y)\rangle|$ as a function of coordinate for OBC and $\Omega_I = 0$ MeV (left), $\Omega_I = 24$ MeV (right). Points with $x \neq 0; y = 0$ from the lattice $8 \times 24 \times 49^2$ are shown.

- The local Polyakov loop $|\langle L(x;y)\rangle|$ is equal three on the boundary in both phases.
- The boundary is screened.

The linear velocity on the boundary $v_l = \; _l (N_s \quad 1)\, a(\; _c)=2$

$$\frac{T_c(v_l)}{T_c(0)} = 1 \quad B_2 \frac{v_l^2}{c^2} \qquad =) \qquad \frac{T_c(v)}{T_c(0)} = 1 + \; B_2 \frac{v^2}{c^2}$$

- The critical temperature increases with the angular velocity.
- The coe cient B_2 slightly depends on the transverse lattice size (N_s=N_t), but it is almost independent of both the lattice spacing and the lattice size along the rotation axis (N_z=N_t).
- For lattices with su ciently large N_s and OBC the coe cient is $B_2 \quad 0{:}7$.

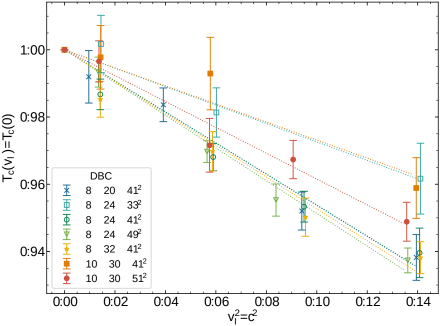
Rotating gluodynamics: Periodic boundary conditions

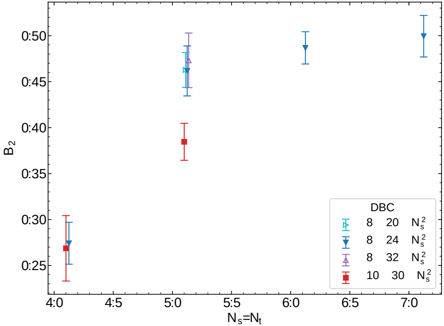
The linear velocity on the boundary $v_I = \ _I (N_s \ \ 1)\, a(\ _c)=2$

$$\frac{T_c(v_I)}{T_c(0)} = 1 \quad B_2 \frac{v_I^2}{c^2} \qquad =) \qquad \frac{T_c(v)}{T_c(0)} = 1 + B_2 \frac{v^2}{c^2}$$

The critical temperature increases with the angular velocity.

The results for the nest lattices with $N_t = 10; 12$ are close to each others, and for PBC the coe cient is $B_2 \quad 1{:}3$.





The linear velocity on the boundary $v_l = \Omega_l (N_s - 1) a(\beta_c)/2$

$$\frac{T_c(v_l)}{T_c(0)} = 1 - B_2 \frac{v_l^2}{c^2} \quad \Longrightarrow \quad \frac{T_c(v)}{T_c(0)} = 1 + B_2 \frac{v^2}{c^2}$$

- The critical temperature increases with the angular velocity.
- For lattices with sufficiently large N_s and DBC the coefficient goes to plateau $B_2 \approx 0.5$.