

# Thermal phase transition in rotating QCD with dynamical quarks

A. A. Roenko,

in collaboration with

V. V. Braguta, A. Yu. Kotov, D. A. Sychev

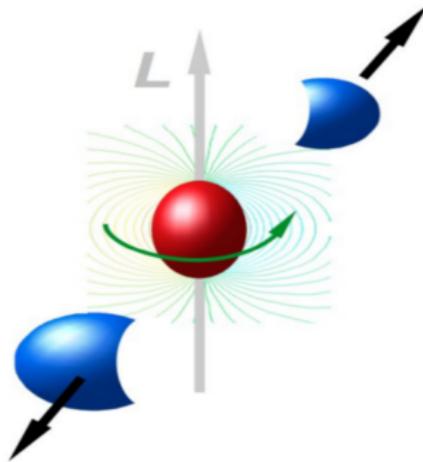
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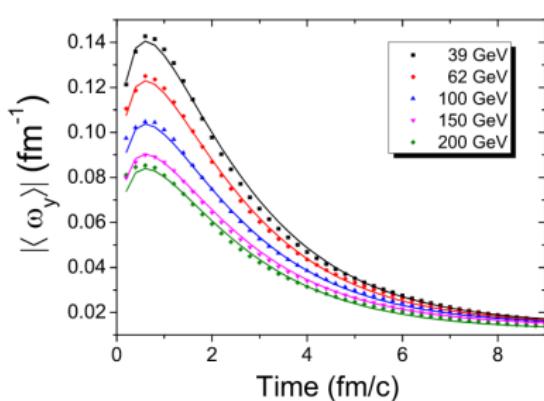
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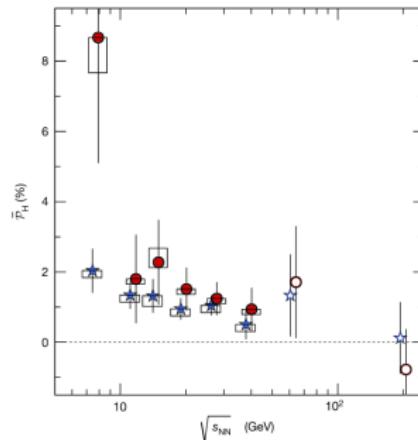


Au+Au,  $b = 7$  fm

[Y. Jiang et al., Phys. Rev. C **94**, 044910

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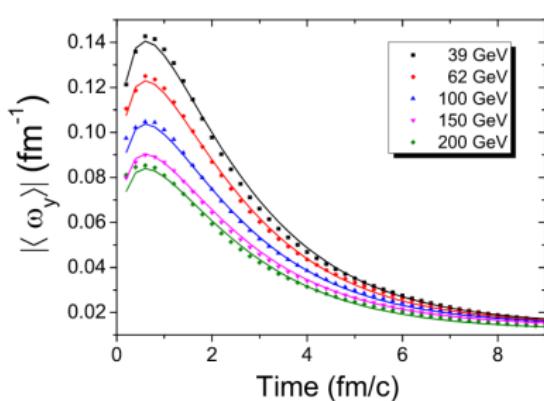
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[ L. Adamczyk et al. (STAR), Nature **548**, 62–65 (2017), arXiv:1701.06657  
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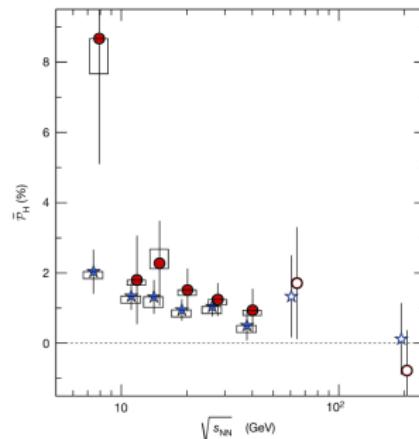
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- How does the rotation affect to phase transitions in QCD?

## Related papers

Rotation on the lattice (phase transitions were not considered):

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- S. M. A. Tabatabaee Mehr and F. Taghinavaz, (2022), arXiv:2201.05398 [hep-ph]
- H. Zhang et al., Chin. Phys. C **44**, 111001 (2020), arXiv:1812.11787 [hep-ph]
- X. Wang et al., Phys. Rev. D **99**, 016018 (2019), arXiv:1808.01931 [hep-ph]
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Rotation **suppresses the chiral condensate** ( $S = 0$ ), states with  $S \neq 0$  are preferable.

⇒ Critical temperature **decreases** due to the rotation.

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- Holography: N. R. F. Braga et al., Phys. Rev. D **105**, 106003 (2022), arXiv:2201.05581 [hep-th], A. A. Golubtsova et al., Nucl. Phys. B **979**, 115786 (2022), arXiv:2107.11672 [hep-th], X. Chen et al., JHEP **07**, 132 (2021), arXiv:2010.14478 [hep-ph], ...
- Compact QED in 2+1-D M. N. Chernodub, Phys. Rev. D **103**, 054027 (2021), arXiv:2012.04924 [hep-ph]
- HRG model: Y. Fujimoto et al., Phys. Lett. B **816**, 136184 (2021), arXiv:2101.09173 [hep-ph]
- Instantons in rotating YM: M. N. Chernodub, (2022), arXiv:2208.04808 [hep-th]
- Polyakov loop potential in YM with  $\Omega_I$  (perturbatively, finite  $T$ ): S. Chen et al., (2022), arXiv:2207.12665 [hep-ph]
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Our lattice results for gluodynamics is opposite: critical temperature increases with rotation.

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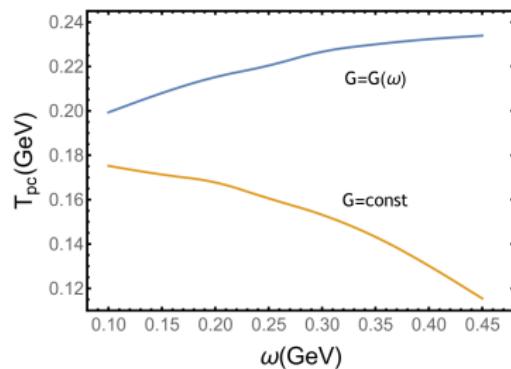
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The running effective coupling  $G(\omega)$  is introduced.

⇒ Critical temperature **increases** due to the rotation.

## Rotating reference frame

- QCD (at thermal equilibrium) is investigated in the reference frame which rotates with the system with angular velocity  $\Omega$ .
- In this reference frame there appears an **external gravitational field**

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

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- **Tolman-Ehrenfest effect:** In gravitational field the temperature isn't a constant in space at thermal equilibrium:

$$T(r)\sqrt{1 - r^2\Omega^2} = \text{const} \equiv T,$$

One could expect, that **the rotation effectively warm up the periphery** and as a result, from kinematics, the critical temperature should **decreases**.

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- The partition function is<sup>1</sup>

$$Z = \int D\psi D\bar{\psi} DA \exp \left( -S_G[A, \Omega] - S_F[\bar{\psi}, \psi, A, m, \Omega] \right). \quad (1)$$

---

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# Rotating QCD: continuum action

The Euclidean gluon action can be written as

$$S_G = \frac{1}{4g^2} \int d^4x \sqrt{g_E} g_E^{\mu\nu} g_E^{\alpha\beta} F_{\mu\alpha}^a F_{\nu\beta}^a. \quad (2)$$

And the quark action reads as follows<sup>2</sup>

$$S_F = \int d^4x \sqrt{g_E} \bar{\psi} (\gamma^\mu (D_\mu - \Gamma_\mu) + m) \psi, \quad (3)$$

The covariant derivative  $D_\mu$  and spinor affine connection  $\Gamma_\mu$  is

$$D_\mu = \partial_\mu - iA_\mu, \quad (4)$$

$$\Gamma_\mu = -\frac{i}{4} \sigma^{ij} \omega_{\mu ij}, \quad (5)$$

$$\sigma^{ij} = \frac{i}{2} (\gamma^i \gamma^j - \gamma^j \gamma^i) \quad (6)$$

$$\omega_{\mu ij} = g_{\alpha\beta}^E e_i^\alpha (\partial_\mu e_j^\beta + \Gamma_{\nu\mu}^\beta e_j^\nu) \quad (7)$$

where  $e_i^\mu$  is the vierbein and  $\Gamma_{\mu\nu}^\alpha$  is the Christoffel symbol.

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## Rotating QCD: sign problem

The Euclidean metric tensor can be obtained from  $g_{\mu\nu}$  by Wick rotation  $t \rightarrow i\tau$

$$g_{\mu\nu}^E = \begin{pmatrix} 1 & 0 & 0 & y\Omega_I \\ 0 & 1 & 0 & -x\Omega_I \\ 0 & 0 & 1 & 0 \\ y\Omega_I & -x\Omega_I & 0 & 1 + r^2\Omega_I^2 \end{pmatrix},$$

where **imaginary angular velocity**  $\Omega_I = -i\Omega$  is introduced.

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## Sign problem

- The Euclidean action is **complex-valued function** with real rotation!
- The Monte–Carlo simulations are conducted with **imaginary angular velocity**  $\Omega_I = -i\Omega$ .
- The results are analytically continued to the region of the real angular velocity.

## Lattice setup

The resulting partition function is

$$\begin{aligned} Z = \int D\psi D\bar{\psi} DU \exp(-S_G[U, \Omega_I] - S_F[\bar{\psi}, \psi, m, U, \Omega_I]) &= \\ &= \int DU \det M[m, U, \Omega_I] e^{(-S_G[U, \Omega_I])} \quad (8) \end{aligned}$$

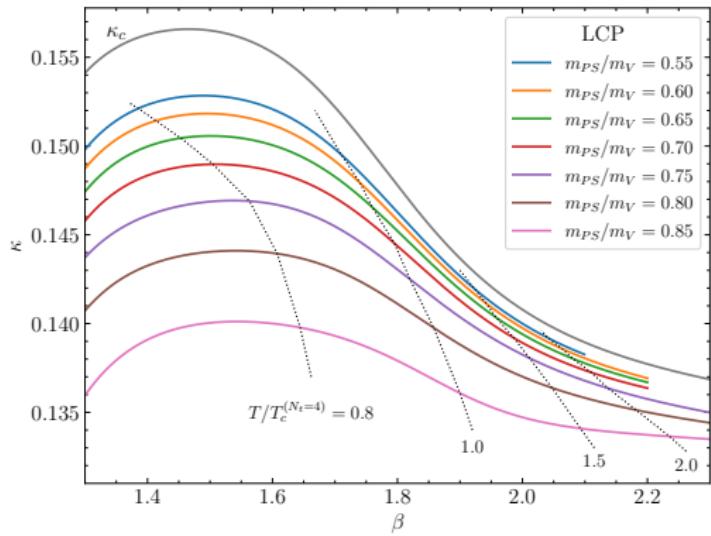
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- The rotation affect both gluon and quark degrees of freedom!
- $N_f = 2$  clover-improved Wilson fermions ( $c_{SW}$  from one-loop) + RG-improved (Iwasaki) gauge action are used.
- We reanalyze data for  $m_{PSA}$  and  $m_{VA}$  at zero temperature from CP-PACS and WHOT-QCD collaborations to restore LCP's more frequently in  $\beta$  and set the scale.
- Simulation is performed on the lattice  $N_t \times N_z \times N_s^2$  ( $N_s = N_x = N_y$ ), which rotates around  $z$ -axis.  
Up to now, only results with  $N_t = 4$  are available, work in progress...

# LCP and scale setting



To set the temperature along the given LCP we use the zero-temperature mass of vector meson ( $m_V$ -input)

$$\frac{T}{m_V}(m_{PS}/m_V, \beta) = \frac{1}{N_t \times m_V a(m_{PS}/m_V, \beta)}. \quad (9)$$

and find

$$\frac{T}{T_{pc}}(\beta) = \frac{m_V a(\beta_{pc, \Omega=0})}{m_V a(\beta)}$$

## Lattice setup: rotation and boundary conditions

- The system should be limited in the directions, which are orthogonal to the rotation axis:  $\Omega_I(N_s - 1)a/\sqrt{2} < 1$

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- The use of periodic/open/Dirichlet BC gives qualitatively the same results for rotating gluodynamics. **PBC in directions  $x, y$  have been imposed.**
- The critical temperature in gluodynamics depends mainly on the linear velocity on the boundary  $v_I = \Omega_I(N_s - 1)a$ . Thus,  **$v_I$  is fixed in simulations** instead of angular velocity  $\Omega_I$  in physical units (e.g., MeV).

## Observables

The Polyakov loop is

$$L(\vec{x}) = \text{Tr} \left[ \prod_{\tau=0}^{N_t-1} U_4(\vec{x}, \tau) \right], \quad L = \frac{1}{N_s^2 N_z} \sum_{\vec{x}} L(\vec{x}). \quad (10)$$

The pseudo-critical temperature  $T_{pc}$  of the confinement/deconfinement phase transition is determined using the Polyakov loop susceptibility

$$\chi_L = N_s^2 N_z (\langle |L|^2 \rangle - \langle |L| \rangle^2), \quad (11)$$

by means of the Gaussian fit and as inflection point of Polyakov loop.

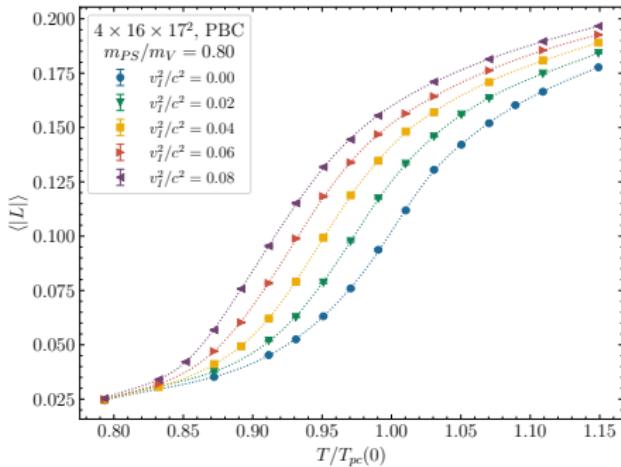
The (bare) chiral condensate is

$$\langle \bar{\psi} \psi \rangle^{bare} = -\frac{N_f T}{V} \langle \text{Tr}(M^{-1}) \rangle \quad (12)$$

For the chiral transition, pseudo-critical temperature  $T_{pc}$  is determined using peak of the (disconnected) chiral susceptibility:

$$\chi_{\langle \bar{\psi} \psi \rangle}^{bare} = \frac{N_f T}{V} \left[ \langle \text{Tr}(M^{-1})^2 \rangle - \langle \text{Tr}(M^{-1}) \rangle^2 \right] \quad (13)$$

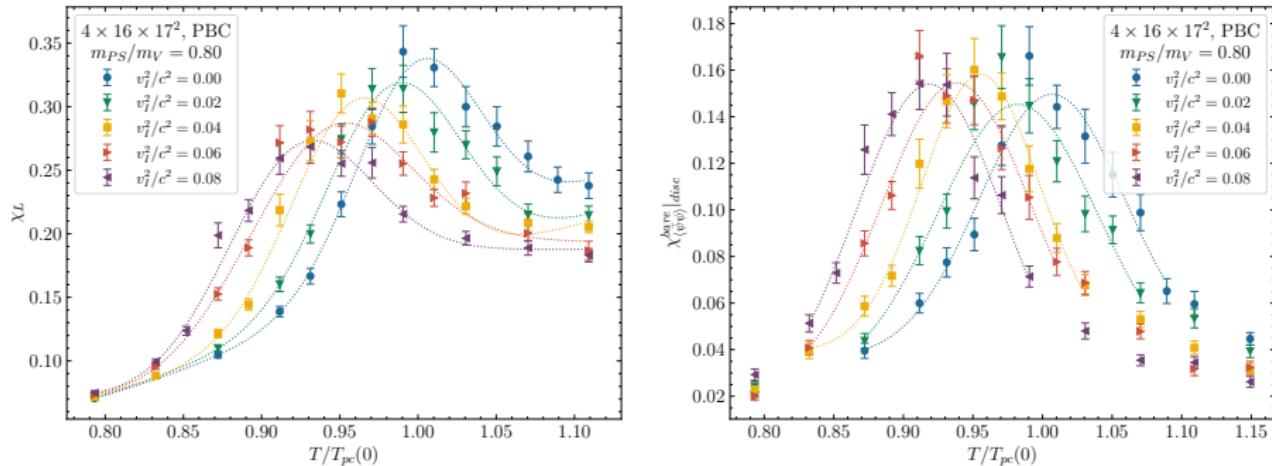
# Rotating QCD: Periodic boundary conditions



**Figure:** The Polyakov loop as a function of  $T/T_{pc}(\Omega = 0)$  for different values of **imaginary** linear velocity on the boundary  $v_I$ . Lattice  $4 \times 16 \times 17^2$ , LCP  $m_{PS}/m_V = 0.80$ .

- Pseudo-critical temperature **decreases** due to **imaginary** rotation (same as in gluodynamics).

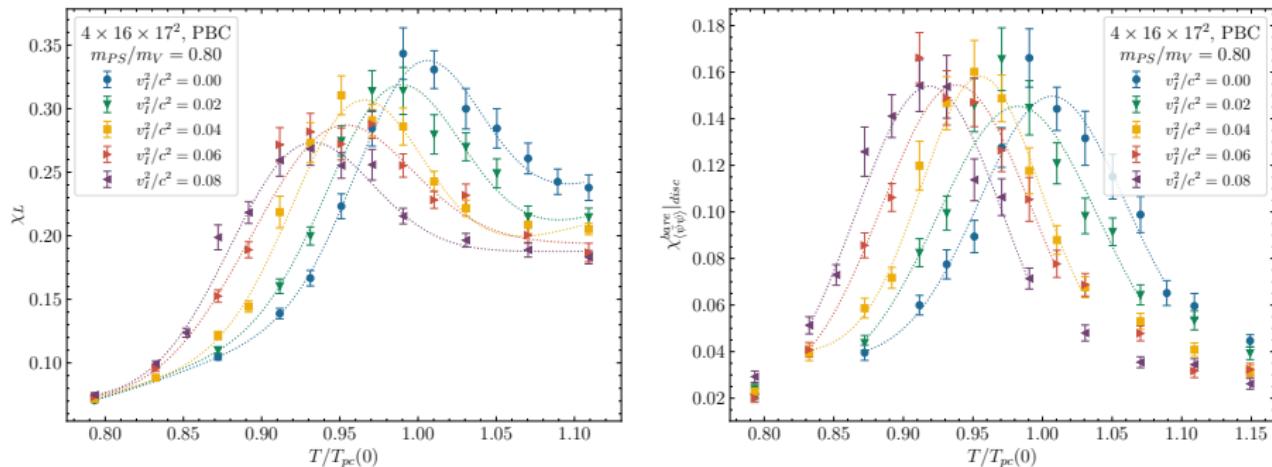
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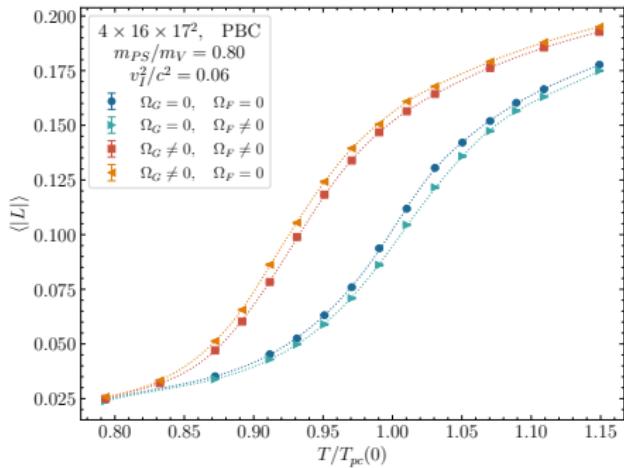


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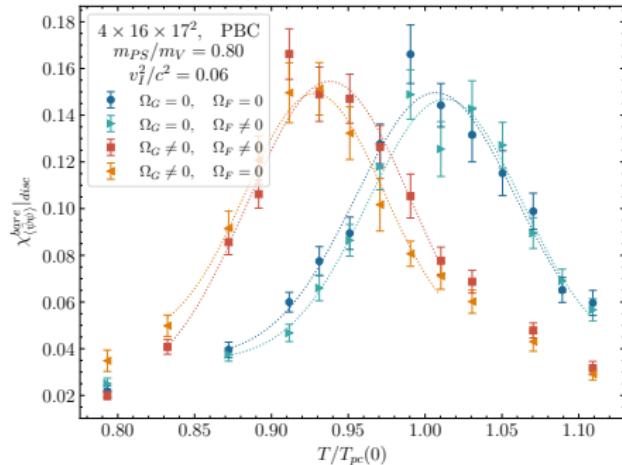
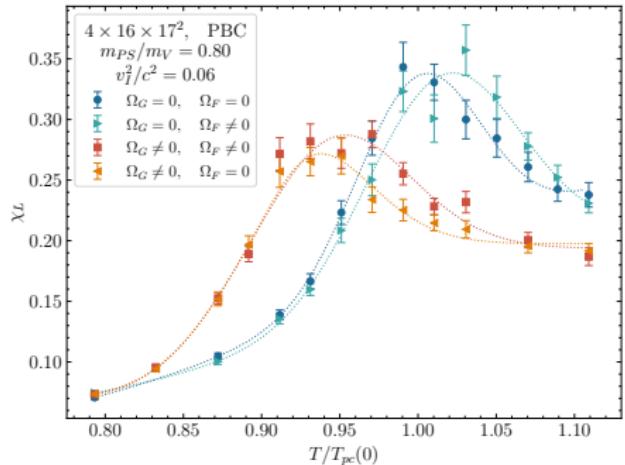
In order to disentangle the effect of the rotation on fermions and gluons, the separate angular velocities are introduced:  $S_G(\Omega_G) + S_F(\Omega_F)$ .

# Rotating QCD: various rotation regimes



**Figure:** The Polyakov loop as a function of  $T/T_{pc}$  for various rotation regimes. Lattice  $4 \times 16 \times 17^2$ ,  $m_{PS}/m_V = 0.80$ .

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**Figure:** The Polyakov loop susceptibility and chiral susceptibility as a function of  $T/T_{pc}$  for various rotation regimes. Lattice  $4 \times 16 \times 17^2$ ,  $m_{PS}/m_V = 0.80$ .

- Rotation of fermions and gluons separately has the **opposite** influence on the critical temperature.

# Rotating QCD: various rotation regimes

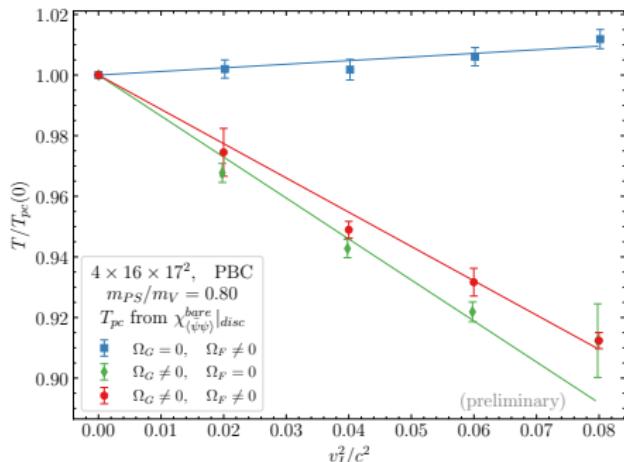
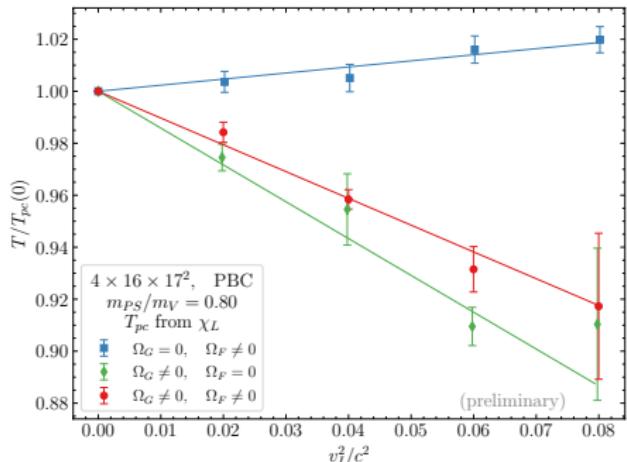


Figure: The pseudo-critical temperature as a function of **imaginary** linear velocity on the boundary for various rotation regimes (full, only gluons, only fermions).

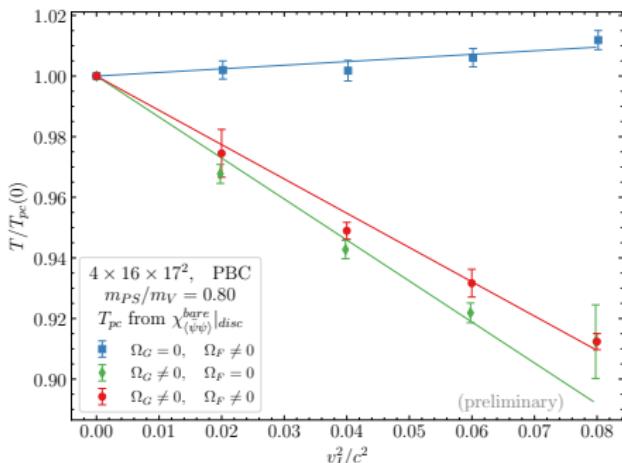
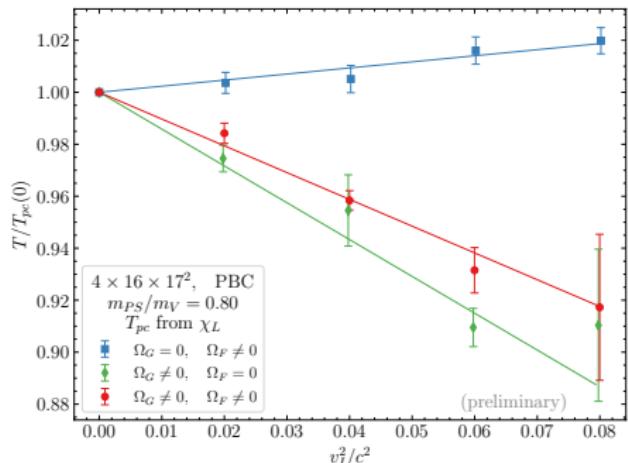
$$\frac{T_{pc}(v_I)}{T_{pc}(0)} = 1 - B_2 \frac{v_I^2}{c^2} \quad (14)$$

$$\begin{aligned} \Omega_G &= \Omega_F \neq 0 \\ B_2 &> 0 \end{aligned}$$

$$\begin{aligned} \Omega_G &\neq 0 \\ B_2^{(G)} &> B_2 \end{aligned}$$

$$\begin{aligned} \Omega_F &\neq 0 \\ B_2^{(F)} &< 0 \end{aligned}$$

## Rotating QCD: various rotation regimes



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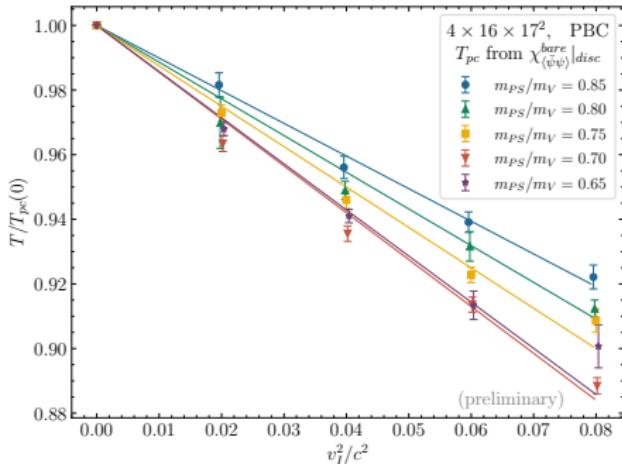
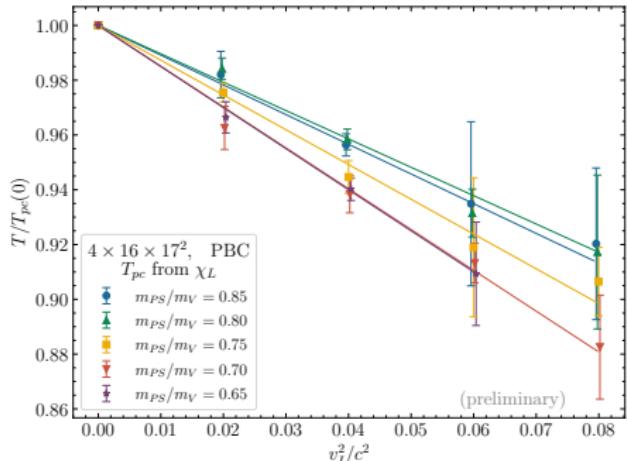
$$B_2^{(G)} > B_2$$

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How do the results depend on  $m_{PS}/m_V$ ?

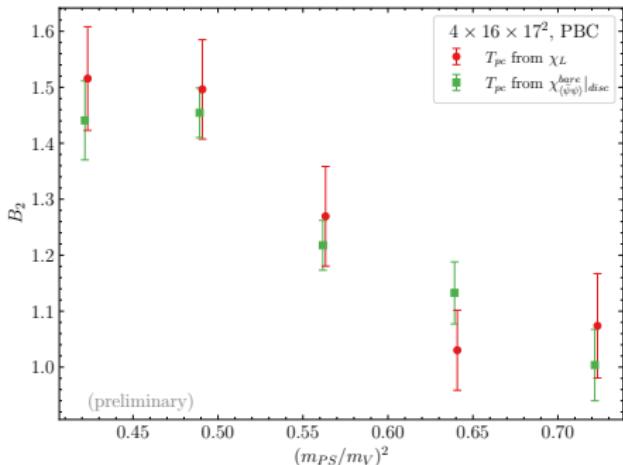
# Rotating QCD: critical temperature



LCP's with  $m_{PS}/m_V = 0.65, 0.70, 0.75, 0.80, 0.85$  were considered;  $v_I/c < 0.3$ .

$$\frac{T_{pc}(v_I)}{T_{pc}(0)} = 1 - B_2 \frac{v_I^2}{c^2}$$

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$$\frac{T_{pc}(v_I)}{T_{pc}(0)} = 1 - B_2 \frac{v_I^2}{c^2} \quad \implies \quad \frac{T_{pc}(v)}{T_{pc}(0)} = 1 + B_2 \frac{v^2}{c^2}$$

- The pseudo-critical temperature **increases** with the angular velocity ( $v \propto \Omega$ ).
- The coefficient  $B_2$  slightly grows with decreasing pion mass in considered range.
- The chiral transition shifts to the same direction as confinement-deconfinement transition.

## Conclusions

- The separate rotation of quarks and gluons in QCD has the **opposite** influence on the critical temperature.
- The critical temperature in  $N_f = 2$  QCD **increases** with angular velocity ( $v \propto \Omega$ )

$$\frac{T_{pc}(v)}{T_{pc}(0)} = 1 + B_2 \frac{v^2}{c^2}.$$

It's not **Tolman-Ehrenfest effect!**

- The coefficient  $B_2$  slightly grows with decreasing pion mass in considered range ( $m_{PS}/m_V = 0.65 \dots 0.85$ ).
- The (preliminary) results are similar to gluodynamics, where the critical temperature also **increases** with angular velocity.
- It should be noted, that NJL (and other phenomenological models) predicts that critical temperature **decreases** due to the rotation. But taking into account the contribution of rotating gluons leads to an **increase** in  $T_c$ .
- Future plans: increase statistics; simulations with smaller pion mass, on finer lattices ( $N_t = 6, 8$ ), with an open BC.

Thank you for your attention!

## Rotating QCD: continuum gluon action

The Euclidean metric tensor can be obtained from  $g_{\mu\nu}$  by Wick rotation  $t \rightarrow i\tau$

$$g_{\mu\nu}^E = \begin{pmatrix} 1 & 0 & 0 & y\Omega_I \\ 0 & 1 & 0 & -x\Omega_I \\ 0 & 0 & 1 & 0 \\ y\Omega_I & -x\Omega_I & 0 & 1 + r^2\Omega_I^2 \end{pmatrix},$$

where **imaginary angular velocity**  $\Omega_I = -i\Omega$  is introduced. Substituting the  $(g_E)_{\mu\nu}$  to formula (15) one gets

$$\begin{aligned} S_G = \frac{1}{2g^2} \int d^4x & \left[ (1 + r^2\Omega_I^2)F_{xy}^a F_{xy}^a + (1 + y^2\Omega_I^2)F_{xz}^a F_{xz}^a + (1 + x^2\Omega_I^2)F_{yz}^a F_{yz}^a + \right. \\ & + F_{x\tau}^a F_{x\tau}^a + F_{y\tau}^a F_{y\tau}^a + F_{z\tau}^a F_{z\tau}^a - \\ & \left. + 2y\Omega_I(F_{xy}^a F_{y\tau}^a + F_{xz}^a F_{z\tau}^a) - 2x\Omega_I(F_{yx}^a F_{x\tau}^a + F_{yz}^a F_{z\tau}^a) + 2xy\Omega_I^2 F_{xz}^a F_{zy}^a \right]. \end{aligned}$$

## Rotating QCD: continuum quark action

The covariant Dirac operator depends on the choice of the vierbein. We choose the vierbein in the form<sup>3</sup>

$$e_1^x = e_2^y = e_3^z = e_4^\tau = 1, \quad e_4^x = -y\Omega_I, \quad e_4^y = x\Omega_I, \quad \text{and other } e_i^\mu = 0$$

As the result, the Euclidean quark action is

$$S_F = \int d^4x \bar{\psi} \left( \gamma^x D_x + \gamma^y D_y + \gamma^z D_z + \gamma^\tau \left( D_\tau + i\Omega_I \frac{\sigma^{12}}{2} \right) + m \right) \psi, \quad (15)$$

where the gamma matrices are given by  $\gamma^\mu = \gamma^i e_i^\mu$

$$\gamma^x = \gamma^1 - y\Omega_I \gamma^4, \quad \gamma^y = \gamma^2 + x\Omega_I \gamma^4, \quad \gamma^z = \gamma^3, \quad \gamma^\tau = \gamma^4. \quad (16)$$

The quark action contains orbit-rotation coupling term  $\gamma^\tau \Omega_I (xD_y - yD_x)$  and spin-rotation coupling term  $i\gamma^\tau \Omega_I \sigma^{12}/2$ .

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<sup>3</sup>A. Yamamoto and Y. Hirono, Phys. Rev. Lett. **111**, 081601 (2013), arXiv:1303.6292 [hep-lat].

## Rotating QCD: gluon lattice action

We use RG-improved (Iwasaki) lattice gauge action (for non-rotating part):

$$S_G = \beta \sum_x \left( (c_0 + r^2 \Omega_I^2) W_{xy}^{1 \times 1} + (c_0 + y^2 \Omega_I^2) W_{xz}^{1 \times 1} + (c_0 + x^2 \Omega_I^2) W_{yz}^{1 \times 1} + \right. \\ \left. + c_0 (W_{x\tau}^{1 \times 1} + W_{y\tau}^{1 \times 1} + W_{z\tau}^{1 \times 1}) + y \Omega_I (W_{xy\tau}^{1 \times 1 \times 1} + W_{xz\tau}^{1 \times 1 \times 1}) - \right. \\ \left. - x \Omega_I (W_{yx\tau}^{1 \times 1 \times 1} + W_{yz\tau}^{1 \times 1 \times 1}) + xy \Omega_I^2 W_{xzy}^{1 \times 1 \times 1} + \sum_{\mu \neq \nu} c_1 W_{\mu\nu}^{1 \times 2} \right), \quad (17)$$

with  $\beta = 6/g^2$ , and  $c_0 = 1 - 8c_1$ , and  $c_1 = -0.331$ , where

$$W_{\mu\nu}^{1 \times 1}(x) = 1 - \frac{1}{3} \text{Re Tr } \bar{U}_{\mu\nu}(x), \quad (18)$$

$$W_{\mu\nu}^{1 \times 2}(x) = 1 - \frac{1}{3} \text{Re Tr } R_{\mu\nu}(x), \quad (19)$$

$$W_{\mu\nu\rho}^{1 \times 1 \times 1}(x) = -\frac{1}{3} \text{Re Tr } \bar{V}_{\mu\nu\rho}(x), \quad (20)$$

$\bar{U}_{\mu\nu}$  denotes clover-type average of 4 plaquettes,

$R_{\mu\nu}$  is a rectangular loop,

$\bar{V}_{\mu\nu\rho}$  is asymmetric chair-type average of 8 chairs.

## Rotating QCD: quark lattice action

The lattice quark action has the following form ( $N_f = 2$  clover-improved Wilson fermions are used)

$$S_F = \sum_f \sum_{x_1, x_2} \bar{\psi}^f(x_1) \left\{ \delta_{x_1, x_2} - \kappa \left[ (1 - \gamma^x) T_{x+} + (1 + \gamma^x) T_{x-} + (1 - \gamma^y) T_{y+} + (1 + \gamma^y) T_{y-} + (1 - \gamma^z) T_{z+} + (1 + \gamma^z) T_{z-} + (1 - \gamma^\tau) \exp\left(\textcolor{red}{ia}\Omega_I \frac{\sigma^{12}}{2}\right) T_{\tau+} + (1 + \gamma^\tau) \exp\left(-\textcolor{red}{ia}\Omega_I \frac{\sigma^{12}}{2}\right) T_{\tau-} \right] - \delta_{x_1, x_2} c_{SW} \kappa \sum_{\mu < \nu} \sigma_{\mu\nu} F_{\mu\nu} \right\} \psi^f(x_2), \quad (21)$$

where  $\kappa = 1/(8 + 2am)$ ,  $T_{\mu+} = U_\mu(x_1) \delta_{x_1+\mu, x_2}$ ,  $T_{\mu-} = U_\mu^\dagger(x_1) \delta_{x_1-\mu, x_2}$  and

$$\gamma^x = \gamma^1 - \textcolor{red}{y}\Omega_I \gamma^4, \quad \gamma^y = \gamma^2 + \textcolor{red}{x}\Omega_I \gamma^4, \quad \gamma^z = \gamma^3, \quad \gamma^\tau = \gamma^4.$$

The clover coefficient is taken as  $c_{SW} = (1 - W^{1 \times 1})^{-3/4} = (1 - 0.8412/\beta)^{-3/4}$  (one-loop result for the plaquette are used).

The spin-rotation coupling term is exponentiated like chemical potential.

## Rotating QCD: quark lattice action

The lattice quark action has the following form ( $N_f = 2$  clover-improved Wilson fermions are used)

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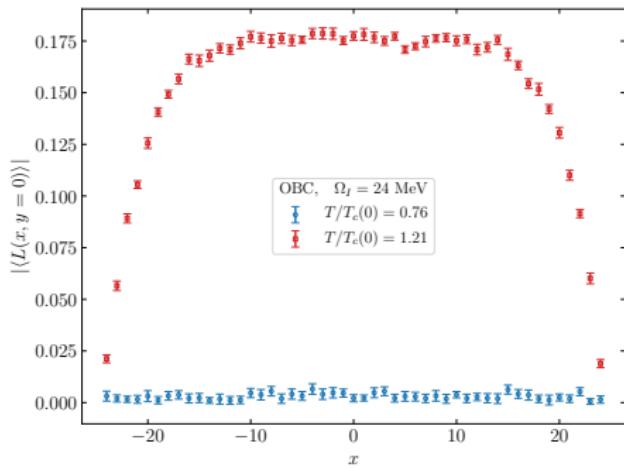
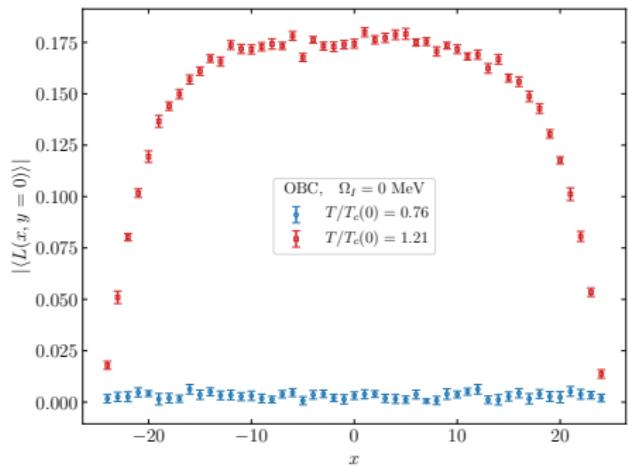
where  $\kappa = 1/(8 + 2am)$ ,  $T_{\mu+} = U_\mu(x_1) \delta_{x_1+\mu, x_2}$ ,  $T_{\mu-} = U_\mu^\dagger(x_1) \delta_{x_1-\mu, x_2}$  and

$$\gamma^x = \gamma^1 - y\Omega_I \gamma^4, \quad \gamma^y = \gamma^2 + x\Omega_I \gamma^4, \quad \gamma^z = \gamma^3, \quad \gamma^\tau = \gamma^4.$$

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The spin-rotation coupling term is exponentiated like chemical potential.

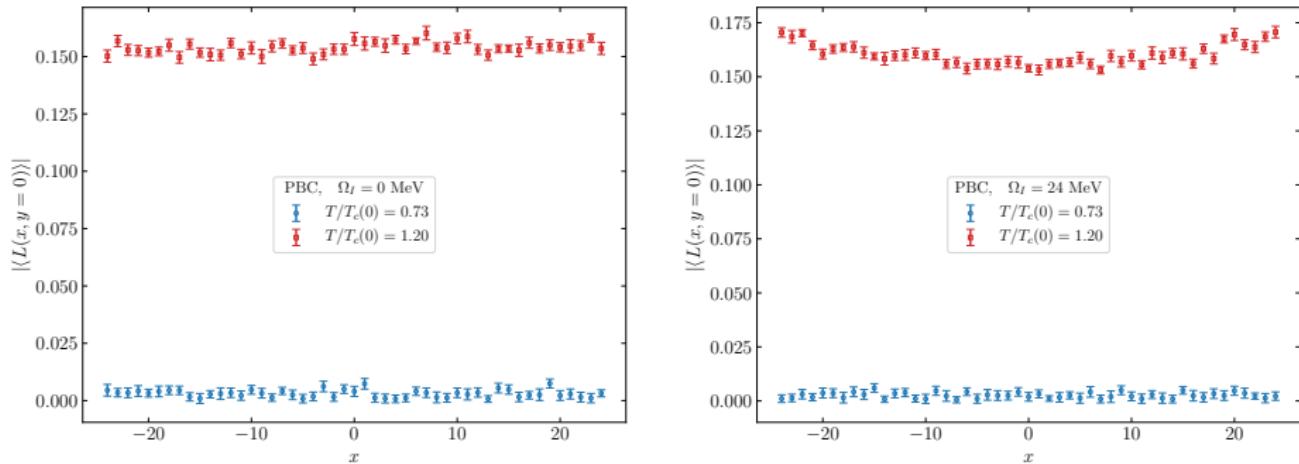
# Rotating gluodynamics: OBC, Polyakov loop distribution



**Figure:** The local Polyakov loop  $|\langle L(x, y) \rangle|$  as a function of coordinate for OBC and  $\Omega_I = 0$  MeV (left),  $\Omega_I = 24$  MeV (right). Points with  $x \neq 0, y = 0$  from the lattice  $8 \times 24 \times 49^2$  are shown.

- The local Polyakov loop  $|\langle L(x, y) \rangle|$  is zero for all spatial points in the confinement phase, both with and without rotation  $\Rightarrow$  Polyakov loop still acts as the order parameter.
- In deconfinement phase the boundary is screened.

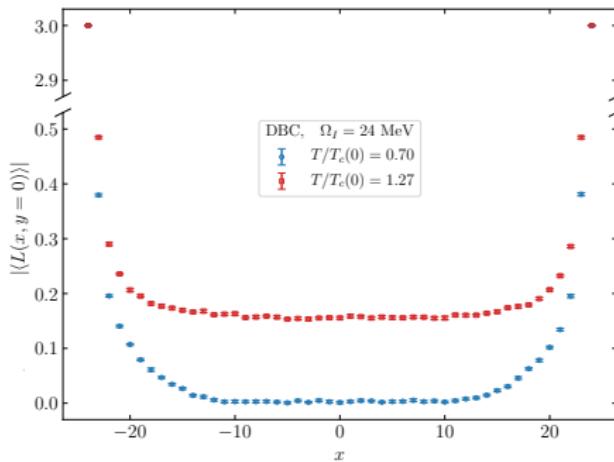
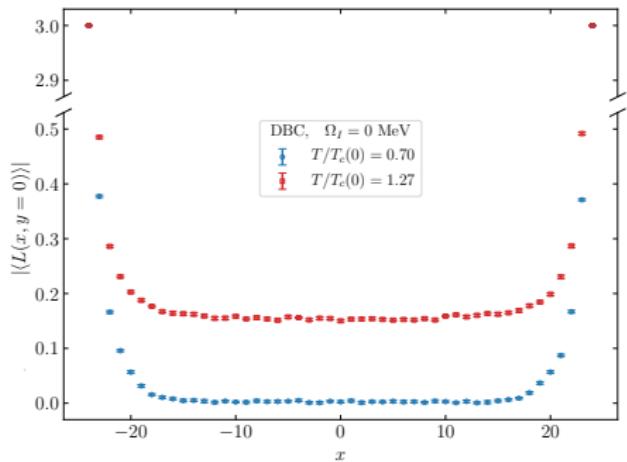
# Rotating gluodynamics: PBC, Polyakov loop distribution



**Figure:** The local Polyakov loop  $|\langle L(x, y) \rangle|$  as a function of coordinate for OBC and  $\Omega_I = 0$  MeV (left),  $\Omega_I = 24$  MeV (right). Points with  $x \neq 0, y = 0$  from the lattice  $8 \times 24 \times 49^2$  are shown.

- The local Polyakov loop  $|\langle L(x, y) \rangle|$  is zero for all spatial points in the confinement phase, both without rotation and with nonzero angular velocity.
- The local Polyakov loop demonstrates weak dependence on the coordinate in the deconfinement phase.

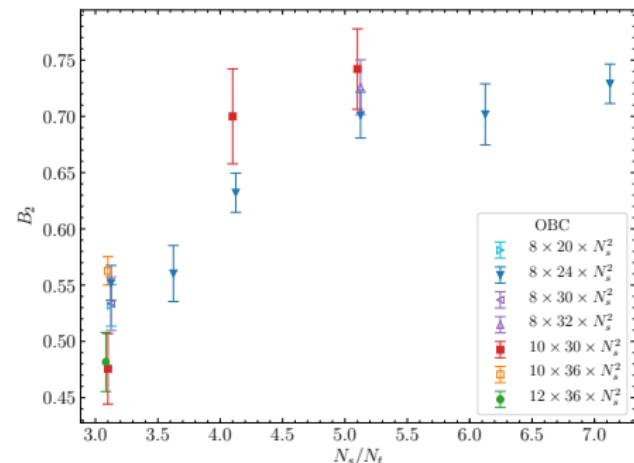
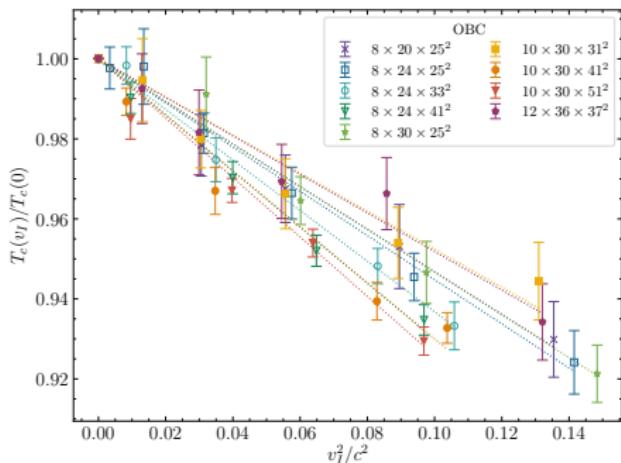
# Rotating gluodynamics: DBC, Polyakov loop distribution



**Figure:** The local Polyakov loop  $|\langle L(x, y) \rangle|$  as a function of coordinate for OBC and  $\Omega_I = 0$  MeV (left),  $\Omega_I = 24$  MeV (right). Points with  $x \neq 0, y = 0$  from the lattice  $8 \times 24 \times 49^2$  are shown.

- The local Polyakov loop  $|\langle L(x, y) \rangle|$  is equal three on the boundary in both phases.
- The boundary is screened.

# Rotating gluodynamics: Open boundary conditions

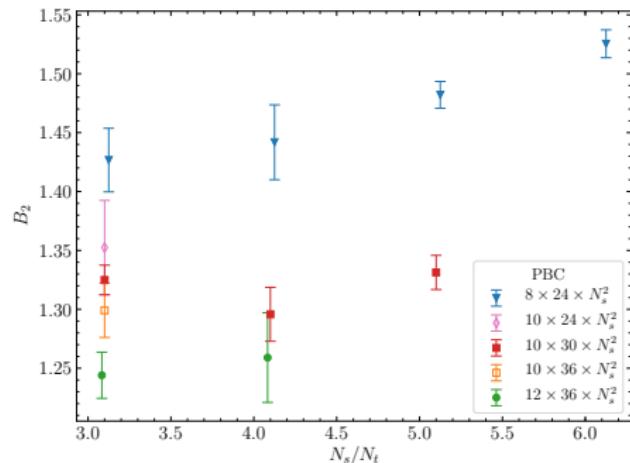
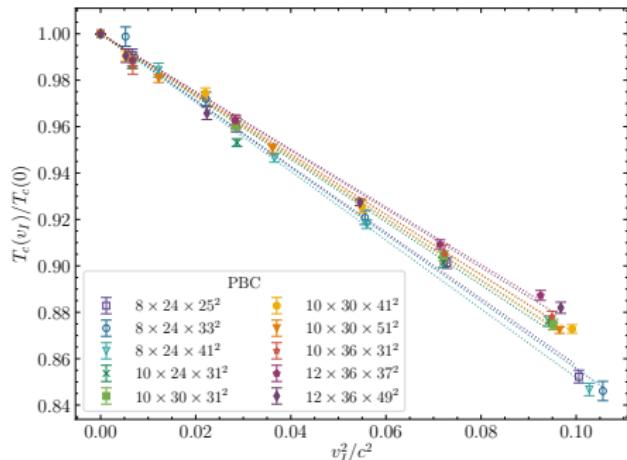


The linear velocity on the boundary  $v_I = \Omega_I (N_s - 1) a(\beta_c)/2$

$$\frac{T_c(v_I)}{T_c(0)} = 1 - B_2 \frac{v_I^2}{c^2} \quad \Rightarrow \quad \frac{T_c(v)}{T_c(0)} = 1 + B_2 \frac{v^2}{c^2}$$

- The critical temperature **increases** with the angular velocity.
- The coefficient  $B_2$  slightly depends on the transverse lattice size ( $N_s/N_t$ ), but it is almost independent of both the lattice spacing and the lattice size along the rotation axis ( $N_z/N_t$ ).
- For lattices with sufficiently large  $N_s$  and OBC the coefficient is  $B_2 \approx 0.7$ .

# Rotating gluodynamics: Periodic boundary conditions

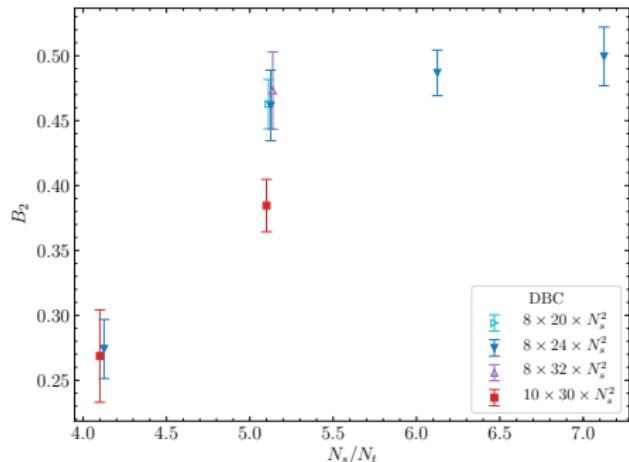
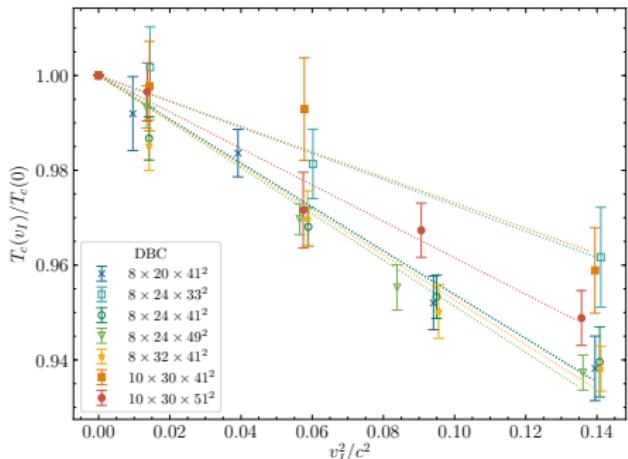


The linear velocity on the boundary  $v_I = \Omega_I (N_s - 1) a(\beta_c)/2$

$$\frac{T_c(v_I)}{T_c(0)} = 1 - B_2 \frac{v_I^2}{c^2} \quad \Rightarrow \quad \frac{T_c(v)}{T_c(0)} = 1 + B_2 \frac{v^2}{c^2}$$

- The critical temperature **increases** with the angular velocity.
- The results for the finest lattices with  $N_t = 10, 12$  are close to each others, and for PBC the coefficient is  $B_2 \sim 1.3$ .

# Rotating gluodynamics: Dirichlet boundary conditions



The linear velocity on the boundary  $v_I = \Omega_I (N_s - 1) a(\beta_c)/2$

$$\frac{T_c(v_I)}{T_c(0)} = 1 - B_2 \frac{v_I^2}{c^2} \quad \Rightarrow \quad \frac{T_c(v)}{T_c(0)} = 1 + B_2 \frac{v^2}{c^2}$$

- The critical temperature **increases** with the angular velocity.
- For lattices with sufficiently large  $N_s$  and DBC the coefficient goes to plateau  $B_2 \sim 0.5$ .