Topology and the Dirac spectrum in hot QCD

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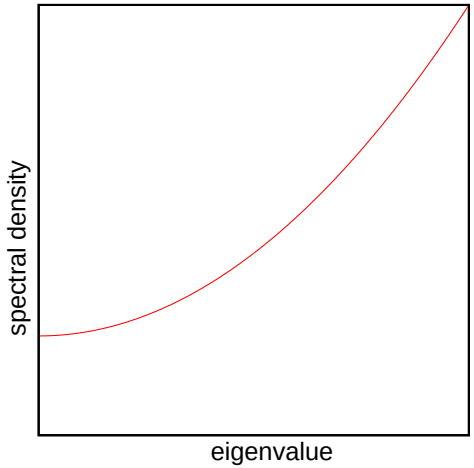
The Dirac spectrum across the transition

Standard lore

Below T_c

- Chiral symmetry broken
- Order parameter: $\rho(0) \neq 0$ (spectral density at zero)



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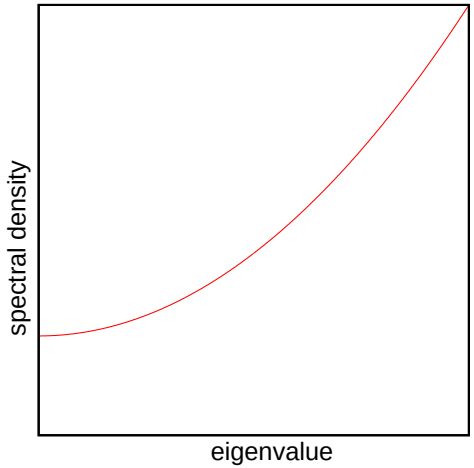
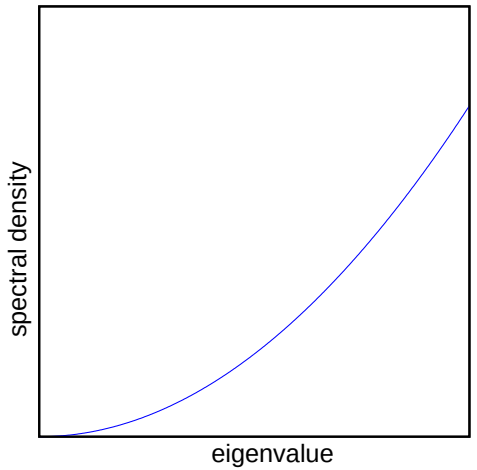


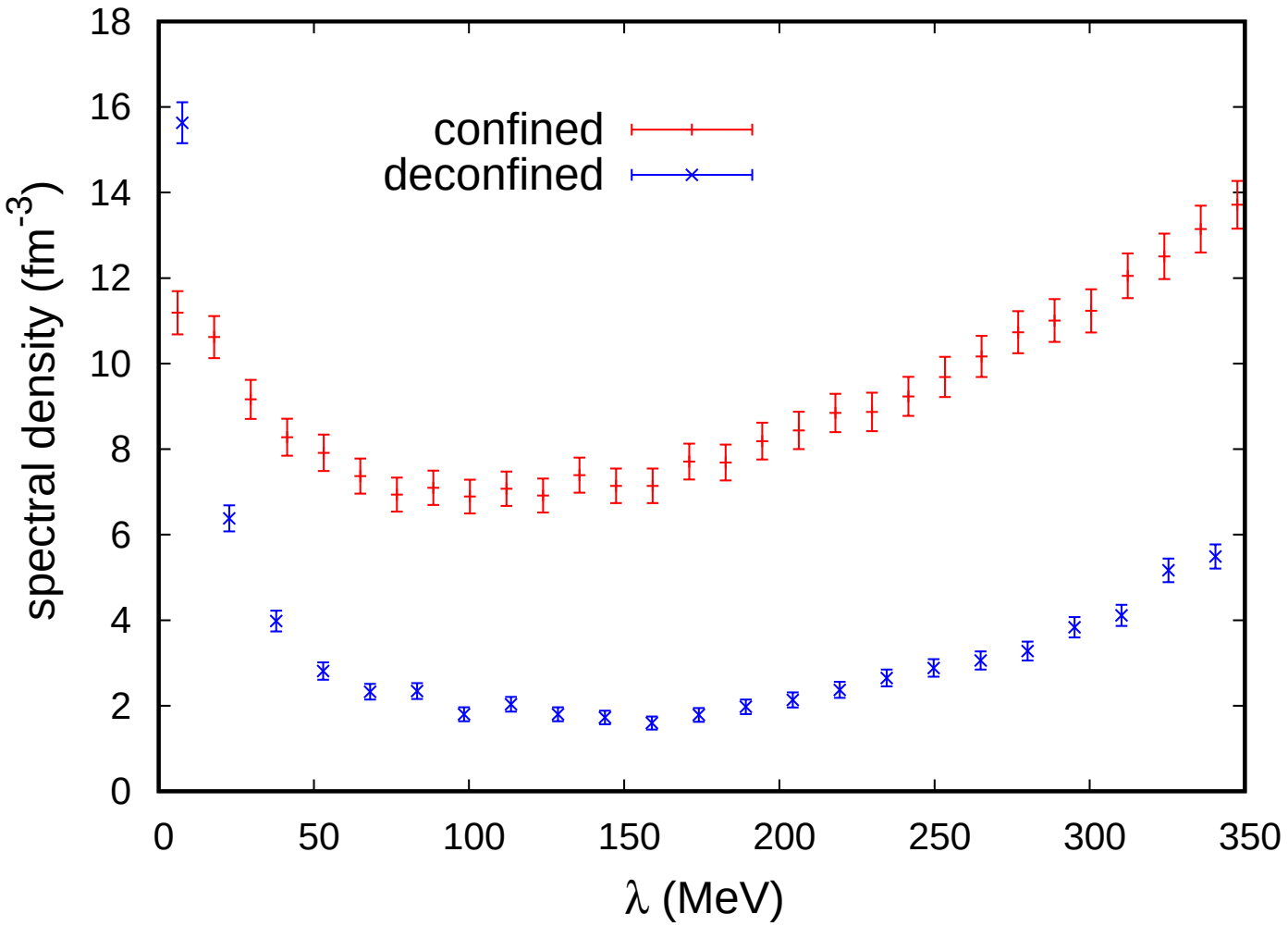
Above T_c

- Chiral symmetry restored
- Order parameter $\rho(0) = 0$



Overlap spectral density in the two phases at T_c

Lattice quenched, $N_t = 8$, $T = T_c$; exact zero modes not shown

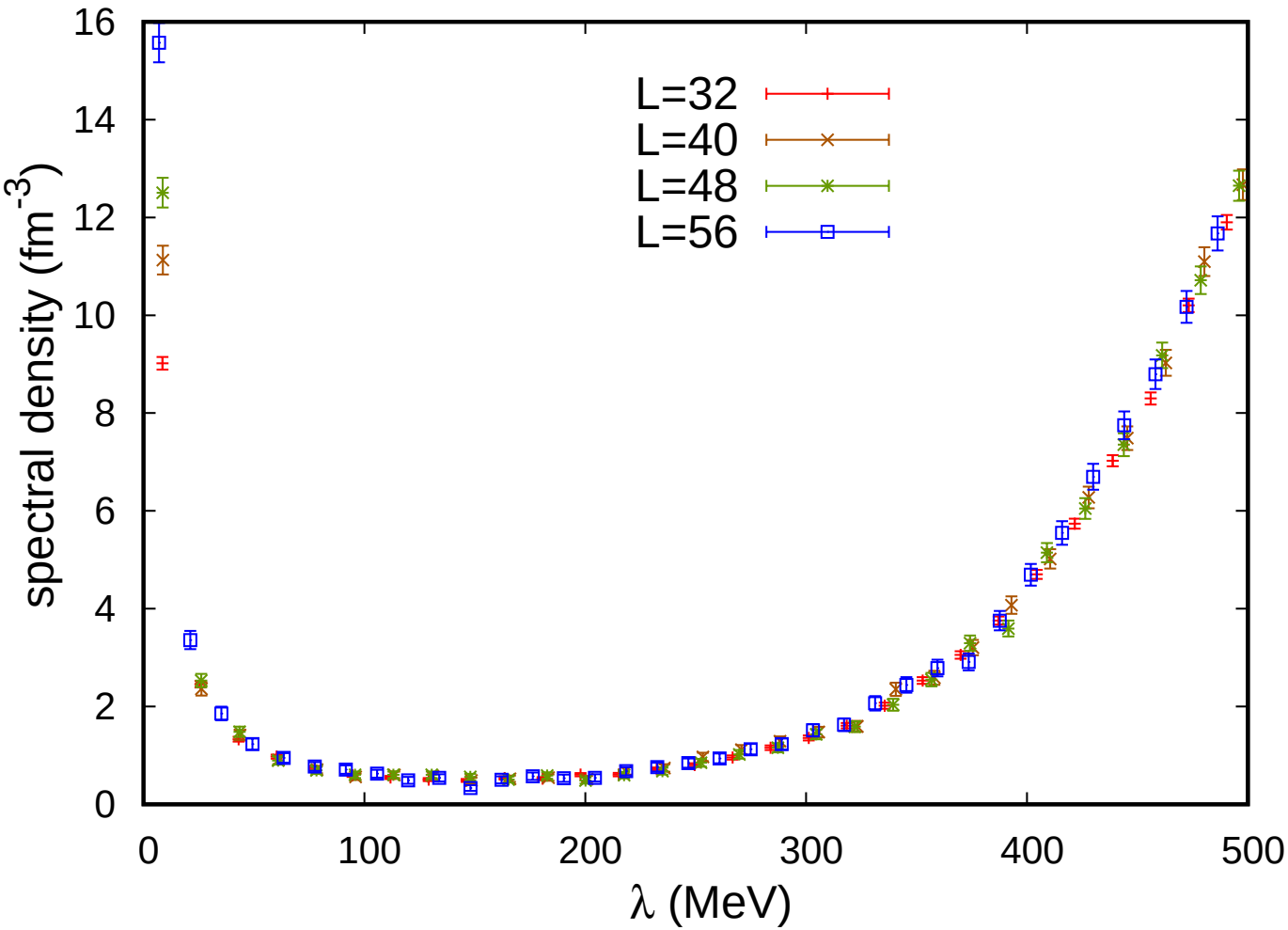


Spectral peak at $\lambda = 0$ above T_c

- Spectral peak first noticed Edwards et al. 2000
  - Consistent with mixing instanton-antiinstaton zero modes?
  - But: strong coupling, small statistics
- Spectral peak is not a quenched artifact
  Alexandru & Horvath 2015 Kaczmarek, Mazur, Sharma 2021
- Spectral density at $\lambda = 0$ nonzero
  $\Rightarrow$ chiral symmetry not restored
- Possible consequences for $U(1)_A$ breaking

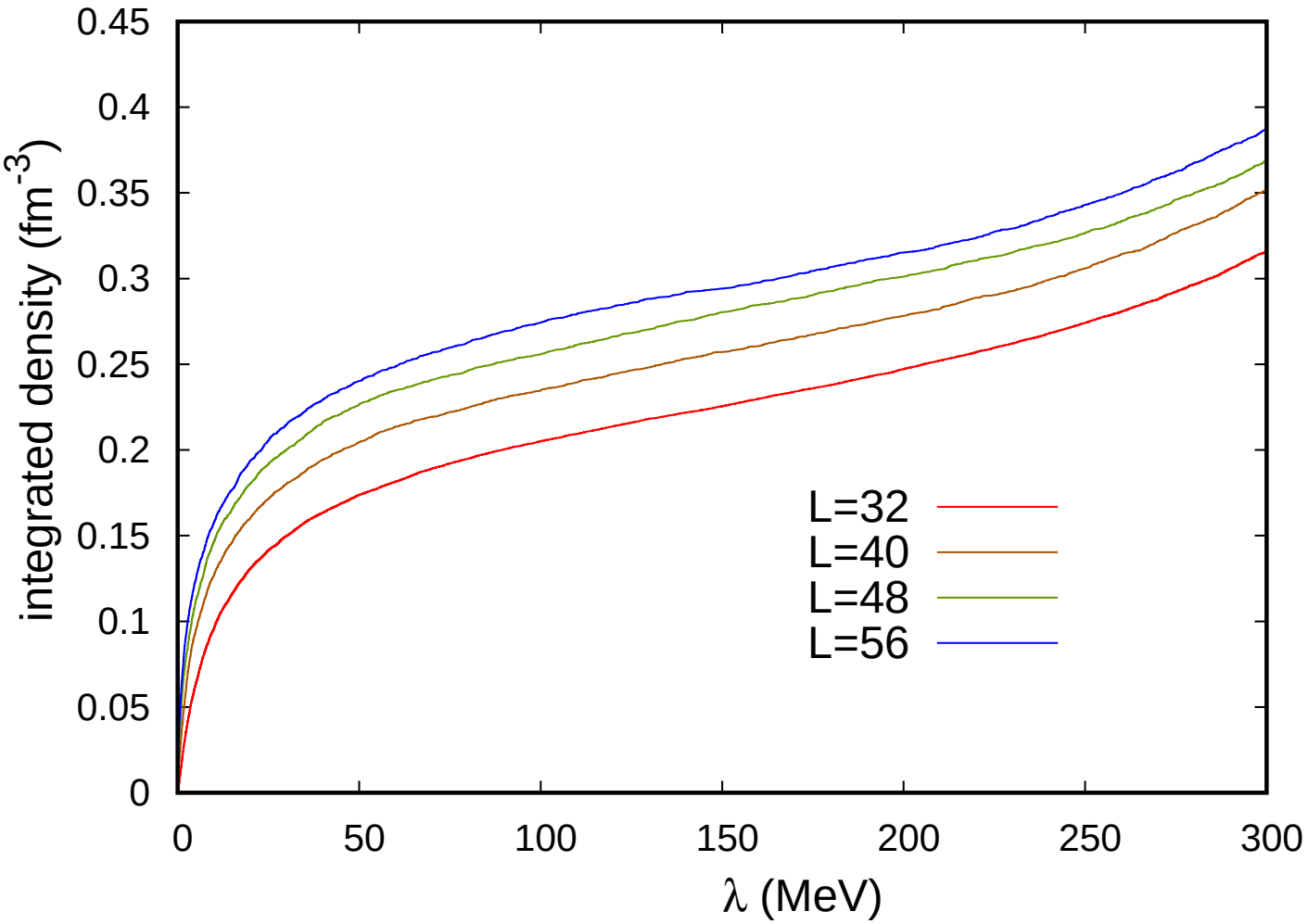
Volume dependence of the spectral density

Lattice $N_t = 8$, $T = 1.045\,T_c$; exact zero modes not shown



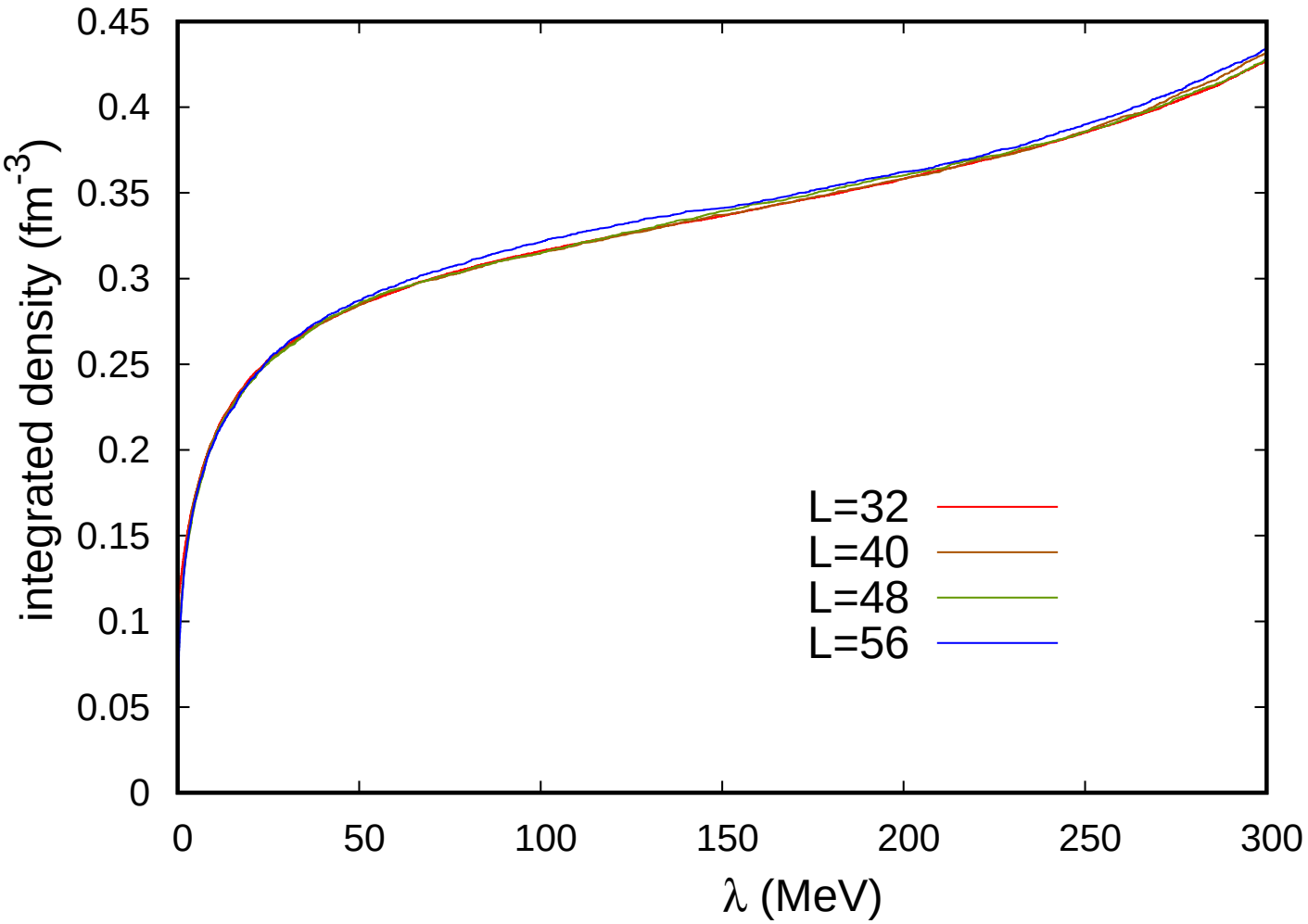
Integrated spectral density

Lattice $N_t = 8$, $T = 1.045\,T_c$



Integrated spectral density including zero modes

Lattice $N_t = 8$, $T = 1.045\,T_c$



Possible explanation of volume dependence

- Above T_c "dilute instanton gas"
  (weakly interacting lumps of charge ± 1)

- n_+ instantons, n_- antiinstantons

  $\Rightarrow$ $|n_+ - n_-|$ chiral zero modes

  $\Rightarrow$ rest of the "zero modes" mix, become nonchiral near zero modes

- $\langle n_+ + n_- \rangle \quad \propto V$

- $\langle |n_+ - n_-| \rangle \quad \propto V^{1/2}$

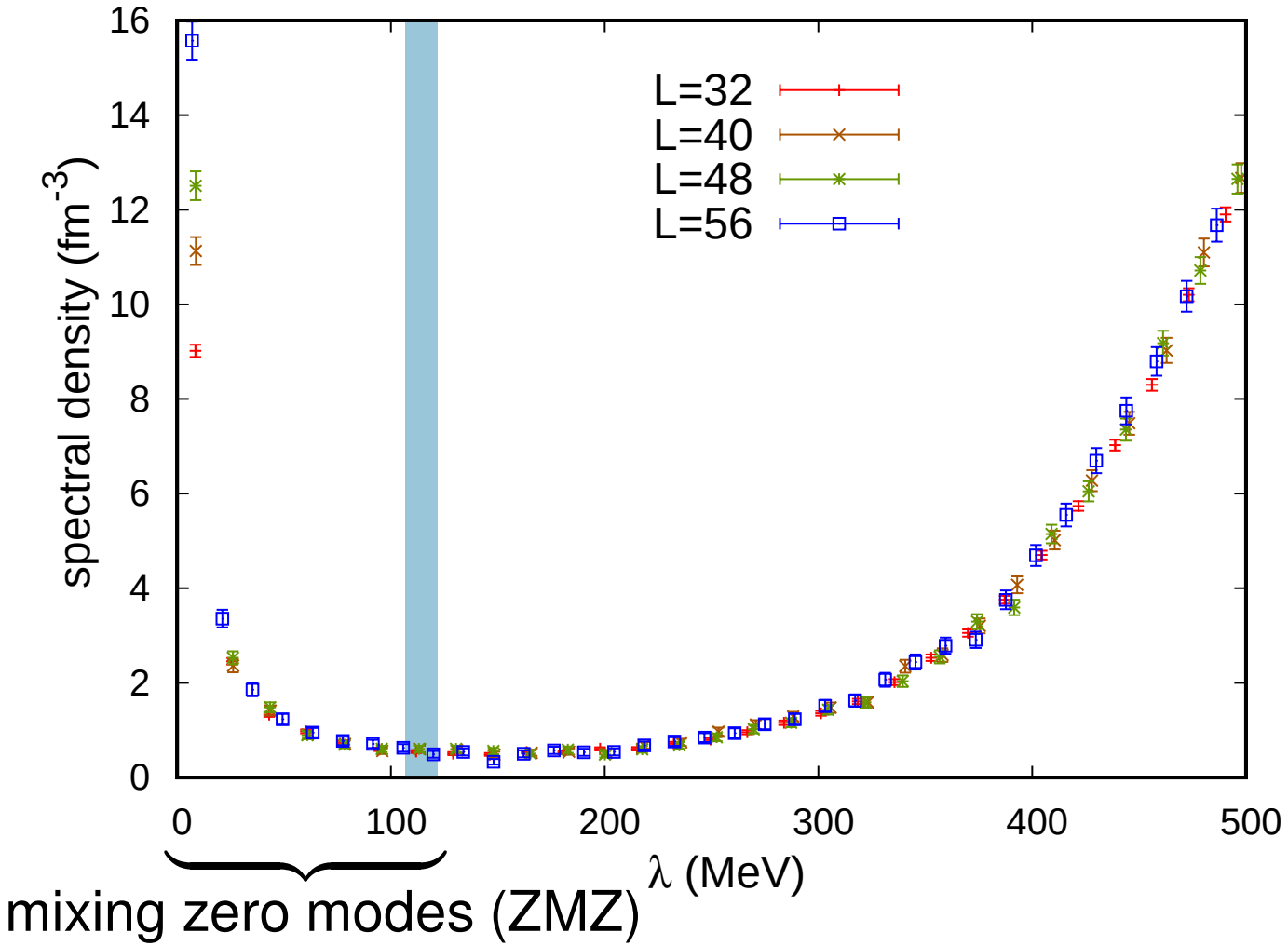
- Spectral peak $\equiv$ near zero modes (ZMZ)?

Assume noninteracting instantons

- n_+, n_- independent identical Poisson distributed
- Poisson d. has one parameter: χV
  topological susceptibility $\times$ volume
- $\langle (n_+ - n_-)^2 \rangle = \langle Q^2 \rangle = \chi V$ definition of top. susceptibility
- $\langle n_+ + n_- \rangle = \chi V$ simple consequence of Poisson distributions
- If the lowest Dirac modes up to λ_{zmz} come from 0-modes

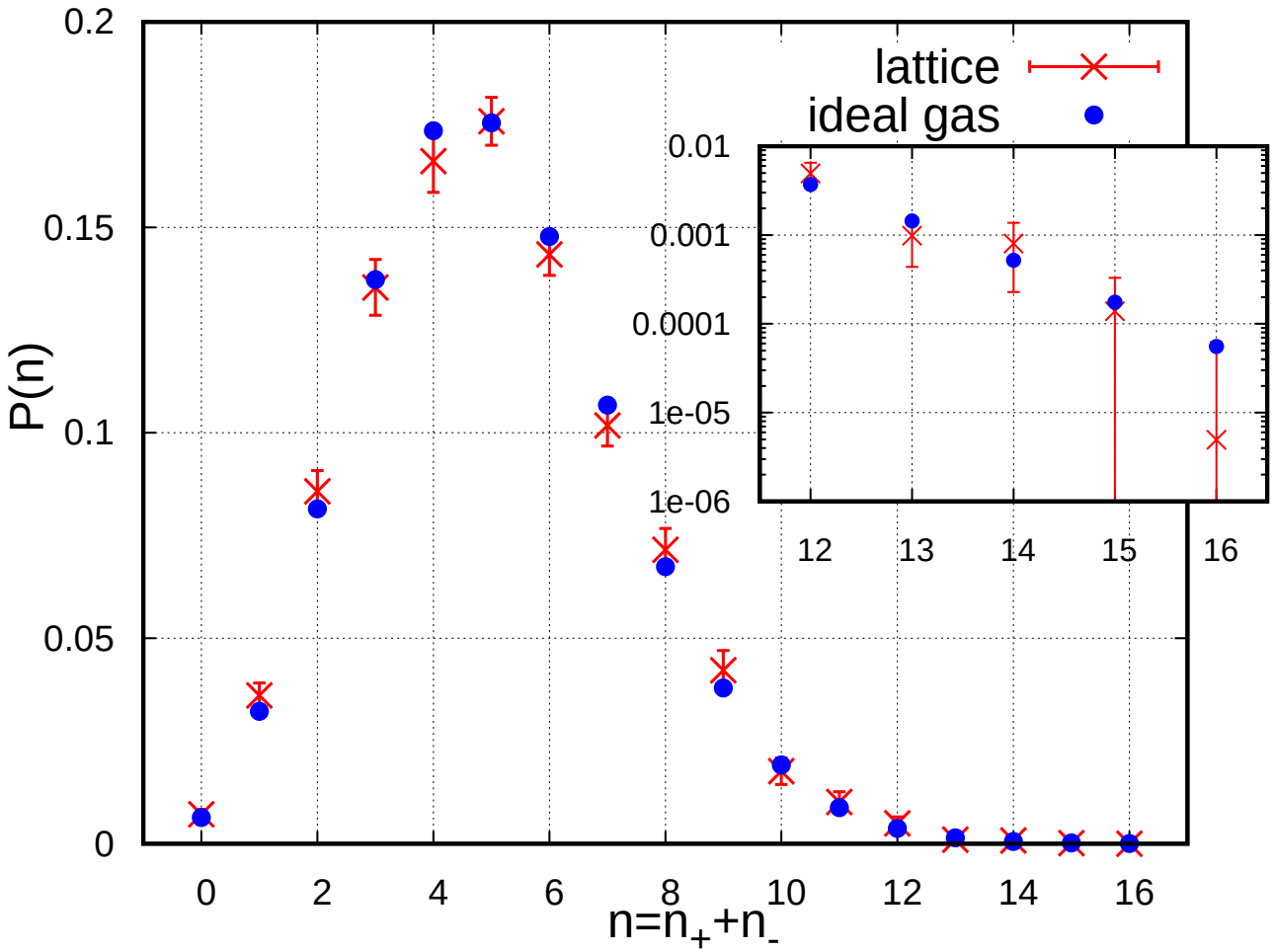
  $\Rightarrow$ then $\langle Q^2 \rangle = \langle n_+ + n_- \rangle$ determines λ_{zmz}

The zero mode zone obtained from $\langle Q^2 \rangle = \langle n_+ + n_- \rangle$



Distribution of the number of eigenvalues $|\lambda| < \lambda_{zmz}$

lattice: quenched, $N_t = 8$, $T = 1.04\,T_c$ vs. $n_+ + n_-$ in ideal gas

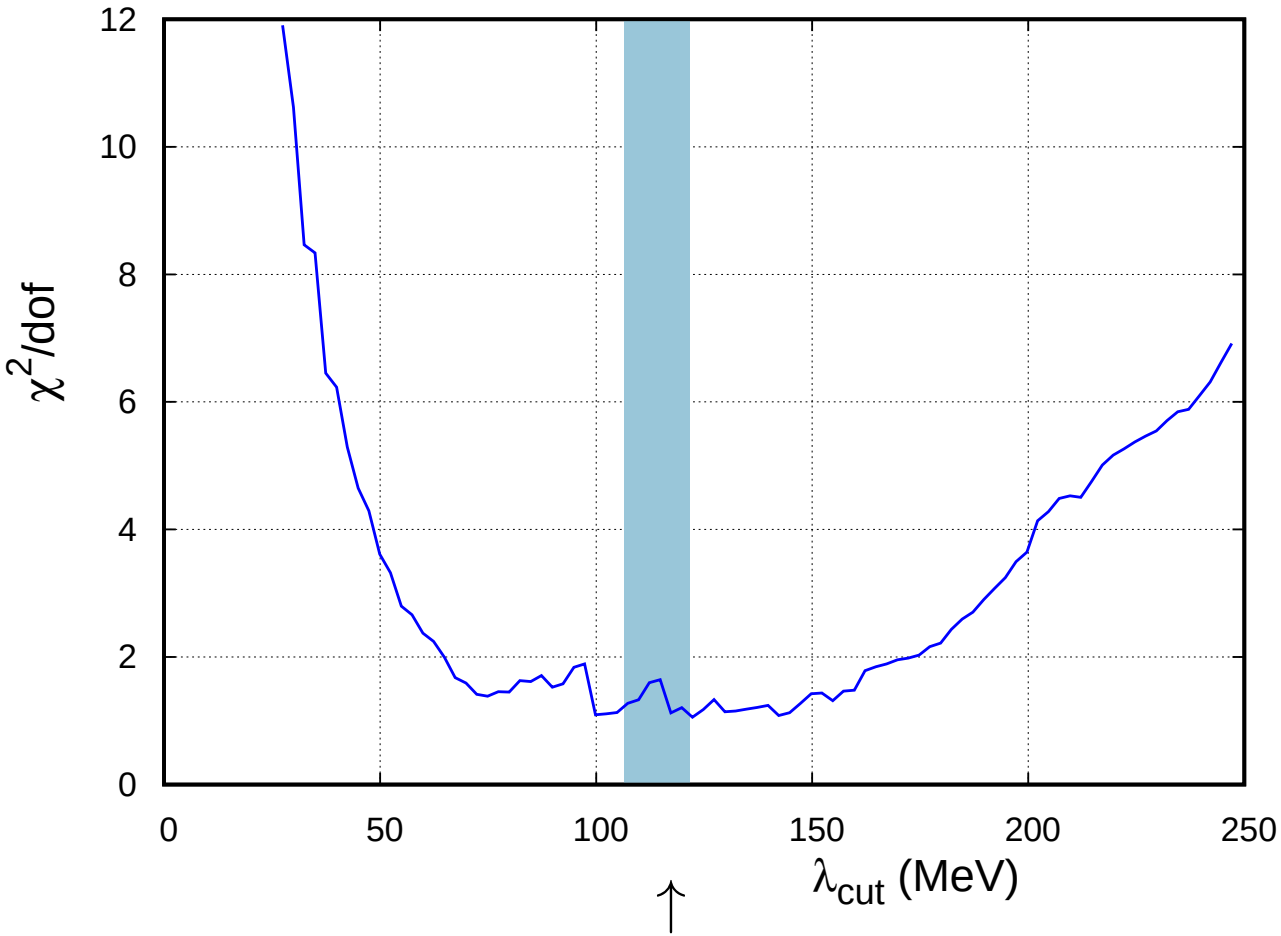


Is λ_{zmz} a special point in the spectrum?

- Distribution of number of $|\lambda| < \lambda_{\text{zmz}}$ eigenvalues Poisson
- Is this only true for λ_{zmz} obtained from $\langle Q^2 \rangle$?

λ_{zmz} is indeed special

Fit distribution of the number of $|\lambda| < \lambda_{\text{cut}}$ eigenvalues with Poisson



λ_{zmz} obtained from $\langle Q^2 \rangle = \langle n_+ + n_- \rangle$

Dynamical quarks

- Quenched $\Rightarrow$ noninteracting instantons
- Light quarks suppress small eigenvalues of Dirac op.

  $\Rightarrow$ instanton interactions?
- Zero modes more suppressed than near zero modes

  $\Rightarrow \langle Q^2 \rangle < \langle n_+ + n_- \rangle$ (I-s and AI-s more likely to come in pairs)
- Nonzero top. susceptibility implies no spectral gap

$$\lim_{\lambda \to 0} \rho(\lambda) \neq 0$$

Conclusions

- Above T_c lowest part of the Dirac spectrum
  $\Longrightarrow$ mixing would be zero modes (ZMZ)
- Spectral peak at zero generic feature,
  not a quenched artifact
- Chiral symmetry restoration might not be trivial,
  depends on the scales m_q and λ_{ZMZ}

  Role of localization in possible fate of the pion: Giordano 2009.00486
- Nonzero topological susceptibility $\Rightarrow$ no spectral gap