

Lattice QCD results for the heavy quark diffusion coefficient

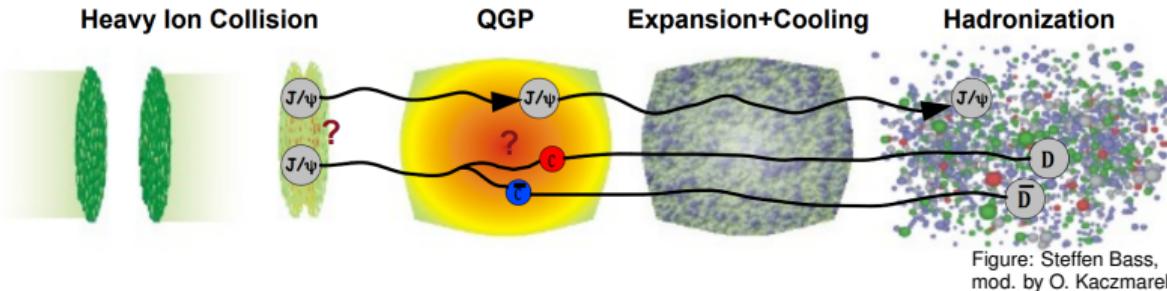
How fast do heavy quarks thermalize in the QGP?

1. Review of quenched QCD at $1.5 T_c$ 10.1103/PhysRevD.103.014511 (2021)

Bielefeld U.: Altenkort, Kaczmarek, Mazur, Shu
TU Darmstadt: Eller, Moore

2. First results from 2 + 1 flavor QCD

Bielefeld U.: Altenkort, Kaczmarek, Shu
Brookhaven NL: Petreczky, Mukherjee
U. of Stavanger: Larsen
(HotQCD collaboration)



Why heavy quark diffusion?

- direct window into strong in-medium QCD force:
- Exp. data (v_2 , R_{AA}) → considerable collective motion! →

$\tau_{\text{heavy}} \stackrel{?}{\approx} \frac{1}{T}$
- Naive hydro: $\tau_{\text{heavy}} \simeq \frac{M}{T} \tau_{\text{light}}$ → $\tau_{\text{light}} \stackrel{?}{\ll} \frac{1}{T}$
- varying results for T -dep. across theoretical models \nearrow Dong, Lee, Rapp (2019)
- input for quarkonium production models \nearrow Brambilla et al. (2021)

Calculate τ_{heavy} from first principles?

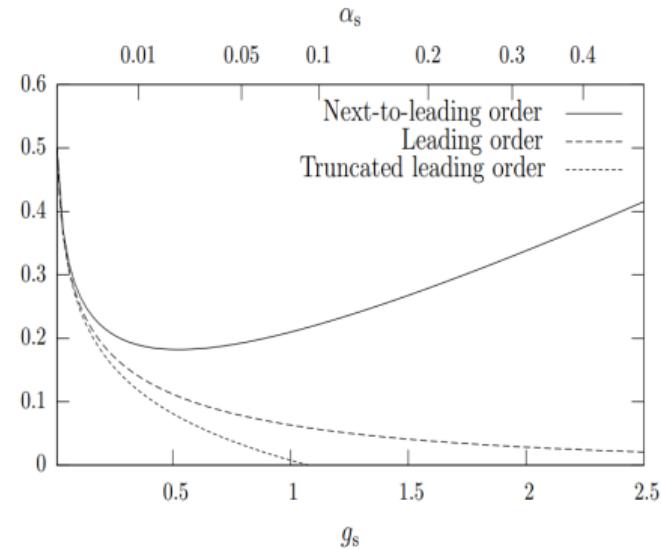
- nonrel. limit $M \gg \pi T \Rightarrow$ Langevin dynamics

\circlearrowleft Moore, Teaney (2005) \circlearrowleft Casalderrey-Solana, Teaney (2006)

\Rightarrow (momentum) diffusion coefficient

$$\tau_{\text{heavy}} = \frac{M}{T} D = \frac{2MT}{\kappa}$$

- Perturbation theory unreliable! \circlearrowleft Caron-Huot, Moore (2008)
- \Rightarrow nonpert. lattice QCD



Diffusion coefficients from the lattice?!

∅ Caron-Huot, Laine, Moore (2009)

∅ Petreczky, Teaney (2005)

- Linear response theory

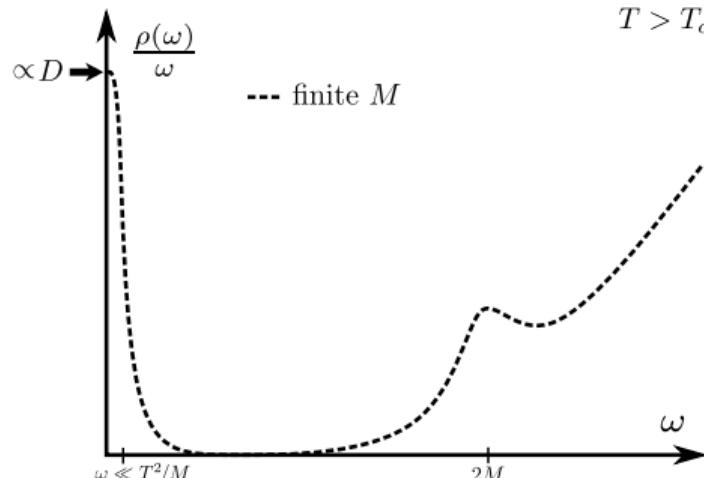
⇒ **in-eq. spectral functions (SPF)**

$$D \sim \lim_{\omega \rightarrow 0} \frac{\rho^{ii}(\omega)}{\omega} \quad \text{with} \quad \rho^{ii}(\omega) = \int_{t,x} e^{i\omega t} \left\langle \frac{1}{2} [\hat{\mathcal{J}}^i(x,t), \hat{\mathcal{J}}^i(0,0)] \right\rangle$$

► HQ vector current

- reconstruct from **Euclidean correlators**:

$$G(\tau) = \int_0^\infty d\omega \rho(\omega) \frac{\cosh(\omega(\tau - \frac{\beta}{2}))}{\sinh(\omega \frac{\beta}{2})}$$



$T > T_c$

1. instead of D , consider κ (encoded in the tail)

2. use HQET:
expand in $1/M$,
replace $\hat{\mathcal{J}}^i$ with LO version,
...

⇒ $G(\tau)$ = color-electric two-point function (force-force correlator)

$$\Rightarrow \kappa = \lim_{\omega \rightarrow 0} 2T \frac{\rho(\omega)}{\omega}$$

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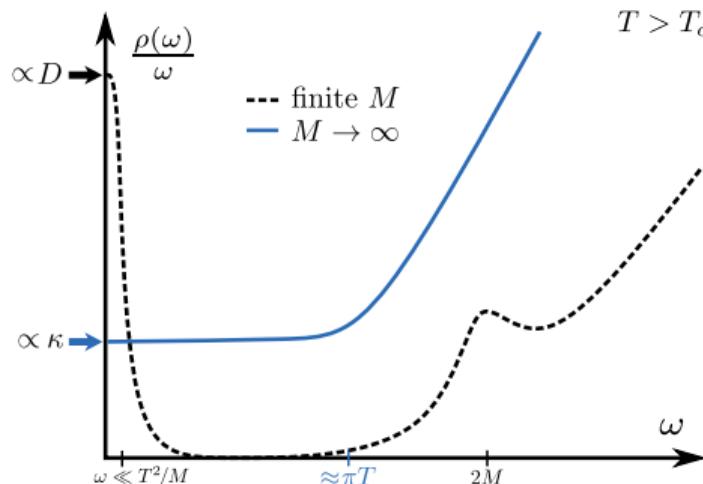
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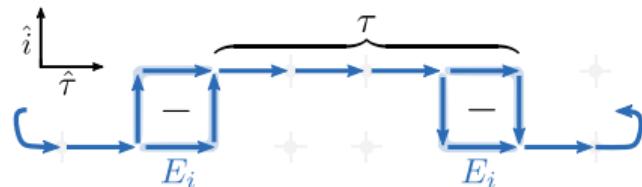
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Gluonic color-electric correlator ✓ Caron-Huot, Laine, Moore (2009)

$$G(\tau) \equiv \frac{1}{3} \sum_{i=1}^3 \frac{-\langle \text{Re tr } U(\beta, \tau) g E_i(\tau) U(\tau, 0) g E_i(0) \rangle}{\langle \text{Re tr } U(\beta, 0) \rangle}$$



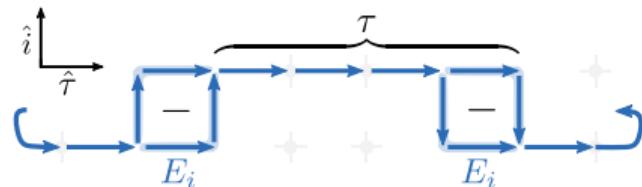
- Weak coupling, small τ : $G(\tau) \sim \tau^{-4}$

Drawback of $M \rightarrow \infty$

- UV gauge fluctuations dominate for large τ
- large τ most sensitive to $\omega \rightarrow 0$
⇒ need noise reduction!

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Noise reduction via gradient flow ✓ Lüscher 2010

- supports nonlocal actions! (e.g. 2+1 flavor HISQ)
- new gaugefield parameter: “flow time” τ_F

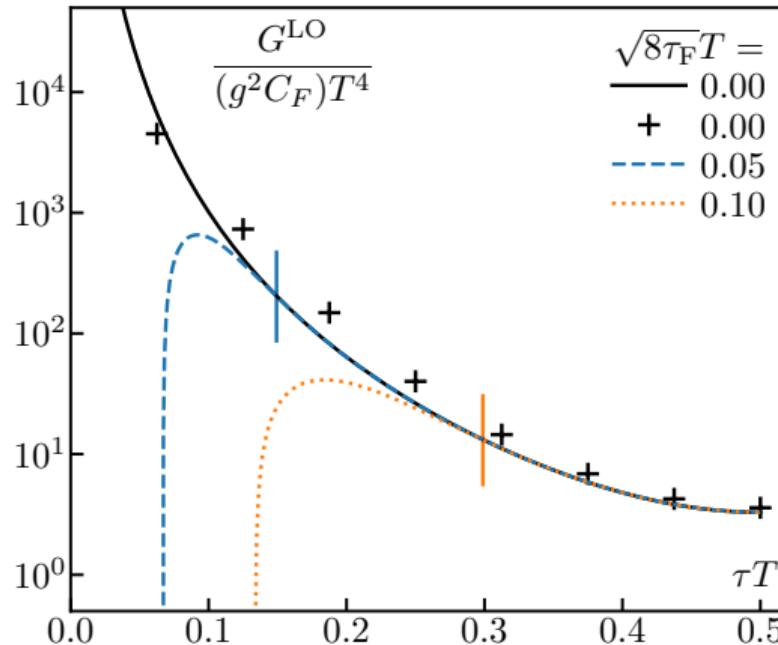
Flow = smooth regulator:

- links continuously smeared, width $\simeq \sqrt{8\tau_F}$, “flow radius”
- lattice renorm. artifacts suppressed if $\sqrt{8\tau_F} \gtrsim a$
- $G(\tau)$ free of distortion if $\sqrt{8\tau_F} \lesssim \tau/3$ (weak-coupling LO ✓ Eller, Moore 2018)

Idea:

1. step-wise smearing + $G(\tau)$ measurement
2. at each τ_F step, extrapolate $a \rightarrow 0$
3. extrapolate $\tau_F \rightarrow 0$, only consider $a < \sqrt{8\tau_F} < \tau/3$

Weak coupling EE correlator (LO) + Wilson flow



Eller, Moore 2018,

Eller, Moore, LA et al. 2021

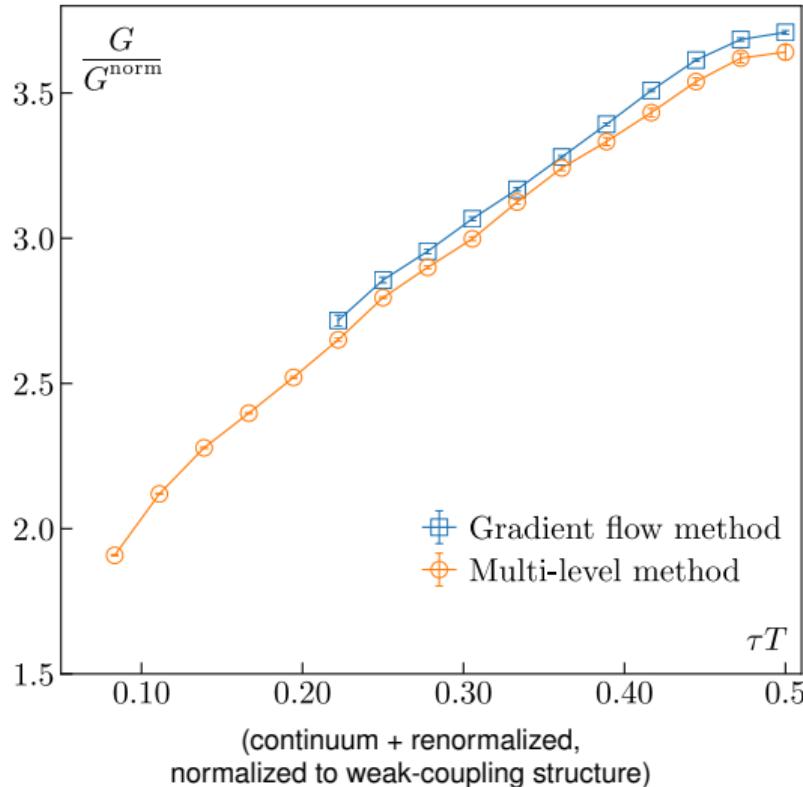
Flow limit = lower bound for τ

- correlator deviates < 1% for $\tau \gtrsim 3\sqrt{8\tau_F}$ (vertical lines)

Enhance nonpert. lattice data:

- normalize to weak-coupling structure $\equiv G^{\text{norm}}$
- remove tree-level discretization errors

Quenched, $1.5T_c$, Wilson action
 $a \rightarrow 0, \tau_F \rightarrow 0$



Comparison to previous method

- shape consistent with Multi-level results
- ∅ Francis et al. 2015 , ∅ Christensen, Laine 2016
- overall shift?
 - ⇒ nonperturbative renormalization
 - ⇒ better statistics

Lattice & flow setup

$N_\sigma^3 \times N_\tau$	a [fm]	10000 conf. each
$80^3 \times 20$	0.0213	■ separation:
$96^3 \times 24$	0.0176	500 sweeps of (1 HB + 4 OR)
$120^3 \times 30$	0.0139	■ $\mathcal{O}(a^2)$ -improved "Zeuthen flow"
$144^3 \times 36$	0.0116	■ 3rd-order RK with adaptive stepsize

Spectral reconstruction = integral inversion problem

- $G(\tau) = \int_0^\infty d\omega \rho(\omega) K(\omega, \tau),$

$$\kappa = \lim_{\omega \rightarrow 0} 2T \frac{\rho(\omega)}{\omega}$$

Strategy: use spectral function models

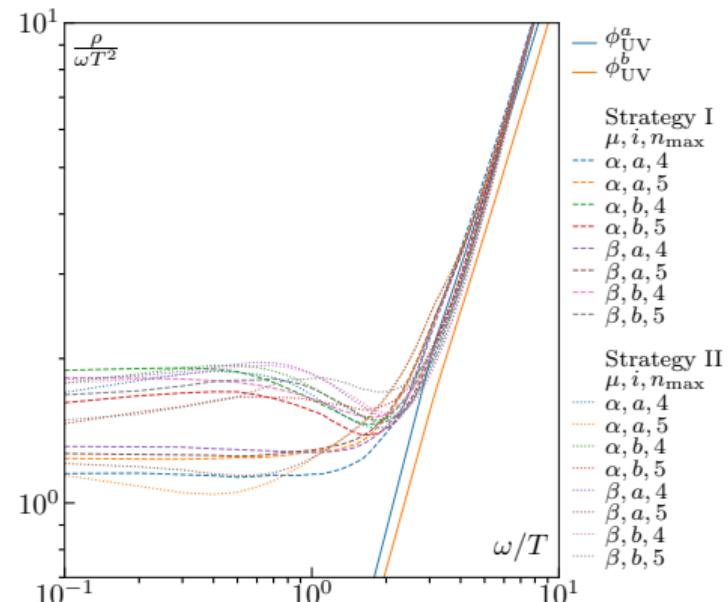
- $\rho_{\text{model}}(\omega) \equiv I(\omega) \sqrt{[\phi_{\text{IR}}(\omega)]^2 + [\phi_{\text{UV}}(\omega)]^2}$

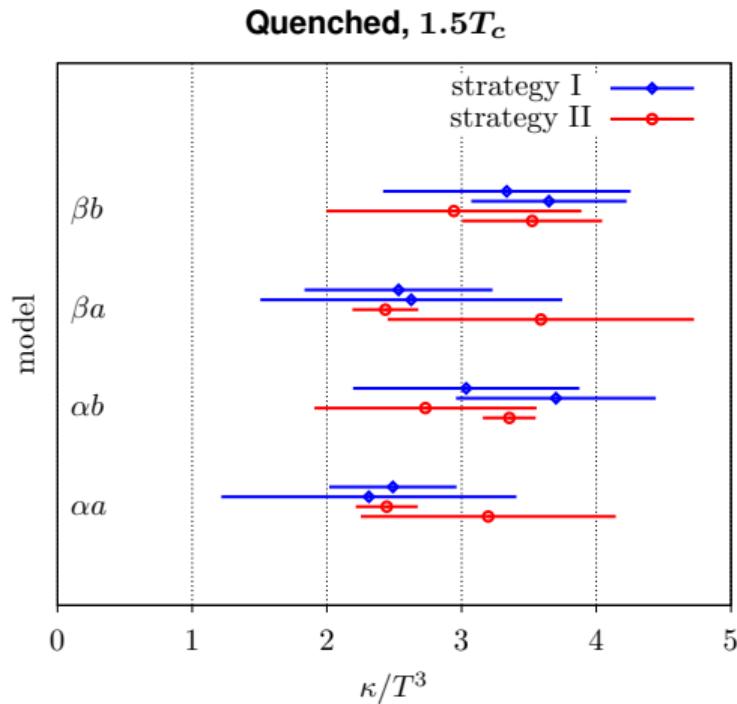
with known $\phi_{\text{IR}}(\omega) \equiv \frac{\kappa}{2T}\omega$, $\phi_{\text{UV}}(\omega) \sim \omega^3$, ...

and various **interpolations** $I(w)$

⇒ obtain κ/T^3 by fitting

$$\chi^2 \equiv \sum_\tau \left[\frac{G(\tau) - G_{\text{model}}(\tau)}{\delta G(\tau)} \right]^2$$





Final estimate:

$$\kappa/T^3 = 2.31 \dots 3.70$$

$$\Leftrightarrow 2\pi TD = 3.40 \dots 5.44$$

$$\Leftrightarrow \tau_{\text{heavy}} = (1.63 \dots 2.61) \left(\frac{T_c}{T}\right)^2 \left(\frac{M}{1.5 \text{GeV}}\right) \text{fm/c}$$

2+1 flavor HISQ, $T \approx 200 \dots 350$ MeV, $m_\pi \approx 310$ MeV

- no $a \rightarrow 0$ and $\tau_F \rightarrow 0$ yet

Intermediate strategy

1. individually fix τ_F relative to τ : $\frac{\sqrt{8\tau_F}}{\tau} \equiv \text{const.} < \frac{1}{3}$

2. use flow-dep. tree-level improvement

↗ Stendebach, Moore 2022 (unpublished)

3. fit simple models to $\tau T \geq 0.35$

↗ nonzero τ_F & a effects = small corrections

↗ tiny add. systematic error for κ/T^3

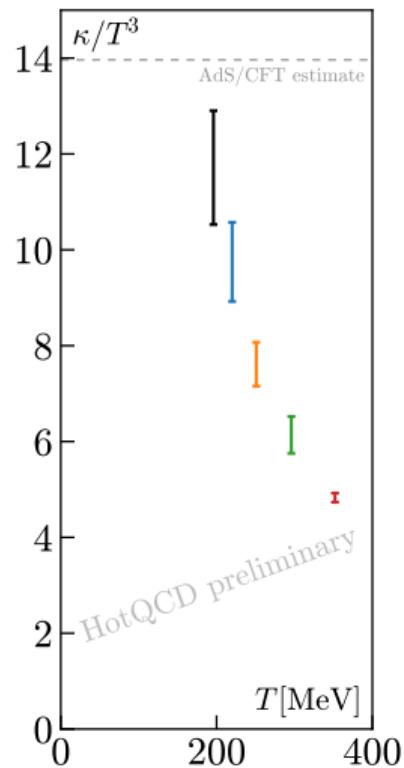
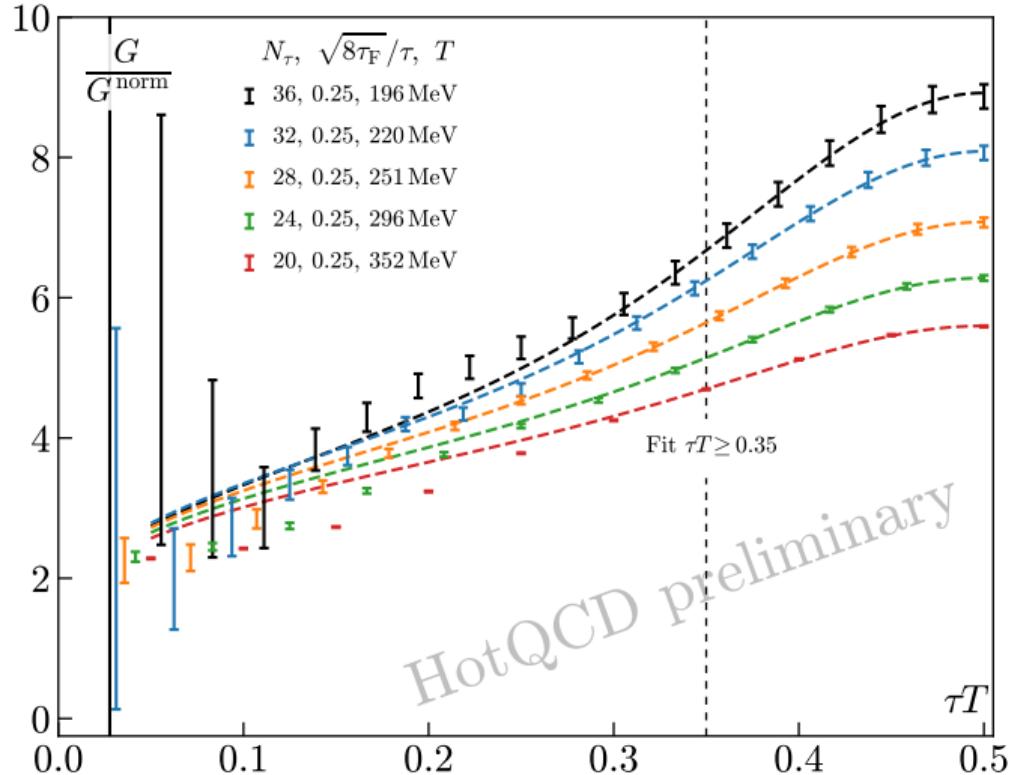
compared to SPF model systematics
(verified through quenched data → backup)

Lattice setup (planned)

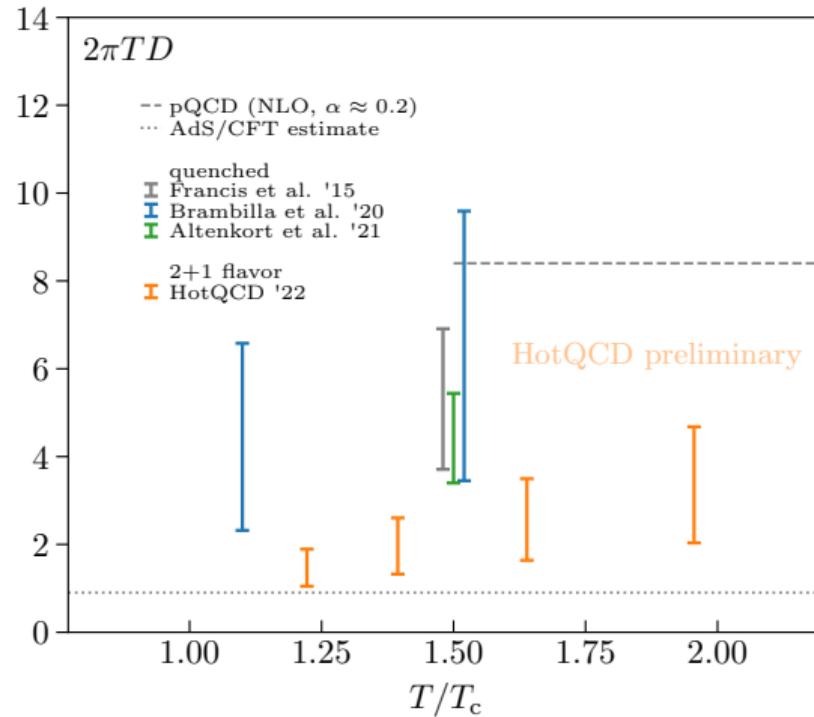
m_l	T [MeV]	$N_\sigma^3 \times N_\tau$	a [fm]
$m_s/5$	195	$96^3 \times 36$	0.028
		$64^3 \times 24$	0.042
		$64^3 \times 20$	0.051
	220	$96^3 \times 32$	0.028
		$64^3 \times 24$	0.037
		$64^3 \times 20$	0.045
	251	$96^3 \times 28$	0.028
		$64^3 \times 24$	0.033
		$64^3 \times 20$	0.039
$m_s/27$	296	$96^3 \times 24$	0.028
		$64^3 \times 22$	0.031
		$64^3 \times 20$	0.034
≤ 195		$64^3 \times 24$	
		...	

2 + 1 flavor

Dashed lines: fit with model $\rho(\omega) = \sqrt{[\kappa\omega/2T]^2 + [c\phi_{\text{UV}}(\mu)]^2}$, $\mu = \sqrt{[2\pi T]^2 + \omega^2}$



Comparison to other results



∅ pQCD: Caron-Huot, Moore (2008)
∅ AdS/CFT: Casalderrey-Solana, Teaney (2006)

■ Reminder:

$$\frac{4\pi}{\kappa/T^3} = 2\pi TD \sim \frac{T^2}{M} \tau_{\text{heavy}}$$

Quenched, $1.5 T_c$: $\mathcal{O}(T/M)$ correction

- $\kappa \simeq \kappa_E + \frac{2}{3} \langle \mathbf{v}^2 \rangle \kappa_B$ ↗ Bouttefoux, Laine (2020) ↗ Laine (2021)

⇒ color-magnetic correlator G_B

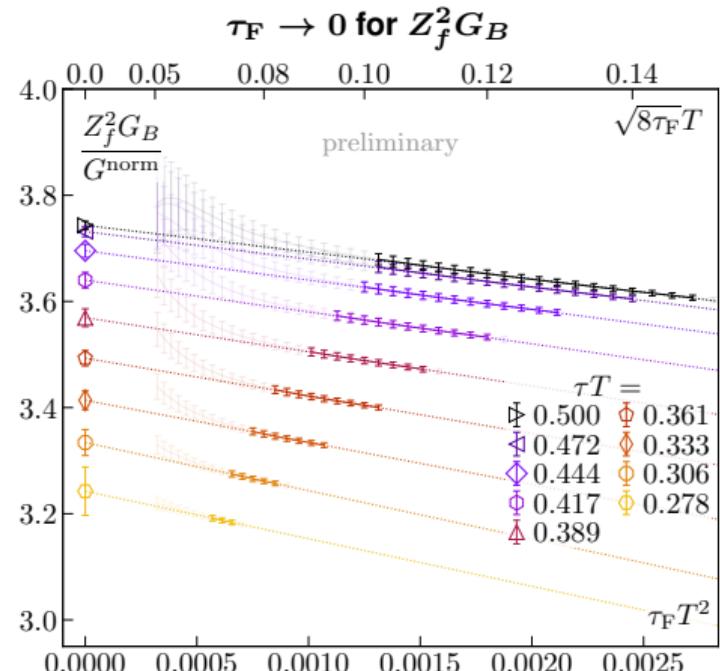
- Problem: anomalous dimension
→ logarithm in $G_B(\tau_F)$

- Solution: consider $Z_B^2(Z_K^2 Z_f^2 G_B)_{\tau_F \rightarrow 0}$
 - cancel scale-dependence
 - match flow to \overline{MS} scheme
 - \overline{MS} renorm. factor

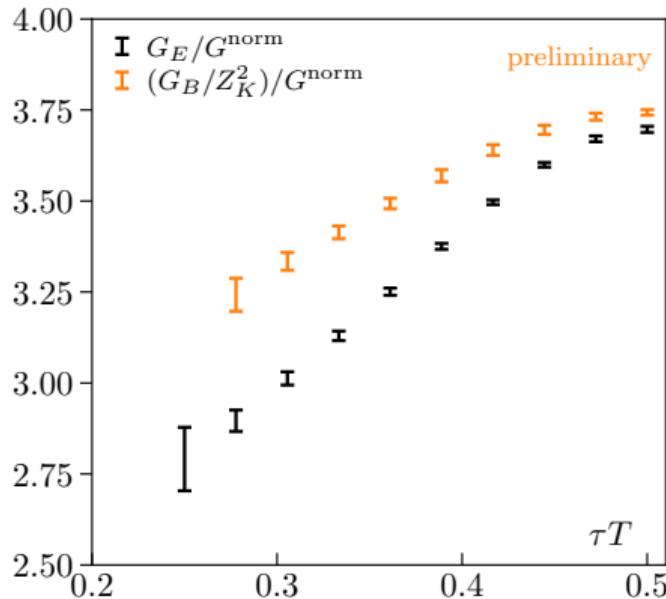
⇒ $Z_f^2(\tau_F, g_{\tau_F}^2)$ obtained by integrating RG

equation, $(g_{\tau_F}^2)$ measured at $T = 0$)

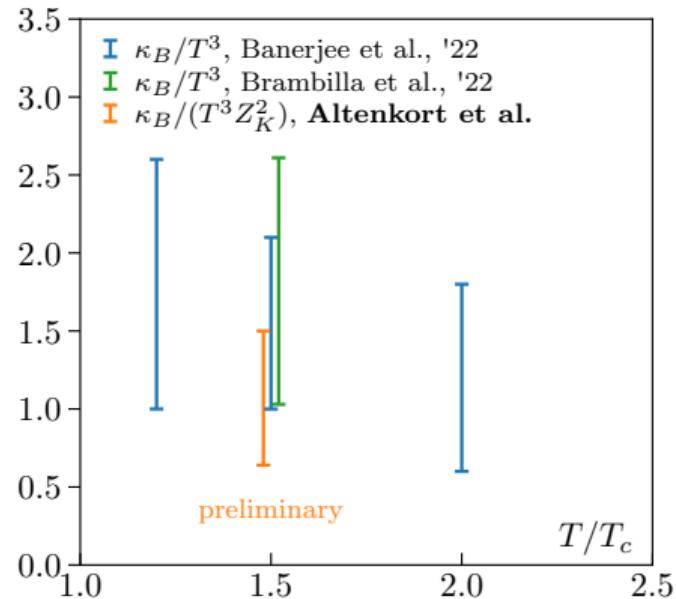
expect $Z_K^2 \sim 1$, calculation in progress



EE vs BB correlator ($a \rightarrow 0$, $\tau_F \rightarrow 0$)

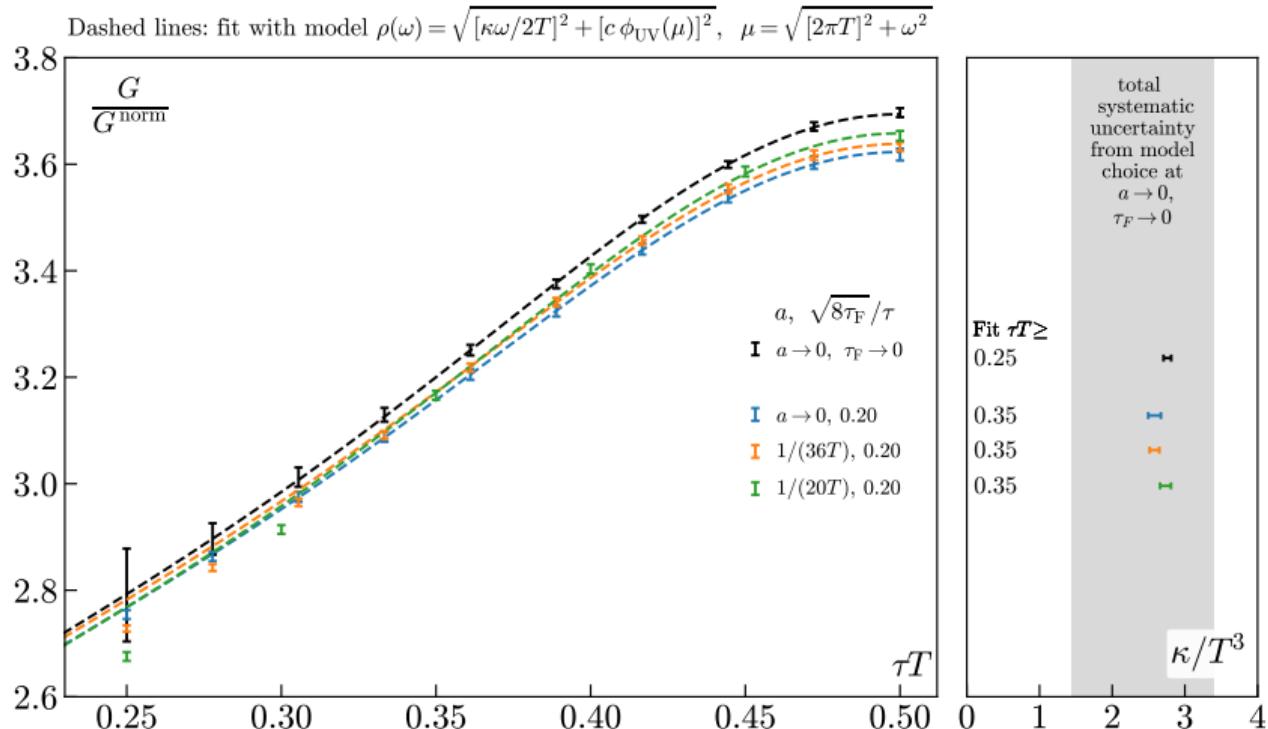


Comparison to other works (quenched)



Backup

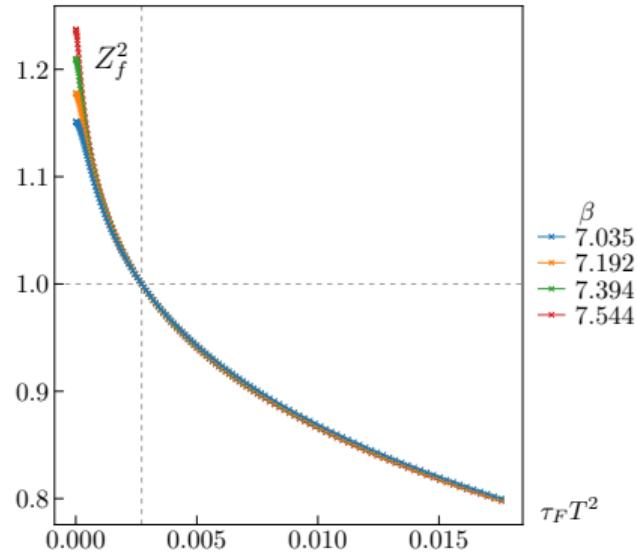
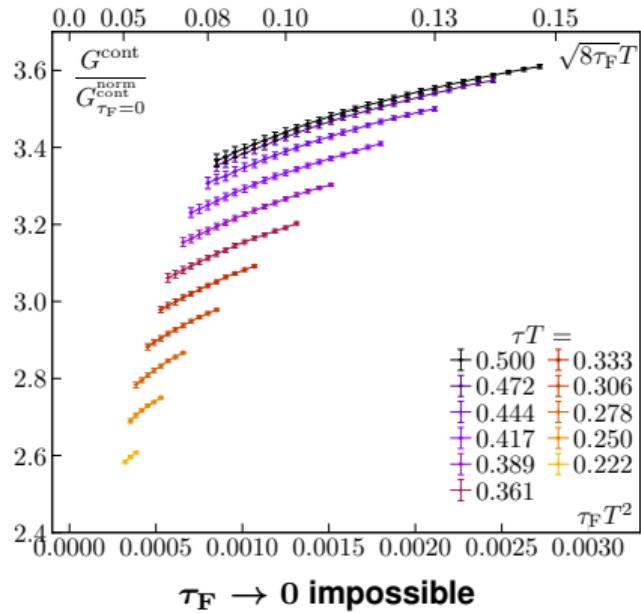
Quenched, $1.5T_c$: systematics of simple model fits



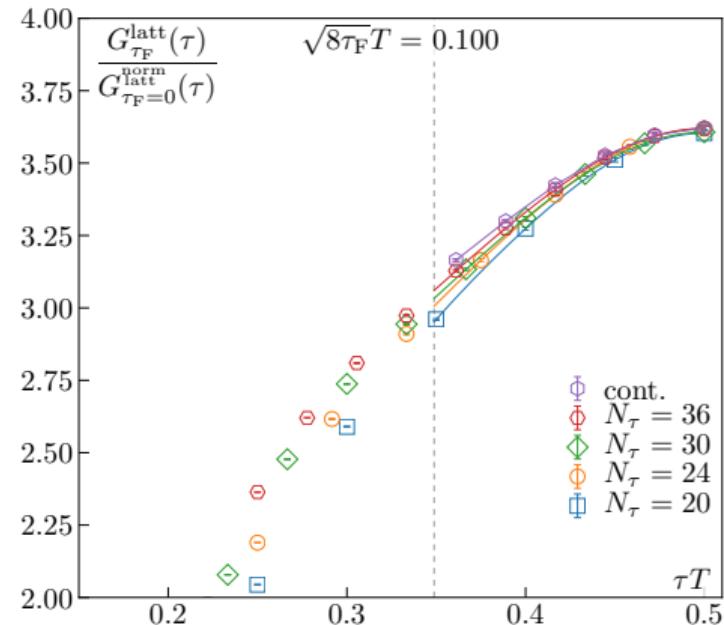
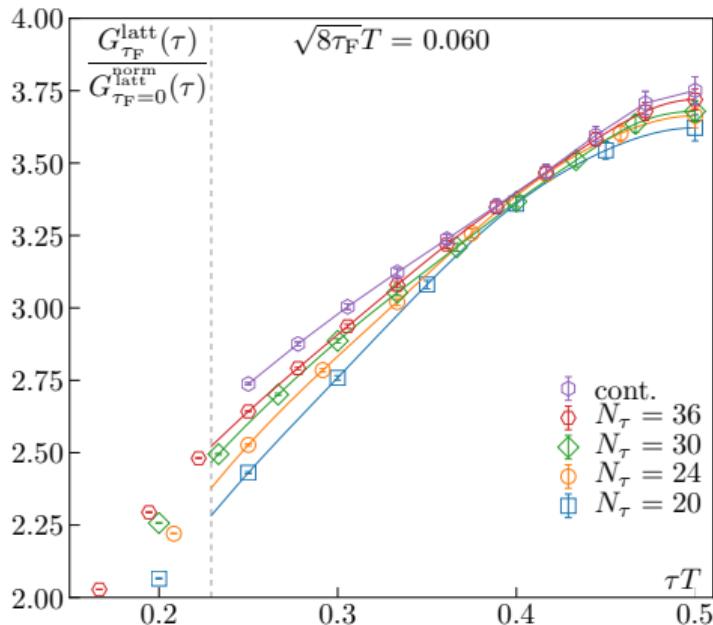
Conclusions

- shape of correlator preserved at fixed small $\sqrt{8\tau_F}/\tau$
- for large τ also preserved at finite a !
- ⇒ sufficient to still constrain κ/T^3 (using simple models)

“bare” BB correlator



Quenched, $1.5T_c$, EE correlator: lattice spacing effects

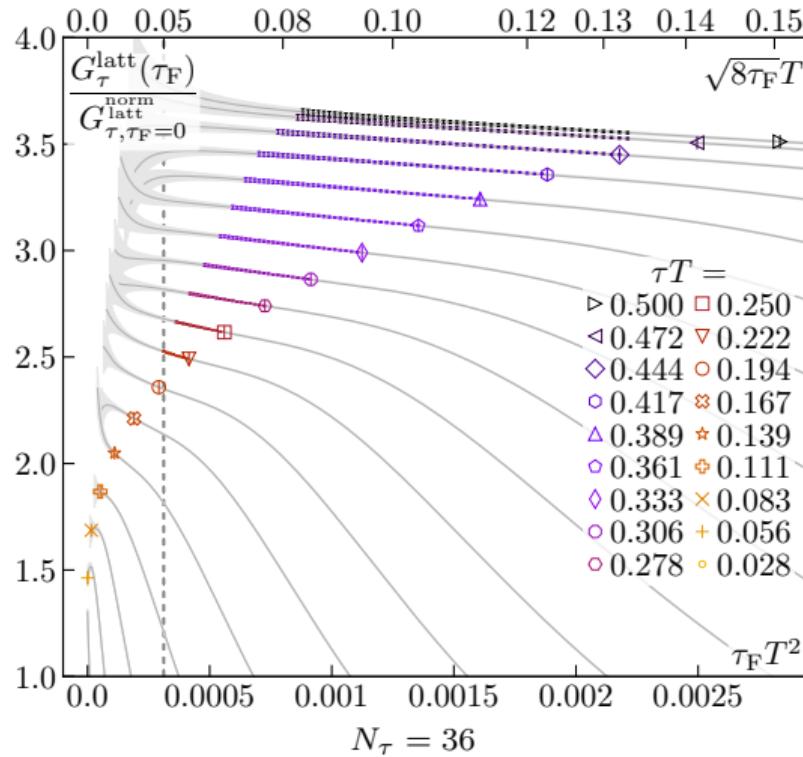


■ dashed lines at $\tau \approx 3\sqrt{8\tau_F}$

⇒ more flow = higher precision,
but smaller window of noncontaminated data

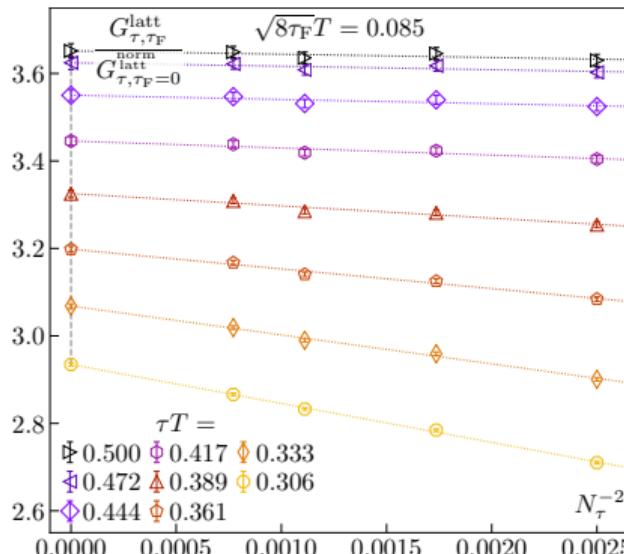
■ interpolation in τ through cubic splines (no smoothing)

EE correlator as a function of flow time (quenched, $1.5T_c$)



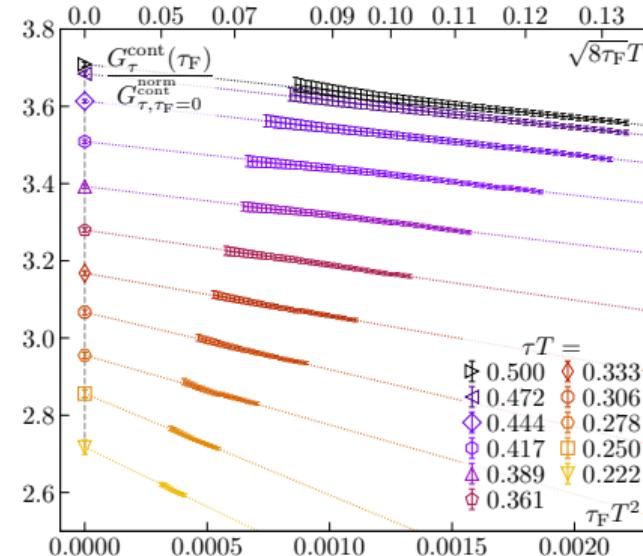
- inside extrapolation window: single colorful data points. outside: data points connected via grey lines.
- markers at $\sqrt{8\tau_F} \approx \tau/3$
- dashed line: minimum flow such that $\sqrt{8\tau_F} \gtrsim a$ for our coarsest lattice
- small τ_F : strong flow dependence (suppress noise and renorm. artifacts)
- intermediate τ_F : minor dependence (for large τ)

■ 1. Continuum extrapolation (linear in a^2)



- ansatz motivated by gauge action discretization
- taken separately for each flow time
- take continuum limit first to control a^2/τ_F -type corrections

■ 2. Flow-time-to-zero extrapolation (linear in τ_F)



- ansatz motivated by NLO pert. theory \nearrow Eller 2021
- flow time window depends on:
 - signal-to-noise
 - $\sqrt{8\tau_F} \gtrsim a$ (suppression of latt. artifacts)
 - $\sqrt{8\tau_F} \lesssim \tau/3$ (flow limit)