New results for thermal interquark bottomonium potentials using NRQCD from the HAL QCD method

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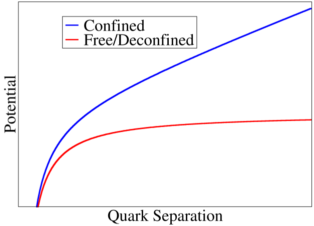
Building upon [1]

[1]TS *et al.* Thermal interquark potentials for bottomonium using NRQCD from the HAL QCD method, PoS LATTICE2021 (2022) 569 [hep-lat/2112.09092]

Confinement

- QCD is a confining theory
- At low temperatures, quarks only exist in hadrons; at high temperatures, quarks can exist unbound in the quark gluon plasma
- Phase transition
- The shape of the interquark potential at large separation can indicate the phase

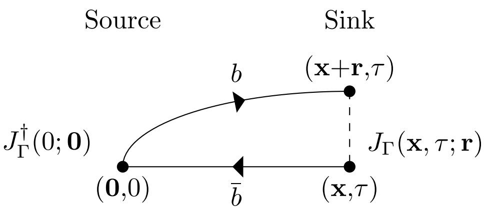


NRQCD

- Effective field theory from the expansion in the heavy quark velocity, v, about $v = 0$ (*i.e.* first approximation beyond static quarks)
- No backwards mover
- FASTSUM $N_f = 2 + 1$ Generation 2 ensembles, anisotropic lattice. More details on the lattice set-up and our NRQCD formalism available here [2]

[2] G. Aarts *et al.*, The bottomonium spectrum at finite temperature from $N_f = 2 + 1$ lattice QCD, JHEP 07 (2014) 097 [1402.6210].

Point-split correlation function



$$J_\Gamma(x;\mathbf{r}) = \bar{q}(x)\Gamma U(x, x+\mathbf{r})q(x+\mathbf{r}), \tag{1}$$

$$C(\mathbf{r},\tau) = \sum_{\mathbf{x}} \langle J_\Gamma(\mathbf{x},\tau;\mathbf{r}) J_\Gamma^\dagger(0;\mathbf{0})\rangle. \tag{2}$$

$\mathcal{O}(\mathrm{Vol}^2_{\mathrm{spatial}})$

Point-split correlation function

If we define a momentum space representation of the quark propagators through the Fourier transform

$$D^{-1}(\mathbf{y}, \tau; \mathbf{0}, 0) = \frac{1}{V} \sum_{\mathbf{q}} \tilde{D}^{-1}(\mathbf{q}) e^{i\mathbf{y}\cdot\mathbf{q}}. \tag{3}$$

Then we can, in a similar fashion to in position space, calculate a correlation function $\tilde{C}(\mathbf{p}, \tau)$.

Point-split correlation function

1D fast Fourier transform turns

$$\sum_{x_1}^{N}\sum_{x_2}^{N} f(x_1, x_2) \tag{4}$$

from $\mathcal{O}(N^2)$ to $\mathcal{O}(N\log(N))$.

Point-split correlation function

$$C_\Gamma(\mathbf{r},\tau) = \sum_{\mathbf{x}} \langle D^{-1}(\mathbf{x}+\mathbf{r},\tau;\mathbf{0},0)\Gamma D^{-1}(\mathbf{0},0;\mathbf{x},\tau)\Gamma^\dagger\rangle$$

$$= \sum_{\mathbf{x}} \langle D^{-1}(\mathbf{x}+\mathbf{r},\tau;\mathbf{0},0)\Gamma\gamma_5 \left(D^{-1}(\mathbf{x},\tau;\mathbf{0},0)\right)^\dagger \gamma_5\Gamma^\dagger\rangle$$

$$= \frac{1}{V}\sum_{\mathbf{p}}\sum_{\mathbf{x}} \langle \tilde{D}^{-1}(\mathbf{p},\tau;\mathbf{0},0)e^{i\mathbf{p}\cdot(\mathbf{x}+\mathbf{r})}\Gamma\gamma_5 \left(\tilde{D}^{-1}(\mathbf{p},\tau;\mathbf{0},0)e^{i\mathbf{p}\cdot\mathbf{x}}\right)^\dagger \gamma_5\Gamma^\dagger\rangle$$

$$= \frac{1}{V}\sum_{\mathbf{p}}\sum_{\mathbf{x}} \langle \tilde{D}^{-1}(\mathbf{p},\tau;\mathbf{0},0)e^{i\mathbf{p}\cdot\mathbf{x}}\Gamma\gamma_5 \tilde{D}^{-1}(-\mathbf{p},\tau;\mathbf{0},0)e^{-i\mathbf{p}\cdot\mathbf{x}}\gamma_5\Gamma^\dagger\rangle e^{i\mathbf{p}\cdot\mathbf{r}}$$

$$C_\Gamma(\mathbf{r},\tau) = \frac{1}{V}\sum_{\mathbf{p}} \tilde{C}_\Gamma(\mathbf{p},\tau)e^{i\mathbf{p}\cdot\mathbf{r}} \tag{5}$$

$\mathcal{O}(\mathrm{Vol}_{\mathrm{spatial}}\log(\mathrm{Vol}_{\mathrm{spatial}}))$

HAL QCD method

The local-extended correlator also has a representation as the sum over eigenstates of the Hamiltonian [3]

$$C(\mathbf{r},\tau) = \sum_j \frac{\psi_j^*(\mathbf{0})\psi_j(\mathbf{r})}{2E_j} e^{-E_j\tau}, \tag{6}$$

which can be written as

$$C(\mathbf{r},\tau) = \sum_j \Psi_j(\mathbf{r}) e^{-E_j\tau}, \tag{7}$$

by introducing an unnormalised wavefunction $\Psi_j(\mathbf{r})$ for brevity.

[3] N. Ishii, S. Aoki and T. Hatsuda, The Nuclear Force from Lattice QCD, Phys. Rev. Lett. 99 (2007) 022001 [nucl-th/0611096].

HAL QCD method

From the Schrödinger equation for S-wave states

$$\left(-\frac{\nabla_r^2}{2\mu} + V_\Gamma\left(r\right)\right)\Psi_j\left(r\right) = E_j\Psi_j\left(r\right), \tag{8}$$

we can take the time derivative to introduce our correlator through

$$\begin{aligned}\frac{\partial C_\Gamma(\mathbf{r},\tau)}{\partial\tau} = -\sum_j E_j\Psi_j(\mathbf{r})e^{-E_j\tau} &= \sum_j\left(-\frac{\nabla_r^2}{2\mu} + V_\Gamma\left(r\right)\right)\Psi_j\left(r\right)e^{-E_j\tau} \\ &= \left(-\frac{\nabla_r^2}{2\mu} + V_\Gamma\left(r\right)\right)C_\Gamma(\mathbf{r},\tau)\end{aligned} \tag{9}$$

HAL QCD method

We can then rearrange this to solve for the potential

$$V_\Gamma(r) = \frac{1}{C_\Gamma(\mathbf{r},\tau)}\left(\frac{\nabla_r^2}{2\mu} - \frac{\partial}{\partial\tau}\right)C_\Gamma(\mathbf{r},\tau) \tag{10}$$

$$V_\mathrm{C} = \frac{1}{4}V_\mathrm{PS} + \frac{3}{4}V_\mathrm{V} \tag{11}$$

Time independence

$$V_\Gamma(r) = \frac{1}{C_\Gamma(\mathbf{r},\tau)}\left(\frac{\nabla_r^2}{2\mu} - \frac{\partial}{\partial\tau}\right)C_\Gamma(\mathbf{r},\tau)$$

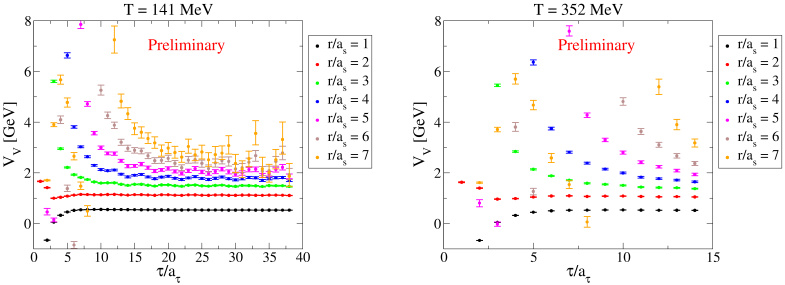


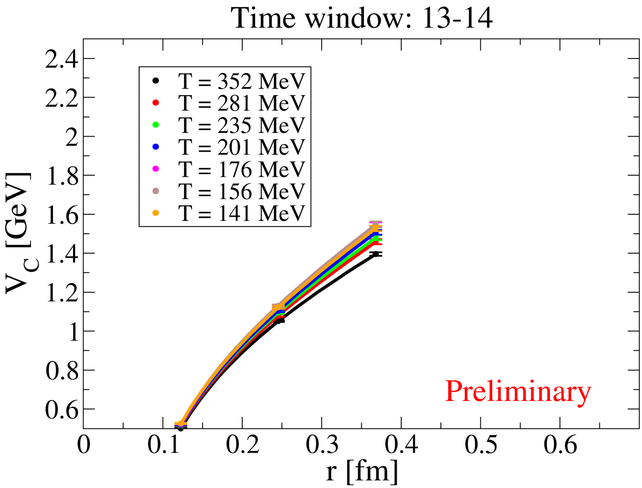
Figure 1: Time dependence in the potential restricting the range of r that we can consider valid.

Time windows

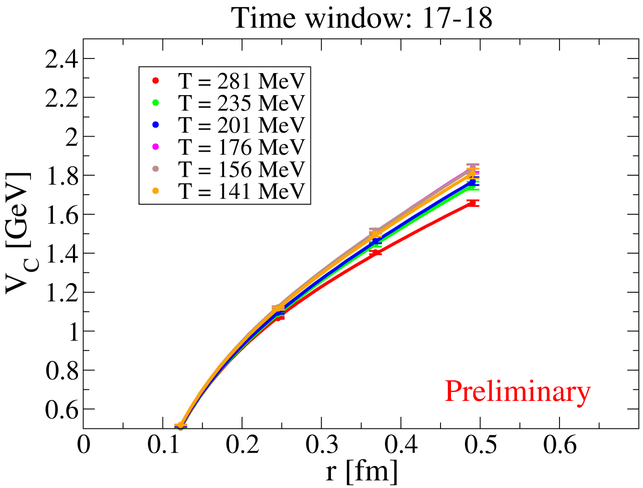
Time window $[a_\tau]$	r range $[a_s]$	r range [fm]	Temperatures [MeV]
13-14	1-3	0.12-0.37	352-141
17-18	1-4	0.12-0.49	281-141
19-22	1-5	0.12-0.61	235-141
21-26	1-5	0.12-0.61	201-141
24-30	1-6	0.12-0.74	176-141
24-33	1-6	0.12-0.74	156-141
24-38	1-6	0.12-0.74	141

Table 1: Range of displacements and temperatures allowed to best approximate time independence in $V(r,\tau)$. Note that $\mathrm{T_{pc}} = 181$ MeV.

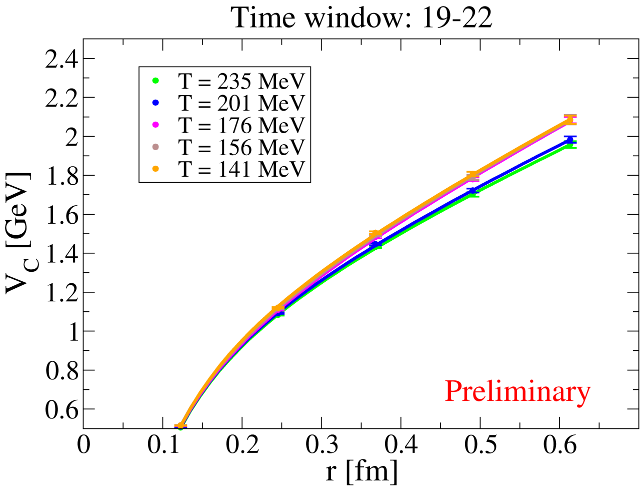
Central potential



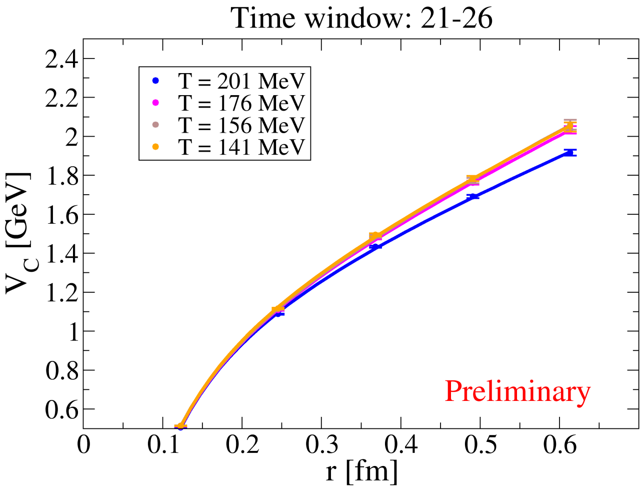
Central potential



Central potential



Central potential



Next steps

- Static quark and relativistic charmonium on the same configurations [4]
- Off-axis separations:
  $C(x, 0, 0, \tau)$, $C(0, y, 0, \tau)$, $C(0, 0, z, \tau) \to C(\mathbf{r}, \tau)$
- Mass spectra from $V(r)$ [5]

[4] C. Allton *et al.* The Charmonium Potential at Non-Zero Temperature [hep-lap/1505.0661]

[5] TS *et al.* A comparison of spectral reconstruction methods applied to non-zero temperature NRQCD meson correlation functions, EPJ Web Conf. 258 (2022) 05011 [hep-lap/1505.0661]