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Complex potential at T>0 from fine lattices

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+ HotQCD COLLABORATION

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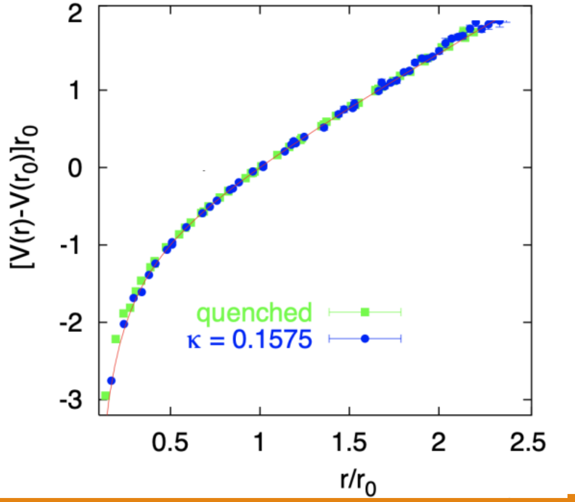
Introduction

Bound states of static quark and anti-quark pair: Probe for existence of Quark Gluon plasma in Heavy ion collisions (Alexander Rothkopf,Heavy quarkonium in extreme conditions,Physics Reports,Volume 858,2020,).

- Time evolution in Real-Time suffers from sign problem. (QFT suffers from sign problem; see Rasmus and Daniel's talk)
- If separation of scales is present: Use EFTs (NRQCD and pNRQCD): describe physics in form of potential?

At T=0 Schrodinger like potential picture has been observed (G. Bali Phys.Rept. 343 (2001) 1-136).

$$i\, \partial_t W_\square(t,r) = \Phi(t,r) W_\square(t,r)$$
$$V(r) = \lim_{t\to\infty} \Phi(t,r)$$



Introduction

- It remains to be seen if the potential picture even holds for T>0, if it does how is it modified?
- Spectral function is a link between real and imaginary time: (A Rothkopf ,T Hatsuda, S Sasaki Phys.Rev.Lett. 108 (2012) 162001).

$$W_{\square}(r,t) = \int d\omega e^{-i\omega t} \rho_{\square}(r,\omega) \quad \Longleftrightarrow \quad W_{\square}(r,\tau) = \int d\omega e^{-\omega\tau} \rho_{\square}(r,\omega)$$

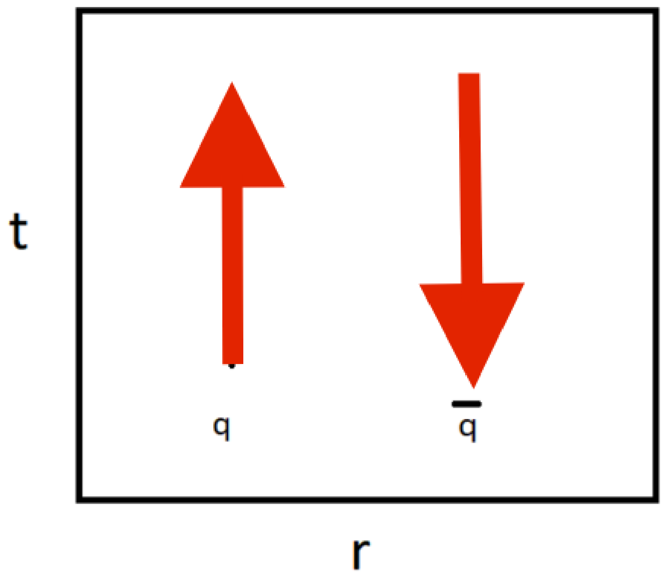
- Computation of Spectral Function : Ill-posed Inverse problem
- Potential linked to the dominant peak position and width of Spectral Function (Yannis Burnier and Alexander Rothkopf 1208.1899).
- In HTL regime there exists a complex potential with screened real part (M.Laine et. al JHEP 03 (2007), 054).

Lattice setup

- (2+1)-flavour QCD - configurations generated by HotQCD and TUMQCD collaborations.
- Using highly improved staggered quark (HISQ) action.

$N_\sigma^3 \times N_\tau$ lattices. $N_\tau = 10, 12, 16$ and $N_\sigma / N_\tau = 4$

- Calculate Wilson Line correlator in Coulomb Gauge.
- Fix box approach; temp range - 140MeV to 2GeV.
- Pion mass 160MeV, Kaon mass physical.

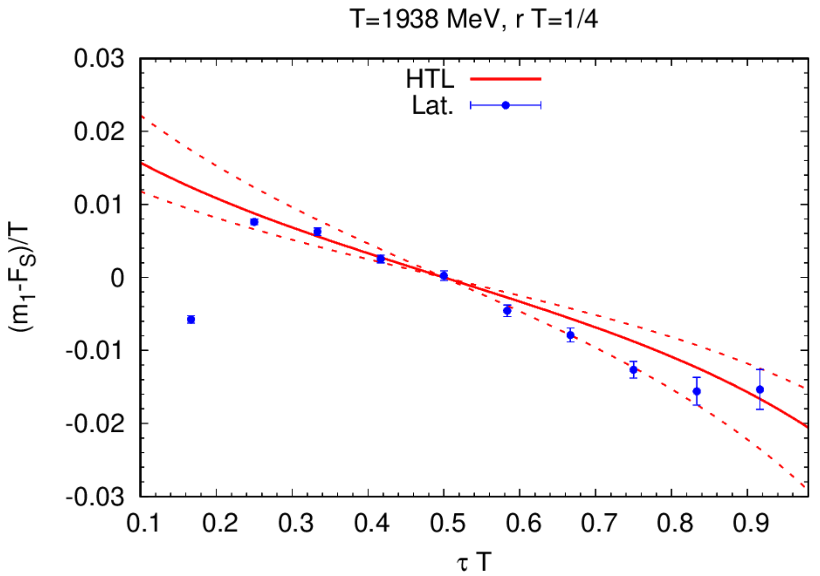


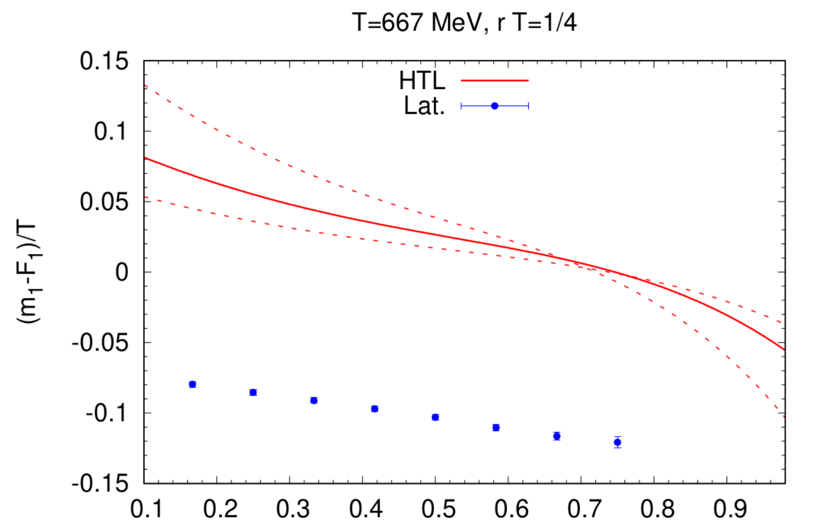
Cumulants and HTL comparison

Define effective mass of the correlation function:

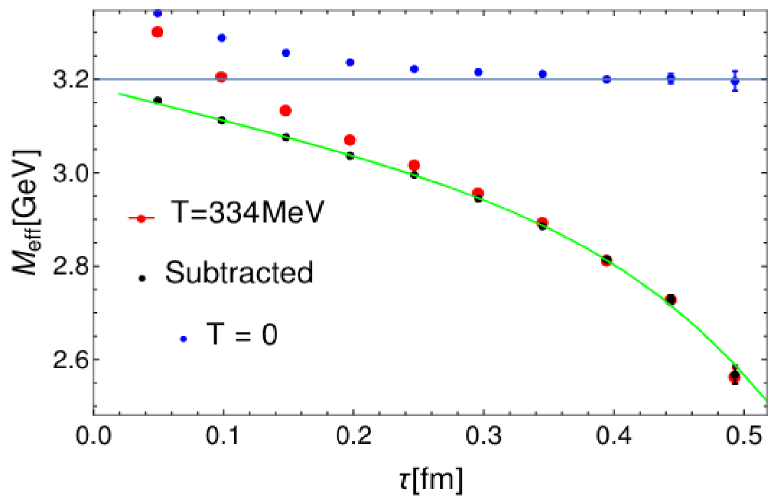
$$m_{eff}(r,\tau,T) = -\partial_\tau \ln W(r,\tau,T)$$

- Subtracting UV part using T=0 correlator results in linear behavior at small tau.
- Plot shows effective mass subtracted from Free Energies.
- HTL ($\mu=2\pi T$) does not quantitatively fit the data except for some specific T and separation distance.





Spectral Function Model Fits



- Lattice data sensitive only to peak position (Ω) and effective width (Γ).
- Parametrize Correlator as: $C_{sub}(\tau, T) \quad \sim \quad \exp(-\Omega\tau + \frac{1}{2}\Gamma^2\tau^2 + O(\tau^3))$

$$\rho_r(\omega, T) \quad = \quad A(T)\exp\left(-\frac{[\omega - \Omega(T)]^2}{2\Gamma^2(T)}\right) + A^{\mathrm{cut}}(T)\,\delta\left(\omega - \omega^{\mathrm{cut}}(T)\right)$$

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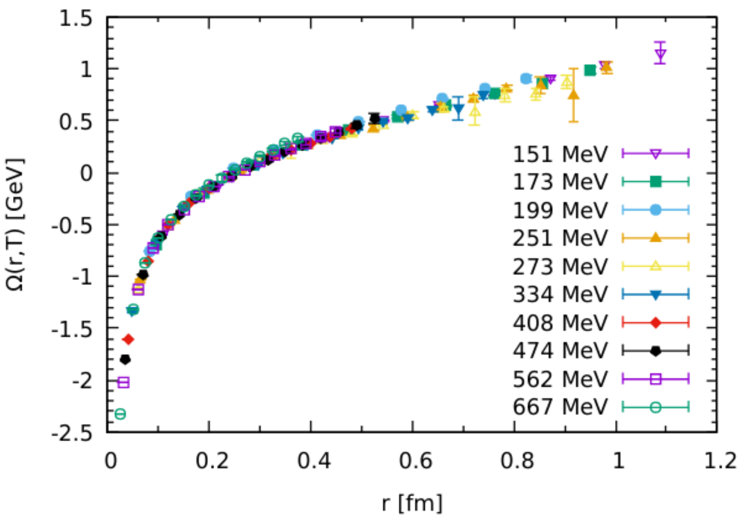

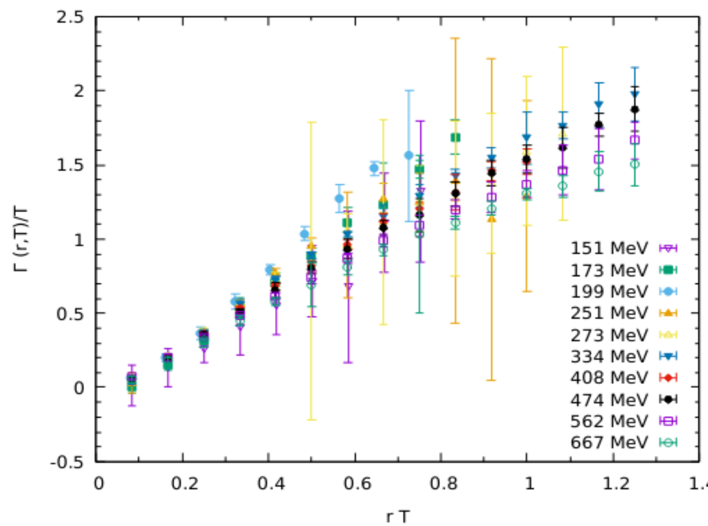

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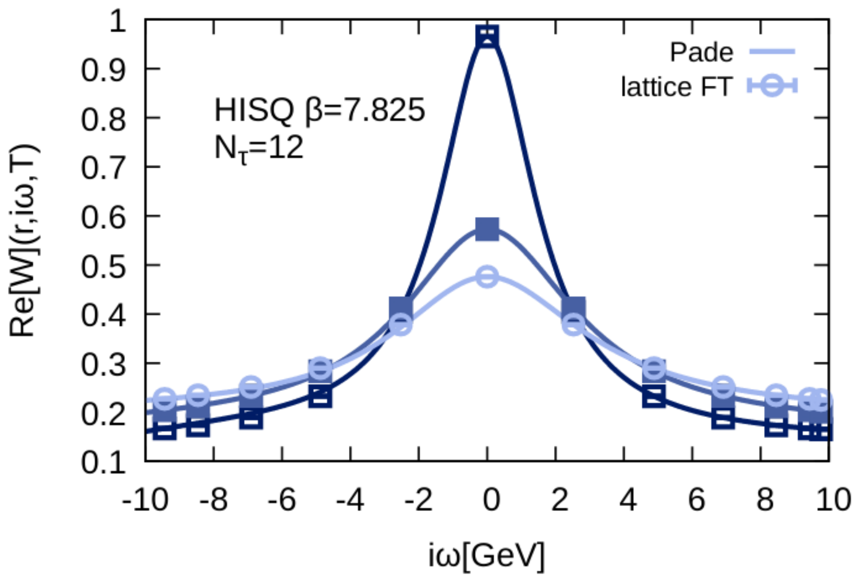
Pade' Interpolation

- Transform the Euclidean correlator into Matsubara frequency space.
- Implement Pade approximation in the form of continued fraction according to Schlessinger prescription (L. Schlessinger, Phys. Rev. 167, 1411 (1968)).
- This is interpolation of data and not fitting. Does not require minimization.
- Obtain pole structure from rational function: Directly related to the peak position (Ω) and width (Γ).



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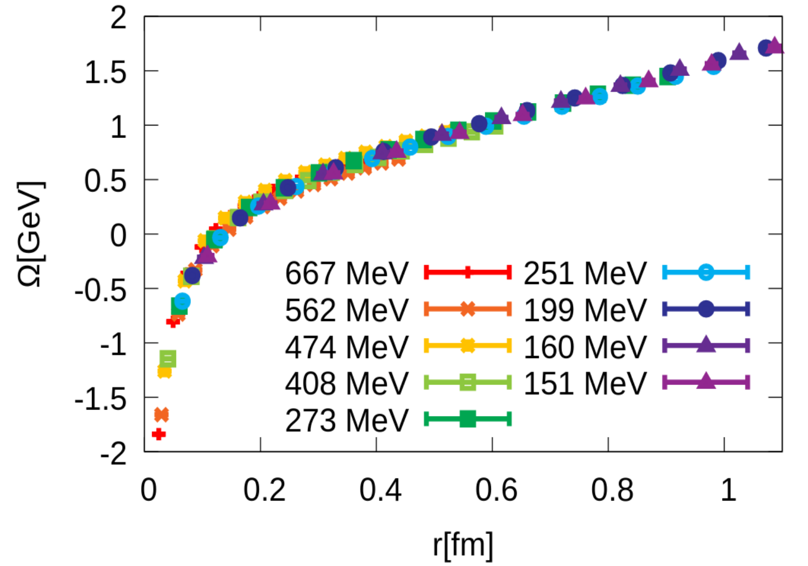
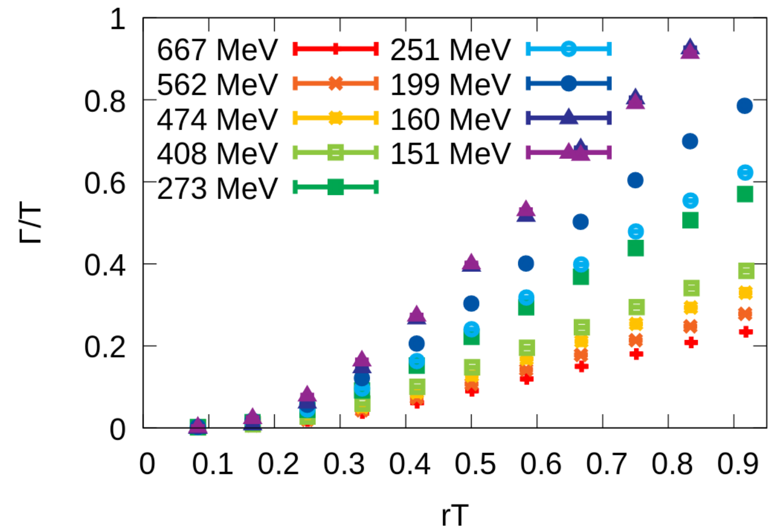




Spectral Function Extraction using Bayesian Method

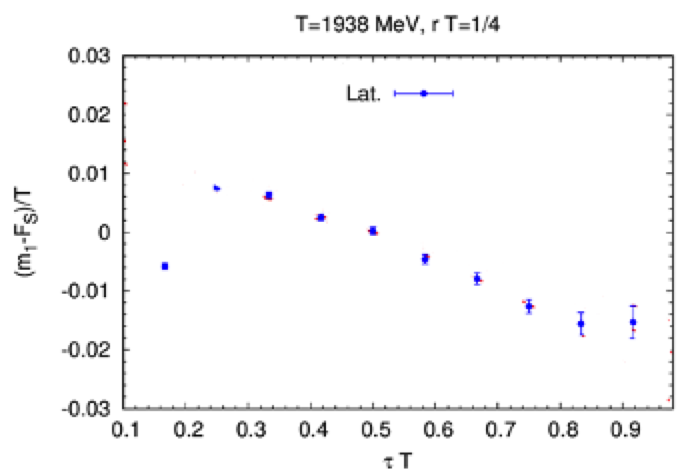
$$P[\rho|D,I] \propto P[D|\rho,I]P[\rho|I] = \exp[-L + \alpha S_{\mathrm{BR}}]$$

L is the usual quadratic distance used in chi-square fitting.

The prior probability $P(\rho|I) = \exp(\alpha S_{BR})$ acts as a regulator

$$\mathrm{S}_{\mathrm{BR}} = \int d\omega \left(1 - \frac{\rho(\omega)}{m(\omega)} + \log\left[\frac{\rho(\omega)}{m(\omega)}\right]\right).$$

- Look for the most probably spectrum by locating the extremum of the posterior.
- Effective masses at high T show non-monotonicity at small tau; non-positive spectral function.--- cannot use Bayesian Methods.



HTL inspired fits

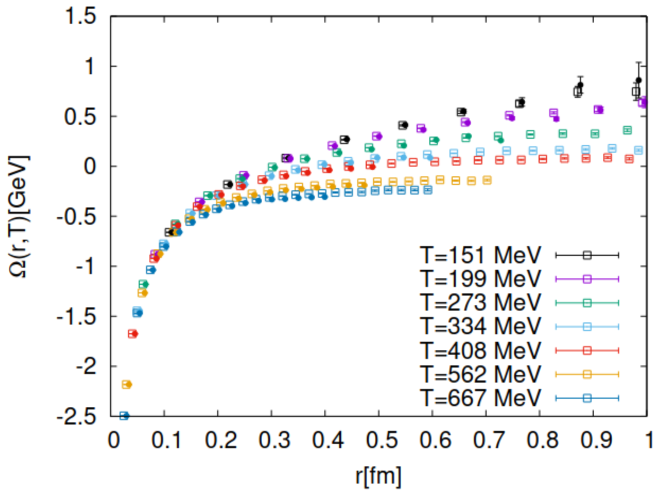
Peak position (Ω) and width (Γ) interpreted as the real and imaginary part of thermal static energy Es (D. Bala and S. Datta, Phys. Rev. D 101, 034507(2020)).

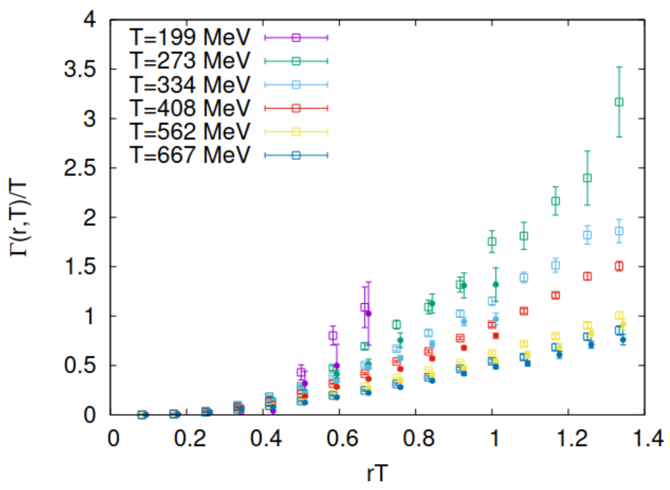
$$E_s(r,T) = \lim_{t\to\infty} i\frac{\partial \log W(r,t,T)}{\partial t} = \Omega(r,T) - i\Gamma(r,T).$$

W (r, t, T) is the Fourier transform of the spectral function ρ_r (r, ω)

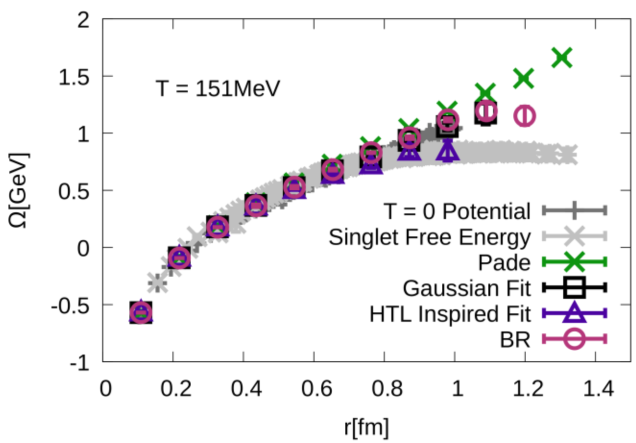
$$m_{eff}(r, n_\tau = \tau/a)a = \log\left(\frac{W(r,n_\tau,N_\tau)}{W(r,n_\tau+1,N_\tau)}\right)$$

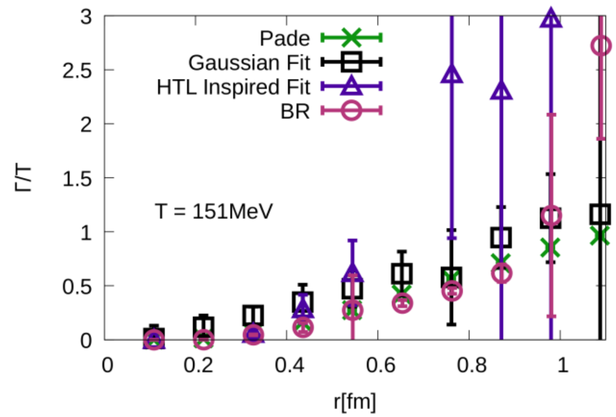
$$= \Omega(r,T)\,a - \frac{\Gamma(r,T)aN_\tau}{\pi}\,\log\left[\frac{\sin(\pi n_\tau/N_\tau)}{\sin(\pi(n_\tau+1)/N_\tau)}\right]$$

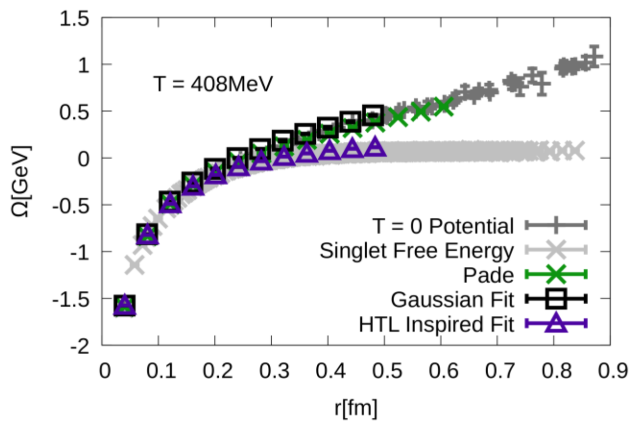


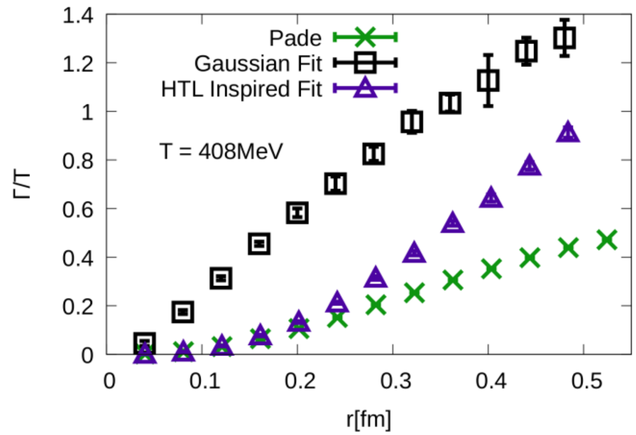


Comparison of Results



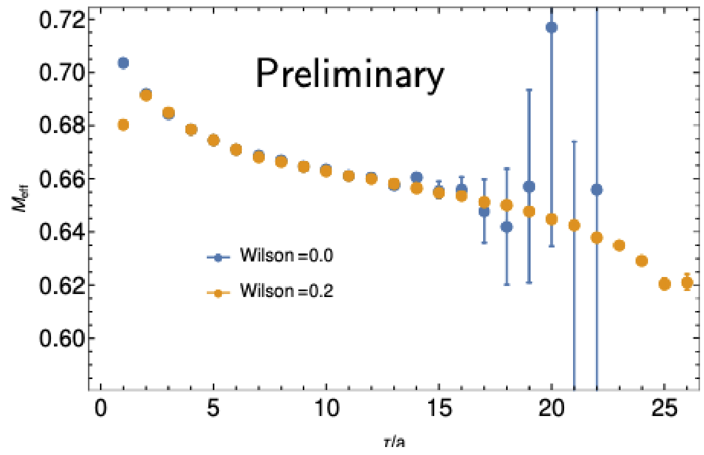


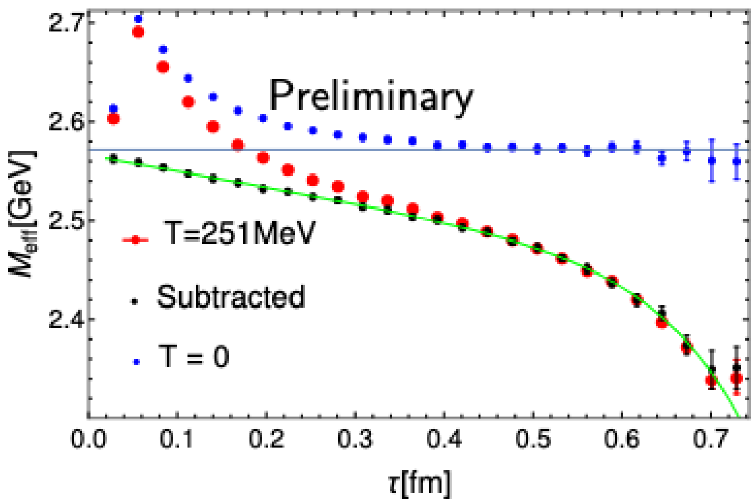




Finer lattices

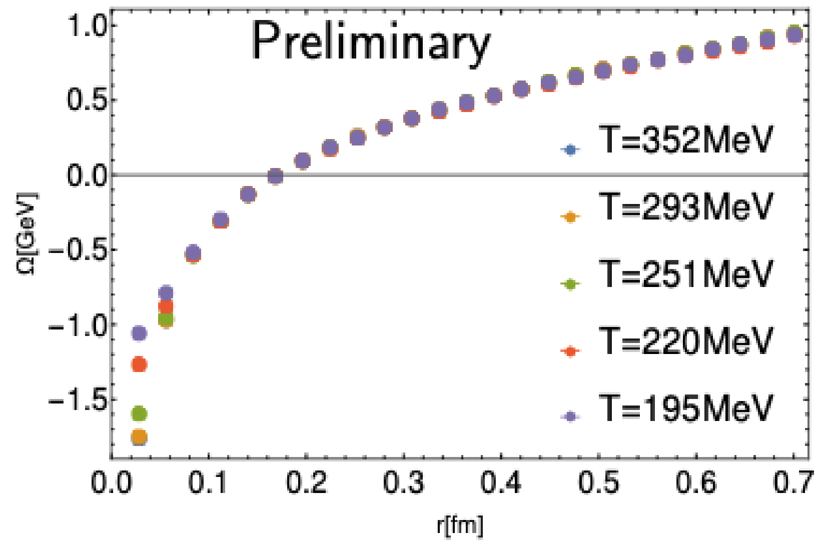
- New finer lattices generated using grant from PRACE.
- Heavy quarks (ms/ml = 5) $96^3 * N_\tau$ lattices used with N_τ =20,24,28,32,36,56.
- High energy fluctuations dominate at large τ.
- Wilson smearing used to remove fluctuations.
- Affects results at small tau and small distances.

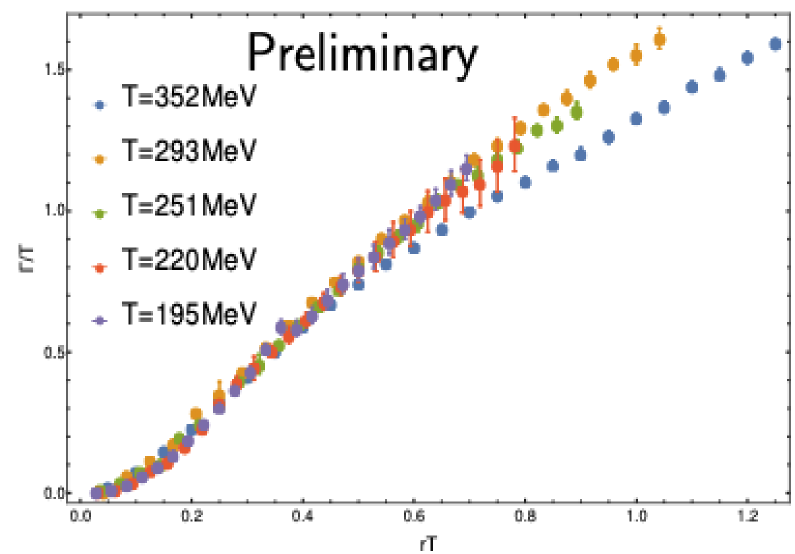




Gaussian Fits on new lattices

- No significant changes seen with changing temperature.
- Smearing affects peak position at small distances.
- Results consistant with $N_x = 48$.





Summary

- Spectral functions of Wilson line correlators encode the real and imaginary part of the complex potential between static quark-antiquark pairs.
- We show analysis of spectral structure with four different methods.
- We see that from the Gaussian fits and Pade' peak position (Ω) is temperature independent in HISQ lattices: Results puzzling and different from previous quenched QCD.
- Width not consistent between different methods.
- Preliminary results from gaussian fits on finer lattices still show no significant change of energy with temperature.
- Attempts to use Pade interpolation and improving fits still a work in progress.