Photon and dilepton production rate in the quark-gluon plasma from lattice QCD

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Motivation: some heavy-ion collision phenomenology

- direct photons (those not produced via hadronic decays) encode information about the environment in which they were created
- at low-p_T: expect a sizeable contribution from thermal photons: quark-gluon plasma and hadronic phase
- RHIC, $\sqrt{s_{NN}} = 200\,\text{GeV}$:
  - PHENIX measurement of $p_T < 3\,\text{GeV}$ photons shows clear excess over N_{coll}-scaled pp measurement [PRL 104, 132301 (2010)]
  - PHENIX: large photon anisotropy wrt reaction plane [PRL 109, 122302 (2012)]
  - STAR: photon yield ~ 3 times smaller than PHENIX: unresolved tension
- LHC, $\sqrt{s_{NN}} = 2760\,\text{GeV}$: ALICE measured photon yield [PLB 754, 235 (2016)].

See recent PHENIX preprint [2203.17187]. Phenomenology: Gale et al. [2106.11216]. Review G. David [1907.08893].

Thermal vector spectral functions: physical significance

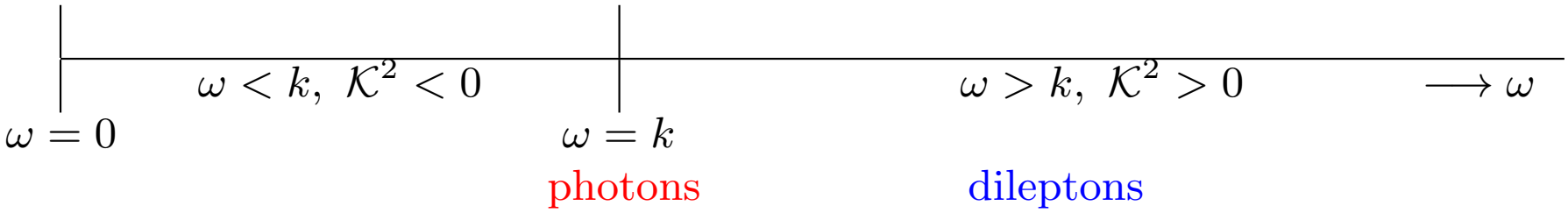
$$\rho^{\mu\nu}(\mathcal{K}) = \int d^4x\, e^{i\mathcal{K}\cdot x}\, \frac{1}{Z} \sum_n e^{-E_n/T} \langle n|[j^\mu(x), j^\nu(0)]|n\rangle$$

- Rate of dilepton production per unit volume plasma:

$$d\Gamma_{\ell^+\ell^-}(\mathcal{K}) = \alpha^2\, \frac{d^4\mathcal{K}}{6\pi^3\mathcal{K}^2}\, \frac{-\rho^\mu{}_\mu(\mathcal{K})}{e^{\beta\mathcal{K}^0} - 1}$$

- Rate of photon production per unit volume plasma:

$$d\Gamma_\gamma(\boldsymbol{k}) = \alpha\, \frac{d^3k}{4\pi^2\, k}\, \frac{-\rho^\mu{}_\mu(k, \boldsymbol{k})}{e^{\beta k} - 1}.$$



Equivalent expressions for the photon rate

- current conservation: $\omega^2 \rho^{00}(\omega, k) = k^i k^j \rho^{ij}(\omega, k)$ implies

$$\rho_L \equiv (\hat{k}^i \hat{k}^j \rho^{ij} - \rho^{00}) = \frac{\mathcal{K}^2}{k^2}\, \rho^{00}$$

  vanishes at real-photon kinematics. NB. $k \equiv |\boldsymbol{k}|, \quad \hat{k}^i = k^i/k$.

- Let

$$\rho_T \equiv \frac{1}{2}(\delta^{ij} - \hat{k}^i \hat{k}^j)\rho^{ij}$$

  and introduce ($\lambda \in \mathbb{R}$)

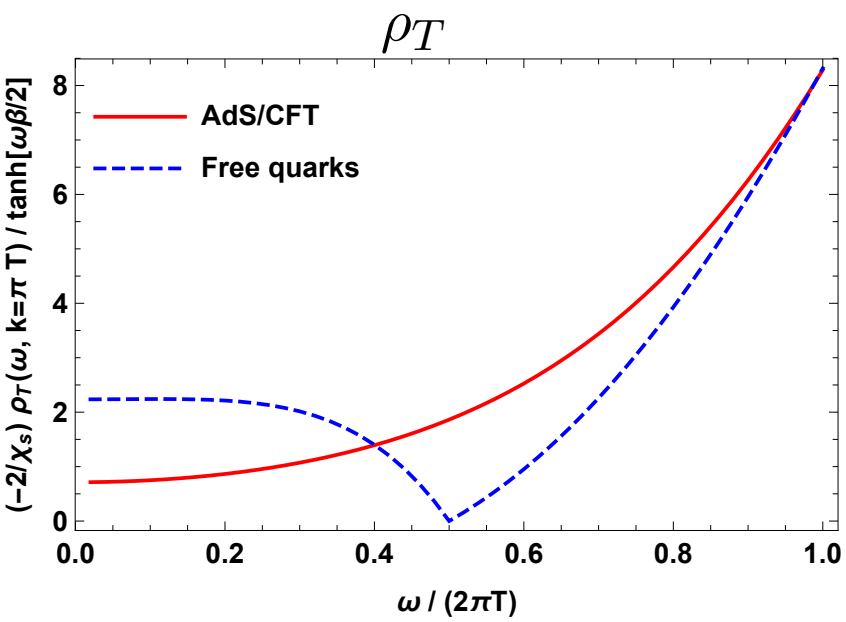
$$\rho(\omega, k, \lambda) = 2\rho_T + \lambda\, \rho_L \qquad \overset{\lambda=1}{=\!=} -\rho^{\mu}{}_{\mu}\ .$$

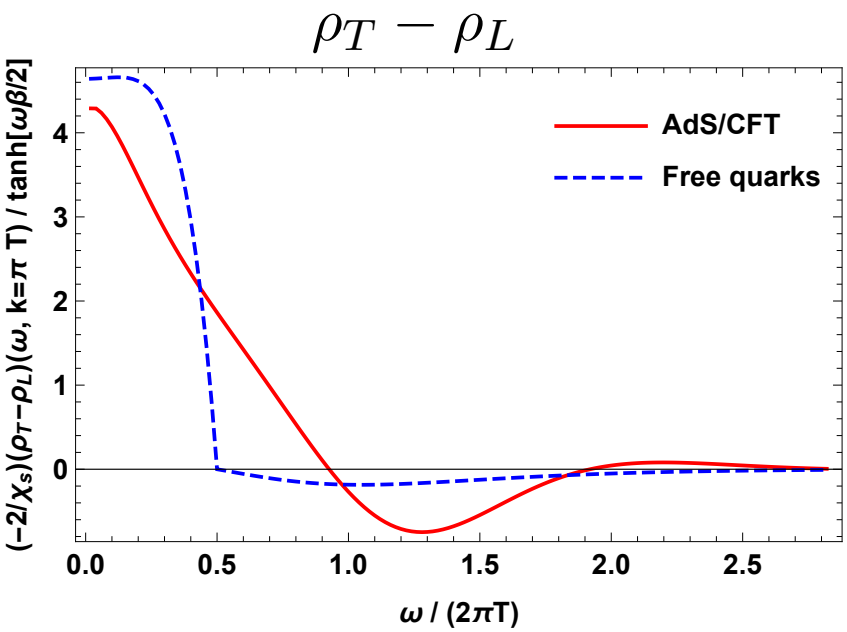
- Photon rate can be written ($\forall \lambda$)

$$d\Gamma_\gamma(\boldsymbol{k}) = \alpha\, \frac{d^3 k}{4\pi^2\, k}\, \frac{\rho(k, k, \lambda)}{e^{\beta k} - 1}.$$

Choosing λ: weak and strong coupling spectral fcts

Spatial momentum $k = \pi T$: (see hep-th/0607237 and 1310.0164)





- ρ_T is positive-definite and free of the diffusion pole
- $(\rho_T - \rho_L)$ is very suppressed at large ω
  and obeys a superconvergent sum-rule.
- At $\omega = k$, the two channels should be equal:
  non-trivial consistency check for lattice-based calculations!

Lattice QCD and vector correlators

Imaginary-time path-integral representation of QFT (Matsubara formalism).

Imaginary-time **vector correlators** ($\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} = 2\mathrm{diag}(1,-1,-1,-1)$),

$$G^{\mu\nu}(x_0, \boldsymbol{k}) = \int d^3x \, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \, \mathrm{Tr}\Big\{ \frac{e^{-\beta H}}{Z(\beta)} j^\mu(x) \, j^\nu(0) \Big\}, \qquad j^\mu = \sum_f Q_f \, \bar{\psi}_f \gamma^\mu \psi_f$$

Spectral representation (u is a real four-vector): $\rightsquigarrow$ **inverse problem**

$$u_\mu G^{\mu\nu} u_\nu(x_0, \boldsymbol{k}) = \int_0^\infty \frac{d\omega}{2\pi} \underbrace{\frac{(u_\mu \rho^{\mu\nu} u_\nu)(\omega, \boldsymbol{k})}{\sinh(\beta\omega/2)}}_{\geq 0} \cosh[\omega(\beta/2 - x_0)].$$

Parameters of the lattice calculations

- $N_f = 2$ flavours of dynamical O(a) improved Wilson fermions with Wilson gauge action (CLS code and action are used)
- $T \simeq 254\,\text{MeV}$, $L = 4/T \simeq 3.1\,\text{fm}$.
- Isovector current correlator is computed.

label	$(6/g_0^2, \kappa)$	$1/(aT)$	N_{conf}	$\frac{\text{MDUs}}{\text{conf}}$	$\tau_{\text{int}}[Q^2(\bar{t})]$
F7	$(5.3, 0.13638)$	12	482	20	11.3(15)
O7	$(5.5, 0.13671)$	16	305	20	19(5)
W7	$(5.685727, 0.136684)$	20	1566	8	81(23)
X7	$(5.827160, 0.136544)$	24	511	10	490(230)

See 2001.03368 ($\rho_T - \rho_L$) and 2205.02821 (ρ_T).

Analysis of the $2(T-L)$ channel

'Hydrodynamics' prediction at small ω, k: with D the diffusion coefficient,

$$\rho(\omega, k, -2)/\omega \approx \frac{4\chi_s\, Dk^2}{\omega^2 + (Dk^2)^2} \qquad \omega, k \ll D^{-1}.$$

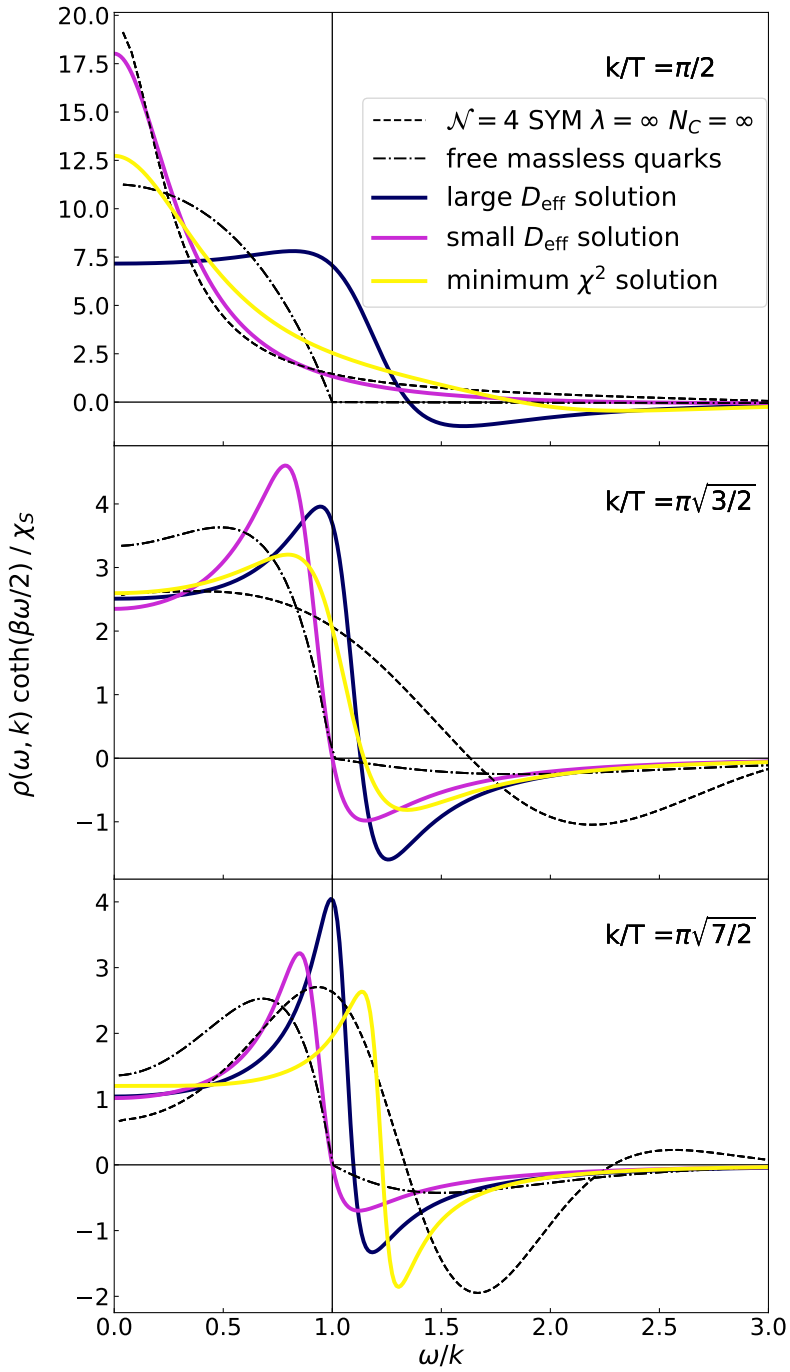
From the operator-product expansion:

$$\rho(\omega, k, \lambda = -2) \overset{\omega \to \infty}{\sim} k^2/\omega^4 \ : \qquad \int_0^\infty d\omega\, \omega\, \rho(\omega, k, -2) = 0.$$

$\rightsquigarrow$ 5-parameter ansatz:

$$\rho(\omega, k, -2) = \frac{A(1 + B\omega^2)\ \tanh(\omega\beta/2)}{[(\omega - \omega_0)^2 + b^2][(\omega + \omega_0)^2 + b^2][\omega^2 + a^2]}.$$

Analysis strategy: always determine B so as to satisfy the sum rule; scan over all other parameters to determine the χ^2 landscape.



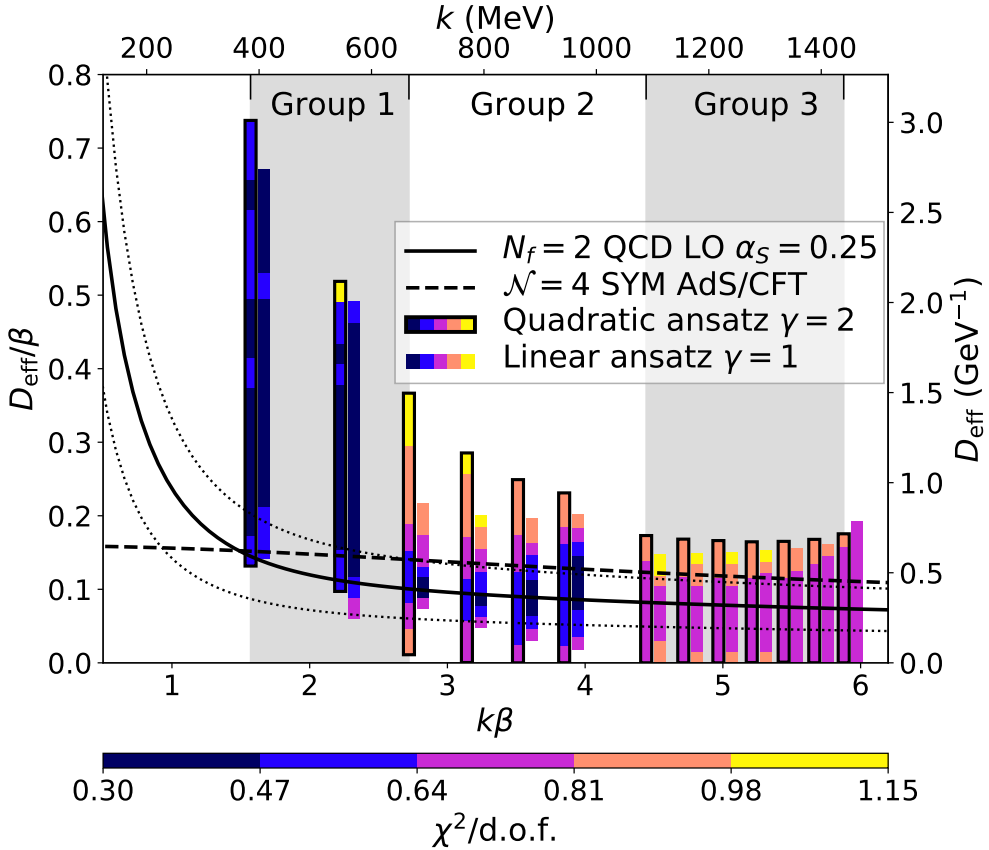
⟵ spec.funct. poorly constrained at small $k \simeq 0.4\,\text{GeV}$

Representative spectral functions describing the lattice $2(T-L)$ correlator

⟵ spec.funct. better constrained at $k \approx 2\pi T \simeq 1.6\,\text{GeV}$

Final result of analysis of $2(T-L)$ channel

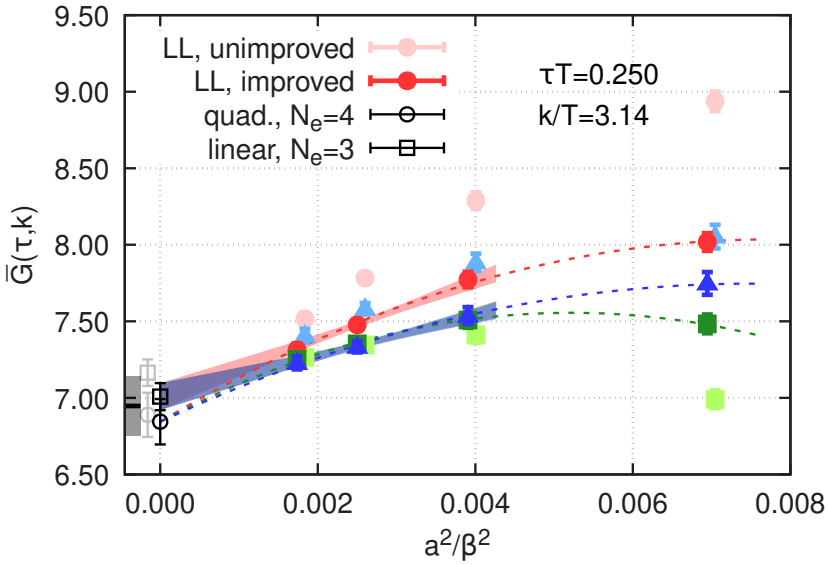
$$D_{\rm eff}(k) \equiv \frac{\rho(\omega = k, k, \lambda)}{4\chi_s k} \xrightarrow{k\to 0} D, \qquad \chi_s = \beta \int d^3x \langle V_0(x) V_0(0)\rangle.$$



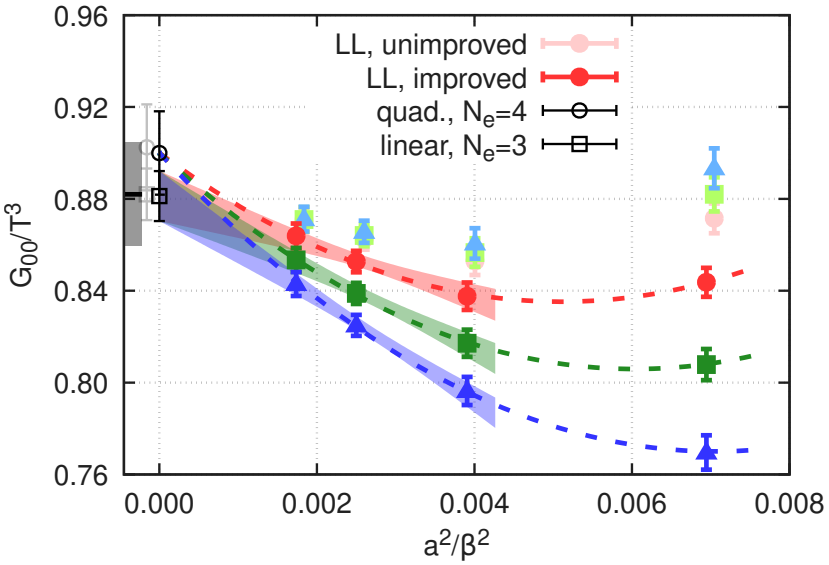
The transverse channel

Taking the continuum limit of $G_T(\tau, k)$

$$\bar{G}_T(\tau, k) = G_T(\tau, k)/(T\chi_s)$$



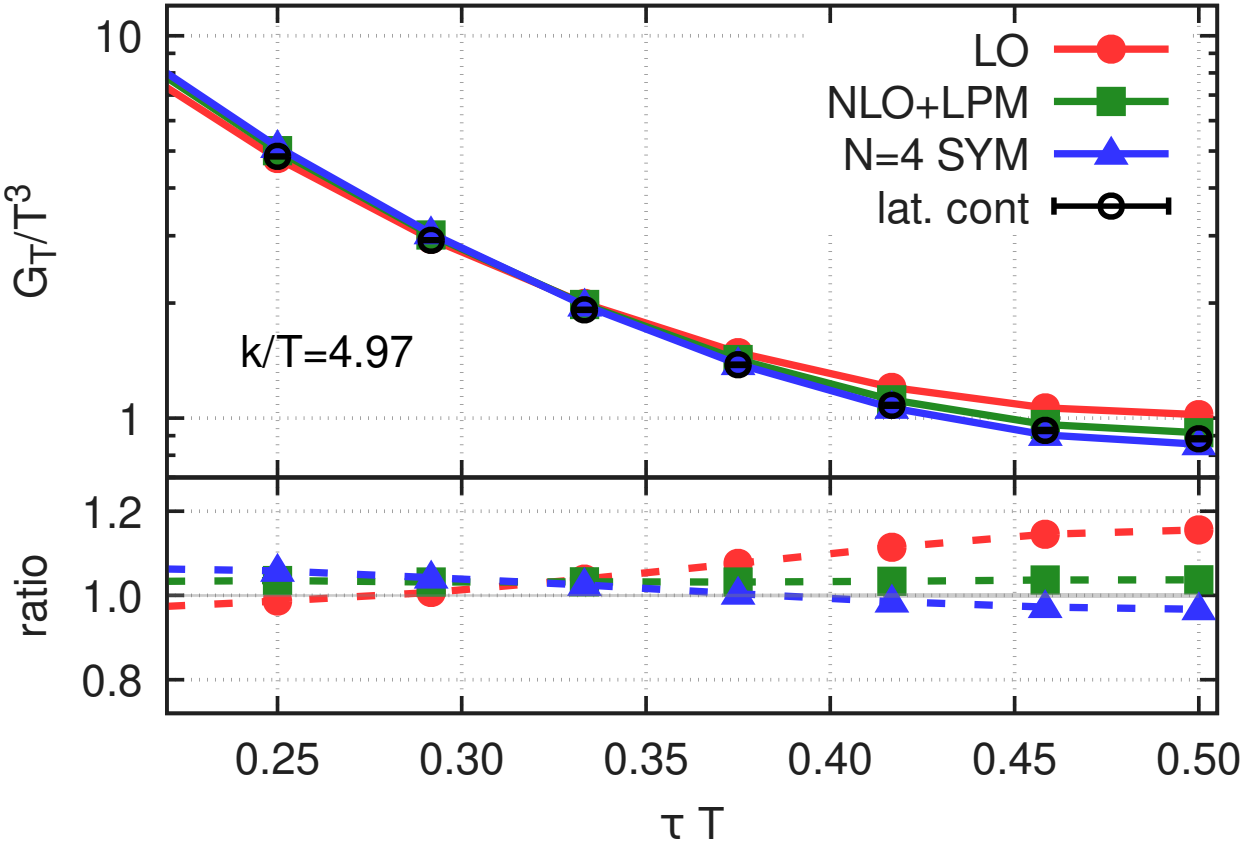
$$\chi_s/T^2$$



Three different discretisations, joint continuum extrapolation.

Use treelevel improvement.

Euclidean correlator: lattice vs. NLO prediction



- Set $\chi_s^{\mathrm{AdS/CFT}} = \frac{N_c^2 T^2}{8} \doteq \frac{9T^2}{8}$.
- All three correlators are within 10% of each other.

Fit ansätze for the spectral functions

$$\rho(\omega) = \rho_{\rm fit}(\omega)(1 - \Theta(\omega, \omega_0, \Delta)) + \rho_{\rm pert}(\omega)\Theta(\omega, \omega_0, \Delta)$$

with $\omega_0 \approx 2.5\,$GeV the matching frequency,

$$\Theta(\omega, \omega_0, \Delta) = (1 + \tanh[(\omega - \omega_0)/\Delta])/2$$

a smooth step function and $\rho_{\rm pert}(\omega)$ from [Jackson, Laine 1910.09567].

A) Polynomial ansatz:

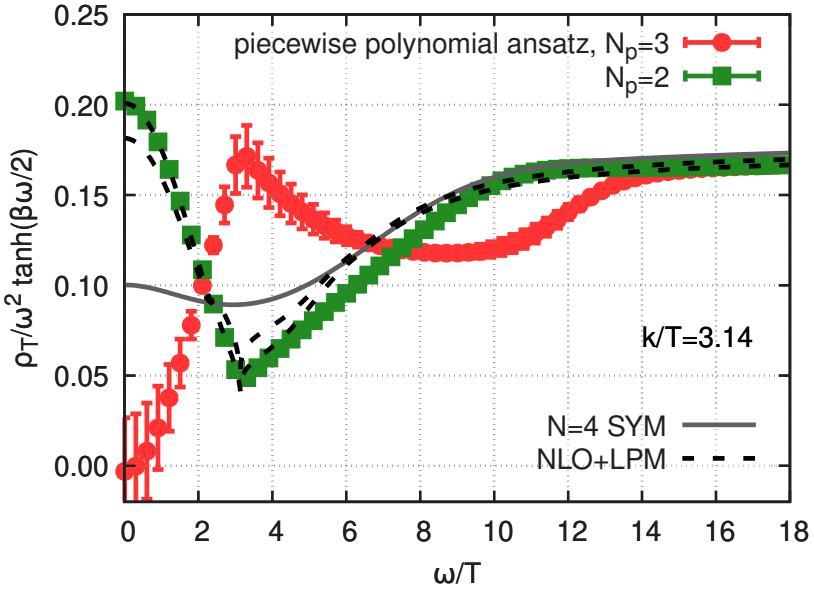
$$\frac{\rho_{\rm fit,1}(\omega)}{T^2} = \sum_{n=0}^{N_{\rm p}-1} A_n \left(\frac{\omega}{\omega_0}\right)^{1+2n},$$

B) Piecewise polynomial ansatz:

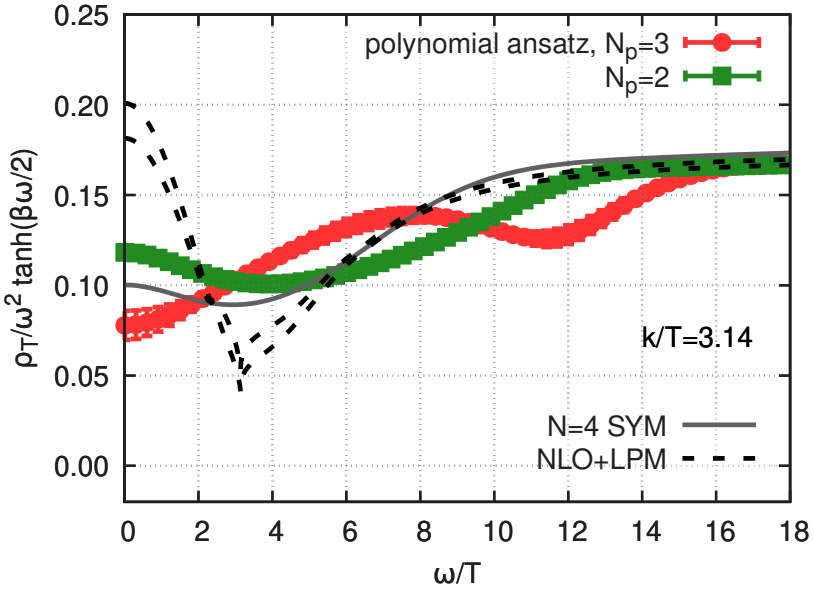
$$\frac{\rho_{\rm fit,2}(\omega)}{T^2} = \begin{cases} A_0\,\dfrac{\omega}{\omega_0} + A_1\left(\dfrac{\omega}{\omega_0}\right)^3, & \text{if } \omega \le k, \\ B_0\,\dfrac{\omega}{\omega_0} + B_1\left(\dfrac{\omega}{\omega_0}\right)^3, & \text{if } \omega > k. \end{cases}$$

Representative lattice-QCD results for the spectral functions

Piecewise polynomial ansatz

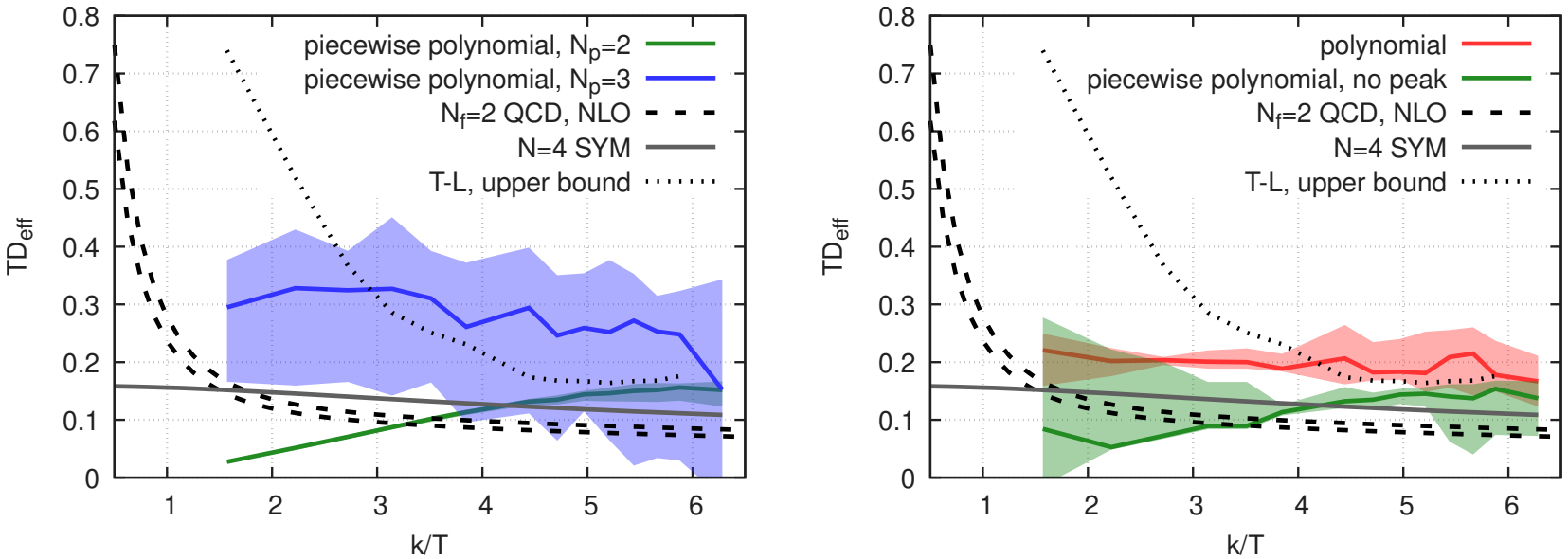


Polynomial ansatz



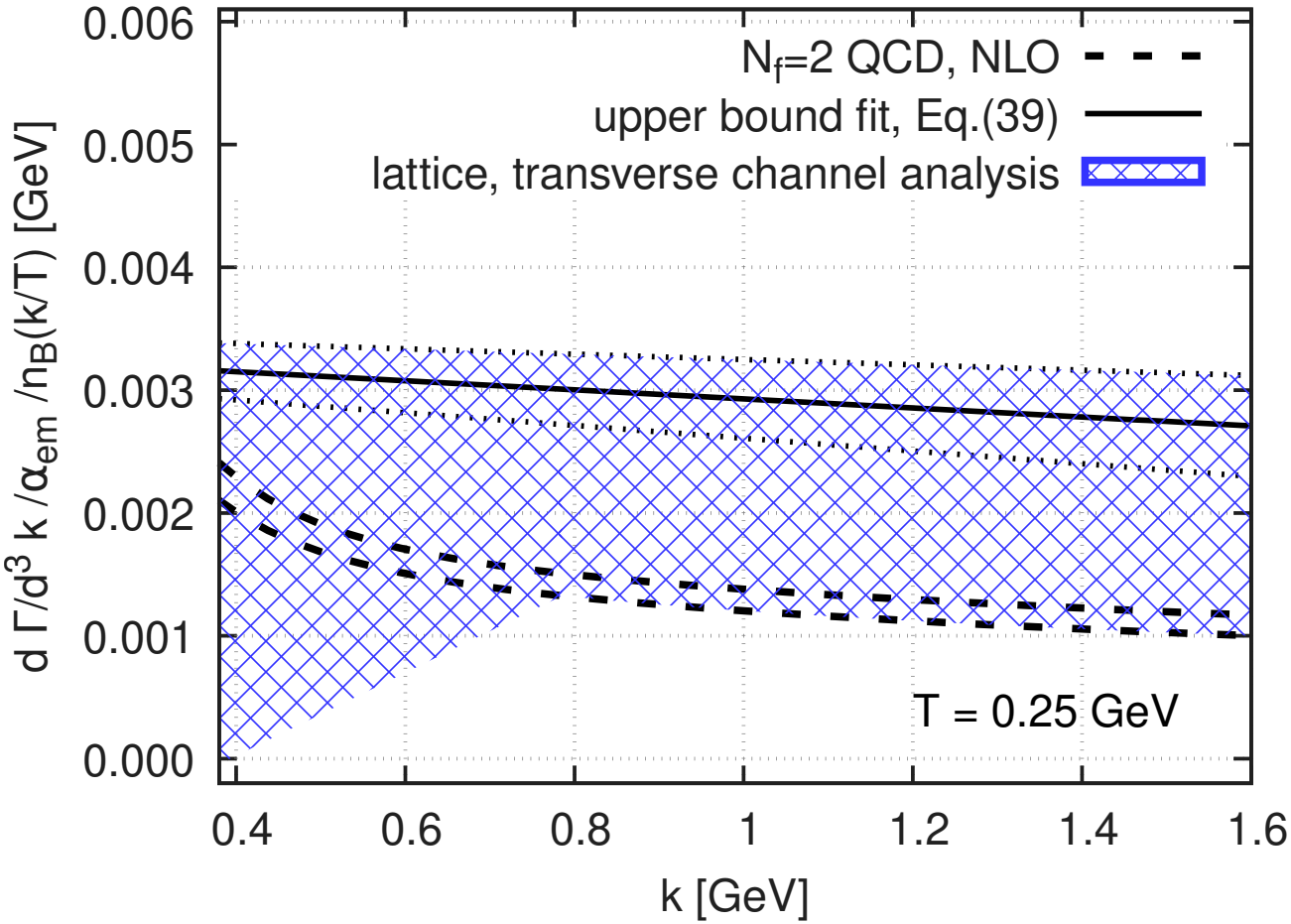
- Piecewise polynomial cannot 'decide' between having a min. or max. at $\omega = k$.
- Qualitatively, both the 'AdS/CFT' type and the 'NLO' type are compatible with the lattice data.

Results for the effective diffusion coefficient $D_{\rm eff} \equiv \frac{\rho_T(\omega{=}k,k)}{2\chi_s k}$



- left panel: comparison with results of $(T-L)$ channel analysis: piecewise-polynomial ansatz with max. at $\omega = k$ is disfavoured for $k \geq \pi T$.
- right panel: forbidding a max. at the 1σ level, predictivity is much stronger.
- polynomial ansatz favours values even larger than the $\mathcal{N}=4$ SYM prediction from AdS/CFT (hep-th/0607237).

Final result for the photon emissivity

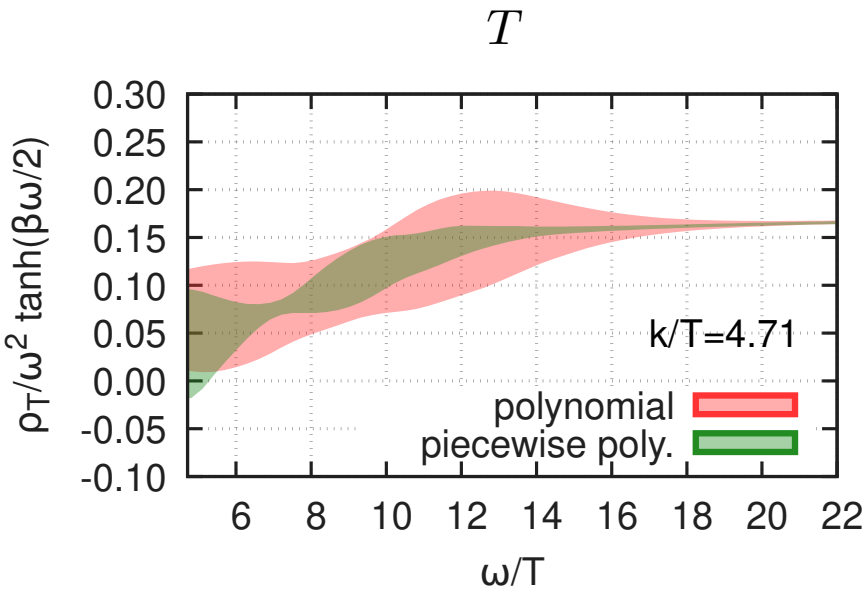


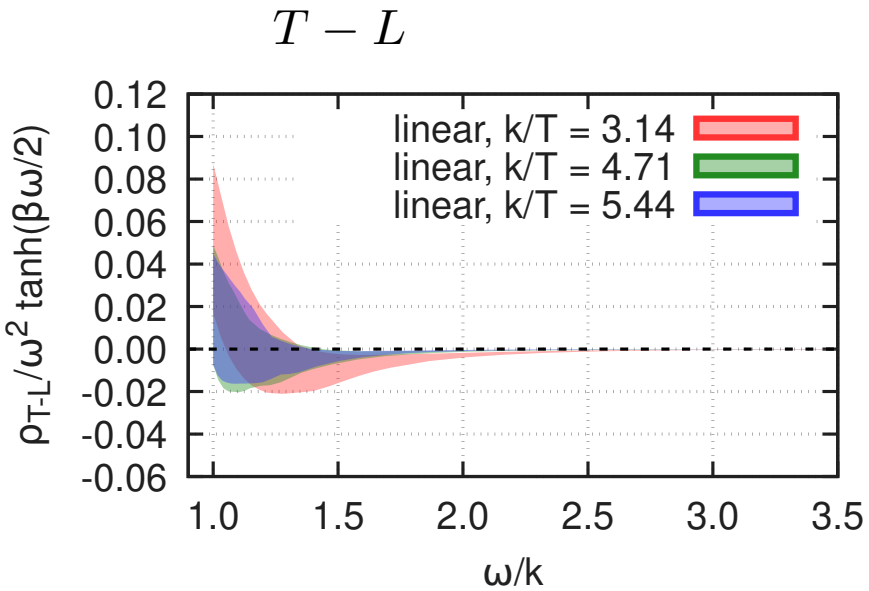
$$k = \pi T \simeq 800\,\mathrm{MeV} \;:\qquad \frac{d\Gamma_\gamma}{d^3k} = \frac{\alpha_{\mathrm{em}}}{e^{k/T}-1}\,(2.2 \pm 0.8) \times 10^{-3}\,\mathrm{GeV}.$$

Dilepton rate: $-\rho^{\mu}{}_{\mu} = 3\rho_T \; - \; (\rho_T - \rho_L)$

Choose e.g. $k = 4.71T = 1.2\,\text{GeV}$ and $\mathcal{K}^2 = (2\pi T)^2 = (1.6\,\text{GeV})^2$

$\Rightarrow \omega = \sqrt{\mathcal{K}^2 + k^2} = 7.85T = 1.33k$





(Free quarks: $\rho_T/\omega^2 \tanh(\omega\beta/2) = 0.093$)

- At these kinematics, contribution of $T - L$ is practically negligible.

Conclusion

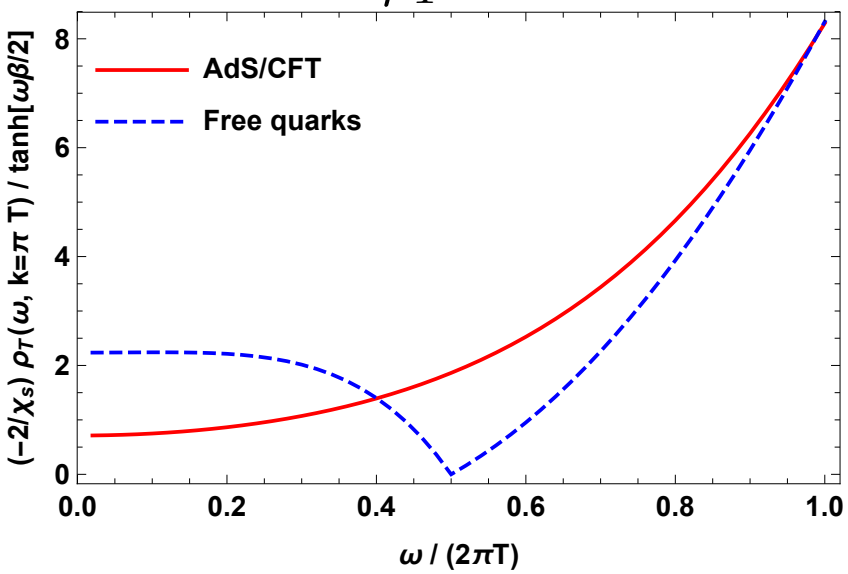
- combining information from T and $(T-L)$ channels proves beneficial: better constraint on the photon emissivity
- lattice results can accomodate a photon emissivity 2.5 times larger than LO weak-coupling prediction
- we have computed the $1/(\omega^2+(2\pi T)^2)$ weighted integral over $\rho_T(\omega,\omega)/\omega$ in lattice QCD without solving an inverse problem: see talk by **Csaba Török** [1807.00781], [2112.00450].

Near future: analyze dilepton rate for $M_{\ell^+\ell^-} \gtrsim 1\,\text{GeV}$.

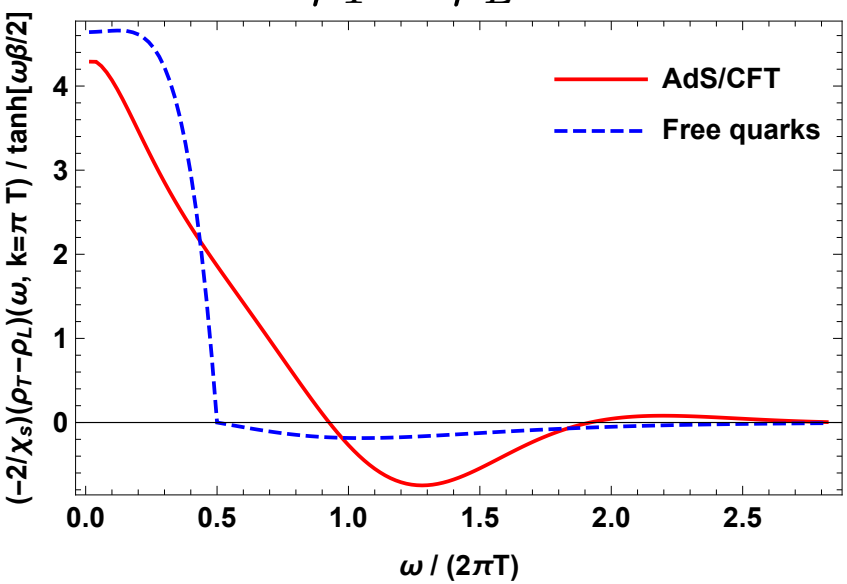
Choosing a favourable λ: weak and strong coupling spectral fcts

Spatial momentum $k = \pi T$:

ρ_T



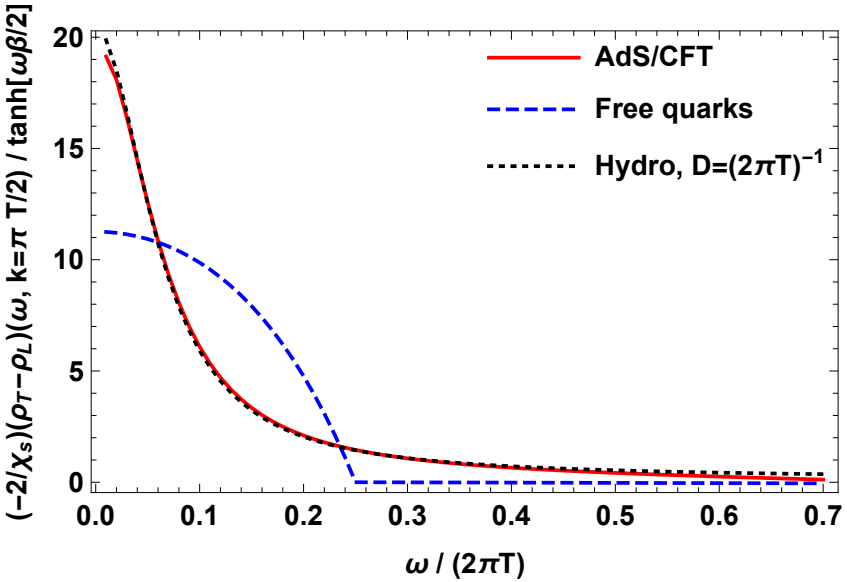
$\rho_T - \rho_L$



Spatial momentum $k = \pi T/2$:
At strong coupling, hydro works:

$$-2(\rho_T - \rho_L)(\omega, k)/\omega \approx \frac{4\chi_s\, D k^2}{\omega^2 + (D k^2)^2},$$

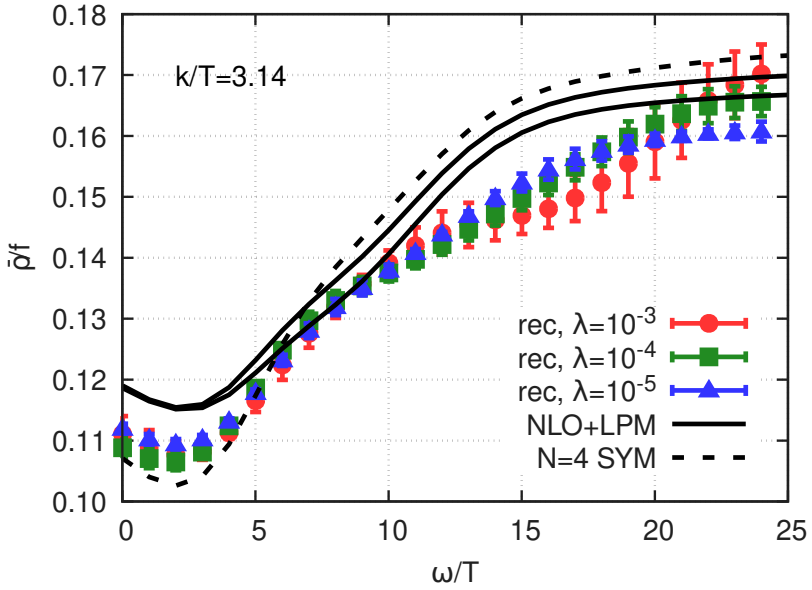
Refs: hep-th/0607237 and 1310.0164.



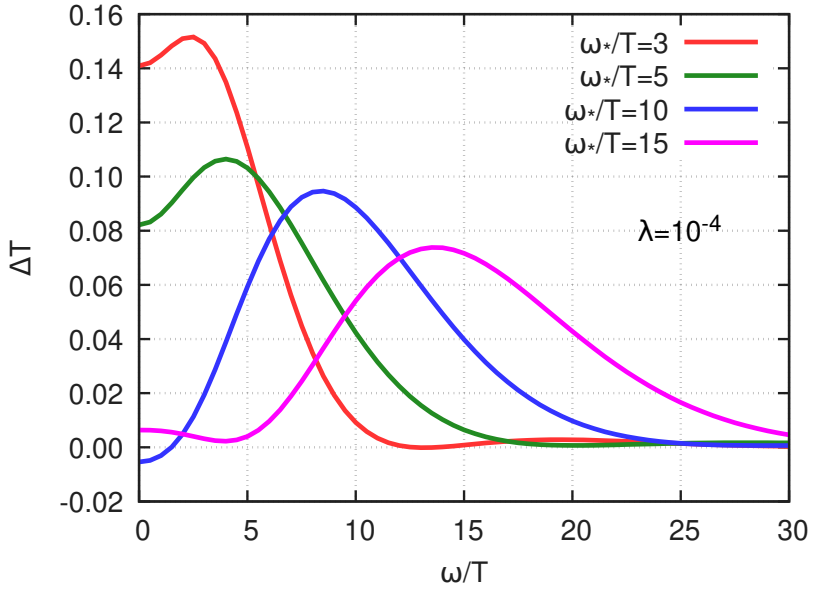
Backus-Gilbert method

$$\frac{\tanh(\omega\beta/2)}{\omega^2}\hat{\rho}(\omega) = \sum_{i=1}^{N_{\rm dat}} g_i(\omega) G_{\rm T}(\tau_i) = \int_0^\infty d\omega' \; \Delta(\omega,\omega') \; \frac{\tanh(\omega'\beta/2)}{\omega'^2}\rho(\omega')$$

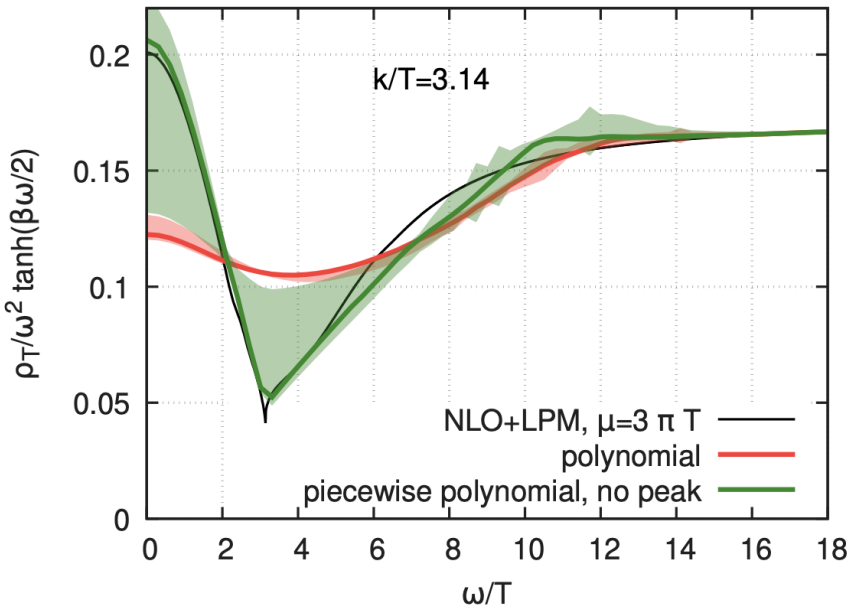
Smeared spectral function

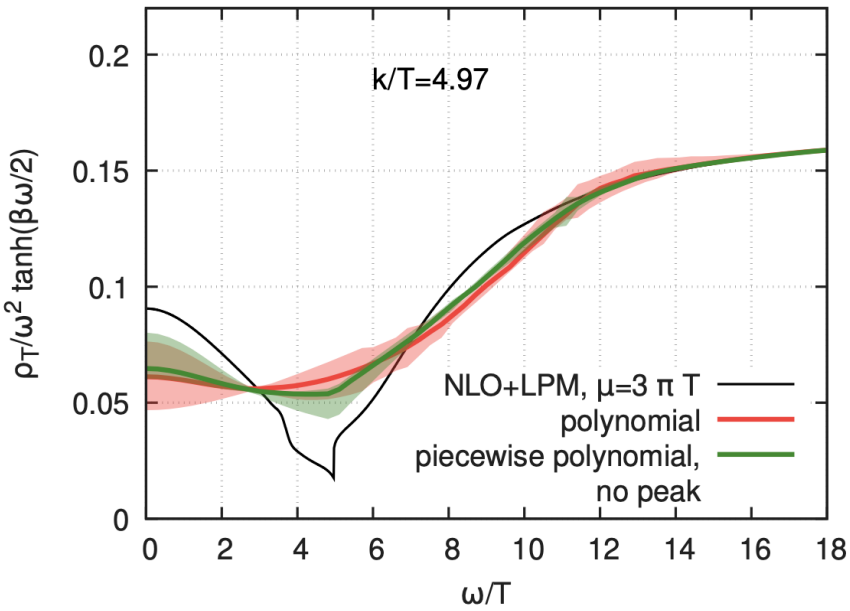


smearing kernel $\Delta(\omega^*,\omega)$



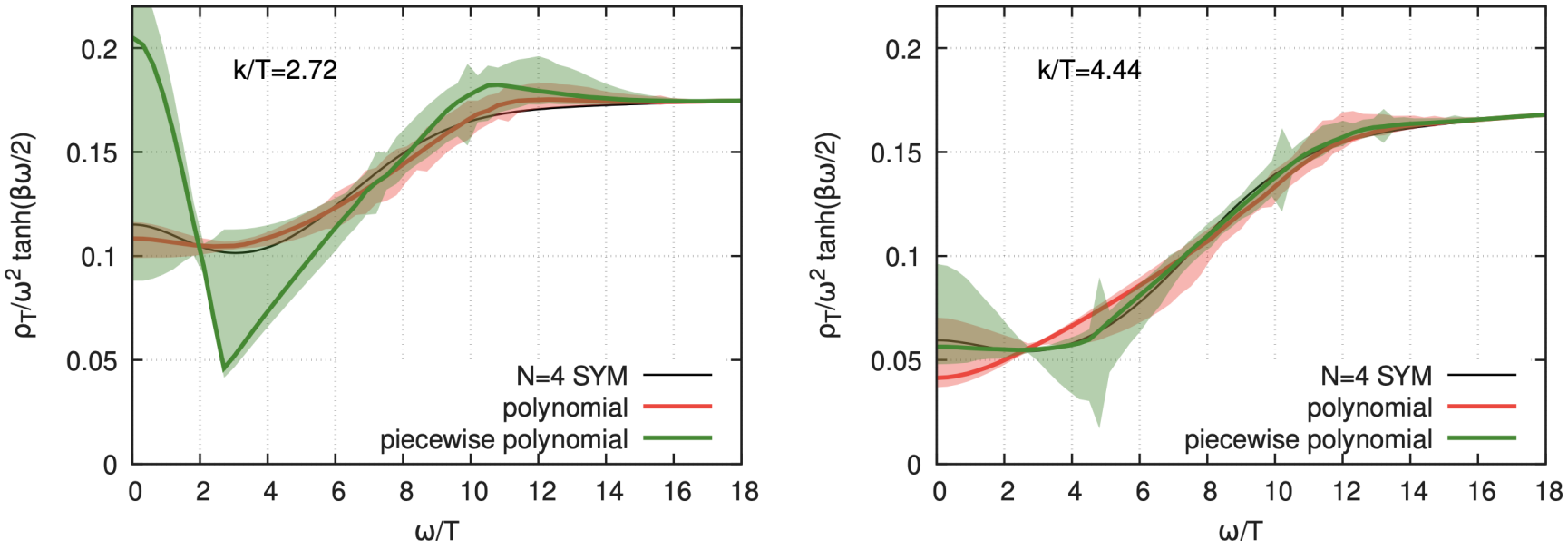
Mock test on NLO+LPM spectral function





- piecewise polynomial ansatz designed to mimic the weak-coupling spectral fct
- the output error band covers the true value of the spectral fct for $k = \pi T$, but not for all momenta (quite strongly dependent on the covariance matrix).

Mock test on $\mathcal{N} = 4$ SYM AdS/CFT spectral fct



- the output error band generously covers the true value of the spectral fct
- polynomial ansatz particularly successful.