Electric charge fluctuations from the lattice

Szabolcs Borsanyi
Z. Fodor, P. Parotto, A. Pásztor, C, Ratti and K. K. Szabó

Lattice 2022, Bonn

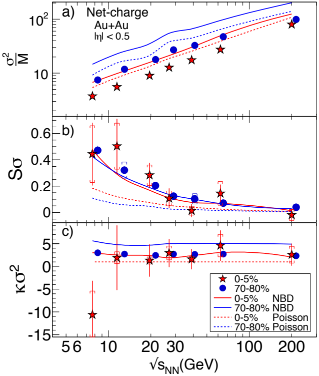
August 9, 2022

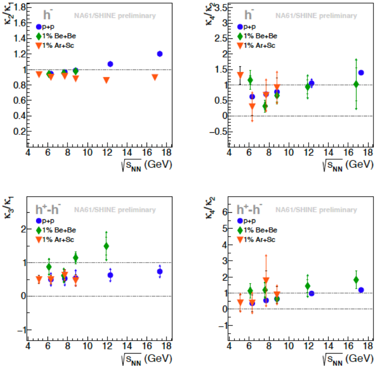
BERGISCHE UNIVERSITÄT WUPPERTAL

Fluctuation data at chemical freeze-out

At RHIC **STAR** has measured the mean, variance, skewness and kurtosis of the event-by-event **net charge** distribution at various energies and centralities. [STAR: 1402.1558]

Other experiment also show interest, here (right) QM22 preliminary plot by NA61/SHINE.





Fluctuations in a grand canonical ensemble

The expectation value of a conserved charge is a derivative with respect to the chemical potential.

$$\langle N_q \rangle = T \frac{\partial \log Z(T, V, \{\mu_q\})}{\partial \mu_q}$$

The response of the system to the thermodynamic force μ_q is proportional to the fluctuation of the conserved charge:

$$\frac{\partial \langle N_i \rangle}{\partial \mu_j} = T \frac{\partial^2 \log Z(T, V, \{\mu_q\})}{\partial \mu_j \partial \mu_i} = \frac{1}{T}(\langle N_i N_j \rangle - \langle N_i \rangle \langle N_j \rangle)$$

The higher derivatives are the generalized quark number susceptibilities:

$$\chi^{u,d,s,c}_{i,j,k,l} = \frac{\partial^{i+j+k+l}(p/T^4)}{(\partial \hat{\mu}_u)^i (\partial \hat{\mu}_d)^j (\partial \hat{\mu}_s)^k (\partial \hat{\mu}_c)^l}$$

with $\hat{\mu}_q = \mu_q / T$.

Translating from quark numbers to B,Q and S

In terms of physical derivatives

$$\begin{aligned} \frac{d}{d\mu_B} &= \frac{1}{3}\partial_u + \frac{1}{3}\partial_d + \frac{1}{3}\partial_s\,, \\ \frac{d}{d\mu_Q} &= \frac{2}{3}\partial_u - \frac{1}{3}\partial_d - \frac{1}{3}\partial_s\,, \\ \frac{d}{d\mu_S} &= -\partial_s \end{aligned}$$

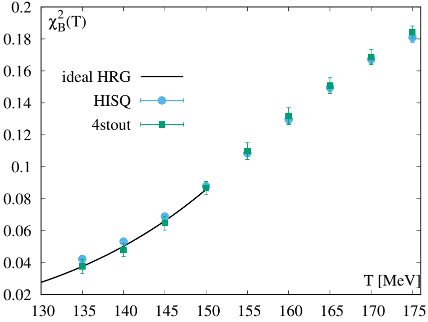
we have

$$\begin{aligned} \chi_2^B &= \frac{1}{VT}\frac{1}{9}\left[2\partial_u^2 + \partial_s^2 + 4\partial_u\partial_s + 2\partial_u\partial_d\right]\log Z\,, \\ \chi_2^Q &= \frac{1}{VT}\frac{1}{9}\left[5\partial_u^2 + \partial_s^2 - 2\partial_u\partial_s - 4\partial_u\partial_d\right]\log Z\,, \\ \chi_2^I &= \frac{1}{VT}\frac{1}{2}\left[\partial_u^2 - \partial_u\partial_d\right]\log Z\,. \end{aligned}$$

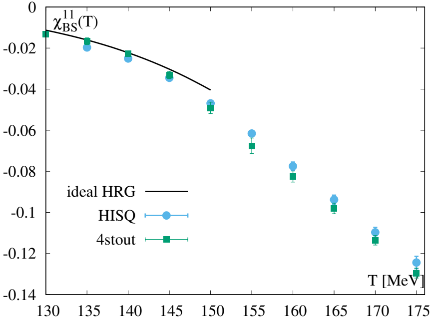
$$\begin{aligned} \chi_4^B &= \tfrac{1}{81} \left[2\chi_4^u + \chi_4^s + 6\chi_{22}^{ud} + 12\chi_{22}^{us} + 8\chi_{13}^{us} + 8\chi_{31}^{us} + 8\chi_{31}^{ud} + 24\chi_{211}^{uds} + 12\chi_{112}^{uds}\right]\,, \\ \chi_4^Q &= \tfrac{1}{81} \left[17\chi_4^u + \chi_4^s + 24\chi_{22}^{ud} + 30\chi_{22}^{us} - 4\chi_{13}^{us} - 28\chi_{31}^{us} - 40\chi_{31}^{ud} + 24\chi_{211}^{uds} - 24\chi_{112}^{uds}\right] \end{aligned}$$

Let us compare the fluctuation results of two groups:

Baryon fluctuations



Baryon-Strange correlator



HISQ results: [HotQCD [2107.10011]]

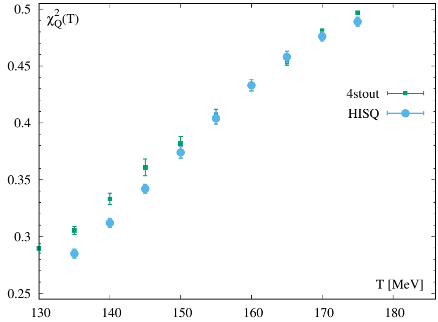
4stout results: [Wuppertal-Budapest [1507.04627,1910.14592]]

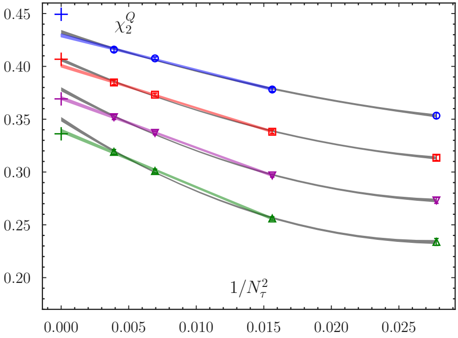
A not so friendly observable: χ_2^Q

The agreement is no longer so nice for the electric charge fluctuations.

HISQ vs 4stout extrapolation

Continuum extrapolation (HISQ/HotQCD)





HISQ results: [HotQCD [2107.10011]]

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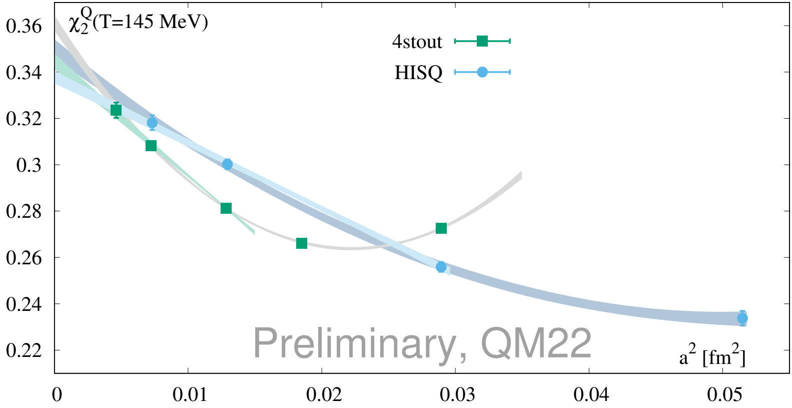
A closer look at $\chi_2^Q(T = 145\ \mathrm{MeV})$

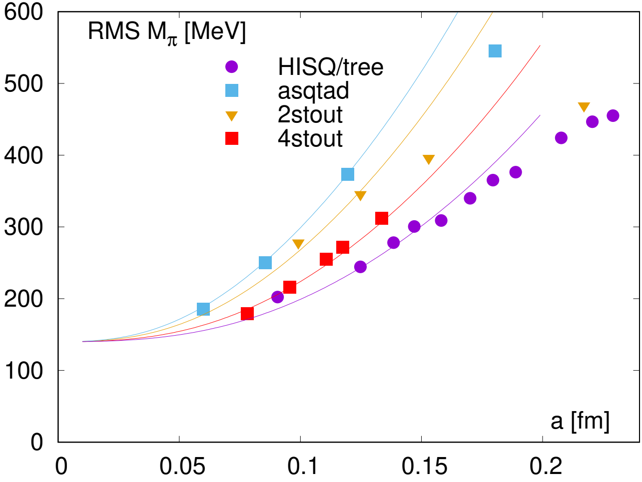
The Nr.1 source of errors:

ambiguities in the continuum extrapolation



Linear and non-linear extrapolations give different results, this persists even in high statistics simulations. Which one should we believe?

Taste symmetry breaking: *(a plot from 2014)*



Several pion operators with unphysical masses – even though the lowest state was tuned to be physical: 135 MeV

Role of the gauge action

$$S_g(U)=\frac{\beta}{3}\left[c_0\sum_{n,\mu<\nu}W^{1\times1}_{\mu\nu}(n)+c_1\sum_{n,\mu\neq\nu}W^{1\times2}_{\mu\nu}(n)\right]$$

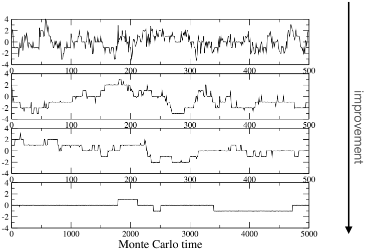
$c_1 = 0$ Wilson

$-1/12$ tree level Symanzik

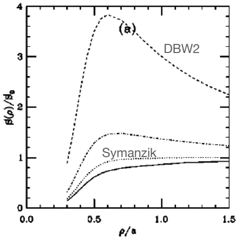
-0.331 Iwasaki

-1.4088 DBW2.

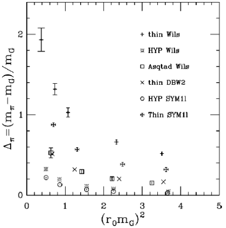
Topological charge history



Action of smooth instantons



Taste breaking: smaller with horrible actions

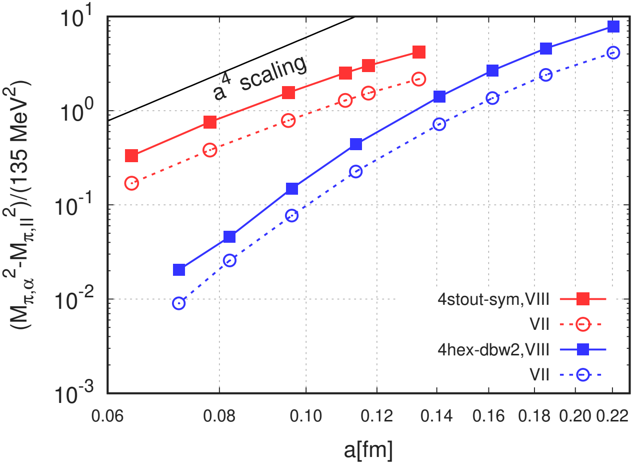


De Grand, Hasenfratz, Kovács PRD 67, 054501 (2003)

The reason of ambiguous continuum scaling

Large discretization errors on the Pion-dominated $\chi_Q^2(T)$:

pions receive mass contamination (taste breaking)



This effect vanishes in the continuum limit,

for some actions this happens faster, for some others this happens slower

Thermodynamics with the 4HEX action

- 4 steps of HEX smearing + DBW2 gauge action to drastically reduce the taste breaking effect
- Physical point defined by $m_\pi/f_\pi = 1.0337$, $m_s/m_{\mathrm{light}} = 27.63$
- m_q tuning and f_π-based scale determination in the a range: $0.22 \ldots 0.072\ \mathrm{fm}$
- Thermodynamics runs
  6 Temperatures $135 \ldots 160\ \mathrm{MeV}$
  4 lattice spacings $32^3 \times 8$, $40^3 \times 10$, $48^3 \times 12$, $64^3 \times 16$
  Statistics: cca 10000 configurations / ensemble
  $N_t = 16$: half statistics so far: $\rightarrow$ results are PRELIMINARY
- Results are less than a month old ...
- A full systematic analysis is yet to be made
  $\rightarrow$ results are PRELIMINARY

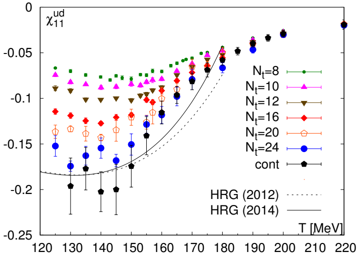
Up-down correlator: the strength of the sign problem

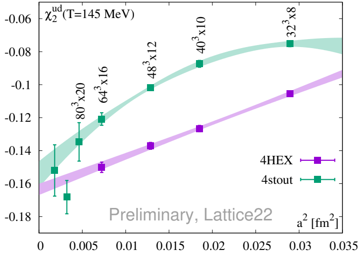
The complex phase of the fermion determinant $\sim$ the light quark density.

[(See formula 5.2 of Allton et al hep-lat/0501030)]

$$\det M = |\det M| e^{i\theta} \qquad \theta = \frac{1}{4} N_f \mathrm{Im} \left[\mu \underbrace{\frac{\partial \ln \det M}{\partial \mu}}_{\text{light quark density}} + \dots \right]$$

$$\langle \theta^2 \rangle = -\frac{1}{9} \mu_B^2 L^3 T N_f^2 \chi_{11}^{ud} \qquad \text{with} \quad \chi_{11}^{ud} \sim \partial^2 \log Z / \partial \mu_u \partial \mu_d \,.$$

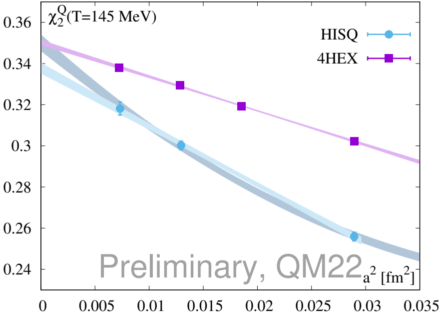




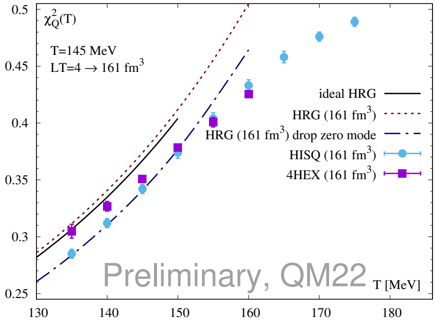
[4stout result: Wuppertal-Budapest 1507.04627]

The 4HEX extrapolation is remarkably linear!

Continuum extrapolation $T = 145$ MeV



Continuum results for all T



HISQ results: [HotQCD [2107.10011]]

4HEX results: [Wuppertal-Budapest **this work**]

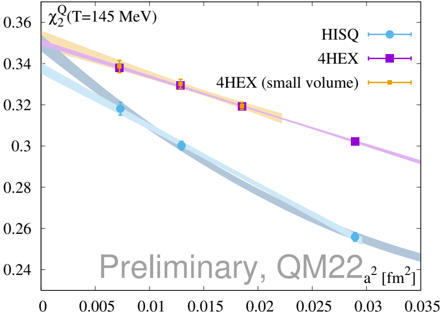
Is the deviation from ideal HRG a volume effect?

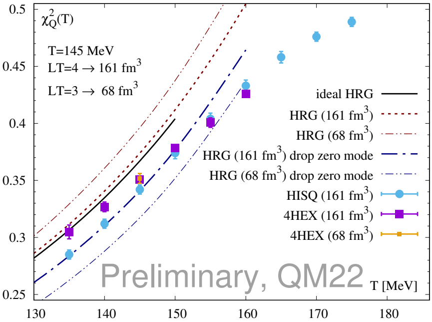
Result for χ_2^Q

For the volume effects: *We rerun the simulations on smaller volumes:* $30^3 \times 10$, $36^3 \times 12$ *and* $48^3 \times 16$

Continuum extrapolation $T = 145$ *MeV*

Continuum results for all T



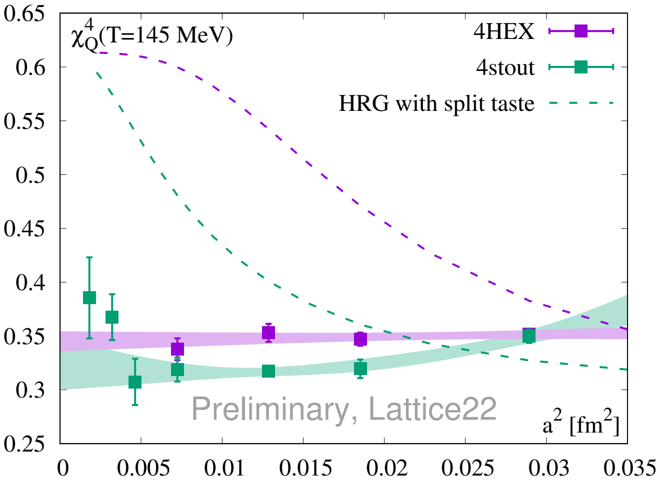


HISQ results: [HotQCD [2107.10011]]

4HEX results: [Wuppertal-Budapest **this work**]

The deviation from ideal HRG is not likely to be a volume effect.

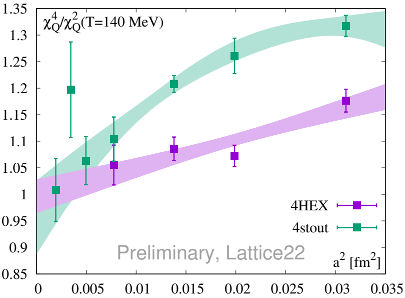
Higher order electric charge correlator

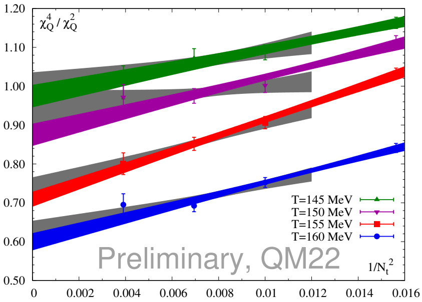


Hadron Resonance Gas model was calculated with a split pion spectrum, as we observed in $T = 0$ lattice simulations with the respective action.

Candidate for chemical freeze-out thermometer:

$$\frac{\chi_4^Q}{\chi_2^Q} = \kappa\sigma^2$$

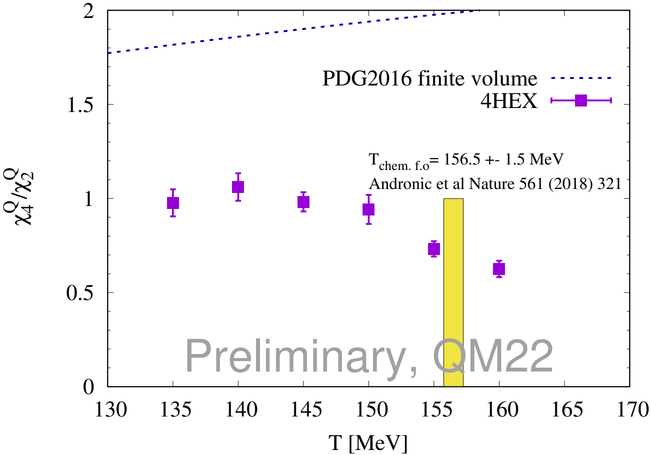




A consistent continuum extrapolation is possible

– the result is far from the ideal HRG

Candidate for chemical freeze-out thermometer: $\frac{\chi_4^Q}{\chi_2^Q} = \kappa\sigma^2$



Monotonic temperature dependence from 140 MeV onwards.

Cca. 0.7 around freeze-out temperature.

Summary

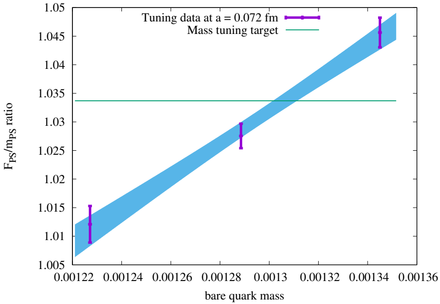
- First thermodynamics results with the new 4HEX staggered action was presented.
- Continuum extrapolation using $N_t = 8, 10, 12, 16$ was linear in most cases.
- Finite volume effects found to be negligible, shown for $\chi^2_Q(T = 145\ \mathrm{MeV})$
- With the precision of the continuum results we see significant deviations from ideal hadron resonance gas prediciton. Improvements can be benchmarked against simulation results.

Thank you!

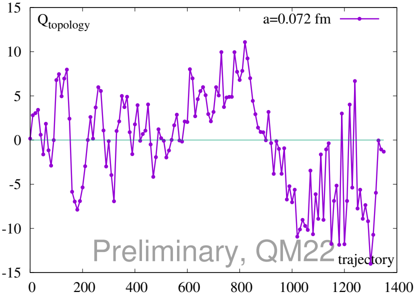
Backup slides

Lattice spacing range: 0.22 .. 0.072 fm
Here: the *finest* lattice (0.072 fm)

Quark mass tuning

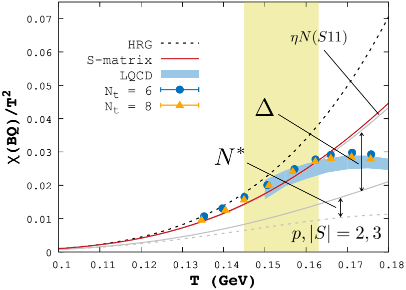


Topological charge history



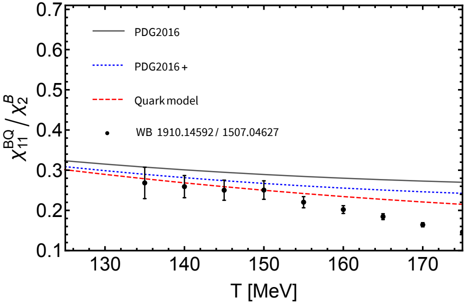
Below T_{pc} QCD is expected to show features of a hadron gas.

Partial wave analysis
in the S-matrix formalism



[Pok Man Lo et al [1710.02711]]

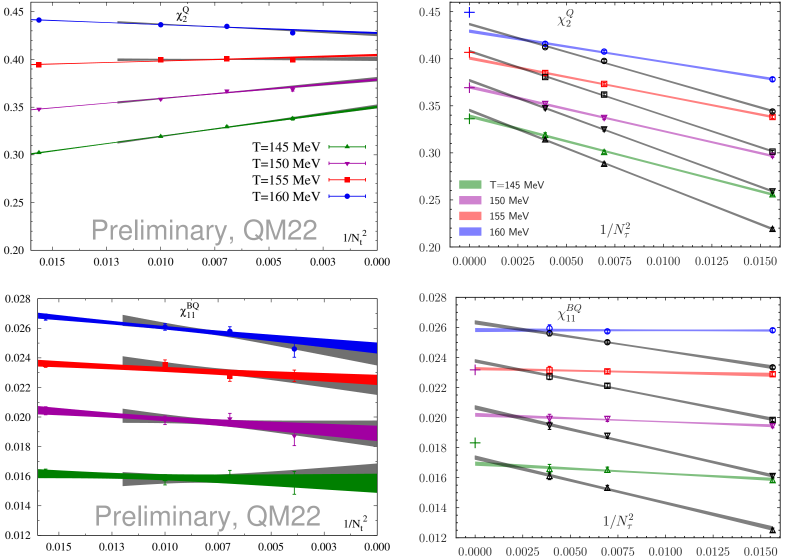
Extensions to ideal HRG
in terms of extended resonance spectrum



[Jamie Karthein et al [2107.00588]]

More continuum scalings 4HEX vs HISQ

Left: 4HEX (this work), Right HISQ scaling [hotQCD [2107.10011]]



The partition function of the lattice gauge theory with staggered fermions is

$$Z = \int \mathcal{D}U \, e^{-S_g} (\det M_u(\mu_u))^{1/4} (\det M_d(\mu_d))^{1/4} (\det M_s(\mu_s))^{1/4} = \int \mathcal{D}U \, e^{-S_{\rm eff}}$$

where S_g is the gauge action. First derivative of the free energy density:

$$\partial_i \log Z = \frac{1}{Z} \int \mathcal{D}U \, \partial_i e^{-S_{\rm eff}} = \langle A_i \rangle \; , \quad A_i = \frac{1}{4} \frac{d \log \det M_i(\mu_i)}{d\mu_i} = \frac{1}{4} \mathrm{Tr} M' M^{-1}$$

When we make further derivatives, the following chain rule applies:

$$\begin{aligned} \partial_j \langle X \rangle &= -\langle X \rangle (\partial_j \log Z) + \left\langle X \partial_j e^{-S_{\rm eff}} \right\rangle + \langle \partial_j X \rangle \\ &= \langle X A_j \rangle - \langle X \rangle \langle A_j \rangle + \langle \partial_j X \rangle \; . \end{aligned}$$

With these rules the 2nd derivative reads

$$\partial_i \partial_j \log Z = \langle A_i A_j \rangle - \langle A_i \rangle \langle A_j \rangle + \delta_{ij} \langle dA_i / d\mu \rangle$$

Isospin and baryon directions:

$$\begin{aligned} (\partial_u - \partial_d)^2 \log Z &= 2 \langle dA/d\mu \rangle \, , \\ (\partial_u + \partial_d)^2 \log Z &= 4[\langle AA \rangle - \langle A \rangle \langle A \rangle] + 2 \langle dA/d\mu \rangle \end{aligned}$$