Heavy quark diffusion coefficient with gradient flow

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Based on:
Nora Brambilla[1], V.L.[1], Julian Mayer-Steudte[1], Péter Petreczky[2]: hep-lat/2206.02861

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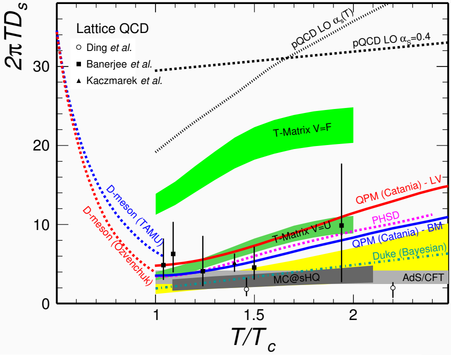
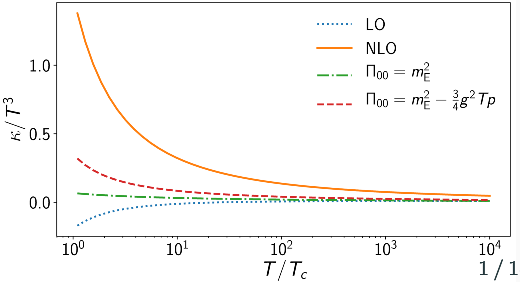
Motivation

- The strongly coupled plasma can be described in terms of transport coefficients
- R_{AA} and v_2 described by spatial diffusion coefficient D_x
- Different models predicting wide range of values
- Perturbative series unreliable
- HTL has too strict assumtion $m_{\mathrm{E}} \ll T$
- Non-perturbative lattice simulations needed





Upper figure: X. Dong CIPANP (2018)

Heavy Quark diffusion

- Heavy quark energy changes only little when colliding with medium

$$E_k \sim T\,, \quad p \sim \sqrt{MT} \gg T$$

- HQ momentum is changed by random kicks from the medium

  $\rightarrow$ Brownian motion; Follows Langevin dynamics

$$\frac{dp_i}{dt} = -\frac{\kappa}{2MT} p_i + \xi_i(t)\,, \quad \langle \xi(t)\xi(t') \rangle = \kappa\delta(t-t')$$

- Heavy quark momentum diffusion coefficient κ related also to:

  Spatial diffusion coefficient $D_\mathrm{s} = 2T^2/\kappa$,

  Drag coefficient $\eta_\mathrm{D} = \kappa/(2MT)$,

  Heavy quark relaxation time $\tau_\mathrm{Q} = \eta_\mathrm{D}^{-1}$

- Considering full Lorentz force:

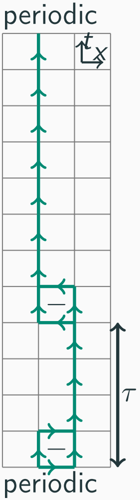
$$F(t) = \dot{p} = q\,(E + v \times B))\,(t)$$

- $\langle v^2 \rangle \sim \mathcal{O}(\frac{T}{M})$ correction to HQ momentum diffusion

$$\kappa_\mathrm{tot} \simeq \kappa_\mathrm{E} + \frac{2}{3}\langle \mathrm{v}^2 \rangle \kappa_\mathrm{B}$$

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)
A. Bouttefeux and M. Laine JHEP 12 (2020) 150, M. Laine JHEP 06 (2021) 139

Heavy quark diffusion from lattice: Euclidean Correlator



- Traditional approach uses HQ current-current correlators:
  Problem: Transport peak at zero
- HQEFT inspired Euclidean correlator is peak free

$$G_{\mathrm{E}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\langle \mathrm{Re\,Tr}\,[U(\beta,\tau)gE_i(\tau,0)U(\tau,0)gE_i(0,0)]\rangle}{\langle \mathrm{Re\,Tr}\,[U(\beta,0)]\rangle}$$

$$G_{\mathrm{B}}(\tau) = \sum_{i=1}^{3}\frac{\langle \mathrm{Re\,Tr}\,[U(1/T,\tau)B_i(\tau,0)U(\tau,0)B_i(0,0)]\rangle}{3\,\langle \mathrm{Re\,Tr}\,U(1/T,0)\rangle}$$

- Field strength tensor components need discretization
- Choose corner discretization for this study
- On lattice there is a self-energy contribution that generates a multiplicative renormalization
- For chromomagnetic fields there is also a finite anomalous dimension and renormalization is required

Caron-Huot *et.al.*JHEP04 (2009) 053, Christensen and Laine PLB02 (2016)

Heavy quark diffusion from lattice: Spectral function

$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho(\omega)\frac{\cosh\left(\frac{\omega}{T}[\tau T - \frac{1}{2}]\right)}{\sinh\frac{\omega}{2T}} \qquad \kappa = \lim_{\omega\to 0}\frac{2T}{\omega}\rho(\omega)$$

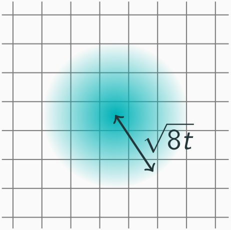
- Euclidean correlator related to spectral function
- Needs inversion of integral equation
- We use simple procedure of modeling $\rho(\omega)$ and comparing to lattice data
    - $\rho(\omega)$ known at IR and UV
    - Connect IR and UV with i) step ii) line
- Related: γ not measured yet

$$\gamma = -\frac{1}{3N_c}\int_0^\infty \frac{d\omega}{2\pi}\frac{\rho(\omega)}{\omega} = -\int_0^\infty d\tau\, G_{\mathrm{E}}(\tau)$$

Lattice simulations

$\frac{T}{T_c}$	N_t	N_s	β	N_{conf}	$\frac{T}{T_c}$	N_t	N_s	β	N_{conf}
1.5	20	48	7.044	4290	10000	20	48	14.635	1890
	24	48	7.192	4346		24	48	14.792	2280
	28	56	7.321	5348		28	56	14.925	2190
	34	68	7.483	3540		34	68	15.093	1830

- Pure gauge simulations, two temperatures
- Gradient flow with Lüscher-Weisz action
  - Smears the initial gauge field with radius $\sqrt{8t}$
  - \+ Automatically renormalizes gauge invariant observables
  - \+ Can be used un-quenched (This work: quenched)
  - Generally needs zero flowtime limit



Lüscher JHEP 1008, 071 (2010)

Renormalization and spectral function: G_{E}

- Normalize the data with perturbative LO result (also tree-level improve)

$$G_{\mathrm{E,B}}^{\mathrm{norm}} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right]$$

- On Lattice E has non-physical self-energy contribution
$Z_{\mathrm{E}} = 1 + g_0^2 \times 0.137718569\ldots + \mathcal{O}(g_0^4)$ (Christensen and Laine PLB02 (2016))
- In practice LO perturbative Z_{E} not enough, we normalize at a single point instead
- Model the spectral function by connecting known IR and UV behavior with ansatz:

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

$$\rho_{\mathrm{QCD,naive}}(\omega) = \frac{g^2 C_F \omega^3}{6\pi} \left\{ 1 + \frac{g^2}{(4\pi)^2} \left[N_c \left(\frac{11}{3} \ln \frac{\mu^2}{4\omega^2} + \frac{149}{9} - \frac{8\pi^2}{3} \right) \right] \right\}$$

- Set scale such that NLO UV contribution vanishes

Renormalization and spectral function: G_{B}

- κ_{B} more complicated, requires renormalization

$$G_{\mathrm{B}}^{\mathrm{flow,UV}}(\tau,\tau_{\mathrm{F}}) = (1+\gamma_0 g^2 \ln(\mu\sqrt{8\tau_{\mathrm{F}}}))^2 Z_{\mathrm{flow}} G_{\mathrm{B}}^{\overline{\mathrm{MS}},\mathrm{UV}}(\tau,\mu) + h_0 \cdot (\tau_{\mathrm{F}}/\tau)\,,$$

- Normalize at finite flow time with G^{flow}
- Assuming $h_0 = 0$ for simplicity. We see very little flow time dependence even with this assumption.
- We know Z_E wasn't enough, so we determine Z_{flow} similarly, i.e, just normalize at a point.
- Use same tree-level improvement as for E-correlator
- IR model the same:

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

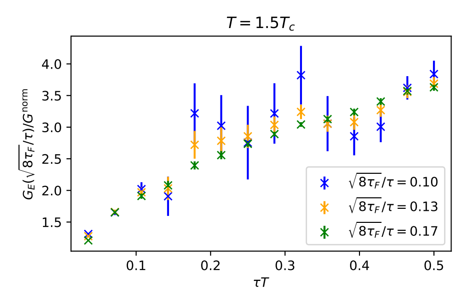
- UV now depends on flow time

$$\rho_{\mathrm{B}}^{\mathrm{UV}}(\omega,\tau_{\mathrm{F}}) = Z_{\mathrm{flow}} \frac{g^2(\mu)\omega^3}{6\pi}(1+g^2(\mu)(\beta_0-\gamma_0)\ln(\mu^2/(A\omega^2))+g^2(\mu)\gamma_0 \ln(8\tau_{\mathrm{F}}\mu^2)$$

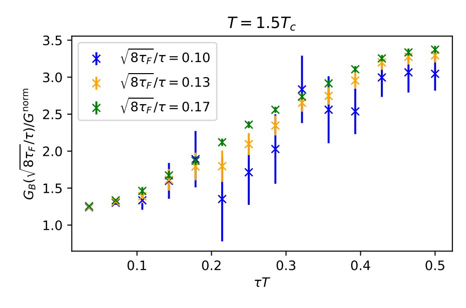
- Again, set the scale such that the second term vanishes

Banerjee et.al. hep-lat/2204.14075

Raw (normalized*, tree-level improved) lattice data

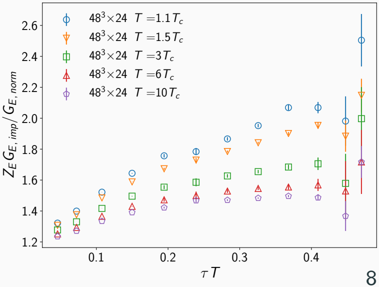


G_{E}

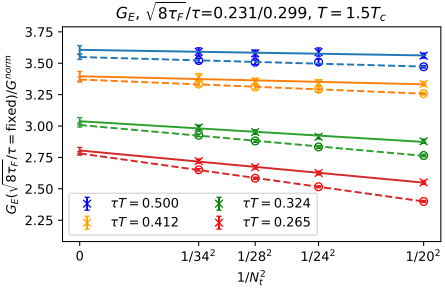


G_{B}

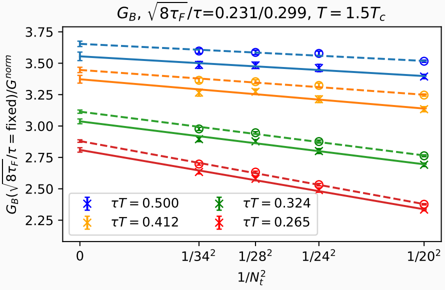
- κ dominant cause for the shape, not flow time
- Very similar shapes for electric and magnetic correlators
- Can see agreement between GF and multilevel (cross-check)



Continuum limit



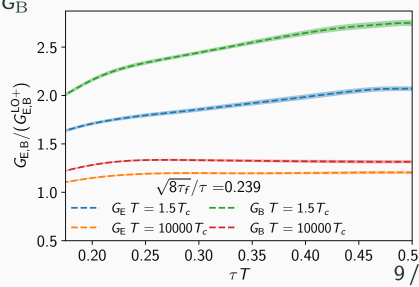
G_{E}



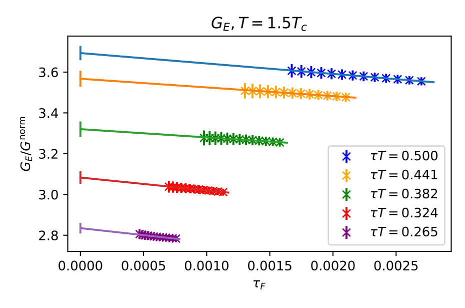
G_{B}

- Good limits for valid ranges of τT and τ_{F}

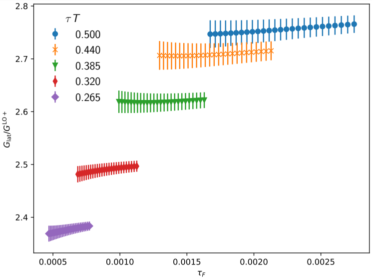
$$\frac{a}{\tau} \leq \frac{\sqrt{8\tau_{\mathrm{F}}}}{\tau} \leq \frac{1}{3}$$



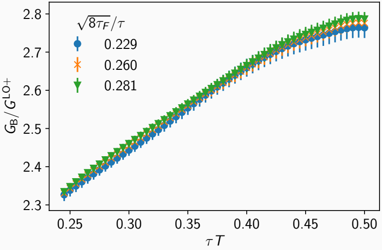
Flow time dependence of G_{E} and G_{B}



G_{E}

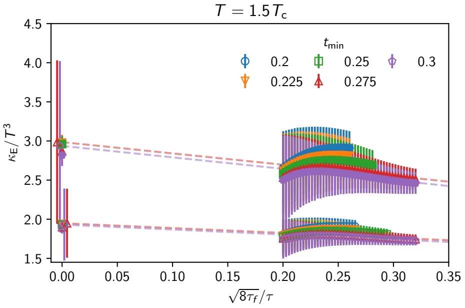


G_{B}

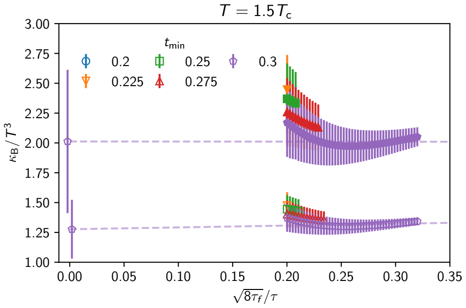


- We observe different small flow time scaling between G_{E} and G_{B}
- Flow dependent spectral function seems to remove most of flow dependence from G_{B}

Flow time κ and order of limits

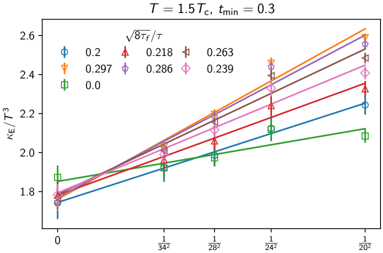


$G_{\rm E}$

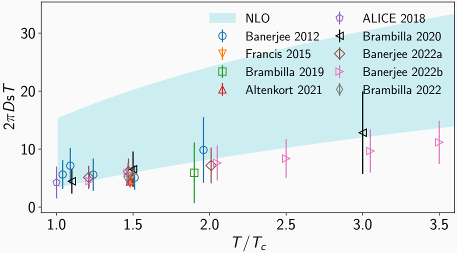


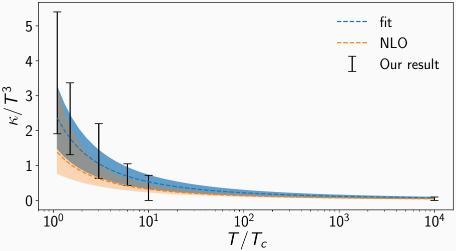
$G_{\rm B}$

- Very little dependence on flow time
- Ordering of Continuum limit, zero flow time limit, and spectral function inversion doesn't seem to matter much



$\kappa_{\rm E}$ results





- Our results:
  (Brambilla *et.al.* hep-lat/2206.02861)

$$1.7 \leq \kappa_{\rm E}/T^3 \leq 3.12\,, \quad T = 1.5$$

$$0.02 \leq \kappa_{\rm E}/T^3 \leq 0.16\,, \quad T = 10^4$$

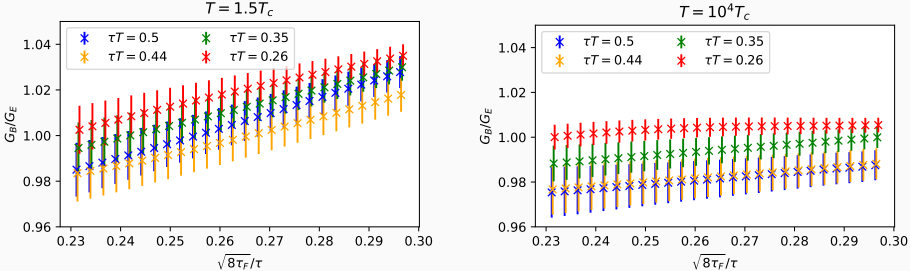
- Can fit temperature dependence:

$$\frac{\kappa^{\rm NLO}}{T^3} = \frac{g^4 C_{\rm F} N_{\rm c}}{18\pi}\left[\ln\frac{2T}{m_{\rm E}} + \xi + C\frac{m_{\rm E}}{T}\right].$$

- Other similar lattice studies

  Meyer NJP13 (2011),
  Ding *et.al.*JPG38 (2011),
  Banerjee *et.al.* PRD85 (2012),
  Francis *et.al.* PRD92 (2015)
  Brambilla *et.al.* PRD102 (2020)
  Altenkort *et.al.* PRD103 (2021)
  Banerjee *et.al.* hep-lat/2204.14075
  Banerjee *et.al.* hep-lat/2206.15471

$\kappa_{\rm B}$ Results



- We get $1.03 \leq \kappa_{\rm B}/T^3 \leq 2.61$ (In agreement: $1.0 \leq \kappa_{\rm B}/T^3 \leq 2.1$)
  Brambilla *et.al.* hep-lat/2206.02861 Banerjee *et.al.* hep-lat/2204.14075

$$\kappa_{\rm tot} \simeq \kappa_{\rm E} + \frac{2}{3}\langle v^2\rangle \kappa_{\rm B}$$

- Using $\langle v^2 \rangle$ from (Petreczky *et.al.* Eur. Phys. J. C62 (2009))
- $\langle v^2_{\rm charm}\rangle \simeq 0.51$ and $\langle v^2_{\rm bottom}\rangle \simeq 0.3$, we get that the mass suppressed effects on the heavy quark diffusion coefficient is 34% and 20% for the charm and bottom quarks respectively.

Conclusions and Future prospects

- Measured in wide range of temperatures
- Fit temperature dependence
- Agreement to perturbation theory at high T
- Agreement to previous results at small T
- Measured $1/M$ corrections
  - Good agreement with other recent study
  - The mass correction indicated to be 20 to 30% for bottom and charm quarks
- Future prospects:
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  - Go un-quenched
  - Check adjoint representation to study quarkonium

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Thank you for your attention!