QCD topology with electromagnetic fields and the axion-photon coupling

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CRC-TR 211 Strong-interaction matter under extreme conditions

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Outline

- Topology in QCD
- Axions and topology
- $g_{a\gamma\gamma}$, on the lattice?
- Preliminary results: $g_{a\gamma\gamma}^{QCD}$ and χ_{top}
- Conclusions and further work

Topology in QCD

Topology in QCD

- We usually expect that our gluon fields vanish at the boundary $|x| \to \infty$,

$$A_\mu(x) = 0.$$

- But we need to consider all possible gauge transformations:

$$A_\mu = i\Omega\partial_\mu\Omega^\dagger, \;\; \Omega \in SU(3).$$

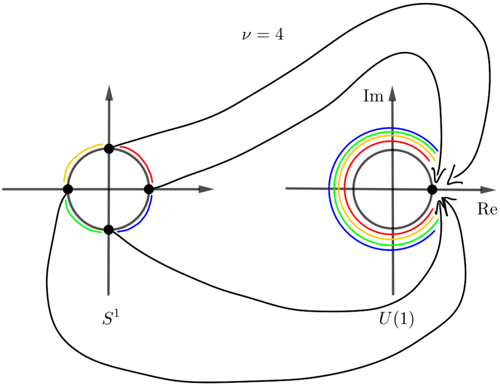
Hence we have an infinite set of solutions.

- They can be classified by an integer label, the "winding number".

$$Q_{top} = \int d^4x \, q_{top}(x), \;\; q_{top} = \frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\, G_{\mu\nu} G_{\rho\sigma}.$$

In general, $Q \propto \int F\widetilde{F}$ for any gauge group.

Winding numbers in $U(1)$



Axions and topology

Strong CP problem

- In principle, the QCD Lagrangian should include an extra term:

$$\mathcal{L}_{\mathrm{QCD}+\theta} = \mathcal{L}_{\mathrm{QCD}} + \theta\, q_{top}.$$

  - This term is CP and T odd.
  - Induces an electric dipole moment in n: $|\theta| < 10^{-10}$ Abel et al 2020.
- Why don't we see it 😞? $\longrightarrow$ Axions 🤔? Peccei, Quinn '77

$$\mathcal{L}_a = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{f_a} q_{top} + \mathcal{L}_{\mathrm{int}}.$$

- Essence of the idea: new pseudoscalar a whose minimum is $\langle a \rangle = -\theta f_a$.

Topological susceptibility χ_{top}

- Is the second moment of Q_{top}: $\chi_{top} = \frac{T}{V}\langle Q_{top}^2 \rangle$
- It is also the mass of the axion:

$$m_a^2 = \frac{T}{V}\frac{\delta^2}{\delta a^2}\log \mathcal{Z}\left(\frac{a}{f_a}\right)\Bigg|_{a=0} = \frac{1}{f_a^2}\frac{T}{V}\frac{\delta^2}{\delta\theta^2}\log \mathcal{Z}(\theta)\Bigg|_{\theta=0} = \frac{\chi_t}{f_a^2}.$$

- Hence, an analysis of χ_t gives information on m_a.
- Current estimates from ChPT. : $m_a = 5.70(6)(4)(10^{12}\mathrm{GeV}/f_a)$ μeV Cortona et al 2016.
  - Lattice calculations give almost the same central value but with a bigger error Borsanyi et al 2016.
- It also gives us cosmological information about a.

The photon-axion coupling $g_{a\gamma\gamma}^{QCD}$

- The axion couples directly and indirectly to photons.
- ChPT calculations show:

$$\mathcal{L}_{a\gamma\gamma} = \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \widetilde{F}^{\mu\nu} = g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B}.$$

- Current estimates from ChPT.: $g_{a\gamma\gamma} = g^0_{a\gamma\gamma} + g^{QCD}_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a} \left(\frac{E}{N} - 1.92(4) \right)$ Cortona et al 2016.

Q_{top} and $g_{a\gamma\gamma}^{QCD}$

- If we include both electric and magnetic background fields, the only CP odd operators in our theory are:

$$\mathrm{Tr}\, G_{\mu\nu}\widetilde{G}^{\mu\nu} \quad \& \quad \mathbf{E}\cdot\mathbf{B}.$$

So by symmetry arguments, Q_{top} can only be (for weak fields):

$$Q_{top} \propto \mathbf{E}\cdot\mathbf{B} + \mathcal{O}\left([\mathbf{E}\cdot\mathbf{B}]^3\right).$$

- By looking at $\mathcal{Z}$:

$$\left.\frac{\delta \log \mathcal{Z}(a)}{\delta a}\right|_{a=0} = \frac{\langle Q_{top}\rangle_{E,B}}{f_a} \longrightarrow g_{a\gamma\gamma}^{QCD} f_a = \frac{T}{V}\frac{\partial^2}{\partial\mathbf{E}\partial\mathbf{B}}\langle Q_{top}\rangle_{E,B}\Big|_{\mathbf{E},\mathbf{B}=0}$$

- So for homogeneous, static and weak EM fields, we can assume:

$$\frac{T}{V}\langle Q_{top}\rangle_{E,B} \approx \frac{g_{a\gamma\gamma}^{QCD}\cdot f_a}{e^2} e^2\mathbf{E}\cdot\mathbf{B} \quad \textit{and} \; g_{a\gamma\gamma}^{QCD} < 0.$$

Computation of $g_{a\gamma\gamma}^{QCD}$ on the lattice

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- Three ways come to mind for obtaining $g_{a\gamma\gamma}^{QCD}$:
  - Measuring the index of D and fitting to $\mathbf{E}\cdot\mathbf{B}$. Recent idea.
  - Computing $\frac{T}{V}\frac{\partial^2 \log Z}{\partial E \partial a}\Big|_{a,E=0}$ and fitting to $\mathbf{B}$. In implementation.
  - Measuring $\langle Q_{top}\rangle_{E,B}$ and fitting to $\mathbf{E}\cdot\mathbf{B}$. Today's talk!

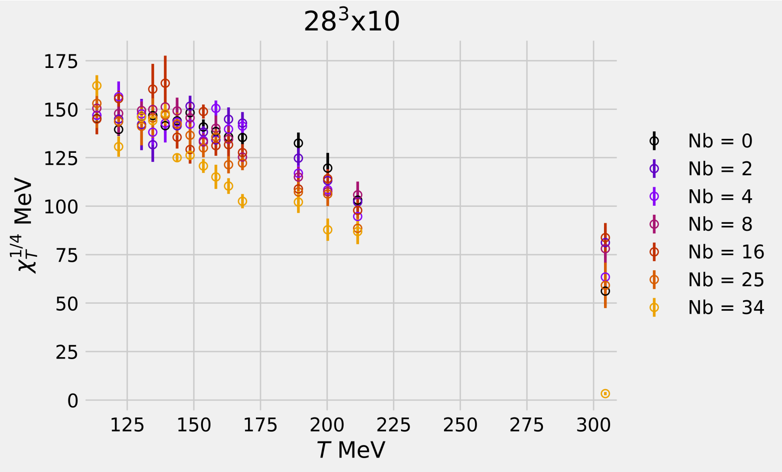
To take into account...

- We are going to simulate gluons with background (homogeneous) EM fields. Thus, we have to deal with two issues:
  1. The sign problem. We can't simulate real electric fields, so $\mathbf{E} \longrightarrow i\mathbf{E}$
  2. UV fluctuations of the gluon fields $\longrightarrow$ *Wilson flow*.

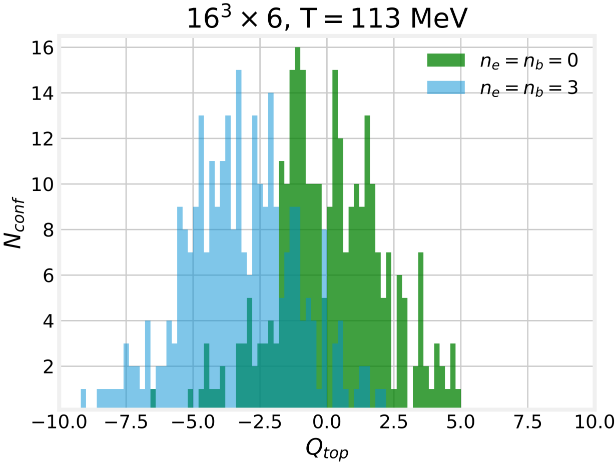
Preliminary results

$\chi_t(B)$ vs T: preliminary results

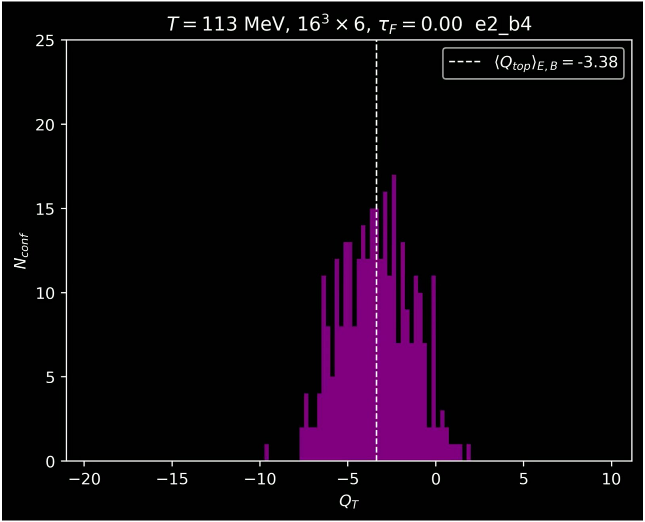
$\chi_t(B)$ as a function of T. Note that $\sqrt{\chi_t} = m_a f_a$.



Shift of Q_{top} at non-zero $\mathbf{E}_I \cdot \mathbf{B}$. Effect also shown in D'Elia et al 2016.

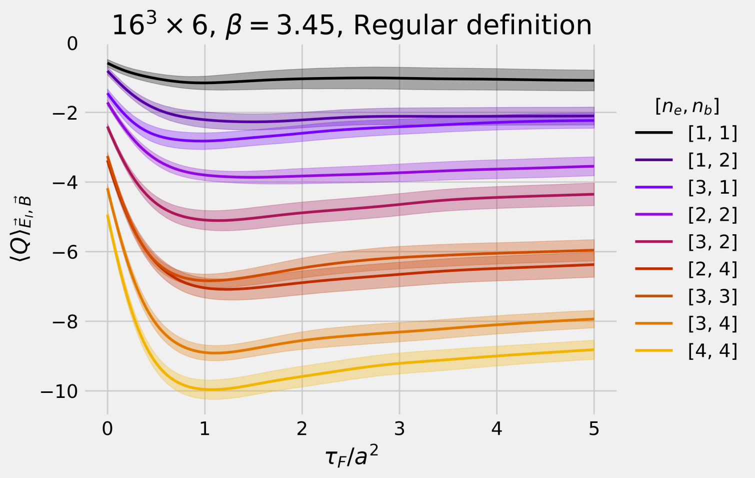


Wilson evolution of $\langle Q_{top} \rangle_{E_I,B}$ for a certain value of $\mathbf{E}_I \cdot \mathbf{B}$.



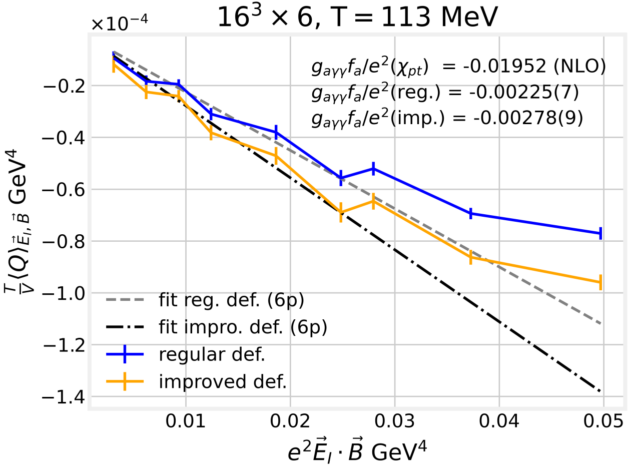
$\langle Q_{top} \rangle_{E_I,B}$ flow: preliminary results (II)

Wilson evolution of $\langle Q_{top} \rangle_{E_I,B}$. Note the plateaus.



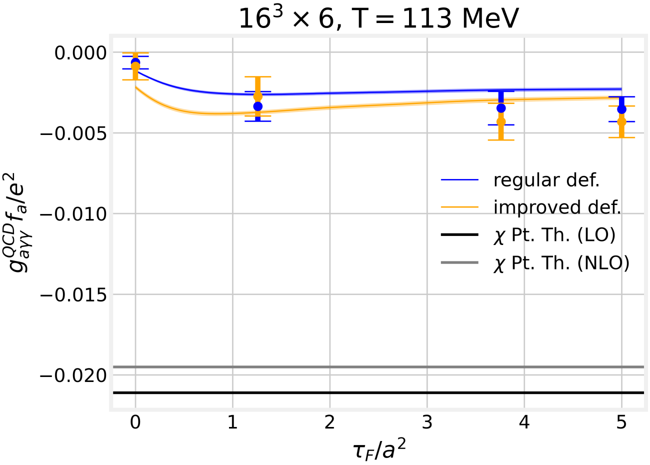
$\langle Q_{top}\rangle_{E_I,B}$ vs $\mathbf{E}_I \cdot \mathbf{B}$: preliminary results

$\langle Q_{top}\rangle_{E_I,B}$ as a function of $\mathbf{E}_I \cdot \mathbf{B}$.



$g_{a\gamma\gamma}^{QCD}$ flow: preliminary results

$g_{a\gamma\gamma}^{QCD}$ as a function of flow time for two different methods.



Conclusions and further work

Conclusions and further work

- We have shown:
  - how there is an interplay between EM fields and $SU(3)$ topology.
  - that there is a linear response of Q_{top} with $\mathbf{E}_I \cdot \mathbf{B}$ for weak fields.
  - preliminary results for χ_{top} as a function of $\mathbf{B}$, T as well as for $g_{a\gamma\gamma}^{QCD}$.
- Further work:
  - Explore the other methods for computing $g_{a\gamma\gamma}^{QCD}$, compare results and study the systematic errors.
  - Generate more statistics and continuum limit.w
  - Eventually $\longrightarrow$ Use experimental bounds and lattice results to constrain axion models.
- For more about EM fields on the lattice $\longrightarrow$ *previous talks by A. D. Marques Valois, Fri. 14:10 and E. Garnacho Velasco, Wed. 17:50!*

Thank you for your attention!

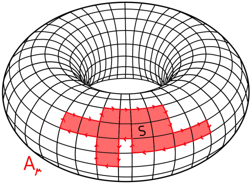
We use $U(1)$ links for the magnetic field: $u_\mu = e^{iaqA_\mu} \in U(1)$.

$$\mathbf{B} = B\hat{z}$$

$$\mathbf{B} = \vec{\nabla} \times \mathbf{A}$$

$$A_y = Bx \qquad A_x = A_z = 0$$

Stoke's theorem must hold in the torus.



$$\oint A_\mu dx_\mu = SB$$

$$\oint A_\mu dx_\mu = (L_x L_y - S)B$$

Unambiguous phase:

$$e^{-iqBS} = e^{iqB(L_x L_y - S)} \qquad qB = \frac{2\pi N_b}{L_x L_y}, \quad N_b \in \mathbb{Z}$$

$$u_x = \begin{cases} e^{-iqBL_x y} & \text{if } x = L_x - a \\ 1 & \text{if } x \neq L_x - a \end{cases}$$

$$u_y = e^{iaqBx} \quad 0 \leq x \leq L_x - a$$

$$u_z = u_t = 1$$

Backup slide: The Atiyah-Singer index theorem

- EM fields can induce topologies in the gluon sector. But how? $\longrightarrow$ Index theorem.
- The index theorem says (for QCD) Atiyah, Singer '71:

$$\text{Index}(D) \equiv n_- - n_+ = Q_{top}$$

Since in QCD $\langle Q_{top} \rangle = 0$, we don't see imbalances in chirality.

- But after including electromagnetic fields the situation is different:

$$Q_{top} \longrightarrow Q_{top} + Q_{U(1)}.$$

We have two different topological contributions to the zero modes.

- Path integral favours as little zero modes as possible: $\det M \uparrow\uparrow$.
- Hence, it selects gluon field configurations such that:

$$Q_{U(1)} \uparrow \iff Q_{top} \downarrow .$$