Topological features of the deconfinement transition in quenched QCD

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Topology at the transition region

In the crossover region of the QCD transition several observables go through rapid changes.
Some known phenomena:

- The absolute value of the Polyakov loop $|P|$ increases,
- Dirac operator spectrum: $\rho(\lambda)$ drops to zero at the origin
- Restoration of chiral symmetry, marked by the drop of $\left\langle \bar{\Psi}\Psi \right\rangle$
- Suppression of topological charge Q fluctuations

In this presentation we focus on two parametars of the topological charge density $P(Q)$:

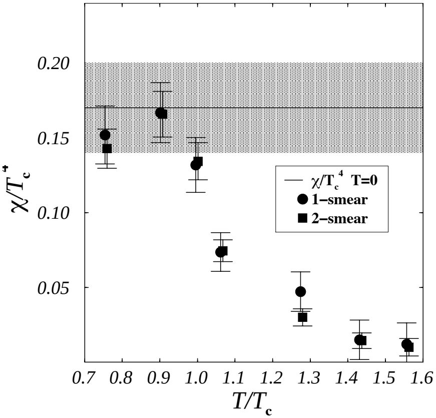
- The topological susceptibility χ
- The b_2 coefficient (from the expansion of the free energy density)

Model

Quenched theory, the limit of large quark masses
→ pure SU(3) gauge theory
→ genuine phase transition

$|P|(T)$ has a discontinuity at the transition.
Is there a discontinuity in $\chi(T)$ also?
What happens with $b_2(T)$?

There are evidences that χ goes through a sudden drop at T_c [Allés et al., 1996]



Topological charge

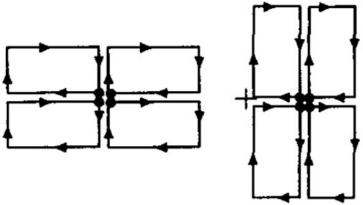
Measured Symanzik improved topological charge on the lattice (from ensemble simulated with Symanzik improved gauge action) :

$$Q = \sum_{m,n\in\{11,12\}} c_{nm} Q_{mn}$$

$$c11 = 10/3, \quad c12 = -1/3$$

$$Q_{mn} = \frac{1}{32\pi^2}\frac{1}{m^2 n^2}\sum_x \sum_{\mu,\nu,\rho,\sigma} \epsilon_{\mu\nu\rho\sigma} Tr(\hat{F}_{\mu\nu}(x;m,n)\hat{F}_{\rho\sigma}(x;m,n))$$

$$\hat{F}_{\mu\nu}(x;m,n) = \frac{1}{8}\,\mathrm{Im}\Big\{$$



$$\Big\}$$

[de Forcrand et al. (1997)]

Moments of the topological charge distribution

The topological susceptibility and b_2 are proportional to the second and fourth cumulants of $P(Q)$:

$$\chi = \frac{\langle Q^2 \rangle}{V_4} \qquad (\langle Q \rangle = 0),$$

$$b_2 = -\frac{\langle Q^4 \rangle - 3 \langle Q^2 \rangle^2}{12 \langle Q^2 \rangle}$$

They also appear in the expansion of $f(\theta \approx 0)$:

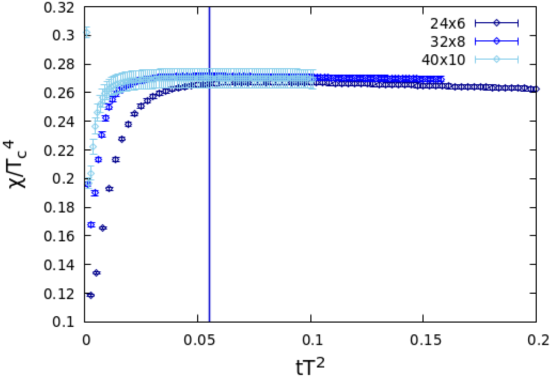
$$f(\theta) = f(0) + \frac{\chi \theta^2}{2} + \frac{\chi b_2 \theta^4}{2} + \mathcal{O}(\theta^6)$$

Topological susceptibility

To make χ at different lattice spacing comparable:

$$\frac{\chi}{T_c^4} = \left(\frac{T}{T_c}\right)^4 \left\langle Q^2 \right\rangle \frac{N_t^4}{V_4}$$

Q is defined throgh the gradient flow.



Flow time is fixed on the physical scale as we go to the continuum

$$tT^2 = 1/18$$

$$t = 1/18 \cdot a^2 \cdot N_t^2$$

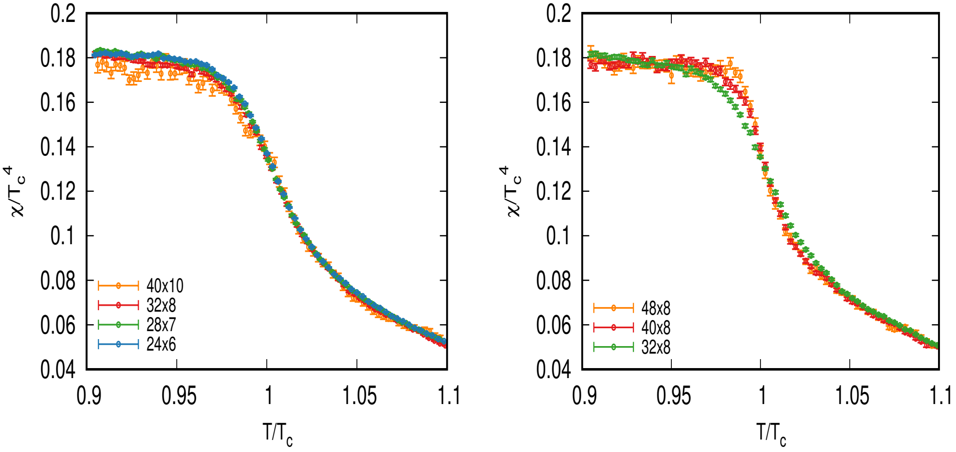
Simulation details

Generation of gauge ensemble with tree level Symanzik improved gauge action. Configurations were generated with parallel tempering around β_c. We set β_c with precision $10^{-4}\frac{T}{T_c}$ and determined gradient flow only at the critical coupling.

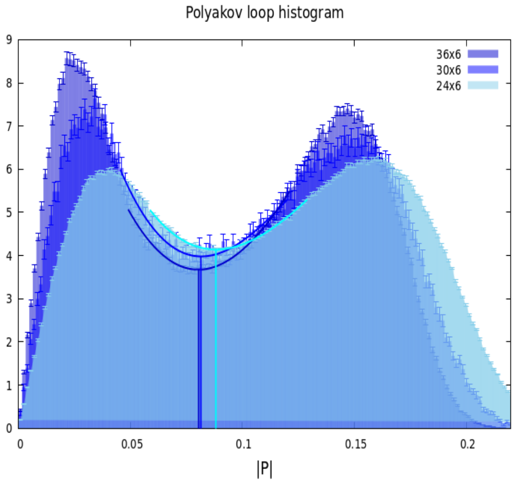
		N_t			
		6	7	8	10
	4	497478	64901	41832	37915
LT	5	20038	6522	36606	13469
	6	67185	7875	53325	6314

We also generated broader temperature scans, in those runs a sequence of stout smearings was used to define Q, with a smearing radius matching our earlier definition.

Topological susceptibility



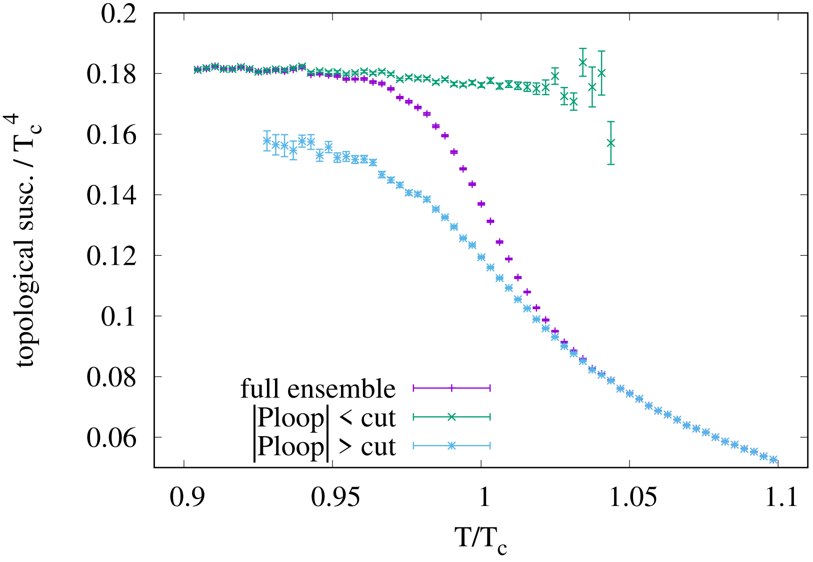
Discontinuity in the Polyakov loop



$|P|$ is the order parameter

Distinguishing hot and cold phases:
finding the minimum between the two peaks of $|P|$

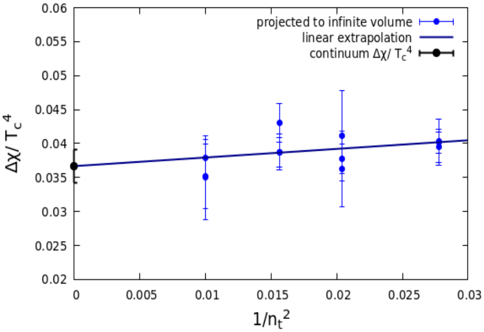
Discontinuity of χ

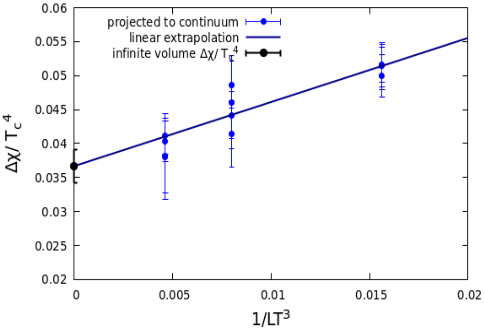


Discontinuity of χ

χ / T_c^4 were calculated in both phases with different values of N_t and LT

We determined the continuum extrapolated values of $\Delta\chi / T_c^4$ in the infinite volume limit.





Result:

$$\Delta\chi / T_c^4 = 0.0367(24)$$

Curvature of the phase line

Clausius-Clapeyron-like equation [D'Elia & Negro 1205.0538]:

$$\frac{T_c(\theta)}{T_c(0)} = 1 - R_\theta \theta^2 + \mathcal{O}(\theta^4)$$

$$R_{\theta,T} = \frac{\Delta\chi}{2\Delta\epsilon}$$

Continuum limit of the curvature $R_\theta = 0.0178(5)$ [D'Elia & Negro 1306.2919] together with continuum limit of the latent heat $\Delta\epsilon/T_c^4 = 1.025(21)_{stat}(27)_{sys}$ [Borsanyi et al. (2022)] gives

$$\Delta\chi/T_c^4 = 0.0365(18)$$

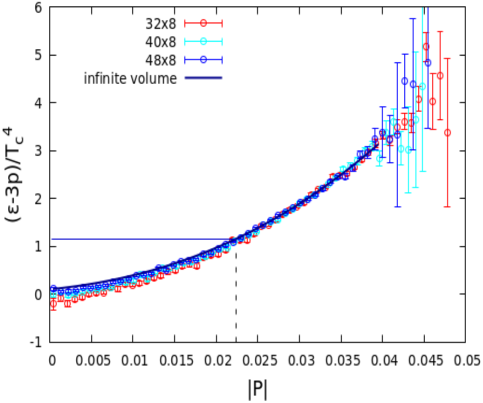
Connection to latent heat

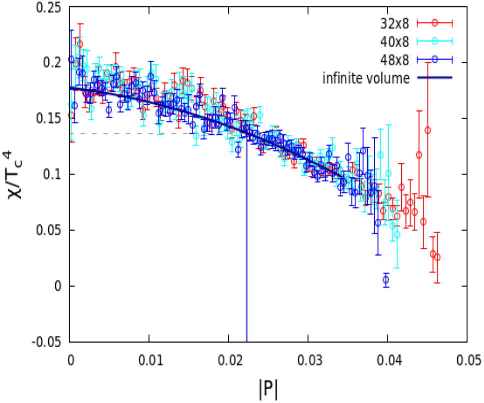
From $\Delta\epsilon$ the infinite volume value of $|P|$ can be determined

Results for $N_t = 8$:

From $\chi(|P|)/T_c^4$: $\Delta\chi/T_c^4 = 0.0408(58)$

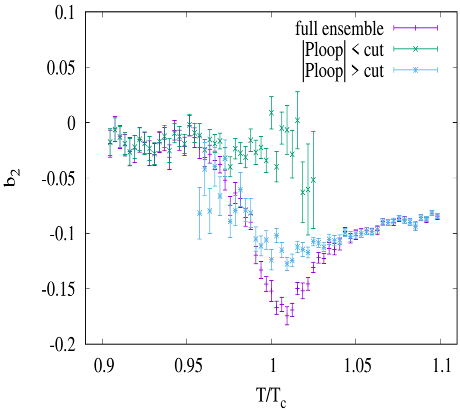
Direct method: $\Delta\chi/T_c^4 = 0.0400(24)$

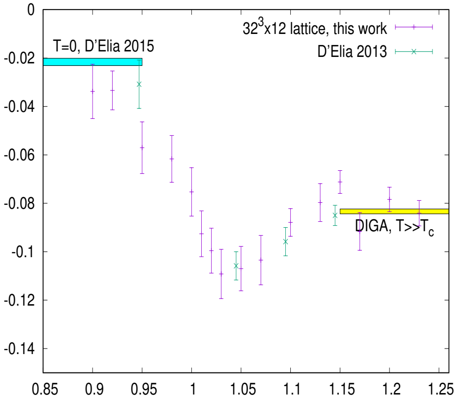




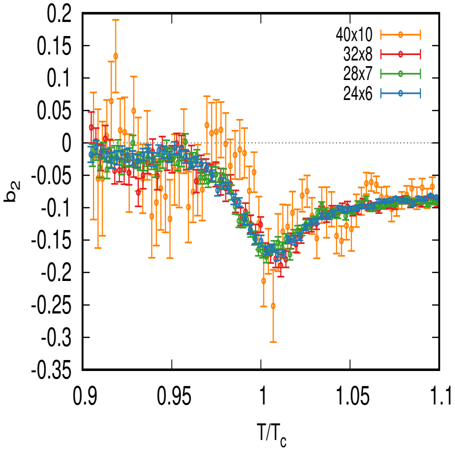
b_2 coefficient

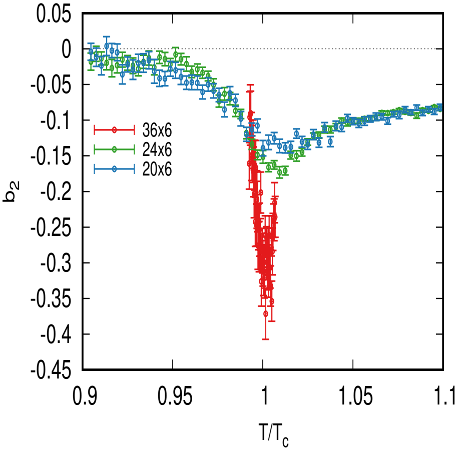
b_2 has a dip where the two phases coexist i.e. it is a sign that the transition is first order.





b_2 coefficient





Summary

We studied the topological features of the 1st order quenched QCD phase transition.
We calculated the discontinuity of $\chi(T)/T_c^4$ at the transition temperature.

Continuum limit	$\Delta\chi/T_c^4$
Direct method	0.0367(24)
From R_θ	0.0365(18)

$N_t = 8$	
Direct method	0.0400(24)
From $\chi(\|P\|)/T_c^4$	0.0408(58)

We also studied the b_2 cumulant at the temperature region around T_c. The shape of the $b_2(T)$ curve around T_c signals the coexistence of the two phases.

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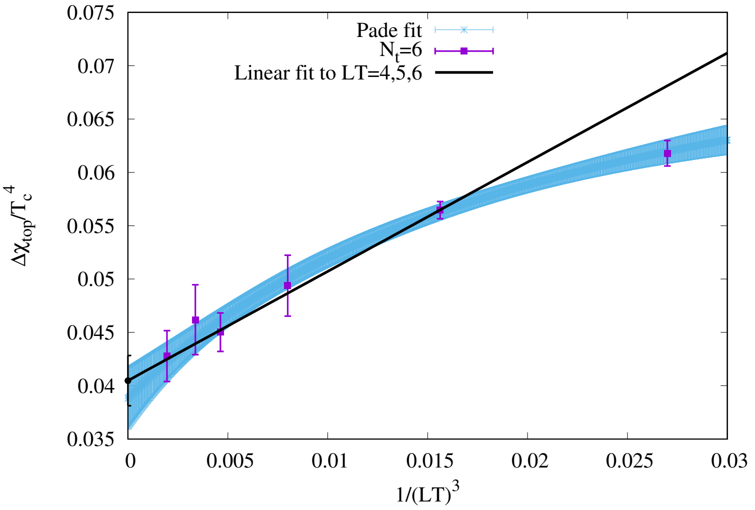
Volumes included in the fit



Imaginary theta

$$\mathcal{L}_\theta = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - i\theta q(x) \;\; \texttt{with} \;\; q(x) = \frac{g^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

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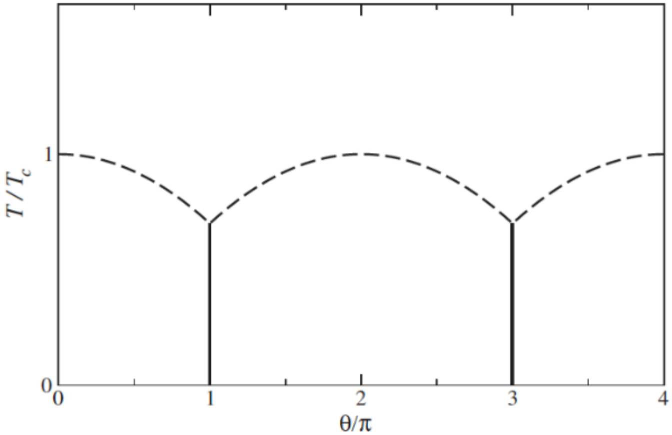

Curvature of the Phase diagram

$$\epsilon = \frac{T^2}{V_3} \partial_T log(Z) \quad Z = \exp\left(-\frac{-V_3 f(T)}{T}\right)$$

$$f_c(T_c) = f_d(T_c)$$

$$f_{c/d}(T) \approx A_{c/d} \frac{T - T_c}{T_c} + \frac{\chi_{c/d}}{T} \frac{\theta^2}{2}$$

$$\Delta\epsilon = T_c(A_c - A_d)$$

At $T_c(\theta)$:

$$(A_c - A_d)(\frac{T_c(\theta) - T_c(0)}{T_c(0)}) = \frac{\Delta\chi\theta^2}{2T_c(\theta)} + \mathcal{O}(\theta^4)$$

$$\frac{T_c(\theta)}{T_c(0)} = 1 - \frac{\Delta\chi\theta^2}{2\Delta\epsilon} + \mathcal{O}(\theta^4)$$