Towards glueball masses of large-N SU(N) Yang-Mills theories without topological freezing via parallel tempering on boundary conditions

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Towards glueball masses of large-N SU(N) pure-gauge theories without topological freezing

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Physical motivations

Glueball states are predicted on the basis of **QCD confinement** and are currently searched in collider experiments. Refinement of QCD theoretical predictions about glueball masses is thus of utmost importance in this respect.

Determining glueball masses from numerical lattice QCD simulations is a long-standing problem that has been widely investigated.
Main computational framework: large-N SU(N) pure-gauge theories:

- large-N is "close" to $N = 3$, as corrections are suppressed as powers of $1/N$
- **no quarks** $+\ 1/N = 0 \implies$ all glueballs are exactly non-interacting and with ∞ lifetime

Overall, this framework provides acceptable approximation of real-world QCD, and an interesting theoretical ground to provide useful predictions.

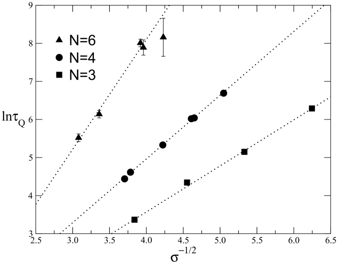
(See also plenary talk by D. Vadacchino on glueball hunting.)

Topological critical slowing down at large N

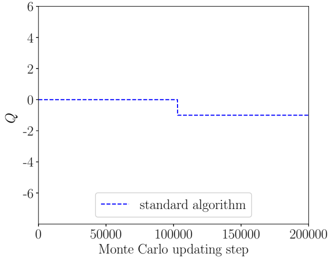
Extracting glueball masses from lattice gauge configurations highly-non trivial $\rightarrow$ several sources of systematic errors

Huge efforts to refine glueball mass extraction methods from the lattice in recent years. Serious systematic that has never been checked in a satisfactory way: topological freezing.

As $a \to 0$ or $N \to \infty$, standard local algorithms become less and less effective in changing the global topological charge Q of a gauge configuration $\implies$ Markov chain remains trapped in a fixed topological sector.



(Fig. from Del Debbio et al., 2002)



Example for $N = 6$, $a \simeq 0.0842$ fm

Plenty of numerical evidence that $\tau(Q)$ diverges as $\sim \exp N$ at fixed a and vice versa

Glueball masses and topology

Computing a glueball mass M on a finite volume and in a fixed topological sector leads to a bias (Brower et al., 2002; Aoki et al., 2007)

$$M_Q \quad = M + \frac{1}{2}\frac{d^2 M}{d\theta^2}\bigg|_{\theta=0}\frac{1}{V\chi} = M + O\left(\frac{1}{N^2 V}\right)$$

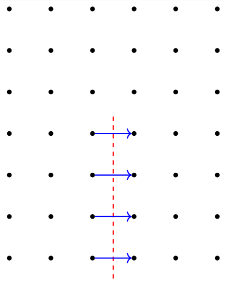
- $S_{\mathrm{YM}}(\theta) = S_{\mathrm{gluons}} + i\theta Q$
- $E_{\mathrm{YM}}(\theta, N) \underset{N\to\infty}{\sim} N^2 f\left(\frac{\theta}{N}\right)$ $\longrightarrow$ θ-dependent vacuum energy
- M_Q = Glueball mass in fixed topological sector Q
- M = Glueball mass averaged over all topological sectors
- $\chi \equiv \dfrac{\langle Q^2 \rangle}{V}$ $\longrightarrow$ **Topological Susceptibility**

No satisfactory check of possible systematics related to fixed topology due to topological freezing at large N so far

Solution: parallel tempering on boundary conditions

Proposed for $2d$ CP^{N-1} models (Hasenbusch, 2017; Berni, CB et al., 2019), recently implemented for $4d$ $\mathrm{SU}(N)$ pure-gauge theories (CB et al., 2021, 2022)

- consider a collection of N_r lattice replicas
- replicas differ for boundary conditions on **small** sub-region: *the defect*
- each replica is updated with standard methods
- after updates, propose swaps among configurations via Metropolis test
- other ingredients: hierarchic updates + translation of periodic replica



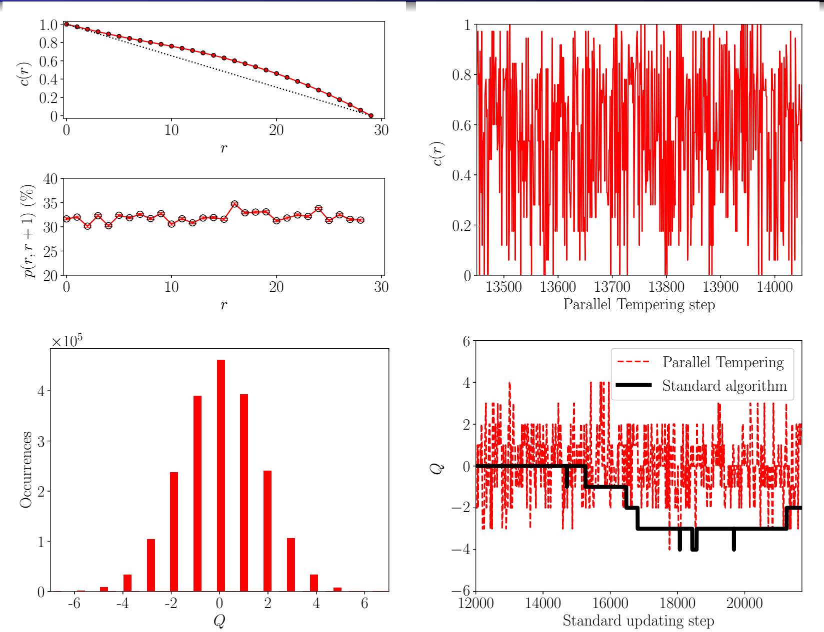
Links crossing the defect: $\beta \to \beta \cdot c(r)$.

Periodic: $c(0) = 1$. Open: $c(N_r - 1) = 0$. Interpolating replicas: $0 < c(r) < 1$.

Decorrelation of Q improved thanks to open boundaries copy, where Q is decorrelated faster.

Observables computed on periodic copy $\to$ easier to have finite-size effects under control.

Results with parallel tempering - SU(6), $a \simeq 0.0938$ fm



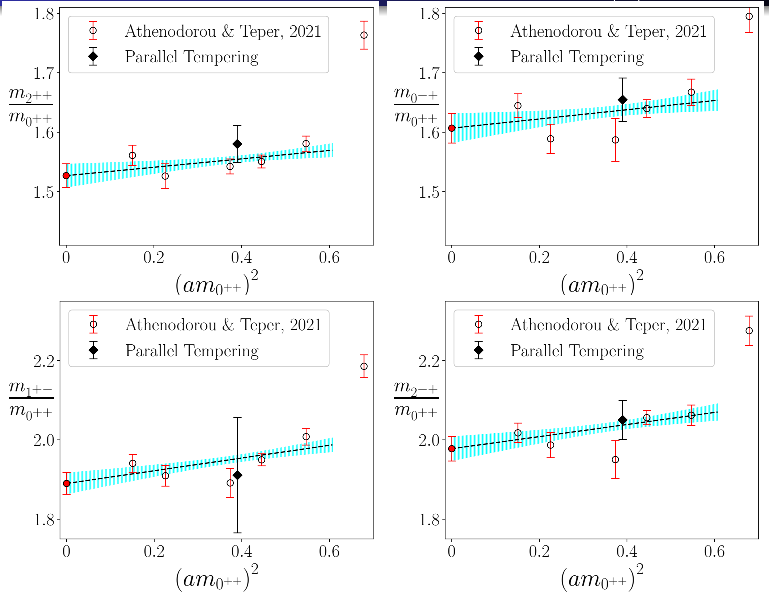
Recap of state-of-the-art methods for glueball masses

(Berg et al., 1983; Teper et al., 1987; Morningstar et al., 1999; Lucini et al., 2001, 2004, 2010; Hong et al., 2017, Bennett et al., 2020; Athenodorou et al., 2020, 2021; and many more...)

- Choose a proper variational basis $\mathcal{B} = \{\mathcal{O}_i(t)\}$ of operators with compatible quantum numbers with respect to the desired channel
- Operators $\mathcal{O}(t) = \sum_{\vec{x}} \mathcal{O}(\vec{x}, t)$: zero-momentum gauge-invariant operators built in terms of traces of product of links along closed spatial paths
- Compute the correlation matrix $C_{ij}(t) = \langle \mathcal{O}_i(t)\mathcal{O}_j(0)\rangle$ and solve the GEVP problem $C_{ij}(t)v_j = \lambda(t,t')C_{ij}(t')v_j$
- For the ground state in the selected channel, it is sufficient to consider $\overline{v}_i$ related to the largest eigenvalue $\lambda(t,t')$
- The best overlapping operator between the vacuum and the desired glueball state is $C_{\text{best}}(t) \equiv C_{ij}(t)\overline{v}_i\overline{v}_j \underset{t\to\infty}{\sim} \exp\{-amt\}$
- Extract the glueball state mass looking for a plateu in

$$am_{\text{eff}}(t) \equiv -\log\left(\frac{C_{\text{best}}(t+1)}{C_{\text{best}}(t)}\right)$$

Results for low-lying glueball masses - SU(6)



Perfect agreement within precision with results of (Athenodorou & Teper, 2021) obtained with standard algorithms, also in channels with same quantum numbers of Q

Conclusions

Take-home messages

- First computation of glueball masses at large-N without topological freezing
- No sizable systematic related to topological freezing observed within our level of precision
- Parallel tempering on boundary conditions is an affordable and viable solution to fight topological freezing: can be easily adopted in future more extensive studies with larger values of N or finer lattice spacings a

Future outlooks

- Bias on computation of M_{glueball} due to fixed topology related to $\left.\frac{d^2 M_{\mathrm{glueball}}}{d\theta^2}\right|_{\theta=0}$. Direct computations of this quantity?
  Only reported result in the literature: $N = 3$ for 0^{++} state. Possible improvements from imaginary-θ method + parallel tempering.
- What happens if topological charge density $Q(t) = \sum_{\vec{x}} q(\vec{x}, t)$ is included in variational basis for channels with PC $= -+$? Would be interesting to investigate.