# Isospin Breaking Effects in the 2-Flavor Schwinger Model

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## Abstract

The automatic fine-tuning of isospin breaking effects by conformal coalescence found by Howard Georgi [1] in the 2-flavor Schwinger model is studied. Numerical investigation of meson mass splitting confirms the exponential suppression of symmetry breaking effects.

## Introduction

The bosonized Lagrangian in the 2D Schwinger model is given by

$$\mathcal{L} = \sum_{f=1}^{2}\frac{1}{2}(\partial_\mu \Phi^f)^2 - \frac{\mu^2}{4}\left(\sum_{f=1}^{2}\Phi^f + \frac{\theta}{\sqrt{4\pi}}\right)^2 + \sum_{f=1}^{2} c m_f^2 N_{m_f}\left[\cos\sqrt{4\pi}\Phi^f\right] + const \tag{1}$$

where $\Phi^f$ are pseudoscalar fields, $\mu = \frac{e}{\sqrt{2\pi}}$ is the Schwinger mass, $\theta$ the vacuum angle, $c = \frac{e^\gamma}{2\pi}$ is a constant with Euler constant $\gamma$, $m_f$ the fermion mass for different flavor $f$, and $N_{m_f}$ denotes normal-ordering with respect to a mass $m_f$.

In the strong coupling limit for light quarks ($e \gg m_f$) we can change the field variables by diagonalizing $\Phi^f$

$$\chi^a = O^a_f \Phi^f + \frac{\theta}{\sqrt{8\pi}}\delta^a_1 \tag{2}$$

using the matrices

$$O^1_f = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \quad O^2_f = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \tag{3}$$

Decoupling the heavy field $\chi^1$ which is the $\eta$ meson and renormal-ordering following Coleman [2]

$$N_{m_f}\left[\cos\left(-\frac{\theta}{2} + \sqrt{4\pi}O^2_f\chi^2\right)\right] = \left(\frac{M}{m_f}\right)^{\frac{1}{N_f}} N_M\left[\cos\left(-\frac{\theta}{2} + \sqrt{4\pi}O^2_f\chi^2\right)\right] \tag{4}$$

the resulting Lagrangian is that of the sine-Gordon theory with $\beta = \sqrt{2\pi}$

$$\mathcal{L}^{light} = \frac{1}{2}(\partial_\mu\chi^2)^2 + \frac{1}{2\pi}M^2 N_M\left[\cos\sqrt{2\pi}\chi^2\right] \tag{5}$$

where

$$M = \left(e^\gamma \mu^{1/2}\sqrt{m_1^2 + m_2^2 + 2m_1 m_2\cos\theta}\right)^{2/3}. \tag{6}$$

There are three solutions: The soliton ($Q = 0$, $I_3 = +1$), the antisoliton ($Q = 0$, $I_3 = -1$) and the lighter breather of the two breather solutions ($Q = 0$, $I_3 = 0$) that correspond to the pions $\pi^+, \pi^-$ and $\pi^0$ respectively. All three solutions are the lightest physical states of mass $M$ where $M$ is flavor dependent and the exponent of 2/3 is due to $N_f = 2$ [3].
Following Georgi, the massless composite 1/2 dimension operators

$$O_f = \psi^*_{f1}\psi_{f2}, \quad O^*_f = \psi^*_{f2}\psi_{f1} \tag{7}$$

of flavor $f = 1, 2$ have opposite charge of the isospin 3rd component, $I_3 = +1, -1$.Mixing these operators

$$O_{\pm 1} = e^{i\theta/2}O_1 \pm e^{-i\theta/2}O^*_2 \qquad O^*_{\pm 1} = e^{-i\theta/2}O^*_1 \pm e^{i\theta/2}O_2 \tag{8}$$

all 2-point correlators vanish except for

$$\langle 0|T(O_{\pm 1}(x)O^*_{\pm 1}(0))|0\rangle = \frac{\xi\mu}{2\pi^2}(e^{\kappa_0} \pm e^{-\kappa_0})\frac{1}{\sqrt{-x^2 + i\varepsilon}} \tag{9}$$

where

$$\kappa_0 = K_0\left(\mu\sqrt{-x^2 + i\varepsilon}\right)$$

and $K_0$ is a modified Bessel function of the second kind.

While the $O_{+1}$ operator goes to a conformal operator at long distances, the $O_{-1}$ operator goes to zero exponentially, meaning that $O_{-1}$ disappears while $O_1$ and $O^*_2$ in $O_{+1}$ coalesce.

## Degenerate Masses $m_f$

Going further, we will consider the case of stable bound isotriplets in the 2 flavor Schwinger model given by $\theta = 0$. From the standard bosonization rules and eq.(2) we find the mass term in eq.(5) in leading order

$$\frac{1}{2\pi}M^2 N_M\left[\cos\sqrt{2\pi}\chi^2\right] \propto m_f(\bar\psi_1\psi_1 + \bar\psi_2\psi_2 + \text{mixed terms}). \tag{10}$$

The mixed terms disappear and using eq.(7) we find the mass term to be

$$\frac{1}{2\pi}M^2 N_M\left[\cos\sqrt{2\pi}\chi^2\right] \propto m_f(O_1 + O_2) + h.c \tag{11}$$

Introducing a flavor degenerate mass term at low energies using eq.(8)

$$m_f(O_{+1} + O^*_{+1}) \tag{12}$$

and from the well known asymptotic behavior of $K_0$ and subsequently $\kappa_0$ with long distance behavior

$$\langle 0|T(O_{+1}(x)O^*_{+1}(0))|0\rangle \xrightarrow{x\to\infty} \frac{\xi\mu}{\pi^2}\langle 0T(O_{1/2}(x)O^*_{1/2}(0))|0\rangle = \frac{\xi\mu}{\pi^2}\frac{1}{\sqrt{-x^2 + i\varepsilon}} \tag{13}$$

we find that the mass term may be written as

$$\frac{\sqrt{\xi}}{\pi}m_f\sqrt{\mu}(O_{1/2} + O^*_{1/2}) \tag{14}$$

implying the relevant mass scale (in the deep IR) is given by $M_s = (m_f^2\mu)^{\frac{1}{3}}$.

## Non-Degenerate Masses $m_f + \delta m$

We now consider non-degenerate masses $m_f \pm \delta m$ with an isospin splitting term

$$\delta m(O_{-1} + O^*_{-1}) \tag{15}$$

and find the asymptotic behavior

$$\langle 0|T(O_{-1}(x)O^*_{-1}(0))|0\rangle \xrightarrow{x\to\infty} \xi\sqrt{\frac{1}{8\pi}(m_f\mu^2)^{\frac{1}{3}}e^{-\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}}}}\langle 0|T(O_{1/2}(x)O^*_{1/2}(0))|0\rangle \tag{16}$$

from which the isospin splitting term is

$$\delta m\sqrt{\xi\sqrt{\frac{1}{8\pi}(m_f\mu^2)^{\frac{1}{3}}e^{-\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}}}}}(O_{1/2} + O^*_{1/2}) \tag{17}$$

and the isospin mass splitting scale is given by

$$\Delta M_s^3 = \delta m^2 m_f^{\frac{1}{3}}\mu^{\frac{2}{3}}e^{-\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}}}. \tag{18}$$

The overall mass term in the Lagrangian of half dimensional conformal operators is

$$\frac{\sqrt{\xi}}{\pi}m_f\sqrt{\mu}\left(1 + \delta m\left(\frac{\pi}{2}\right)^{\frac{3}{4}}m_f^{-\frac{5}{6}}\mu^{-\frac{1}{6}}e^{-\frac{1}{2}\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}}}\right)(O_{1/2} + O^*_{1/2}) \tag{19}$$

and knowing from the sine-Gordon solution that $M_\pi \propto m_f^{\frac{2}{3}}$ we find the isospin breaking corrections to leading order in $\delta m$

$$M_\pi \propto m_f^{\frac{2}{3}}\left(1 + \frac{2}{3}\left(\frac{\pi}{2}\right)^{\frac{3}{4}}\frac{\delta m}{m_f^{\frac{5}{6}}\mu^{\frac{1}{6}}}e^{-\frac{1}{2}\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}}}\right). \tag{20}$$

From the pion operator

$$O_{\pi^0}(x) = \frac{1}{2}(\bar\psi_1(x)\gamma_5\psi_1(x) - \bar\psi_2(x)\gamma_5\psi_2(x)) \tag{21}$$

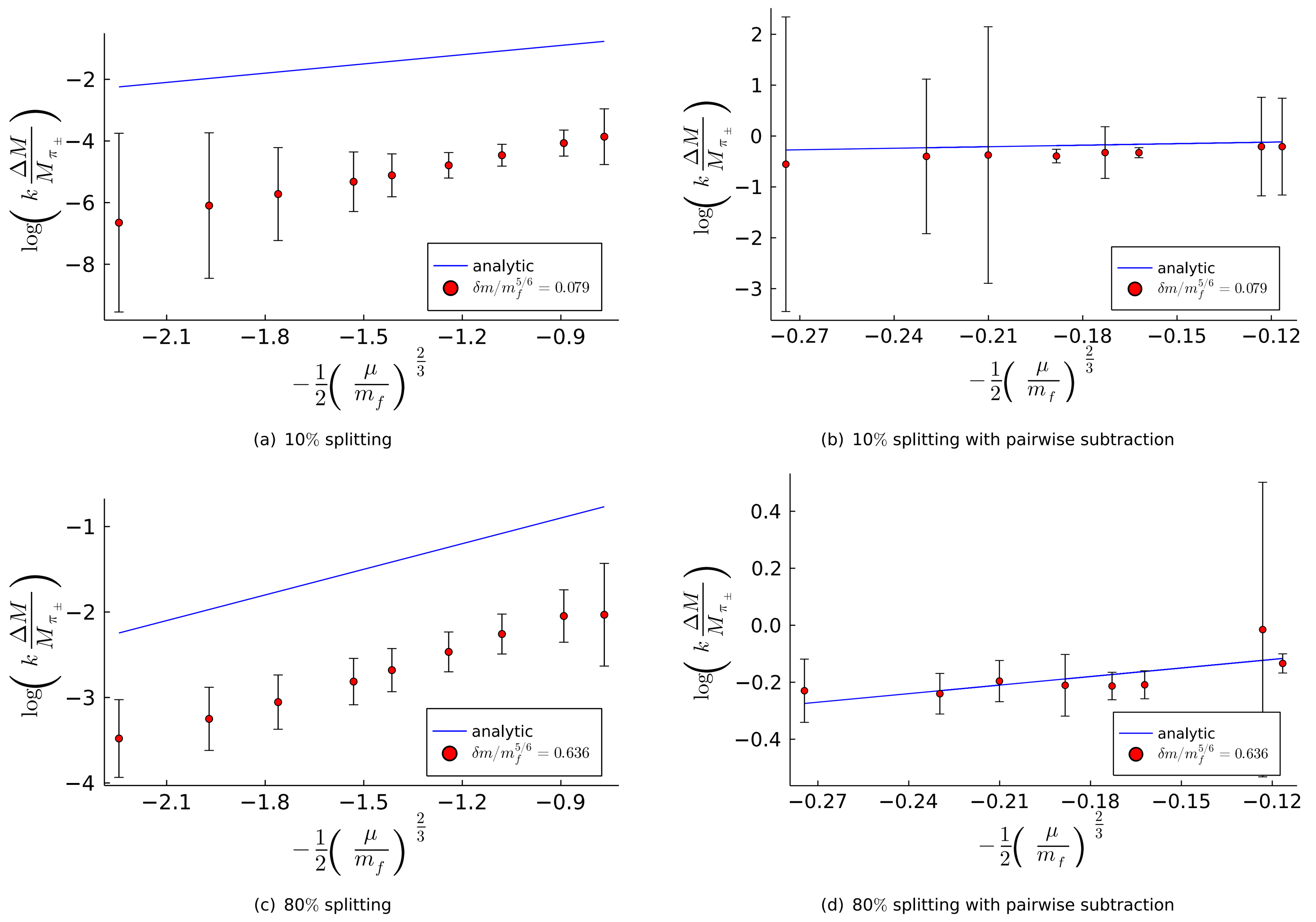
we find the propagator

$$\langle 0|T(O_{\pi^0}(x)O^*_{\pi^0}(0))|0\rangle \propto \langle 0|T(O_{+1}(x)O^*_{+1}(0))|0\rangle \tag{22}$$

from which it follows that an isosplin splitting can be observed in the neutral pion propagator.

## Numerical Results

$$\log\Bigg(\underbrace{\frac{3}{2}\left(\frac{2}{\pi}\right)^{\frac{3}{4}}\mu^{\frac{1}{6}}\frac{m_f^{\frac{5}{6}}}{\delta m}}_{k}\frac{\Delta M}{M_{\pi_\pm}}\Bigg) = -\frac{1}{2}\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}} \tag{23}$$




(a) 10% splitting (b) 10% splitting with pairwise subtraction (c) 80% splitting (d) 80% splitting with pairwise subtraction

Figure 1. Isospin breaking effects for 10% and 80% splitting displayed with and without pairwise subtraction.

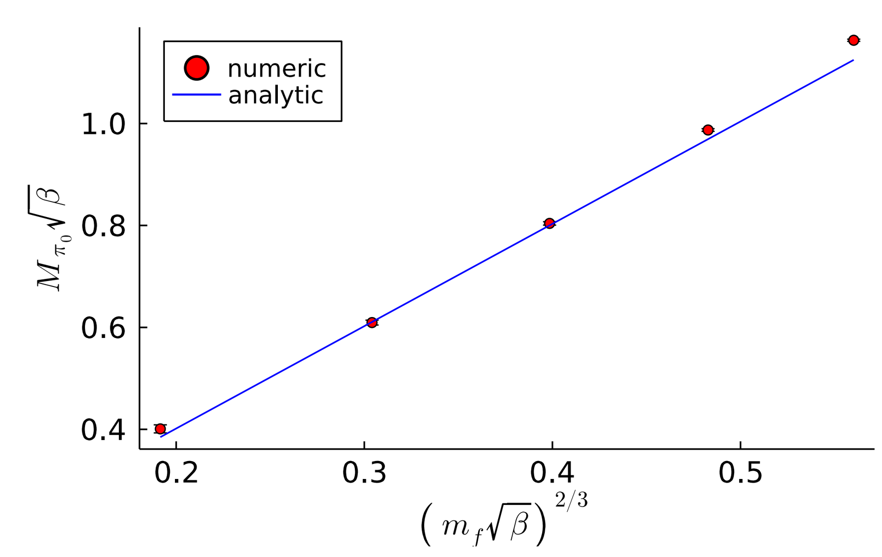



Figure 2. $M_\pi$ vs. $m_f$ for degenerate masses on a fine lattice with $N = 32, \beta = 7.2$ and $M_\pi \sim 2.008 \cdot m_f^{2/3}$ [4].

## References

[1] H.Georgi. arXiv:2007.15965

[2] S.Coleman. ANNALS OF PHYSICS 101, 239-267 (1976)

[3] M. Sadzikowski and P. Wegrzyn. arXiv:hep-ph/9605242

[4] C.Gattringer, I.Hip, C.B.Lang. arXiv:hep-lat/9909025

* Corresponding talk on tue 09.08. 4:30 PM in HS5 (CP1-HSZ)