Topological susceptibility in high temperature full QCD via staggered spectral projectors

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Based on the work in collaboration with:
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Outline

QCD topology and axion

Peccei-Quinn mechanism: one of the most promising solution of strong CP-problem

New symmetry in the SM: $U(1)_{\mathrm{PQ}} \xrightarrow{\mathrm{SSB}}$ axion

- Axion couplings $\propto 1/f_a$ with f_a very large $\implies$ dark matter candidate
- knowledge of axion effective potential $V_{\mathrm{eff}}(a;T) \sim F\left(\theta + \frac{a(x)}{f_a}; T\right)$ fixes all effective parameters which enter the axion field evolution (e.g. mass, couplings) and axion relic abundance
- mass term for the axion given by QCD topology

$$\boxed{m_a^2(T) = \frac{\partial^2 V_{\mathrm{eff}}(a;T)}{\partial a^2}\bigg|_{\langle a \rangle = -\theta f_a} = \frac{\chi(T)}{f_a^2} = \frac{\langle Q^2 \rangle_{T,\theta=0}}{V\, f_a^2}}$$

Chiral symmetry and lattice artifacts

Index theorem:

$$Q = \mathrm{Tr}\{\gamma_5\} = n_+ - n_-$$

Path-integral weight of a $Q \neq 0$ configuration:

$$\det[\not{D} + m_q] = \prod_i [m_q + i\lambda_i]; \qquad \lambda_{min} = m_q$$

Staggered discretization breaks chiral symmetry at finite lattice spacing

$$\lambda_{min} = m_q \longrightarrow m_q + i\lambda$$

On the lattice, $Q \neq 0$ configurations are less suppressed $\implies$ large lattice artifacts for χ

- Problem observed with standard gluonic definition (see, e.g., C. Bonati *et al.*, 2018), faced by using the eigenvalue reweighting technique in (S. Borsányi *et al.*, 2016)
- We adopt a fermionic definition based on **spectral projectors** (L. Giusti and M. Lüscher, 2009; C. Alexandrou *et. al*, 2018; C. Bonanno *et al.*, 2019)

Spectral Projectors

Lattice version of index theorem for staggered fermions ($\mathbb{P}_M$ is the projector on the space spanned by the eigenstates of $\not{D}_{stag}$ with eigenvalues $|\lambda| \leq M$):

$$Q^{(\text{stag})}_{\text{SP, bare}} = \frac{1}{2^{d/2}} \text{Tr}\{\Gamma_5 \mathbb{P}_M\} = \frac{1}{2^{d/2}} \sum_{|\lambda_k| \leq M} u_k^\dagger \Gamma_5 u_k; \qquad \not{D}_{stag} u_k = i\lambda_k u_k$$

Multiplicative renormalization of $Q^{(\text{stag})}_{\text{SP, bare}}$ (C. Bonanno *et al.*, 2019):

$$Q^{(\text{stag})}_{\text{SP}} = Z_{\text{SP}}\, Q^{(\text{stag})}_{\text{SP, bare}} = \sqrt{\frac{\langle \text{Tr}\{\mathbb{P}_M\}\rangle}{\langle \text{Tr}\{\Gamma_5 \mathbb{P}_M \Gamma_5 \mathbb{P}_M\}\rangle}}\, Q^{(\text{stag})}_{\text{SP, bare}}$$

Topological susceptibility: $\chi_{\text{SP}} = \frac{1}{V} \left\langle \left(Q^{(\text{stag})}_{\text{SP}} \right)^2 \right\rangle$

How to choose the cut-off mass M?

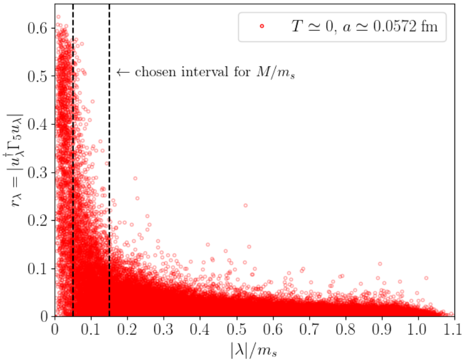
- Continuum limit does not depend on the choice of M, lattice artifacts do
- $M_R = M/Z_S^{(s)}$ has to be fixed when performing the continuum limit
- If a LCP is known $\implies$ $M_R/m_{q,R} = M/m_q$ can be fixed
- Idea! Choose M/m_q to include modes with the highest value of chirality

Example:

run $T \simeq 0$, $a = 0.0572$ fm

$r_\lambda \equiv |u_\lambda^\dagger \Gamma_5 u_\lambda|$ vs λ/m_s for the first 200 eigenmodes

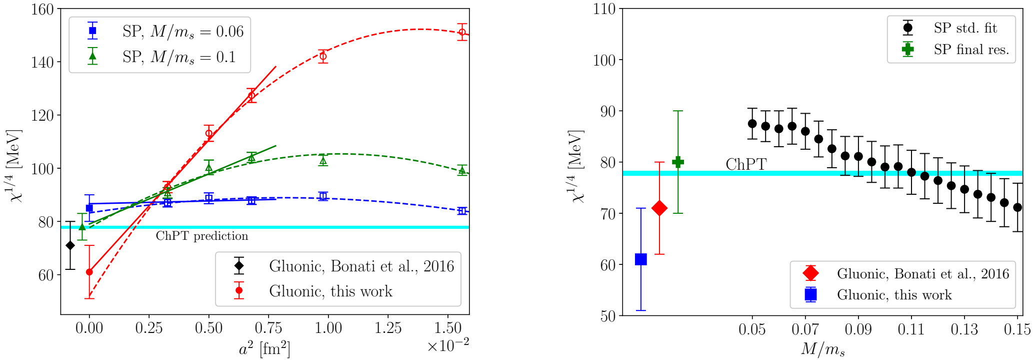
M-range: $M/m_s \in [0.05, 0.15]$



$T \simeq 0$ result

Lattice setup: $N_f = 2+1$, rooted stout staggered discretization, physical point

Cont. scaling: $\chi_{\mathrm{SP}}^{1/4}(a, M_R) = \chi_{\mathrm{SP}}^{1/4} + c_{\mathrm{SP}}(M_R)a^2 + o(a^2)$



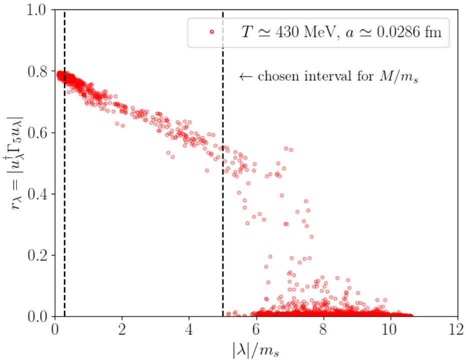
Final estimation at $T \simeq 0$: $\chi_{\mathrm{SP}}^{1/4} = 80(10)$ MeV in agreement with $\chi_{\mathrm{ChPT}}^{1/4} = 77.8(4)$ MeV

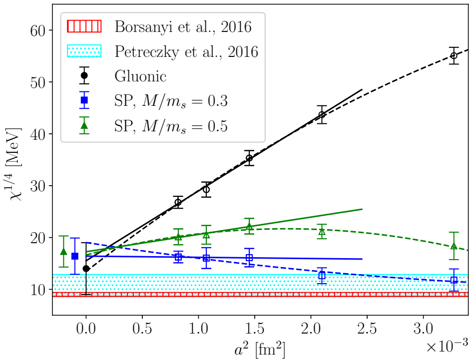
Topological susceptibility at finite temperature

At high T, $\langle Q^2 \rangle = \chi V \ll 1 \implies$ multicanonical approach (Bonati *et al.*, 2018; P. T. Jahn *et al.*, 2018)

Results for $T \simeq 430$ MeV, M-range: $M/m_s \in [0.3, 5]$

Clearer separation between high and low chirality modes

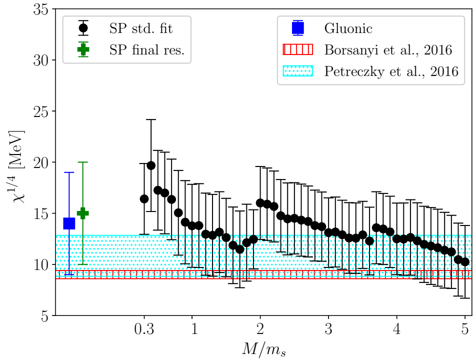


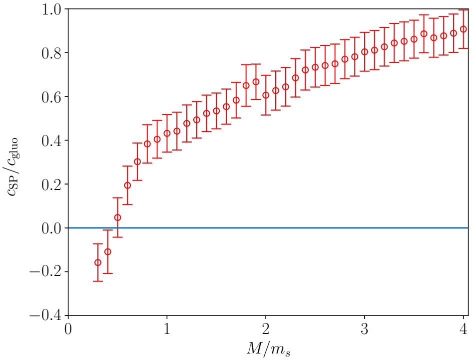


Topological susceptibility at finite temperature

Agreement between SP determinations in the M-range

Lattice artifacts can be reduced if M/m_s is chosen properly





<u>Final estimation at $T \simeq 430$ MeV</u>: $\chi_{\mathrm{SP}}^{1/4} = 15(5)$ MeV, $\chi_{\mathrm{gluo}}^{1/4} = 14(5)$ MeV

Temperature dependence of the topological susceptibility

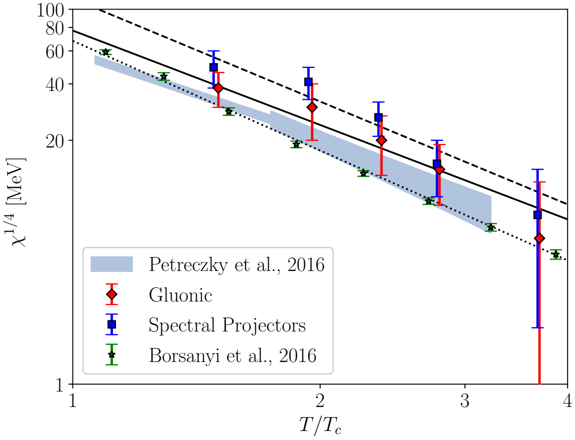
According to DIGA: $\chi^{1/4} \propto T^{-b}$ ($b_{\text{DIGA}} \sim 2$) when $T \gg T_c$

Agreement with DIGA power-law behaviour:

$b_{\text{SP}} = 1.82(43) \qquad b_{\text{gluo}} = 1.67(51)$

With values in (S. Borsányi *et al.*, 2016), for $T \gtrsim 170$ MeV:

$$b_{\text{BW}} = 1.945(23)$$



Conclusions

Summary

- SP method allows to reduce lattice artifacts with respect to the gluonic definition
- The introduction of the free parameter M provides a better control of the systematics in the continuum extrapolation
- 2-3 std. dev. tension with values of (S. Borsányi *et al.*, 2016) and (P. Petreczky *et al.*, 2016) in the range 300 MeV $\lesssim T \lesssim$ 400 MeV
- Agreement with DIGA power-law behaviour

Future perspectives

- Refine our analysis in the range 300 MeV $\lesssim T \lesssim$ 400 MeV by adding a finer lattice spacing
- Exploration of region around $T \sim 1$ GeV $\implies$ very small lattice spacing (~ 0.01 fm) $\implies$ **topological freezing**. A promising solution: *parallel tempering on boundary conditions* (M. Hasenbusch, 2017; C. Bonanno *et al.*, 2021, 2022, see also talk by C. Bonanno)

Backup slides

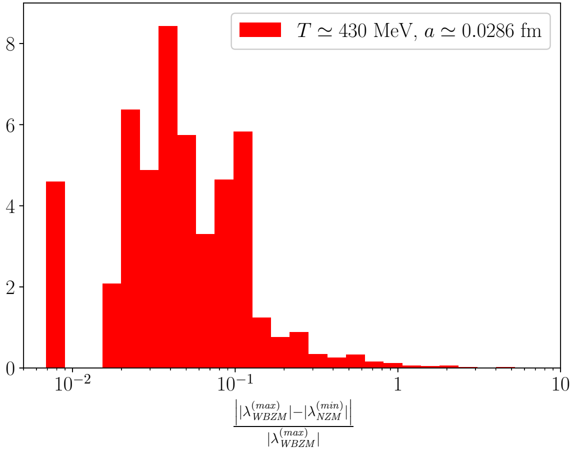
Identification of Would-Be Zero Modes (WBZM)

run $T \simeq 430$ MeV, $a \simeq 0.0286$ fm

$|\lambda^{(\text{max})}_{\text{WBZM}}|$: largest eigenvalue in the set of WBZM candidates, identified with the same criterium in (S. Borsányi *et al.*, 2016)

$|\lambda^{(\text{min})}_{\text{NZM}}|$: smallest eigenvalue in the set of Non-Zero Modes (NZM)

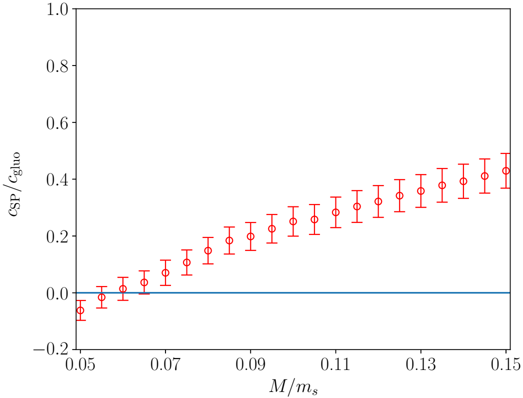
A sharp separation can not unambiguously established already at the level of the single configuration



$T \simeq 0$ SP continuum limit corrections

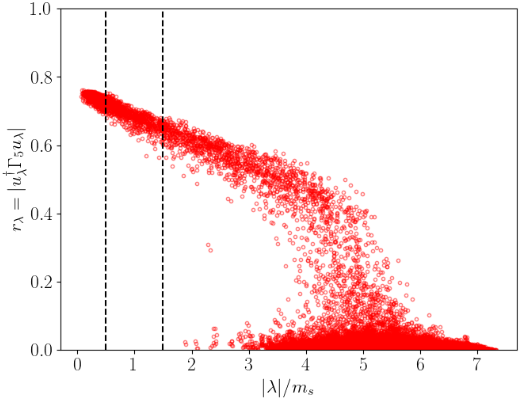
Behavior of $c_{\text{SP}}/c_{\text{gluo}}$ as a function of the cut-off M/m_s in the optimal range

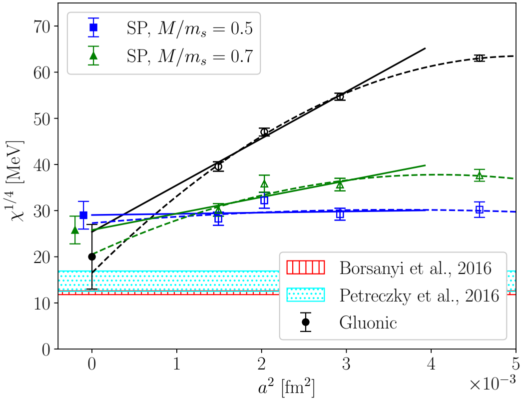
SP corrections grow when M/m_s is increased, getting closer to the gluonic ones



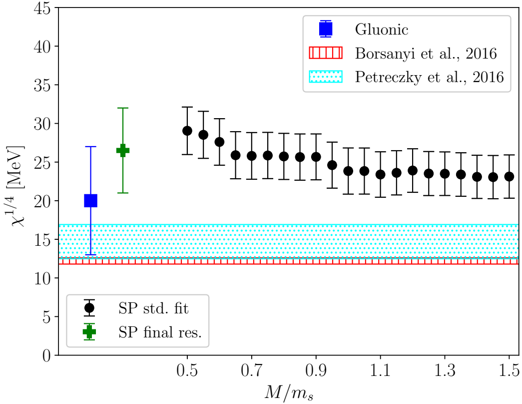
$T \simeq 365$ MeV results

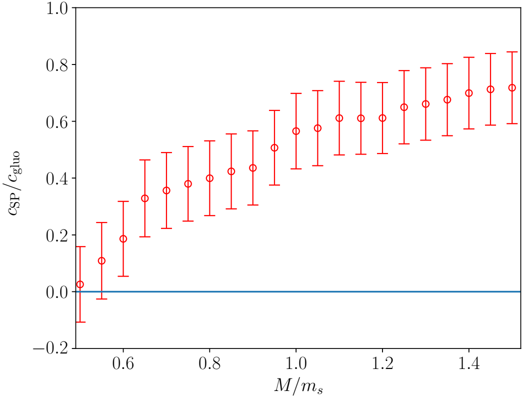
M-range: $M/m_s \in [0.5, 1.5]$





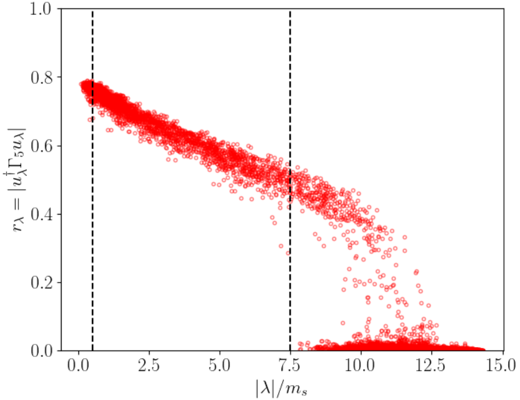
$T \simeq 365$ MeV results

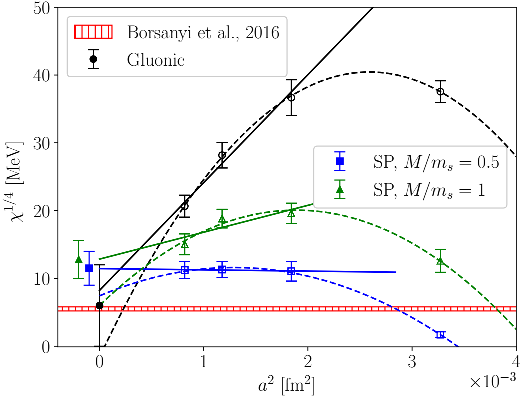




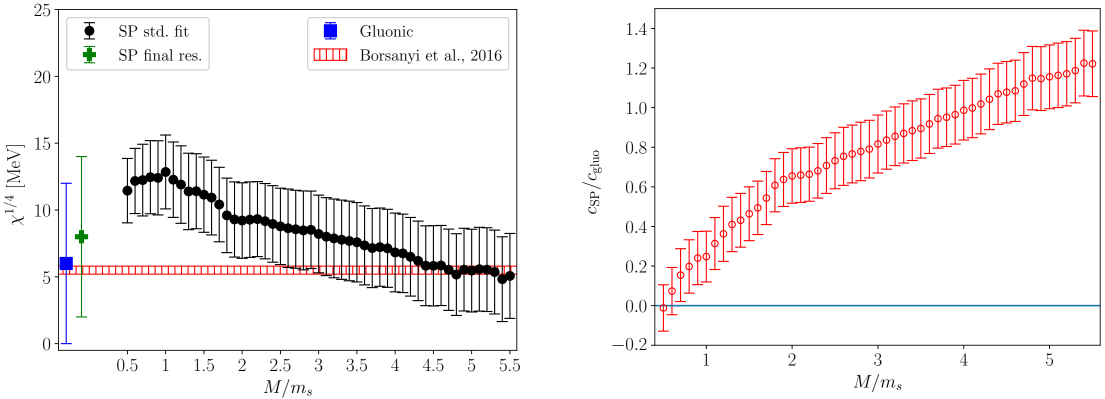
Final estimation at $T \simeq 365$ MeV: $\chi_{SP}^{1/4} = 26.5(5.5)$ MeV, $\chi_{gluo}^{1/4} = 20(7)$ MeV

M-range: $M/m_s \in [0.5,\ 7.5]$





$T \simeq 570$ MeV results



<u>Final estimation at $T \simeq 570$ MeV</u>: $\chi_{\mathrm{SP}}^{1/4} = 8(6)$ MeV, $\chi_{\mathrm{gluo}}^{1/4} = 6(6)$ MeV

Summary of $\chi(T)$ values

T [MeV]	T/T_c	$\chi_{\text{SP}}^{1/4}$ [MeV]	$\chi_{\text{gluo}}^{1/4}$ [MeV]
230	1.48	49(11)	38(8)
300	1.94	41(8)	32(10)
365	2.35	26.5(5.5)	20(7)
430	2.77	15(5)	14(5)
570	3.68	8(6)	6(6)

Results for the fourth root of the topological susceptibility as a function of T. For the crossover temperature T_c we adopted the reference value $T_c = 155$ MeV.