Charged Particles in C-periodic Volumes

Alessandro Mariani, Gurtej Kanwar, Joao Pinto Barros, Uwe-Jens Wiese

Albert Einstein Center for Fundamental Physics
Institute for Theoretical Physics, Bern University

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C-Periodic Boundary Conditions

Vortex in $(2+1)$-d $U(1)$ Scalar Field Theory

Single "Quarks" as Non-Abelian Infraparticles

Conclusions

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A periodic torus T^3 is always neutral

$$Q = \int_{T^3} d^3x \, \vec{\nabla} \cdot \vec{E} = \int_{\partial T^3} d^2\vec{f} \cdot \vec{E} = 0$$

A C-periodic torus has a charge-conjugation twist
and allows the existence of charged particles

$$\begin{aligned}
A_\mu(\vec{x} + L_i\vec{e}_i, t) &= {}^C A_\mu(\vec{x}, t) = -A_\mu(\vec{x}, t) \\
\Phi(\vec{x} + L_i\vec{e}_i, t) &= {}^C \Phi(\vec{x}, t) = \Phi(\vec{x}, t)^* \\
G_\mu(\vec{x} + L_i\vec{e}_i, t) &= {}^C G_\mu(\vec{x}, t) = G_\mu(\vec{x}, t)^* \\
\psi(\vec{x} + L_i\vec{e}_i, t) &= {}^C \psi(\vec{x}, t) = C\bar{\psi}(\vec{x}, t)^\mathsf{T}
\end{aligned}$$

C-periodic boundary conditions are translation invariant.
C-parity determines the momentum quantization

$$\begin{aligned}
C = + \ &: \ \mathrm{Re}\Phi(\vec{x} + L_i\vec{e}_i, t) = \mathrm{Re}\Phi(\vec{x}, t) \ \Rightarrow \ p_i = 2\pi n_i / L_i \\
C = - \ &: \ \mathrm{Im}\Phi(\vec{x} + L_i\vec{e}_i, t) = -\mathrm{Im}\Phi(\vec{x}, t) \ \Rightarrow \ p_i = 2\pi (n_i + \tfrac{1}{2}) / L_i
\end{aligned}$$

L. Polley, UJW, Nucl. Phys. B356 (1991) 629.
A. S. Kronfeld, UJW, Nucl. Phys. B357 (1991) 521.

C-periodic theories have C-periodic gauge transformations

$$A'_\mu(x) = A_\mu(x) - \partial_\mu \alpha(x) \;\Rightarrow\; \alpha(\vec{x} + L_i \vec{e}_i, t) = -\alpha(\vec{x}, t)$$
$$G'_\mu(x) = \Omega(x)(G_\mu(x) + \partial_\mu)\Omega(x)^\dagger \;\Rightarrow\; \Omega(\vec{x} + L_i \vec{e}_i, t) = \Omega(\vec{x}, t)^*$$

C-periodic boundary conditions break the center symmetry:
$\mathbb{Z}(N) \rightarrow \{1\}$ for N odd, $\mathbb{Z}(N) \rightarrow \mathbb{Z}(2)$ for N even

$$\begin{aligned}
\Omega(\vec{x}, t + \beta) &= \Omega(\vec{x}, t) z \;\Rightarrow \\
\Omega(\vec{x} + L_i \vec{e}_i, t + \beta)^* &= \Omega(\vec{x}, t + \beta) = \Omega(\vec{x}, t) z = \Omega(\vec{x} + L_i \vec{e}_i, t)^* z \\
&= \Omega(\vec{x} + L_i \vec{e}_i, t + \beta)^* z^2 \;\Rightarrow\; z^2 = 1
\end{aligned}$$

They also break baryon number: $U(1)_B \rightarrow \mathbb{Z}(2)_B$

G-periodic boundary conditions break chiral symmetry:
$SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L=R}$

$$\begin{aligned}
\psi_R(\vec{x} + L_i \vec{e}_i, t) &= {}^G\psi_R(\vec{x}, t) = \sigma_2 \times \sigma_2 \bar{\psi}_L(\vec{x}, t)^\mathsf{T} \\
\psi_L(\vec{x} + L_i \vec{e}_i, t) &= {}^G\psi_L(\vec{x}, t) = -\sigma_2 \times \sigma_2 \bar{\psi}_R(\vec{x}, t)^\mathsf{T}
\end{aligned}$$

UJW, Nucl. Phys. B375 (1992) 45.

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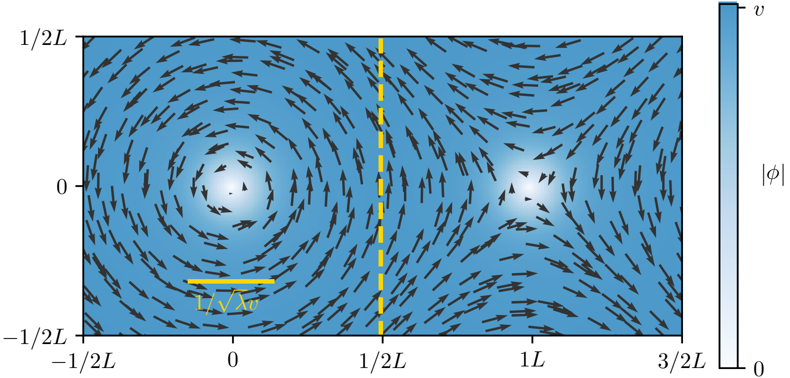
Lagrange density of $(2+1)$-d $U(1)$ scalar field theory

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi^* \partial^\mu \Phi - V(\Phi),\ \ V(\Phi) = \frac{\lambda}{4!}\left(|\Phi|^2 - v^2\right)^2 \ ,\ \Phi(x) \in \mathbb{C}$$

A vortex can exist in a finite twisted C-periodic volume

$$\Phi(\vec{x} + L_1 \vec{e}_1) = i\Phi(\vec{x})^* \ , \quad \Phi(\vec{x} + L_2 \vec{e}_2) = -i\Phi(\vec{x})^* \ .$$

Classical vortex solution in a twisted C-periodic volume

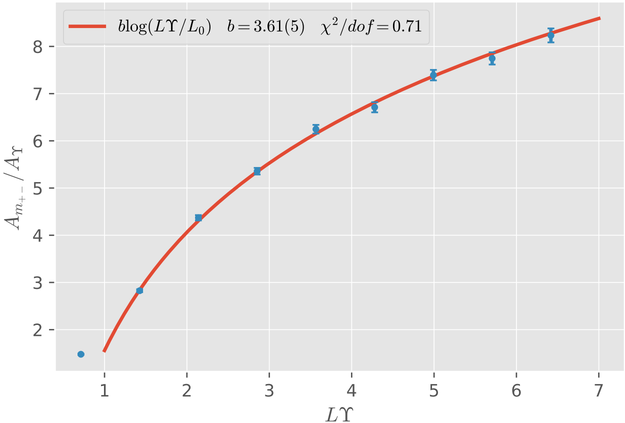


M. Hornung, J. C. Pinto Barros, UJW, arXiv:2106.16191

Duality transformation to $(2+1)$-d scalar QED:
Goldstone boson $\leftrightarrow$ photon, vortex $\leftrightarrow$ charged particle:
an infraparticle surrounded by its non-local Coulomb cloud

$$\chi_x^C = \exp\left(i\Delta^{-1}\delta A\right)\chi_x$$

Logarithmic behavior of the finite-volume vortex mass:



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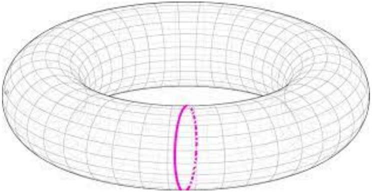
Flux string winding around a C-periodic volume



Spatial Polyakov loop transforms unusually

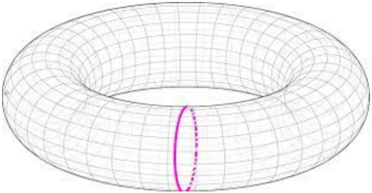
$$U_{\mathcal{C}}(\vec{x}) = \mathcal{P} \exp\left(\int_0^{L_i} dy_i \, G_i(\vec{x} + y_i \vec{e}_i)\right) \Rightarrow U'_{\mathcal{C}}(\vec{x}) = \Omega(\vec{x}) U_{\mathcal{C}}(\vec{x}) \Omega(\vec{x})^{\mathsf{T}}$$

For odd $N \geq 3$ one can construct gauge invariant single-quark states in a C-periodic finite volume:

$$\{3\} \text{ for } SU(3): \ \epsilon_{abc} \psi^a(\vec{x}) U_{\mathcal{C}}^{bc}(\vec{x})$$

The single-quark state can mix with a baryon and is thus unstable against decay via string-breaking in a large volume.

Flux string winding around a C-periodic volume



Spatial Polyakov loop transforms unusually

$$U_{\mathcal{C}}(\vec{x}) = \mathcal{P} \exp \left(\int_0^{L_i} dy_i \; G_i(\vec{x} + y_i \vec{e}_i) \right) \;\Rightarrow\; U'_{\mathcal{C}}(\vec{x}) = \Omega(\vec{x}) U_{\mathcal{C}}(\vec{x}) \Omega(\vec{x})^{\mathsf{T}}$$

For even $N \geq 4$ one can construct gauge invariant single-"quark" states in a C-periodic finite volume:

$$\{6\} \text{ for } SU(4): \; \epsilon_{abcd} \xi^{ab}(\vec{x}) U_{\mathcal{C}}^{cd}(\vec{x})$$
$$\{10\} \text{ for } SU(4): \; \eta^{ab}(\vec{x}) U_{\mathcal{C}}^{ab}(\vec{x})$$

The energy of these states, which diverges linearly with the volume, defines a renormalized proper physical quark mass.

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• Vortices can be quantized fully non-perturbatively.

• They are infraparticles that are surrounded by a cloud of massless Goldstone bosons (dual photons).

• The vortex mass diverges logarithmically with increasing volume.

• Still the vortex mass is finite and well-defined in a C-periodic volume.

• For odd $N \geq 3$ C-periodic boundary conditions break the center symmetry $\mathbb{Z}(N) \to \{1\}$. As a result in C-periodic QCD the single-quark states mix with baryon states.

• For even $N \geq 4$ C-periodic boundary conditions break the center symmetry $\mathbb{Z}(N) \to \mathbb{Z}(2)$. This allows the construction of stable single-"quark" states as non-Abelian infraparticles. Their energy, which diverges linearly with increasing volume, defines a physical "quark" mass that runs with the volume.

• Such states are fermionic strings with a Lüscher term that would be interesting to study.