Non-invertible self-duality defects of Cardy-Rabinovici model and mixed gravitational anomaly

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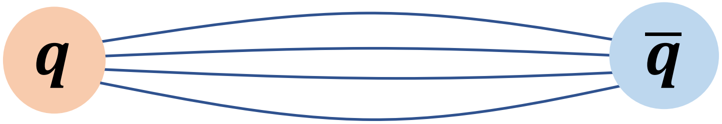
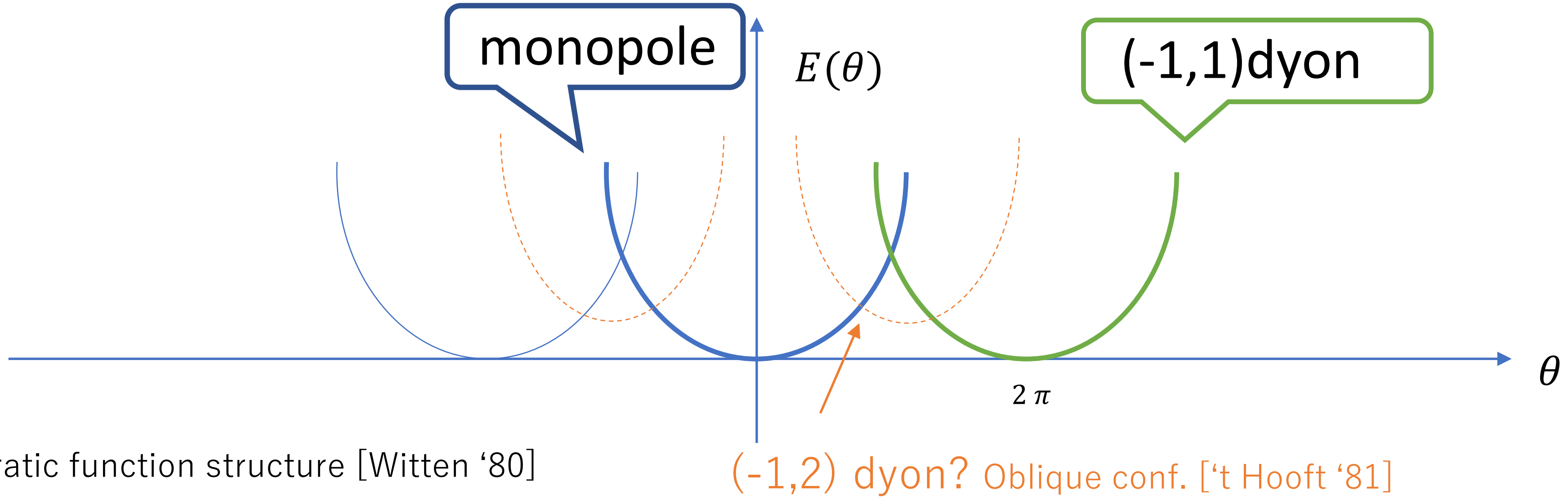
Motivation: confinement & $\boldsymbol{\theta}$ angle

A popular understanding of quark confinement: dual superconductor picture
(monopole condensation)



Witten effect: monopole acquires electric charge $\theta/2\pi$ by increasing θ



cf.) Large-N → multi-branch quadratic function structure [Witten '80]

Overview

- Model: Cardy-Rabinovici model (a toy model for YM with θ angle)
- Method: non-invertible symmetry from “duality” & its anomaly
- Results:
    1. $SL(2,\mathbb{Z})$ transformations of the CR model can be understood as “**dualities**” between the CR model and its (appropriately) $\mathbb{Z}_N^{[1]}$-gauged model.
    2. From these “dualities,” at self-dual parameters, we construct **non-invertible symmetries** and determine their fusion rules.
    3. We find **a mixed gravitational anomaly** of this symmetry for some cases, which rules out the trivially-gapped vacuum.

Model

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Cardy-Rabinovici model [Cardy and Rabinovici '82, Cardy '82]

- **U(1) gauge** + **charge-N Higgs** + **monopole.** Symbolically,

$$Z_{CR} = \int \mathcal{D}a\, e^{-S_{U(1)}[da]} \sum_{C,C':\text{loops}} W^N(C)\, H(C')$$

where $S_{U(1)}[da] = \frac{1}{2g^2} \int da \wedge * da + \frac{iN\theta}{8\pi^2} \int da \wedge da$,
$W(C)$: Wilson loop, $H(C)$: 't Hooft loop

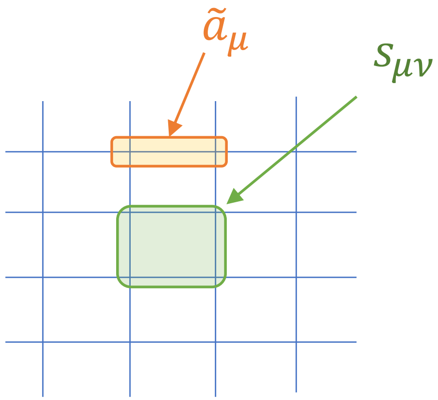
This model gives an interesting playground for studying topological aspects!

- $\mathbb{Z}_N^{[1]}$ symmetry (~ $\mathbb{Z}_N^{[1]}$ center symmetry in $SU(N)$ YM)
  + same CP& $\mathbb{Z}_N^{[1]}$ anomaly structure at $\theta = \pi$ as $SU(N)$ YM [Honda and Tanizaki 2020]
- The conjectured phase diagram (shown later) exhibits θ dependence similar to that of $SU(N)$ YM.

Cardy-Rabinovici model: Lattice description

- Villain-type U(1) gauge theory + θ term: $S_{U(1)} = \sum_x \frac{1}{4g^2} f_{\mu\nu} f_{\mu\nu} + \frac{iN\theta}{2\pi} m_\mu \cdot \tilde{a}_\mu$
  - U(1) gauge field $a \leftrightarrow (\tilde{a}_\mu, s_{\mu\nu})$
    - $\mathbb{R}$-valued link variable: $\tilde{a}_\mu$
    - $\mathbb{Z}$-valued plaquette variable: $s_{\mu\nu}$
  - Field-strength: $f_{\mu\nu} \coloneqq \partial_\mu \tilde{a}_\nu - \partial_\nu \tilde{a}_\mu - 2\pi s_{\mu\nu}$ [∂_μ: lattice derivative]
  - Monopole current: $m_\mu \coloneqq \frac{1}{2} \varepsilon_{\nu\mu\rho\sigma} \partial_\nu s_{\rho\sigma}$ [defined on dual lattice]
  - Gauge transformation $U(1) \simeq \mathbb{R}/\mathbb{Z}$
    - $\mathbb{R}$ -valued 0-form: $\lambda^{(0)}$
    - $\mathbb{Z}$ -valued 1-form: $\lambda^{(1)}_\mu$
    - $\tilde{a}_\mu \to \tilde{a}_\mu + \partial_\mu \lambda^{(0)} + 2\pi\lambda^{(1)}_\mu$, $s_{\mu\nu} \to s_{\mu\nu} + \partial_\mu \lambda^{(1)}_\nu - \partial_\nu \lambda^{(1)}_\mu$

has some problems…
A better lattice θ term was proposed in [Anosova, Gattringer, Sulejmanpasic 2022]



- Matter part: $S_{Mat} = \sum_x iN\, n_\mu \tilde{a}_\mu$
  - Worldline of electric charge-N particle: n_μ

Action: $S_{CR}[\tilde{a}_\mu, s_{\mu\nu}, n_\mu] \coloneqq S_{U(1)}[\tilde{a}_\mu, s_{\mu\nu}] + S_{Mat}[\tilde{a}_\mu, n_\mu]$

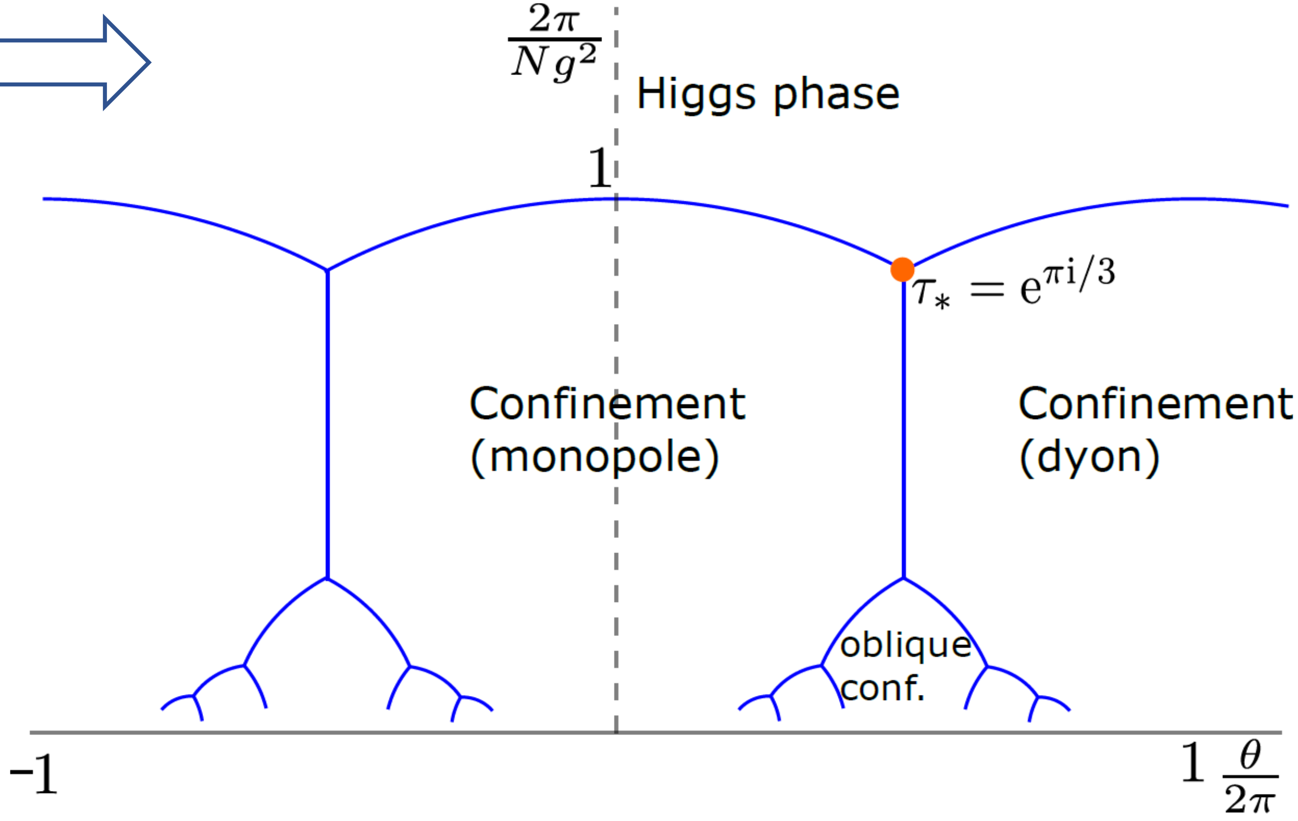
Conjectured phase diagram

Complex coupling

$$\tau := \frac{\theta}{2\pi} + i\,\frac{2\pi}{Ng^2}$$

- Phase diagram by a heuristic free-energy argument

(assuming that there exists some condensate everywhere)



$\boldsymbol{SL(2,\mathbb{Z})}$ **invariance generated by**

$$S: \tau \mapsto -\frac{1}{\tau}, \qquad \begin{pmatrix} e \\ m \end{pmatrix} \mapsto \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} e \\ m \end{pmatrix} = \begin{pmatrix} -m \\ e \end{pmatrix}$$

$$T: \tau \mapsto \tau + 1, \qquad \begin{pmatrix} e \\ m \end{pmatrix} \mapsto \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} e \\ m \end{pmatrix} = \begin{pmatrix} e - m \\ m \end{pmatrix}$$

- S duality? S-transformed model: $U(1)$ gauge + (1,0) matter + (0,N) matter

→ duality between the CR model and its $\mathbb{Z}_N^{[1]}$ gauged model

Non-invertible symmetry

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Generalized Symmetry

Important ~conservation law.

(ordinary symmetry) = (co-dim. 1 topological defect with group-like structure)

Higher-form symmetry

describing symmetry acting on higher-dimensional objects (e.g., Wilson loop)
Many applications to gauge theories

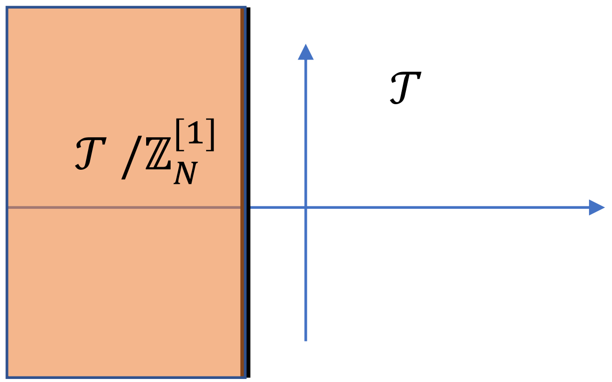
e.g.) $SU(N)$ YM theory has $\mathbb{Z}_N^{[1]}$ symmetry
→ CP & $\mathbb{Z}_N^{[1]}$ mixed "anomaly" @$\theta = \pi$
constrains the phase diagram [Gaiotto, Kapustin, Komargodski, Seiberg '17]

"noninvertible" symmetry

Well-studied in 2d QFTs
less known in higher-dimensions

Non-invertible duality defect

Recently, construction of duality defects in 4d has been developed.
[Koide, Nagoya, Yamaguchi '21; Choi et. al. '21; Kaidi, Ohmori, Zheng '21]



Rough idea:

Famous 2d example: Kramers-Wannier duality in Ising model.

$$\mathcal{T}\ /\ \mathbb{Z}_2 \simeq \mathcal{T} \quad \Longrightarrow$$

KW duality defect line =
"half-space gauging"

Generalization to 4d: **self-duality by 1-form symmetry $\mathbb{Z}_N^{[1]}$ gauging leads to a similar defect**

$$\mathcal{T}\ /\mathbb{Z}_N^{[1]} \simeq \mathcal{T} \quad \Longrightarrow$$

"half-space $\mathbb{Z}_N^{[1]}$ gauging"
: 3-dim topological defect

Note.) Gauging a p-form discrete symmetry causes a dual (d-p-2)-form symmetry
→ When d=4, only 1-form symmetry gauging can be self-dual.

Results

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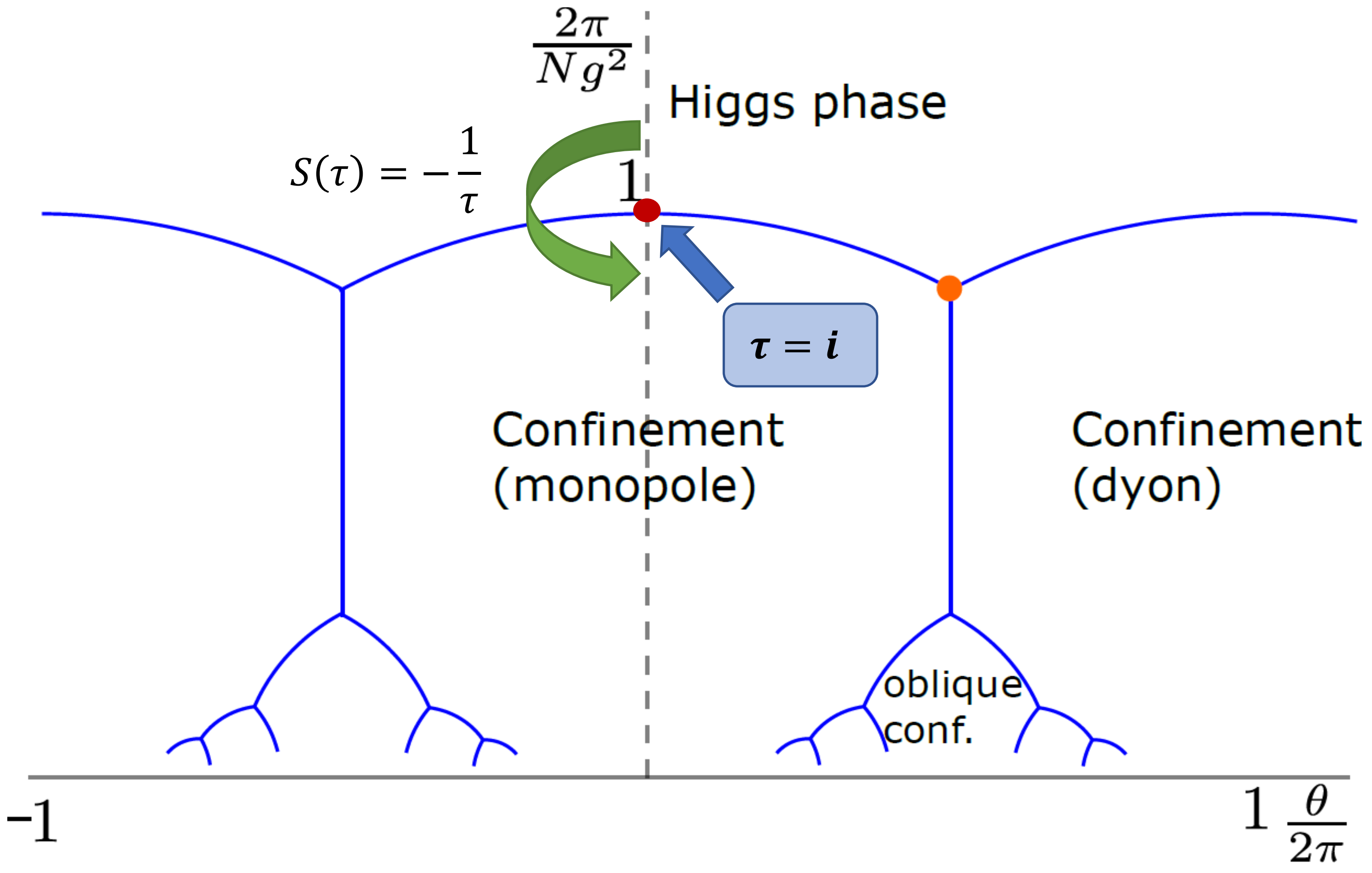
Warm-up: $\boldsymbol{S}$-defect

For Maxwell theory, constructed in [Choi et. al. 2021]

Complex coupling

$$\tau := \frac{\theta}{2\pi} + i\,\frac{2\pi}{Ng^2}$$



The S "self-duality" at $\tau = i$ can be realized as

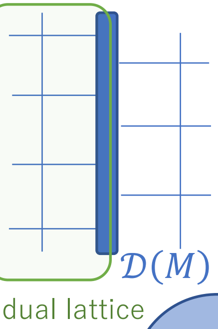
$$CR \,/\, \left(\mathbb{Z}_N^{[1]}\right)_0 \simeq CR$$

[U(1) gauge + (1,0) matter + (0,N) matter] system with coupling $\tau = i/N^2$

[U(1) gauge +(N,0) matter + (0,1) matter] system with coupling $\tau = i$

electromagnetic S transform

CP symmetry



We can construct non-invertible defects by half-space gauging

$$\mathcal{D}(M) \times \mathcal{D}(M) = \mathcal{C}(M)\,\frac{1}{N}\sum_{\Sigma\in H_2(M,\mathbb{Z}_N)} \eta\,(\Sigma)$$

$$\eta\,(\Sigma) \times \mathcal{D}(M) = \mathcal{D}(M) \times \eta\,(\Sigma) = \mathcal{D}(M)$$

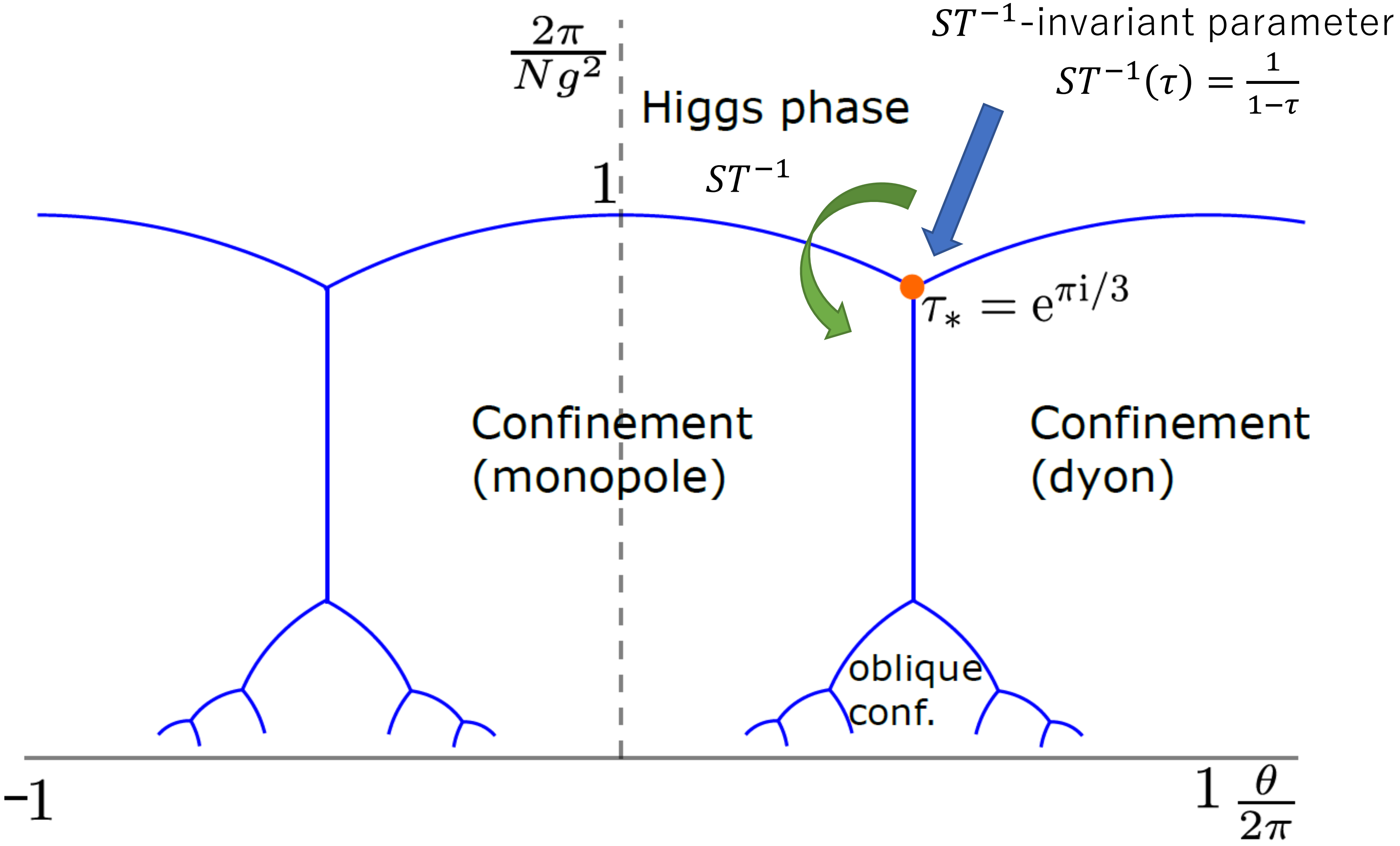
The trivially-gapped phase is ruled out for $N > 2$.

A simple guess $Z_{mono}[B] + Z_{Higgs}[B]$ is consistent with these constraints: consistent with the conjectured phase diagram!

Nontrivial example: ST^{-1} defect

Complex coupling

$$\tau := \frac{\theta}{2\pi} + i\,\frac{2\pi}{Ng^2}$$



The ST^{-1} "self-duality" at $\tau=\tau_*=e^{i\pi/3}$ can be realized as

attach discrete θ-term $e^{-\frac{iN}{4\pi}\int B\wedge B}$ when gauging

$$\boldsymbol{CR}\,/\left(\mathbb{Z}_{\boldsymbol{N}}^{[\mathbf{1}]}\right)_{-\mathbf{1}}\simeq \boldsymbol{CR}$$

[U(1) gauge + (1,0) matter + (0,N) matter] system with coupling $\frac{\tau_*-1}{N^2}$

[U(1) gauge + (N,0) matter + (0,1) matter] system with coupling $-\frac{1}{\tau_*-1}=\tau_*$

$$\left(Z^{\tau_*}_{CR\,/\left(\mathbb{Z}_N^{[1]}\right)_{-1}}[B]=N^{\frac{\chi}{2}}e^{-\frac{i\pi}{3}\sigma}Z^{\tau_*}_{CR}[B]\right)$$

cf.) [Witten '95] for Maxwell theory

We can construct non-invertible defects by half-space gauging

$$\mathcal{D}(M)\times\mathcal{D}(M)\times\mathcal{D}(M)\propto \mathrm{C}(M)\sum_{\Sigma\in H_2(M,\mathbb{Z}_N)}\eta\,(\Sigma)$$

$$\eta\,(\Sigma)\times\mathcal{D}(M)=\mathcal{D}(M)\times\eta\,(\Sigma)=\mathcal{D}(M)$$

The trivially-gapped phase is ruled out.

The constraint can be satisfied by a natural guess from the phase diagram

$$Z_{mono}[B]+e^{\frac{\pi i}{3}\sigma}Z_{dyon}[B]+N^{-\frac{\chi}{2}}\,e^{\frac{2\pi i}{3}\sigma}\,Z_{Higgs}[B]$$

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