Numerical investigation of automatic fine-tuning in the Schwinger model

Lattice22

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Introduction

- Howard Georgi published a paper 2020 where he found automatic fine-tuning in the isospin breaking effects by conformal coalescence
- Three solutions of standard sine-Gordon theory for $\beta = \sqrt{2\pi}$ that are idendified with the pions $\pi_{\pm}$ $(Q = 0,\ I_3 = \pm 1)$ and π_0 $(Q = 0,\ I_3 = 0)$
- Following Georgi [2]: define massless composite 1/2 dimensional operators $O_f = \psi^*_{f1}\psi_{f2}$ and $O^*_f = \psi^*_{f2}\psi_{f1}$
  with opposite isospin component $I_3 = +1, -1$ for flavor $f = 1, 2$
- Mix operators to get rid of θ dependency:
  $O_{\pm 1} = e^{i\theta/2} O_1 \pm e^{-i\theta/2} O^*_2 \qquad O^*_{\pm 1} = e^{-i\theta/2} O^*_1 \pm e^{i\theta/2} O_2$
- All 2-point correlators vanish except for

$$\langle 0 | T(O_{\pm 1}(x) O^*_{\pm 1}(0)) | 0 \rangle = \frac{\xi\mu}{2\pi^2}(e^{\kappa_0} \pm e^{-\kappa_0}) \frac{1}{\sqrt{-x^2 + i\varepsilon}} \tag{1}$$

with: $\kappa_0 = K_0\left(\mu\sqrt{-x^2 + i\epsilon}\right)$ Schwinger mass: $\mu = \sqrt{2/\pi}\, e$ (2)

- Detailed description of calculations → Poster session: Nuha Chreim Tuesday 8:00pm

Degenerate Masses m_f

- Introduce degenerate mass $m_f \ll \mu$ in Lagrangian mass term note $m_f(O_1 + O_2) + \text{h.c.} = m_f(O_{+1} + O^*_{+1})$
  With fermion mass: $m_f = (m_u + m_d)/2$
- The two point correlor sets the mass scale through the long distance behaviour of standard Bessel function K_0
- With the definition of O_{+1} this leaves the Lagrangian mass term

$$\frac{\sqrt{\xi}}{\pi} m_f \sqrt{\mu}(O_{1/2} + O^*_{1/2}) \tag{3}$$

- Implying a mass scale (in the deep IR) of

$$M_s = \left(m_f^2 \mu\right)^{\frac{1}{3}} \tag{4}$$

Non Degenerate Mass $m_f + \delta m$

- Introduce mass difference δm in the isospin splitting term
  $\delta m(O_1 - O_2) + h.c. = \delta m(O_{-1} + O^*_{-1})$
  With fermion mass difference: $\delta m = m_u - m_d$
- The two point correlator again sets the mass scale through the long distance behaviour of the K_0
- With the definition of O_{-1} the Lagrangian mass term reads

$$\delta m \sqrt{\xi \sqrt{\frac{1}{8\pi}} (m_f \mu^2)^{\frac{1}{3}} \underbrace{e^{-\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}}}}_{\text{predicted by Georgi}}} (O_{1/2} + O^*_{1/2}) \tag{5}$$

- Implying a mass scale of the isosplitting term (in the deep IR) of

$$\Delta M_s = \left(\delta m^2 m_f^{\frac{1}{3}} \mu^{\frac{2}{3}} e^{-\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}}}\right)^{\frac{1}{3}} \tag{6}$$

Non Degenerate Mass $m_f + \delta m$

- Overall mass term

$$\frac{\sqrt{\xi}}{\pi} m_f \sqrt{\mu} \left(1 + \left(\frac{\pi}{2}\right)^{\frac{3}{4}} \frac{\delta m}{\mu^{\frac{1}{6}} m_f^{\frac{5}{6}}} e^{-\frac{1}{2}\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}}}\right) (O_{1/2} + O^*_{1/2}) \tag{7}$$

- Isospin breaking corrections to leading order in δm:

$$M_\pi \propto m_f^{\frac{2}{3}} \left(1 + \frac{2}{3}\left(\frac{\pi}{2}\right)^{\frac{3}{4}} \mu^{-\frac{1}{6}} \frac{\delta m}{m_f^{\frac{5}{6}}} e^{-\frac{1}{2}\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}}}\right) \tag{8}$$

- To get rid of proportionality factor normalize with: $M_{\pi\pm} = 2.008 * m_f^{2/3}$ [1]

$$\Rightarrow \frac{M_{\pi\pm} + \Delta M}{M_{\pi\pm}} = 1 + \frac{2}{3}\left(\frac{\pi}{2}\right)^{\frac{3}{4}} \mu^{-\frac{1}{6}} \frac{\delta m}{m_f^{\frac{5}{6}}} e^{-\frac{1}{2}\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}}} \tag{9}$$

$$\Leftrightarrow \log\left(\underbrace{\frac{3}{2}\left(\frac{2}{\pi}\right)^{\frac{3}{4}} \mu^{\frac{1}{6}} \frac{m_f^{\frac{5}{6}}}{\delta m}}_{k} \frac{\Delta M}{M_{\pi\pm}}\right) = -\frac{1}{2}\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}} \tag{10}$$

- By keeping k constant $\delta m \xrightarrow{m_f \to 0} 0$

Non Degenerate Mass $m_f + \delta m$

- Overall mass term

$$\frac{\sqrt{\xi}}{\pi} m_f \sqrt{\mu}\left(1+\left(\frac{\pi}{2}\right)^{\frac{3}{4}}\frac{\delta m}{\mu^{\frac{1}{6}}\, m_f^{\frac{5}{6}}}\, e^{-\frac{1}{2}\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}}}\right)\left(O_{1/2}+O^{*}_{1/2}\right) \tag{7}$$

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- By keeping k constant $\delta m \xrightarrow{m_f \to 0} 0$

- $\pi_\pm$ consists of connected terms, thus in leading order it does not hold isospin mass splitting [3]:

$$\langle O_{\pi^0}(x)\overline{O}_{\pi^0}(y)\rangle = \frac{1}{2}\Big(\underbrace{-\mathrm{tr}(D_u^{-1}(x,y)\gamma_5 D_u^{-1}(y,x)\gamma_5)}_{\langle O_{\pi_\pm}(x)\overline{O}_{\pi_\pm}(y)\rangle}$$

$$+\,\mathrm{tr}(D_u^{-1}(x,x)\gamma_5)\mathrm{tr}(D_u^{-1}(y,y)\gamma_5)$$

$$-\,\mathrm{tr}(D_u^{-1}(x,x)\gamma_5)\mathrm{tr}(D_d^{-1}(y,y)\gamma_5)$$

$$+\,u \leftrightarrow d$$

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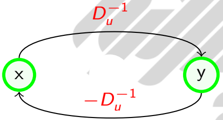

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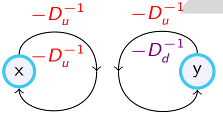


Technical details

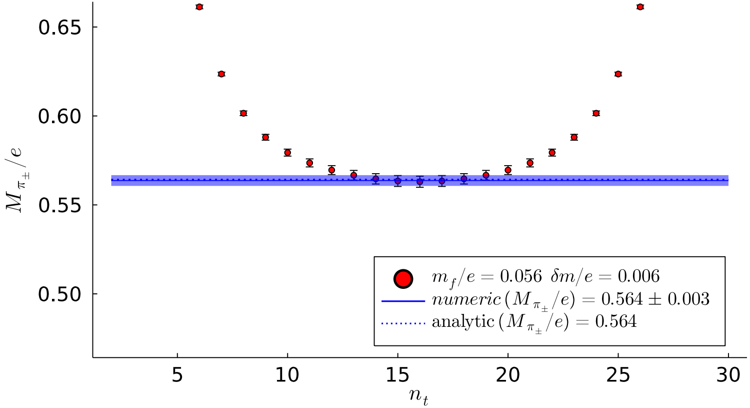
- Overlap operator
- Explicit diagonalization for every configuration
- Metropolis algorithm in quenched theory
  reweighting with fermion determinant for $N_f = 2$
- $N^2 = 32^2 \quad \Big| \quad \beta = 7.2 \quad \Big| \quad V_{ph} = \frac{N^2}{\beta} = 142.2$

  for 1.) $\delta m \approx 0.1 m_f \Leftrightarrow \frac{m_f^{\frac{5}{6}}}{\delta m} = 14.837$

  2.) $\delta m \approx 0.8 m_f \Leftrightarrow \frac{m_f^{\frac{5}{6}}}{\delta m} = 1.855$

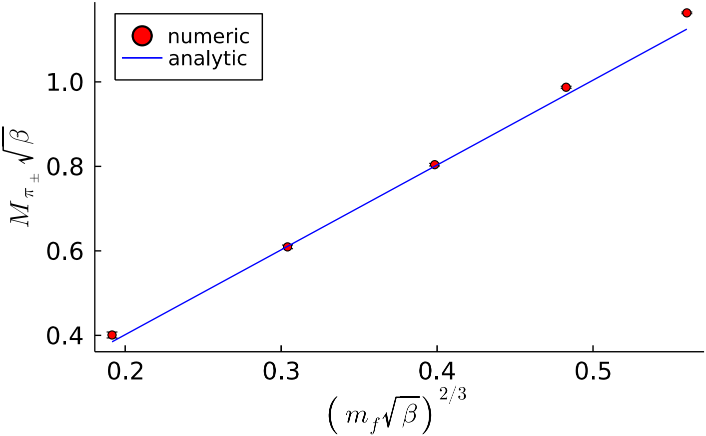
Mass plateaus from propagators

- Example of 3 point mass plateau:
  $N^2 = 32^2 \mid \beta = 7.2 \mid V_{ph} = 142.2$
- Analytical result via approximation $M_{\pi_\pm} = 2.008 * m_f^{2/3}$ [1]



Crosscheck: M_π vs. m_f on fine lattice

- Mass measurements for degenerate fermions for
- Crosscheck via sine-Gordon approximation $M_{\pi_\pm} = 2.008 * m_f^{2/3}$ [1]
  $N^2 = 32^2 \mid \beta = 7.2 \mid V_{ph} = 142.2$

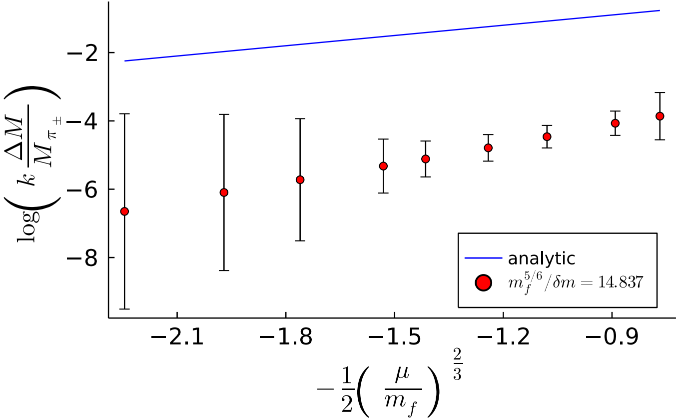


Measuring isospin breaking effects

- Measurement for non degenerate fermions for Lattice config:

$$N^2 = 32^2 \;\Big|\; \beta = 7.2 \;\Big|\; V_{ph} = 142.2 \;\Big|\; \delta m \approx 0.1 m_f \Leftrightarrow \frac{m_f^{\frac{5}{6}}}{\delta m} = 14.837$$

$$-\frac{1}{2}\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}} = \log\left(k\frac{\Delta M}{M_{\pi_\pm}}\right) \qquad \text{with: } k = \frac{3}{2}\left(\frac{2}{\pi}\right)^{\frac{3}{4}} \mu^{\frac{1}{6}} \frac{m_f^{\frac{5}{6}}}{\delta m}$$



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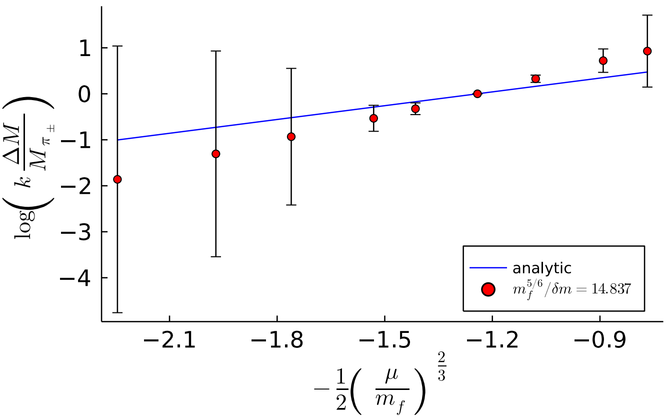
- Noise reduction via subtraction by 6th mass



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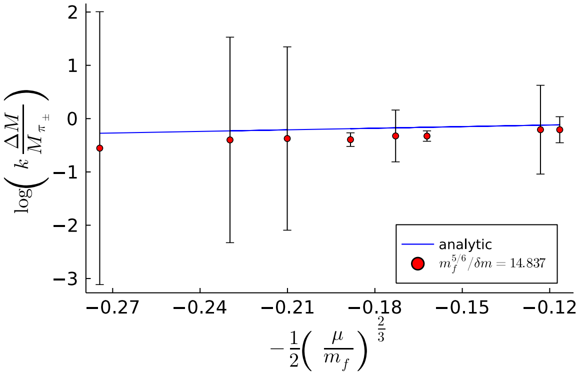
- Noise reduction via subtraction by neighbouring masses



Measuring isospin breaking effects → different ratio

- Measurement for non degenerate fermions for Lattice config:

$$N^2 = 32^2 \;\Big|\; \beta = 7.2 \;\Big|\; V_{ph} = 142.2 \;\Big|\; \delta m \approx 0.8 m_f \Leftrightarrow \frac{m_f^{\frac{5}{6}}}{\delta m} = 1.855$$

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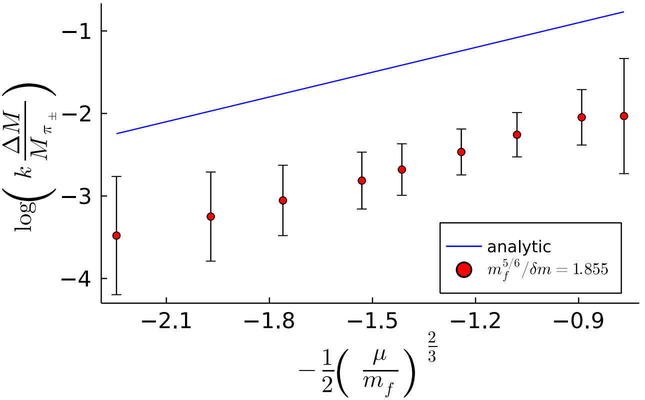

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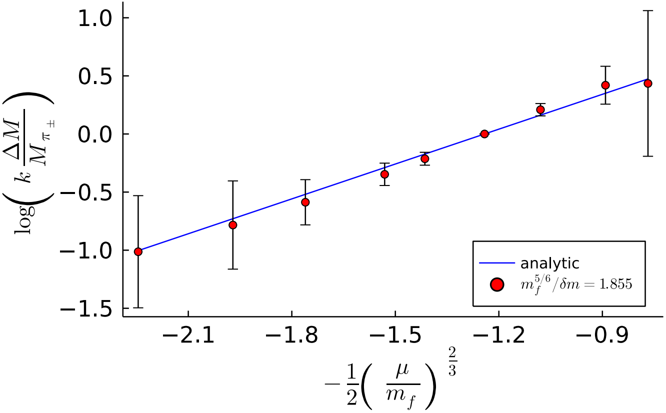
- Noise reduction via subtraction by 6th mass



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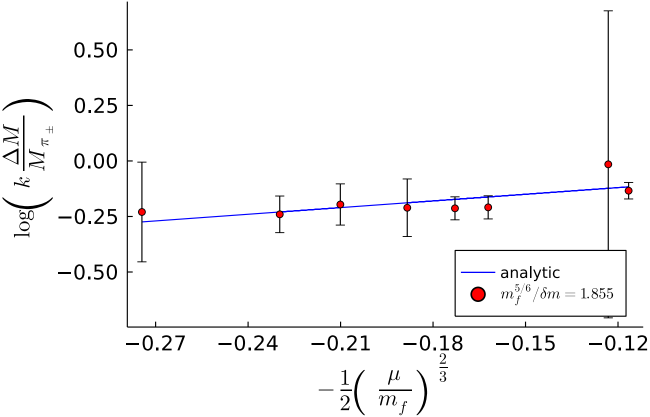

Conclusions & Outlook

What we found:

- Automatic fine tuning for $\delta m \approx m_f$ can be tested numerically in the Schwinger model by using overlap fermions
- Exponential surpression in m_f

For further investigation:

- Systematic expansion in m_f and δm
- Power counting

$\rightarrow$ to get better understanding of prefactor

Further details:

- Description of analytics on poster by Nuha Chreim
  Isospin Breaking Effects in the 2-Flavor Schwinger Model
  Tuesday 8:00pm

Thank you for your attention!

References

Christof Gattringer, Ivan Hip, and C.B. Lang. "The chiral limit of the two-flavor lattice Schwinger model with Wilson fermions". In: *Physics Letters B* 466.2-4 (1999), 287–292. ISSN: 0370-2693. DOI: 10.1016/s0370-2693(99)01116-8. URL: http://dx.doi.org/10.1016/S0370-2693(99)01116-8.

Howard Georgi. "Automatic Fine-Tuning in the Two-Flavor Schwinger Model". In: *Physical Review Letters* 125.18 (2020). ISSN: 1079-7114. DOI: 10.1103/physrevlett.125.181601. URL: http://dx.doi.org/10.1103/PhysRevLett.125.181601.

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