Mass Renormalization of the Schwinger Model with Wilson Fermions and Tensor Networks

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HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

Motivation

Schwinger Model:

- Similarities with QCD (confinement, chiral symmetry breaking)
- Benchmarking model for new lattice methods

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Wilson Fermions: $H \to H - \frac{ra^2}{2} \sum_x \bar{\psi}_x \partial_1^2 \psi_x$

- Completely remove the doublers
- Easily generalised to higher dimensions

Schwinger Model

Hamiltonian Lattice Formulation with Wilson Fermions (r = 1), 1 Fermion Flavour, Open Boundary Conditions in 1+1 dimensions

$$W = 2x \sum_{n=1}^{N-1} \left(\phi^\dagger_{n,1} U_n \phi_{n+1,2} + H.c.\right) + 2\left(\frac{m}{g}\sqrt{x} + x\right) \sum_{n=1}^{N} \left(\phi^\dagger_{n,1}\phi_{n,2} + H.c.\right) + \sum_{n=1}^{N-1} L_n^2 \quad [1]$$

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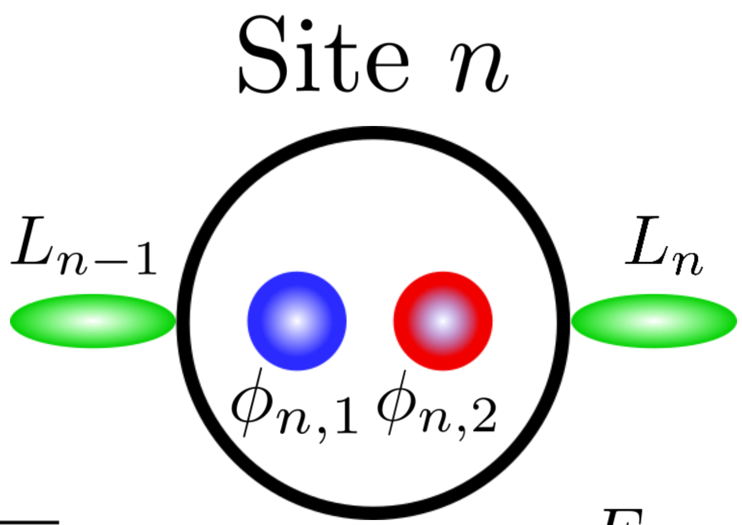

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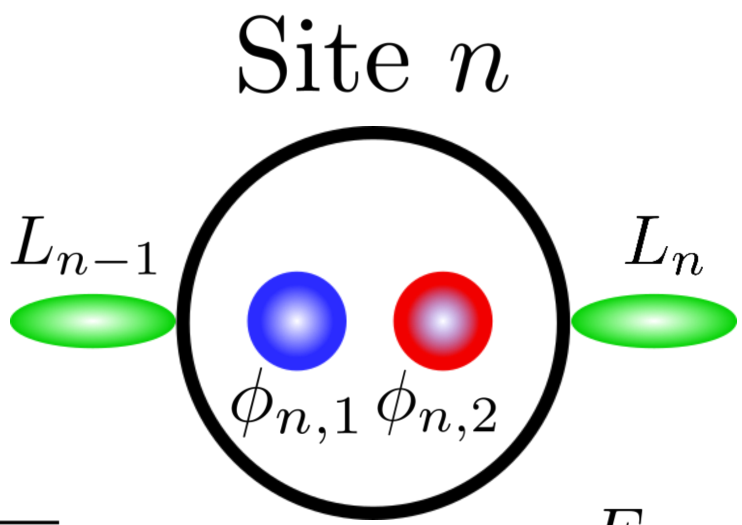

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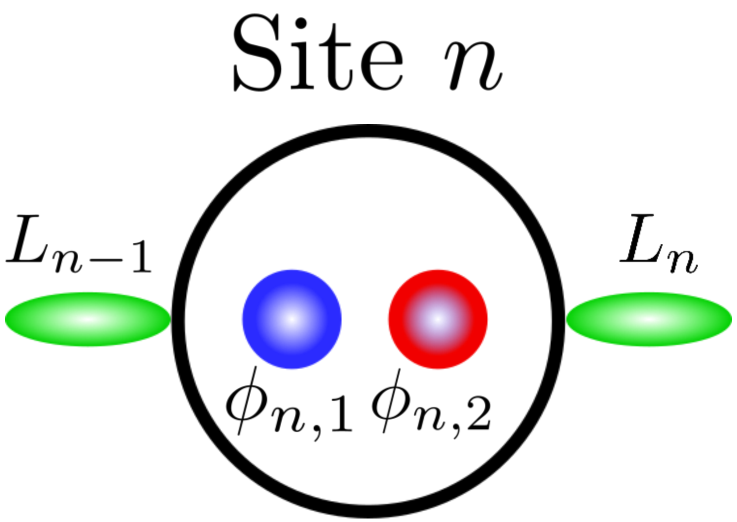
$$L_n - L_{n-1} = Q_n = \phi^\dagger_n \phi_n - 1 \qquad \text{Gauss' Law}$$

Schwinger Model

Eliminating the Gauge Field



For OBC:

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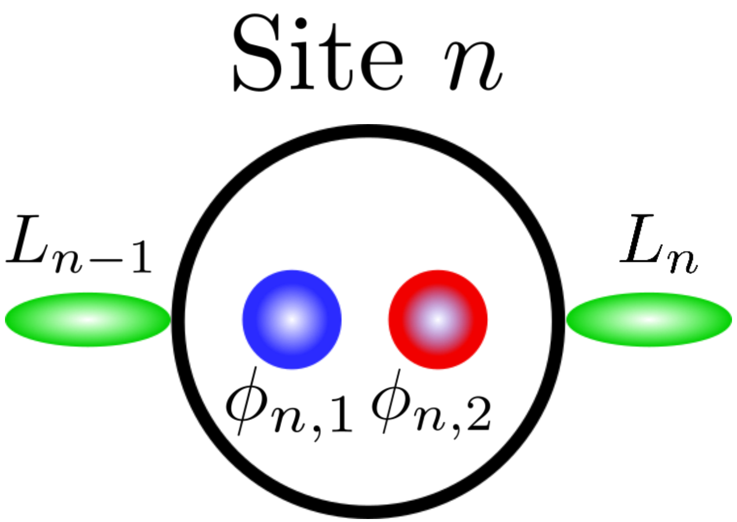

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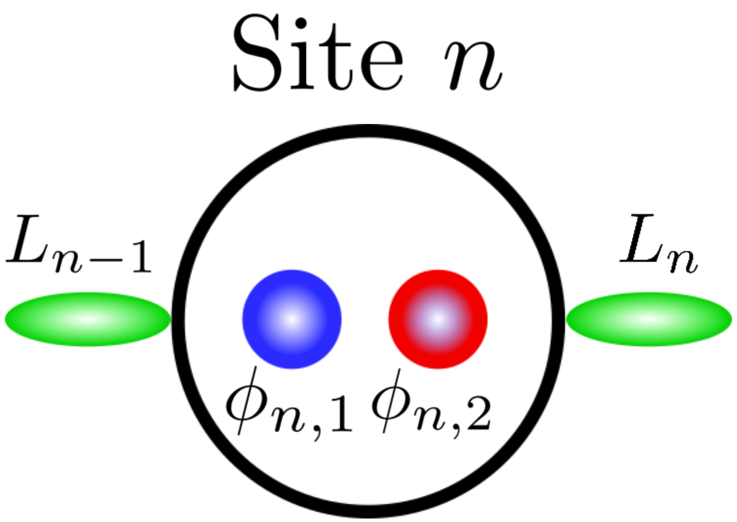

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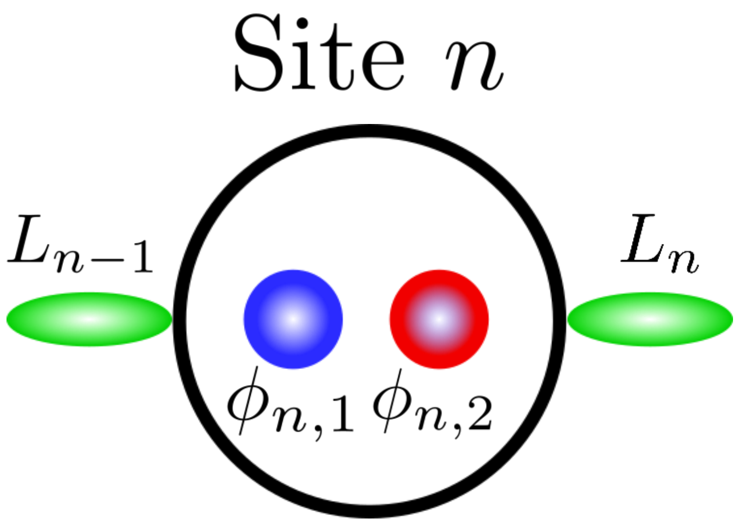

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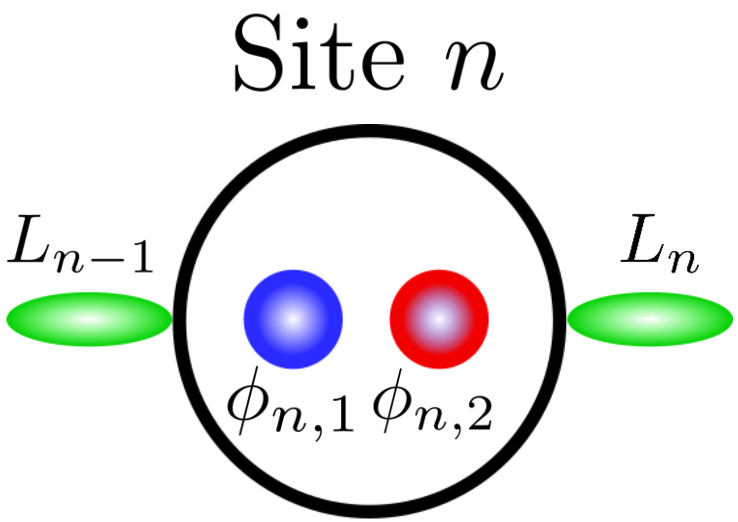

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$\text{Wilson term causes an additive shift to } \frac{m}{g}: \quad \left(\frac{m}{g}\right)_r = \left(\frac{m}{g}\right)_b + f(x)$

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Electric field density of G.S. $= 0$ at $\left(\frac{m}{g}\right)_r = 0$

Plot EFD vs $\left(\frac{m}{g}\right)_b \rightarrow$ x-intercept $= \left(\frac{m}{g}\right)_b^* = -f(x)$

Methods & Goals

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Find ground state as
matrix product states (MPS)
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$$|\psi\rangle = \sum_{i_1,i_2,..,i_N} c_{i_1,i_2,..,i_N} |i_1\rangle \otimes |i_2\rangle \otimes ... |i_N\rangle = \sum_{i_1,i_2,..,i_N} A^{i_1}_{1,\alpha_1} A^{i_2}_{\alpha_1,\alpha_2} ... A^{i_N}_{\alpha_{N-1},1} |i_1\rangle \otimes |i_2\rangle \otimes ... |i_N\rangle$$

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Find ground state as matrix product states (MPS) using variational algorithm

→ Measure the electric field density of the ground state vs bare mass

→ Determine the additive mass shift from the Wilson term

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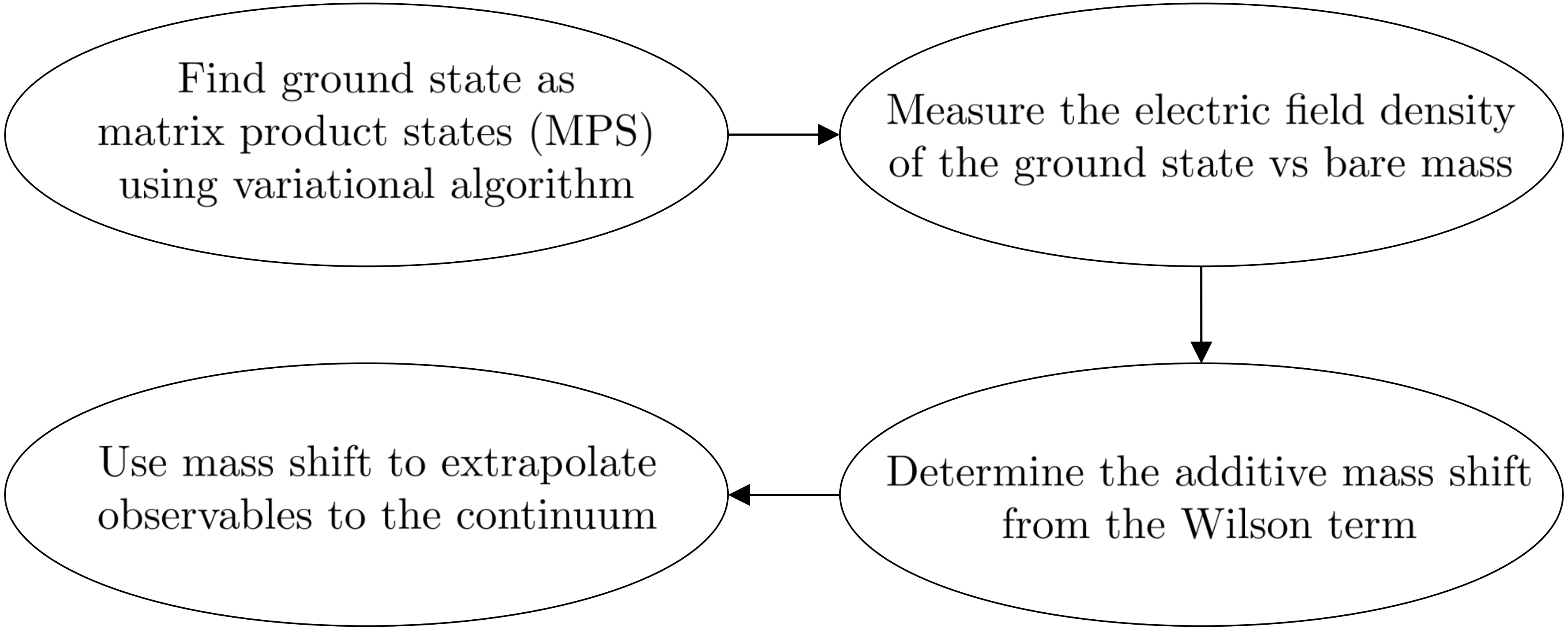

EFD vs bare m/g

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Electric Field Density
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$$\frac{\mathcal{F}(m,\theta)}{g} = A \cdot \frac{m}{g} \sin(\theta) + B \cdot \left(\frac{m}{g}\right)^2 \sin(2\theta) + \mathcal{O}\left(\left(\frac{m}{g}\right)^3\right) \text{ [1]}$$

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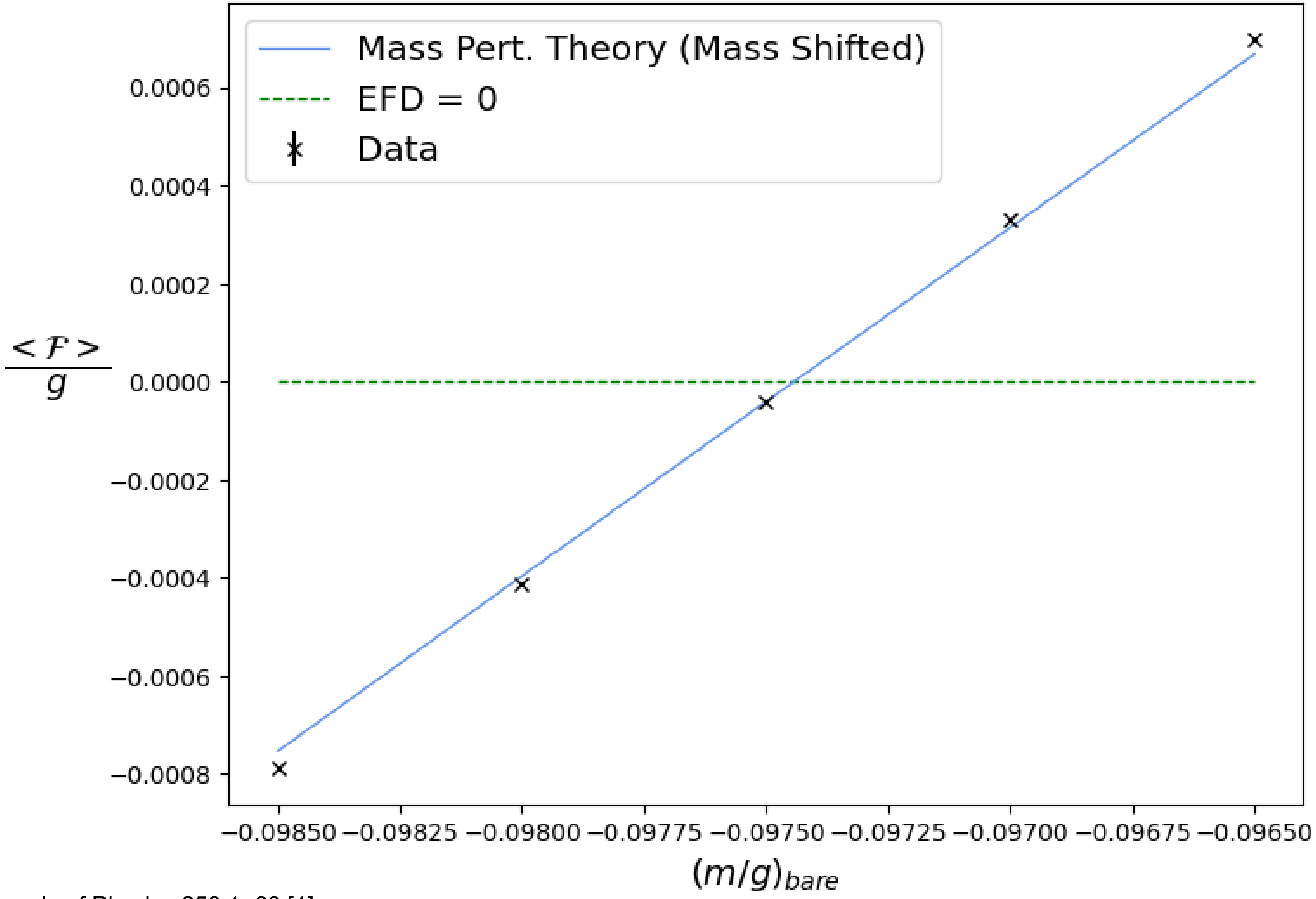

$$N = 64$$

$$x = \frac{1}{(ag)^2} = 10$$

$$l_0 = \frac{\theta}{2\pi} = 0.125$$

Preliminary Results

Mass Shift vs ag

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Fixed dimensionless volume: $\frac{N}{\sqrt{x}} = 20$

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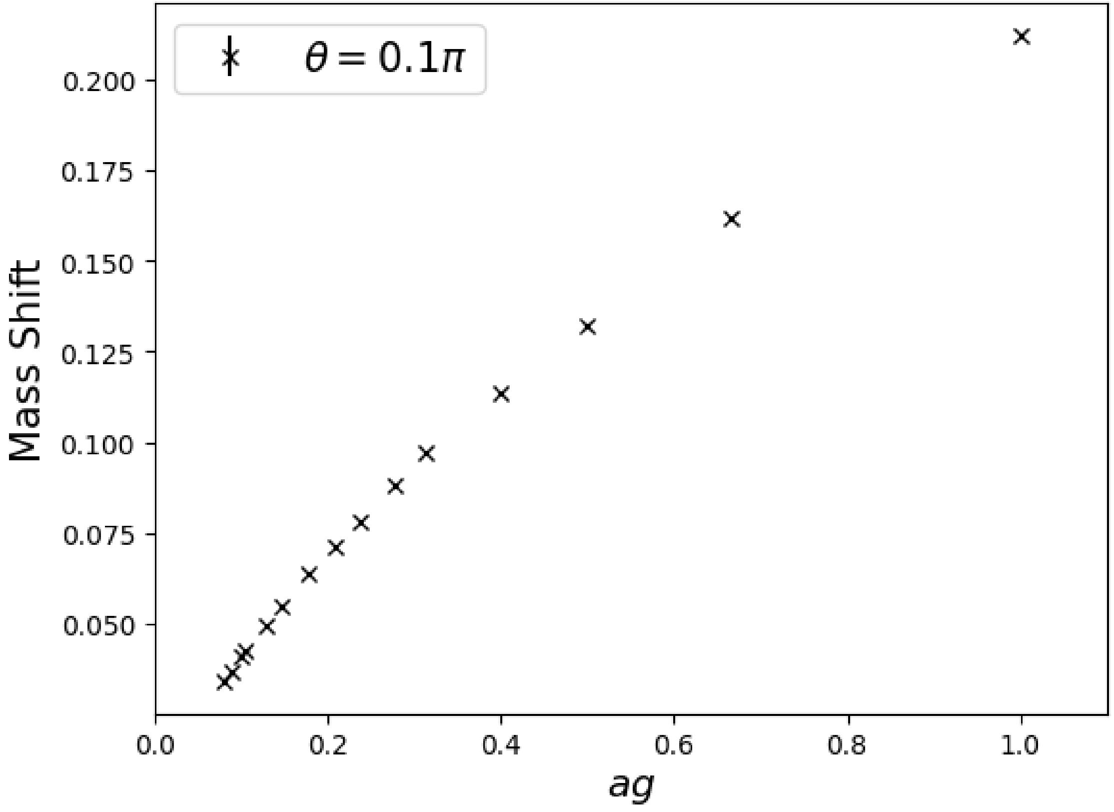

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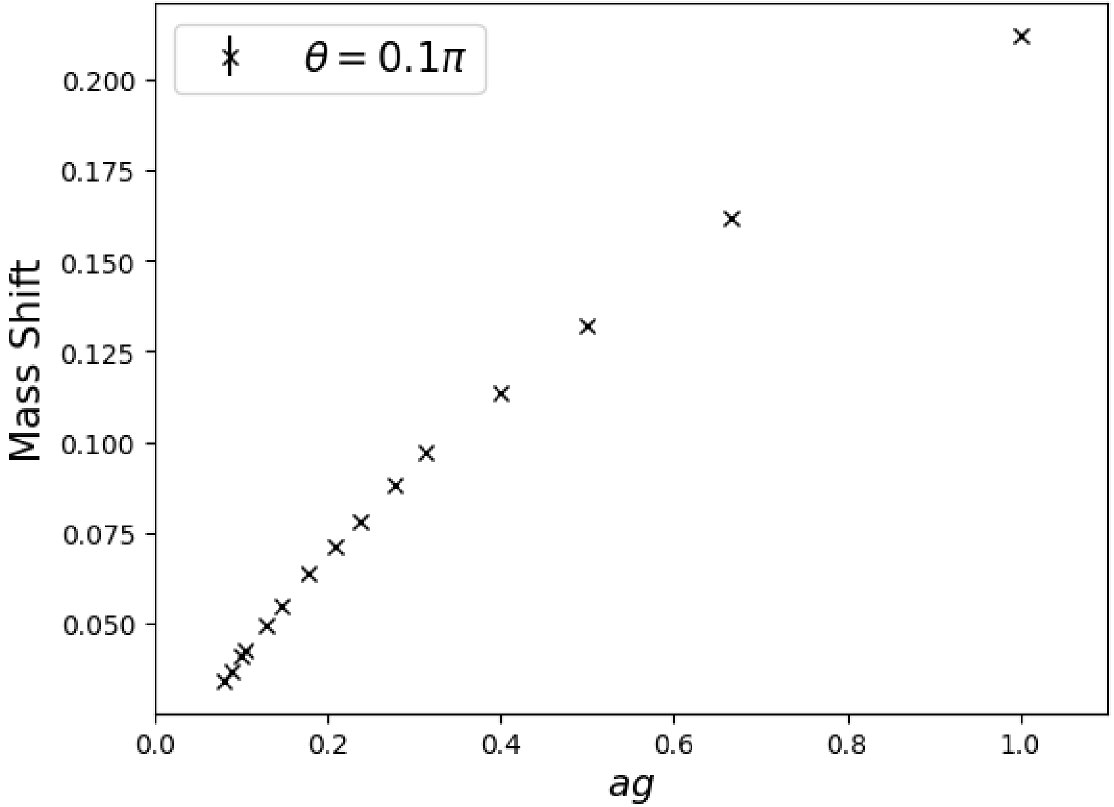

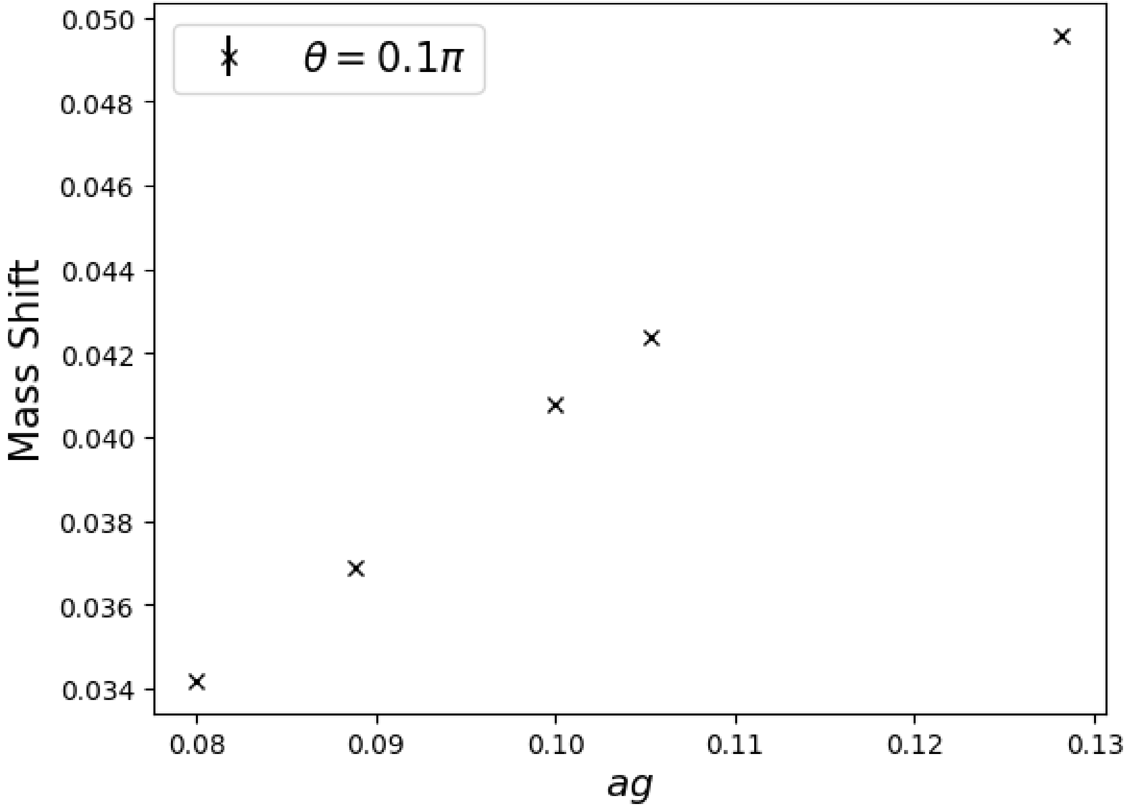

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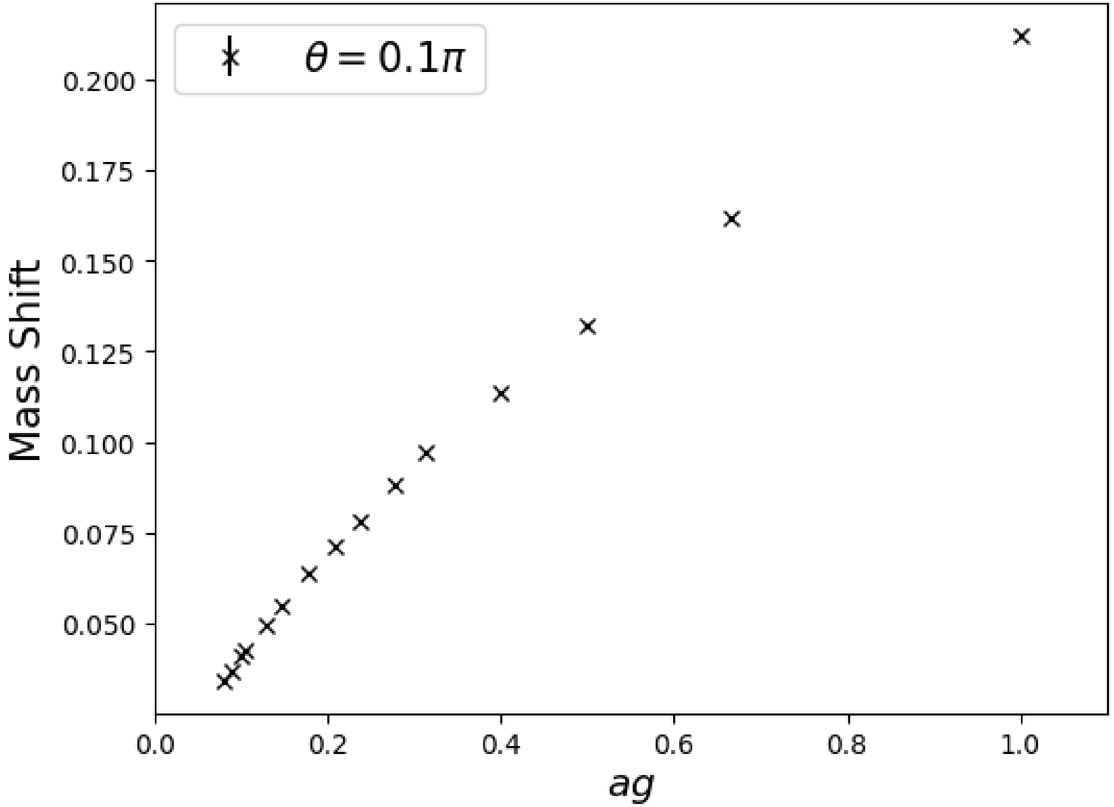

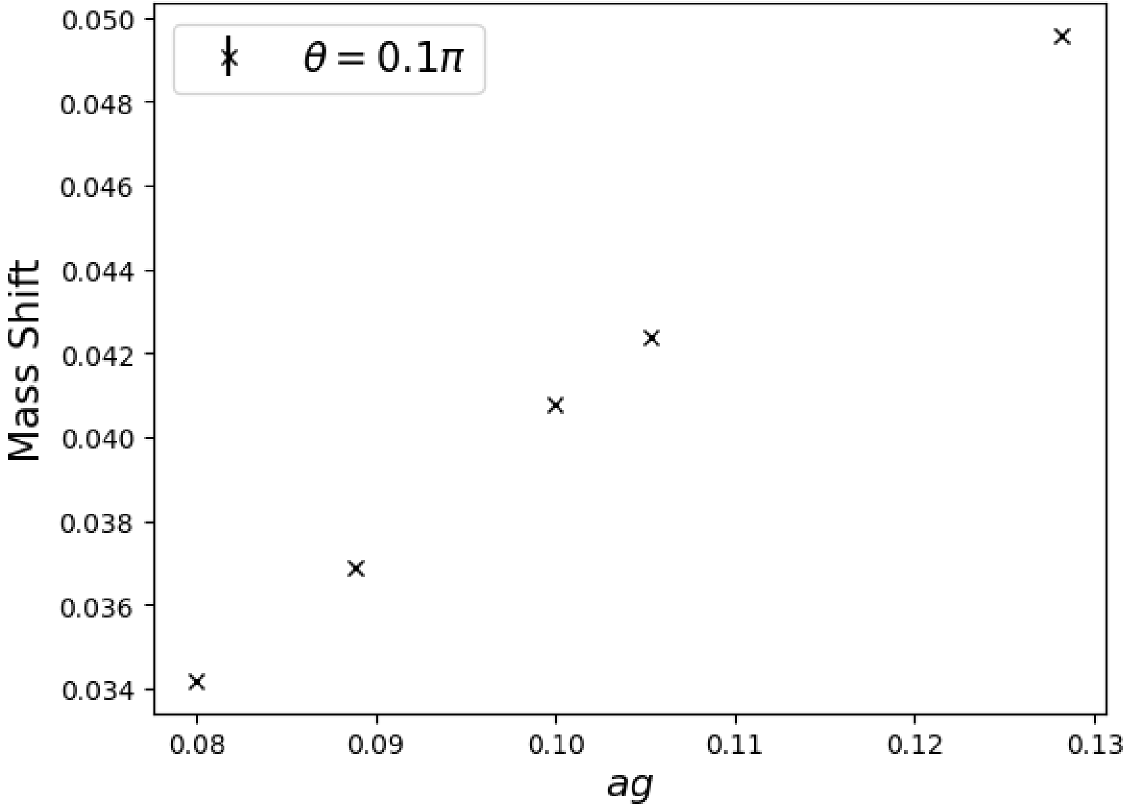

Mass Shift $\propto \mathcal{O}(ag)$

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Mass Shift θ Dependence

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Chiral anomaly at $m_r = 0 \implies \theta$ is unphysical

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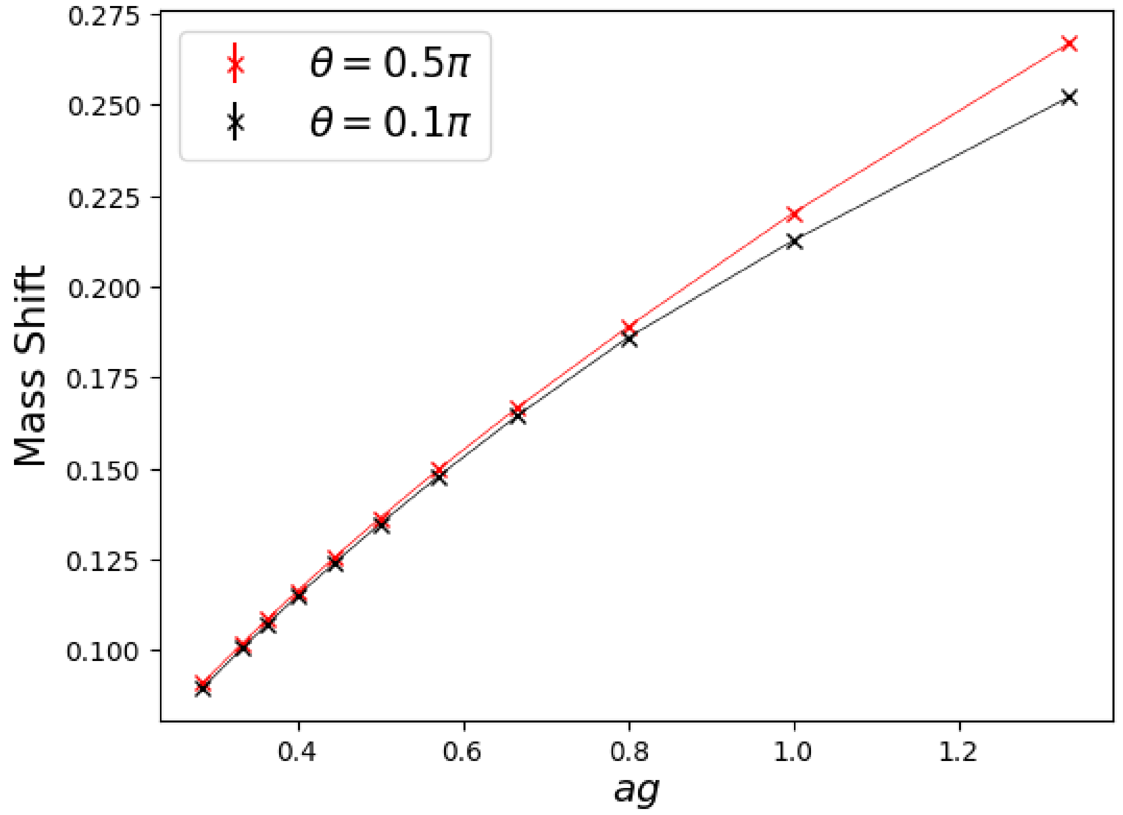
$$\text{Chiral anomaly at } m_r = 0 \implies \theta \text{ is unphysical}$$



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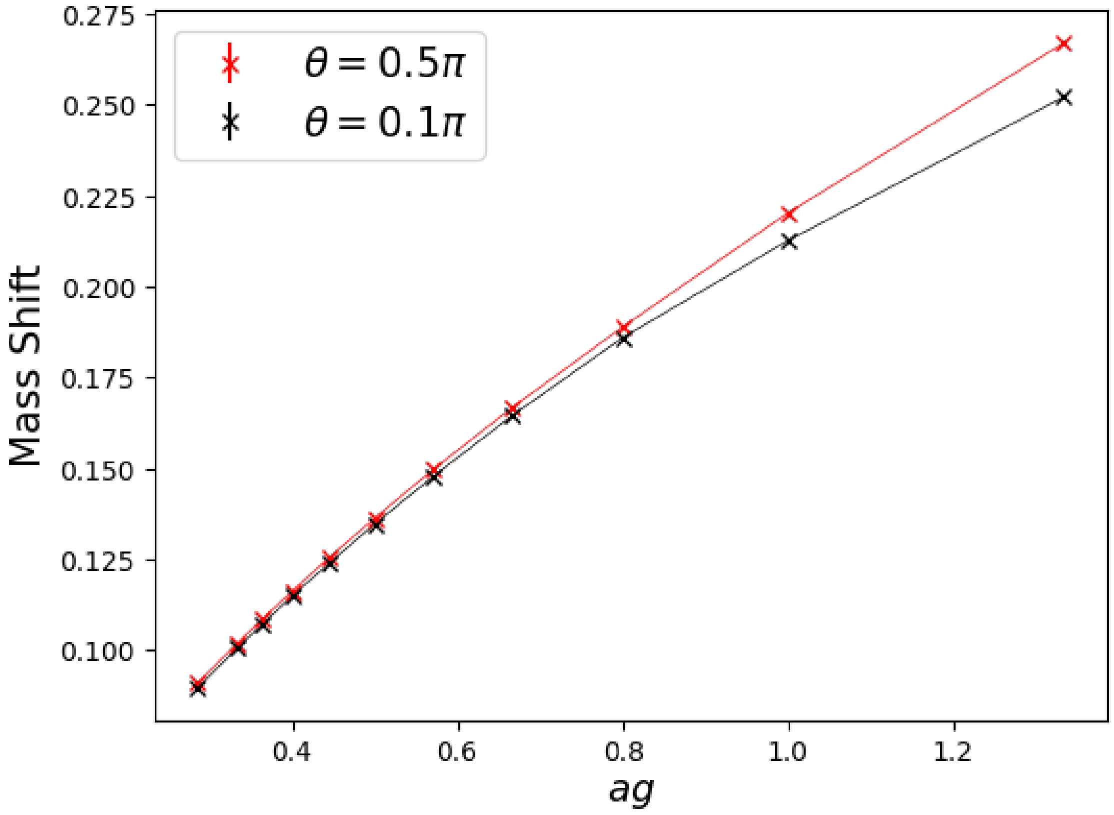

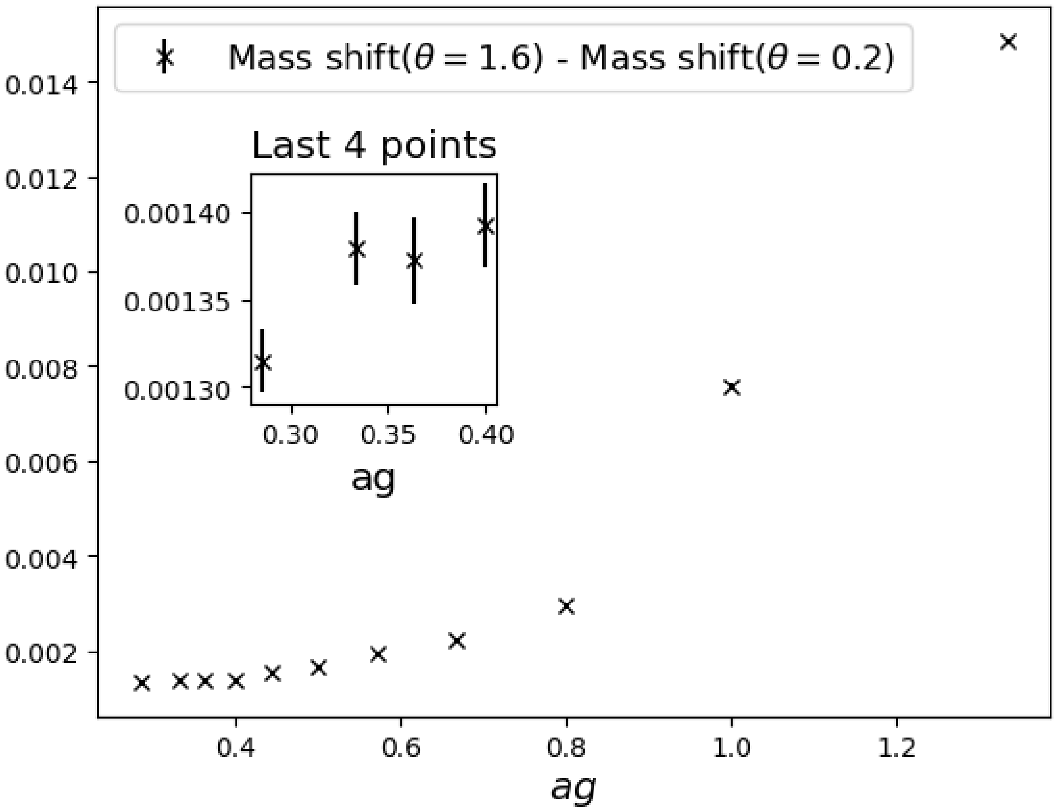

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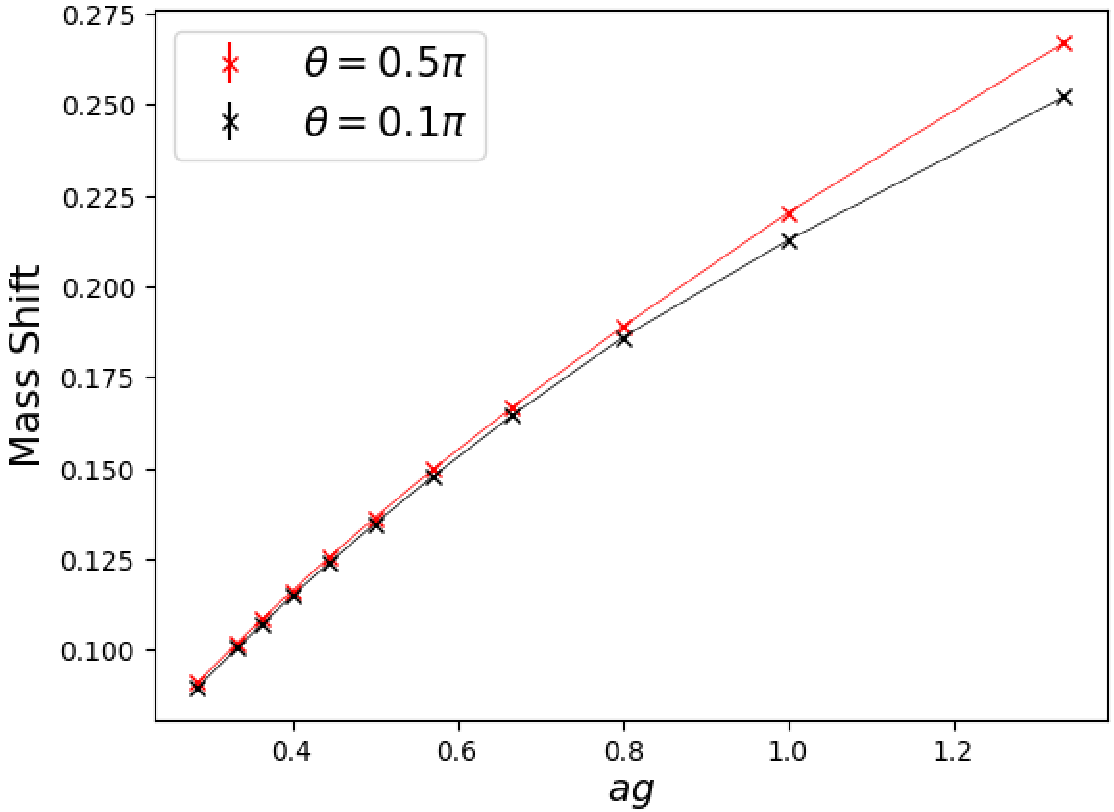

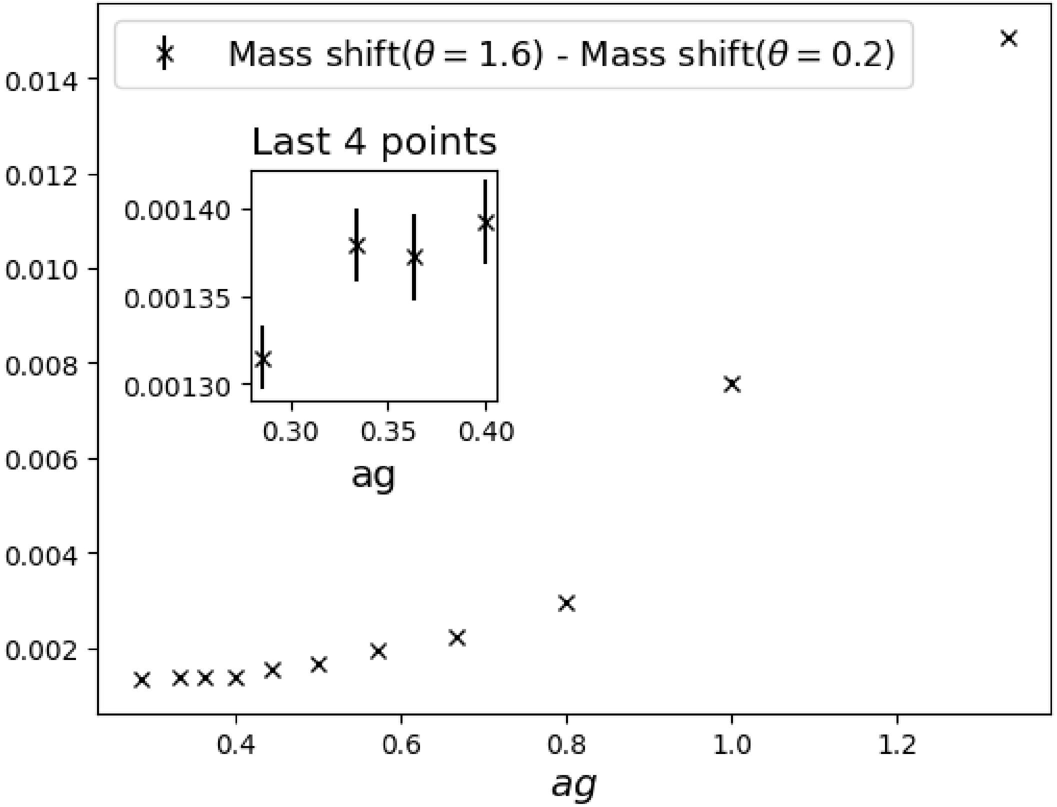

θ dependence of mass shift decays as $ag \to 0$

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EFD vs renormalized mass in continuum extrapolation

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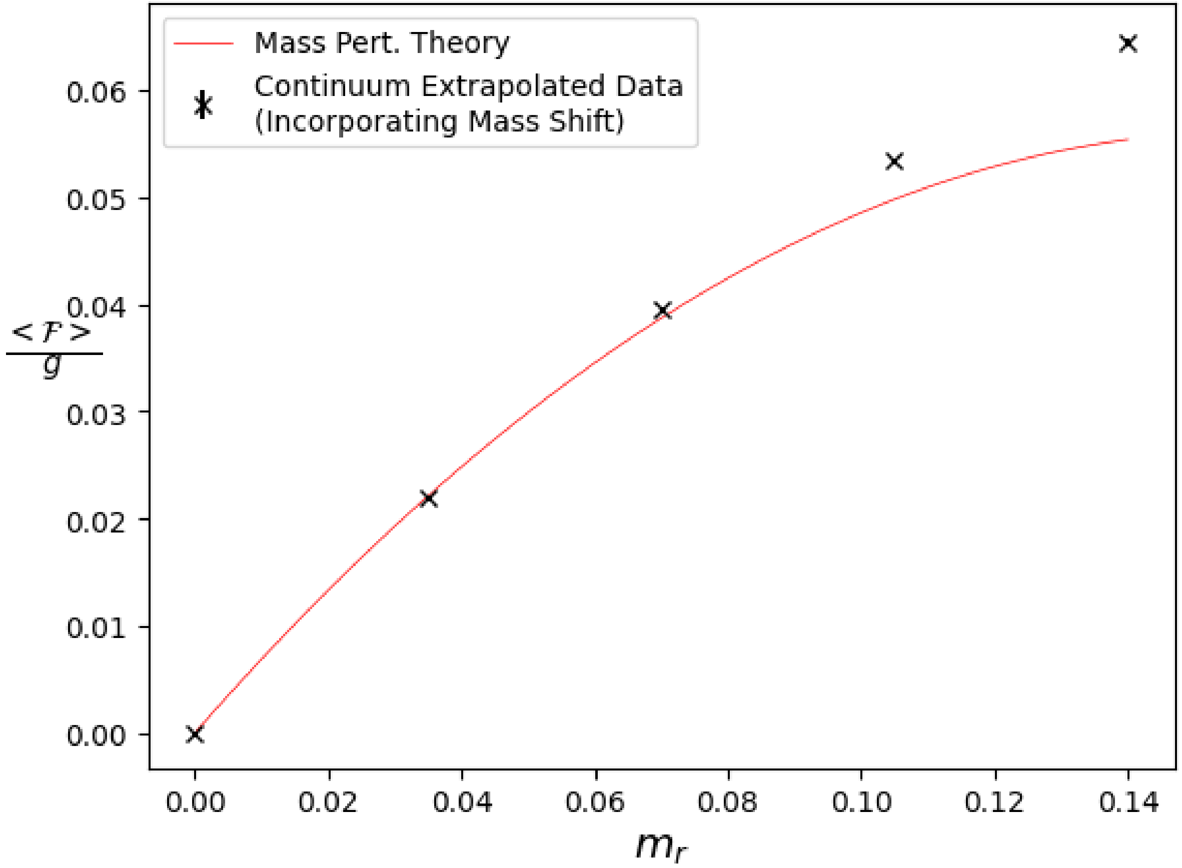

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Mass Shift with Staggered Fermions (Schwinger model, 1 Flavour)

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Prediction: $\left(\frac{m}{g}\right)_r = \left(\frac{m}{g}\right)_b + \frac{1}{8\sqrt{x}}$ [1]

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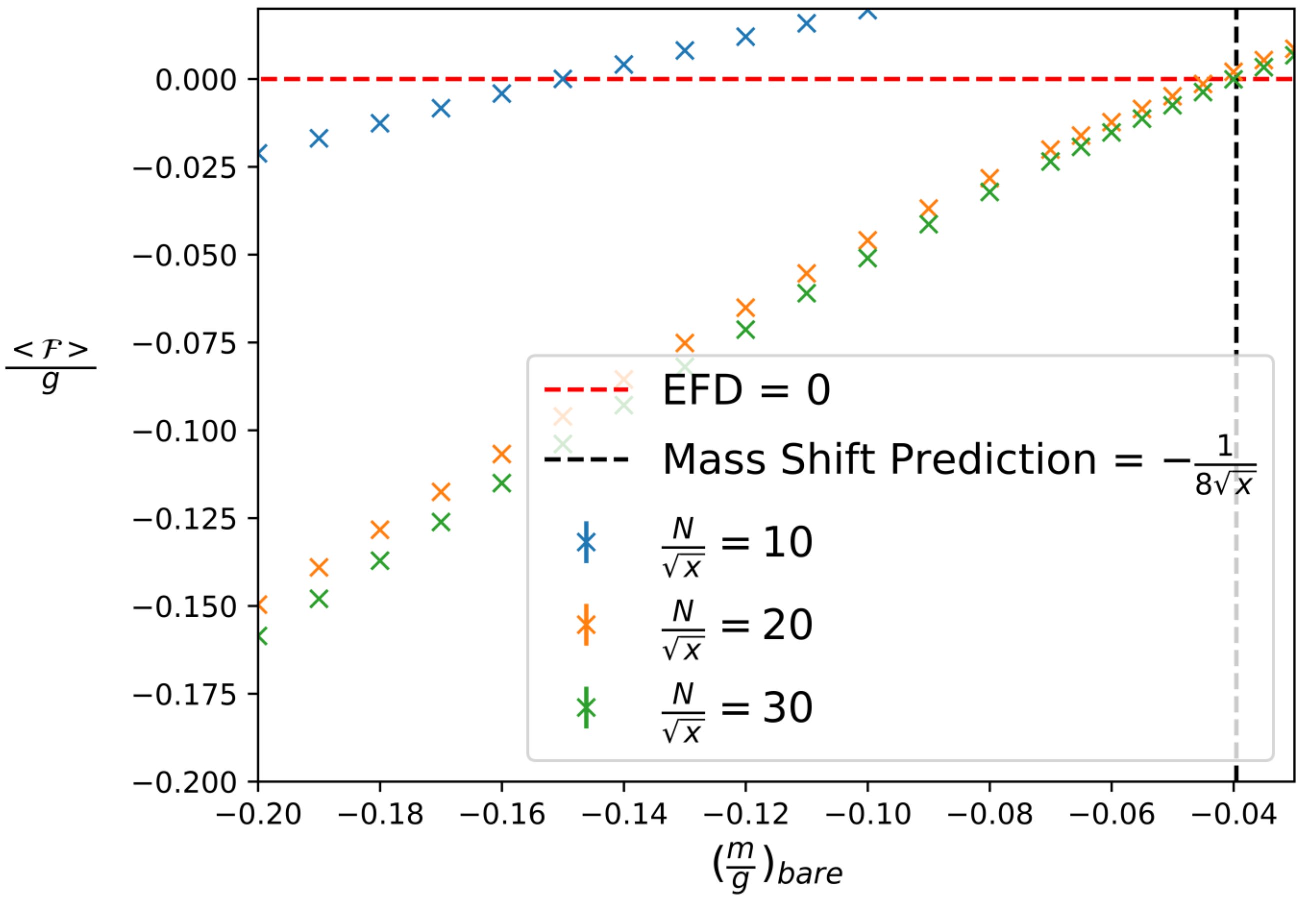
With 1 fermion flavour → chiral symmetry breaking → additive mass shift



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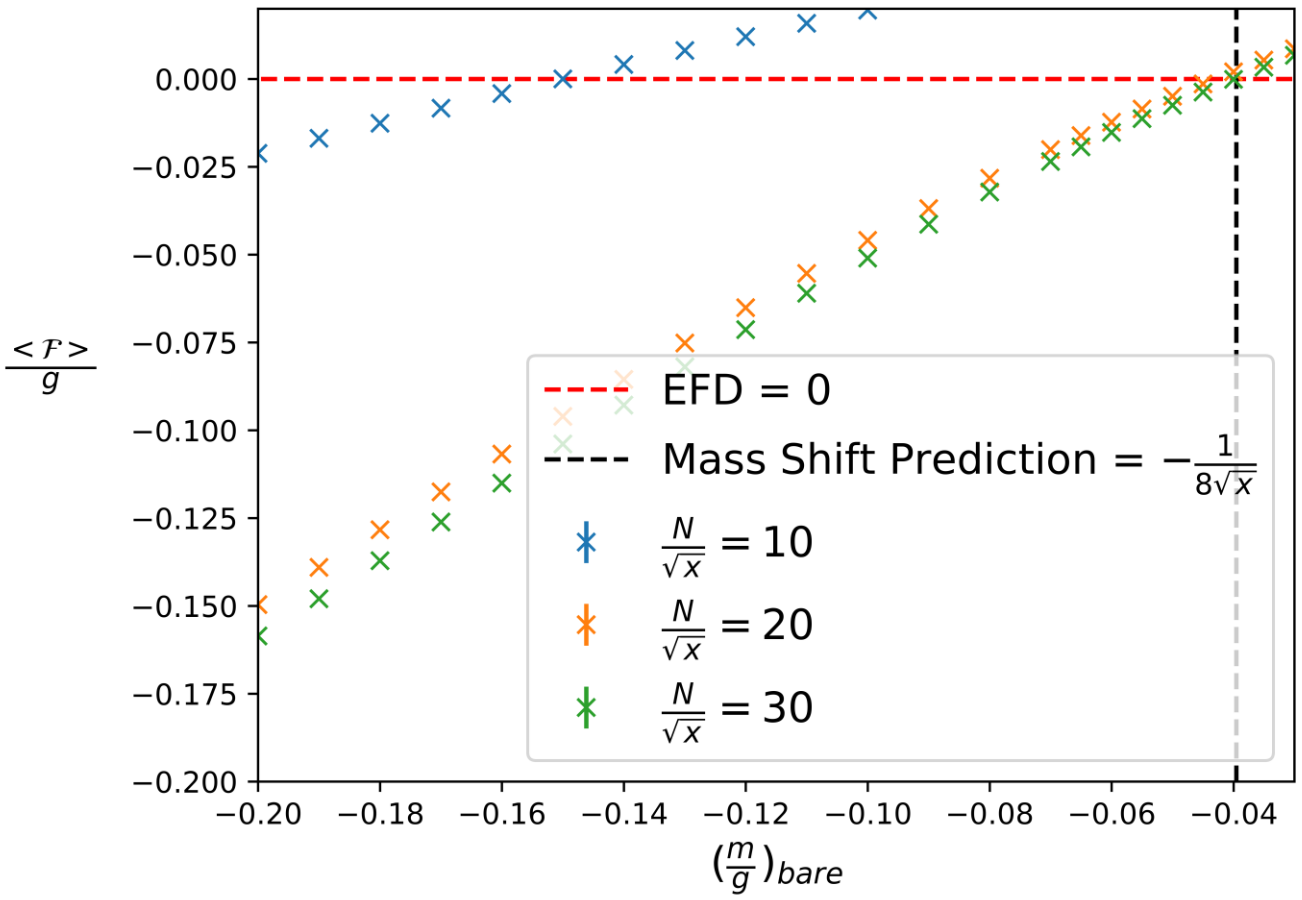

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Mass shift agrees with prediction at sufficiently large volume

Conclusions & Outlook

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Continuum prediction agrees with extrapolated data close to small masses after using mass shift

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Check whether $Z = 1$ in $\left(\frac{m}{g}\right)_r = Z\left(\frac{m}{g}\right)_b + f(x)$

Thank you

Questions?

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Appendix A

Multiplicative Mass Renormalization – Schwinger Model Phase Diagram

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Need a second renormalization condition to fix both Z and $f(x)$ in $\left(\frac{m}{g}\right)_r = Z\left(\frac{m}{g}\right)_b + f(x)$

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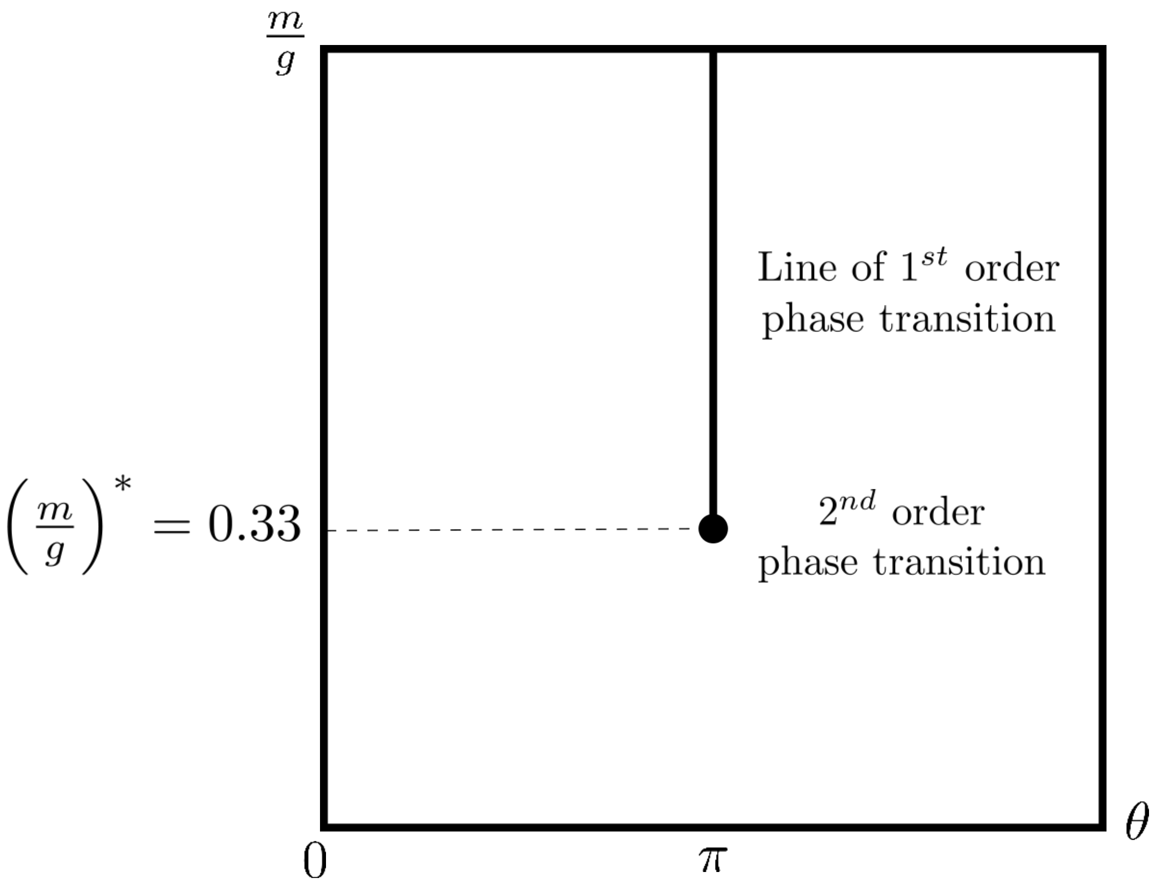

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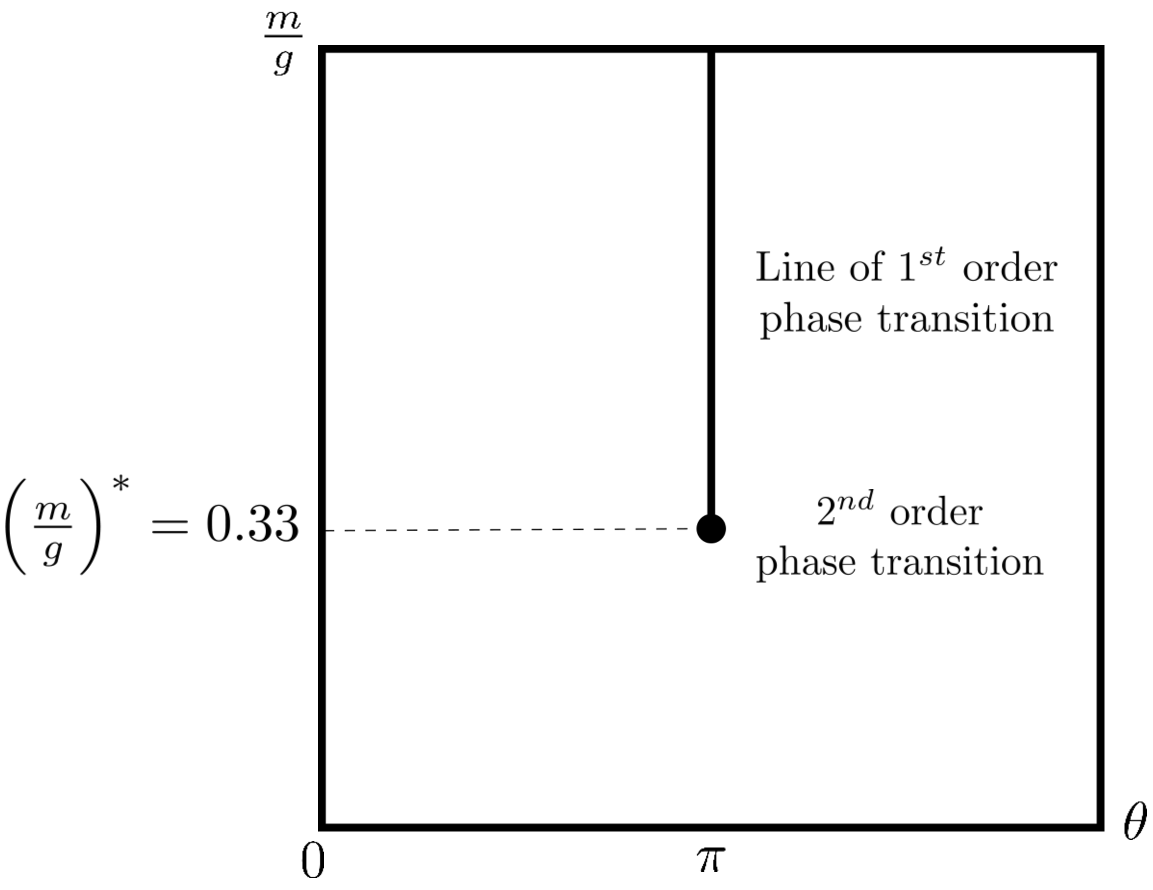

Plot at $\theta = \pi$: Entanglement Entropy vs $\left(\frac{m}{g}\right)_b$

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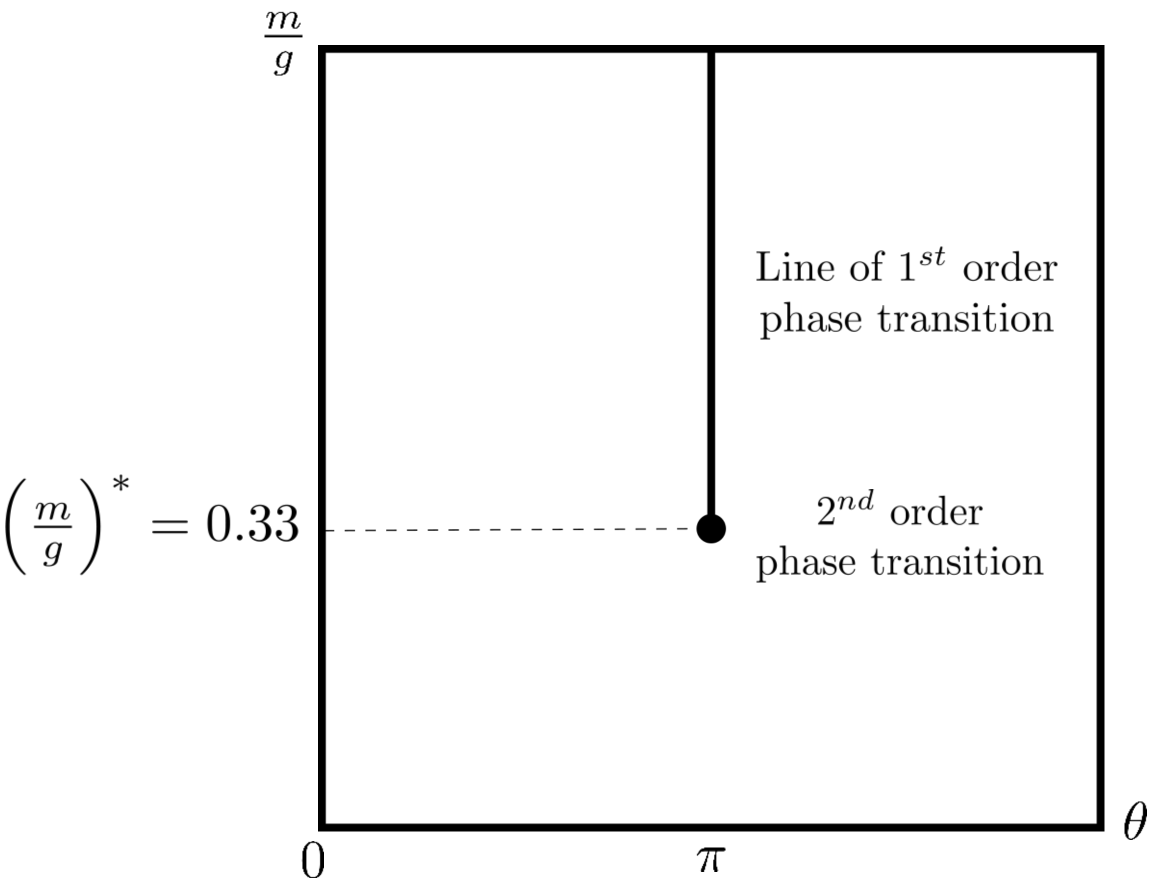

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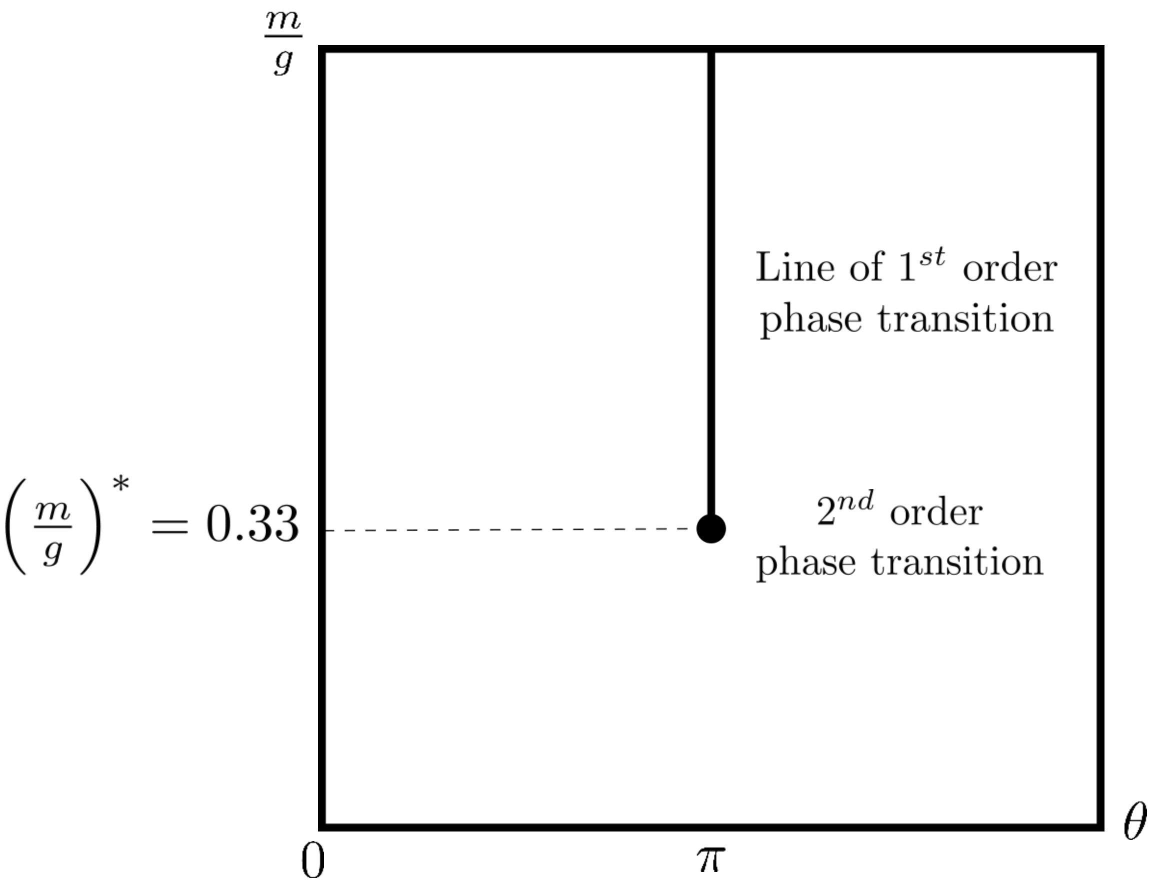

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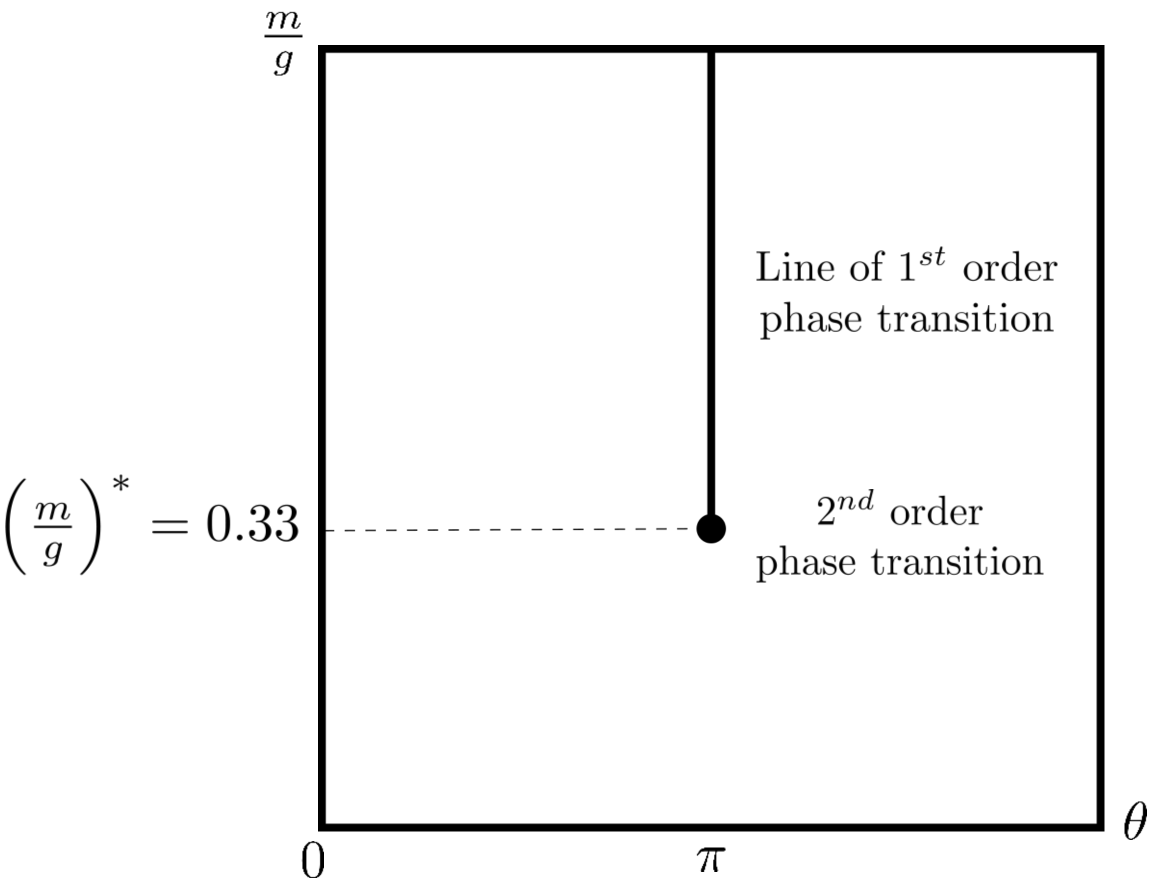

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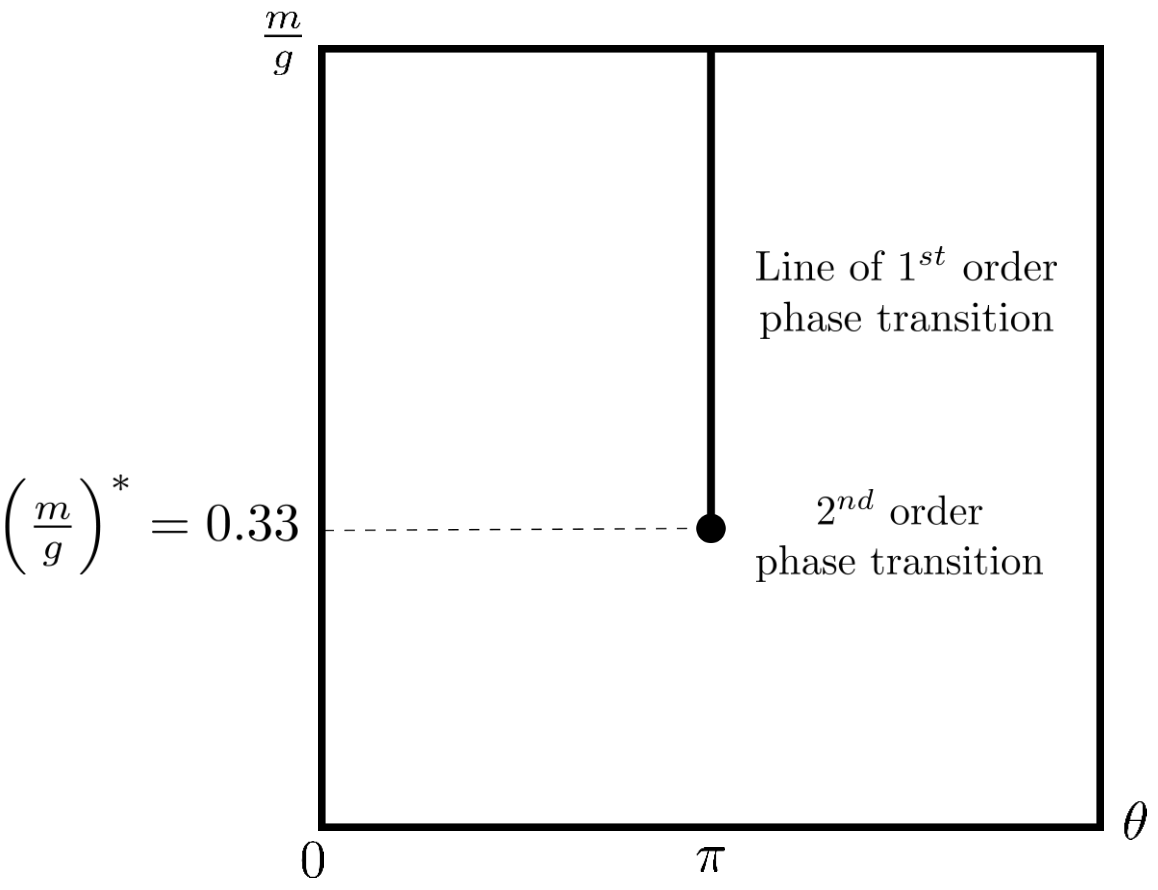

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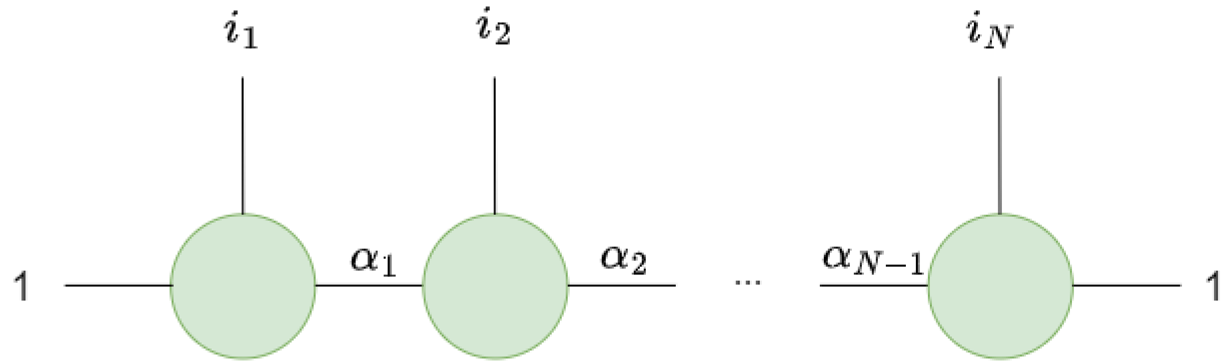
Problems:

- Finite volume effects affect P.T. point position
- Expensive for MPS to simulate 2^{nd} order P.T.

Appendix B

Tensor Networks – Matrix Product State (MPS)

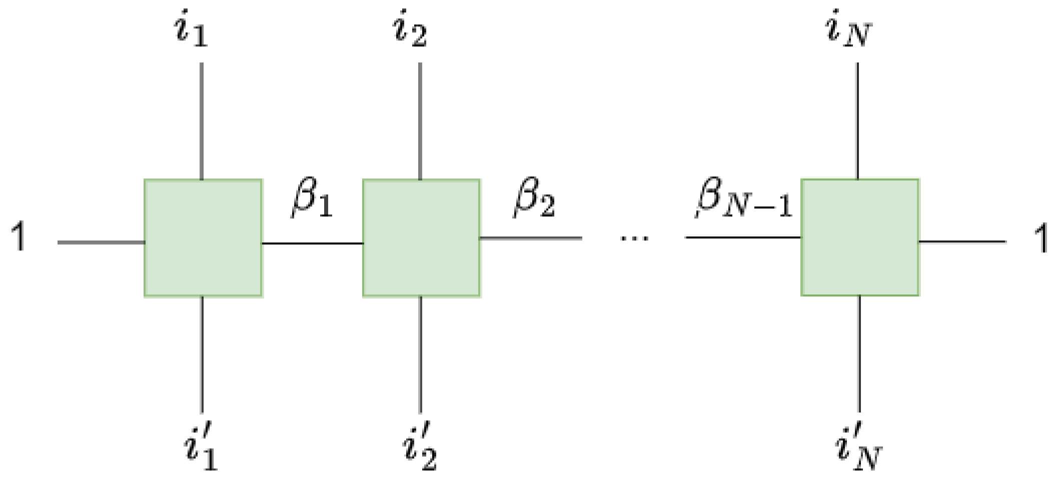
$$|\psi\rangle = \sum_{i_1, i_2, .., i_N} A^{i_1}_{1,\alpha_1} A^{i_2}_{\alpha_1,\alpha_2} \dots A^{i_N}_{\alpha_{N-1},1} |i_1\rangle \otimes |i_2\rangle \otimes \dots \otimes |i_N\rangle$$



Appendix B

Tensor Networks – Matrix Product Operator (MPO)

$$H = \sum_{\substack{i_1, i_2, \ldots, i_N \\ i'_1, i'_2, \ldots, i'_N}} W^{i_1, i'_1}_{1, \beta_1} W^{i_2, i'_2}_{\beta_1, \beta_2} \ldots W^{i_N, i'_N}_{\beta_{N-1}, 1} \left(|i_1\rangle \langle i'_1| \right) \otimes \left(|i_2\rangle \langle i'_2| \right) \otimes \ldots \otimes \left(|i_N\rangle \langle i'_N| \right)$$



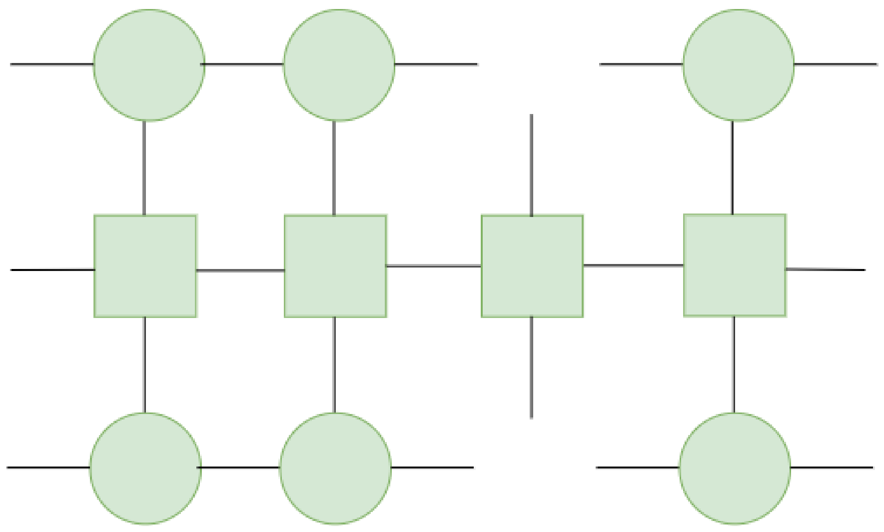
Appendix B

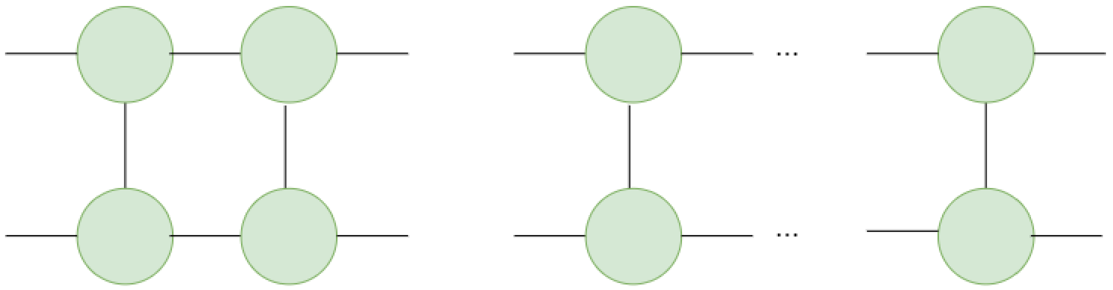
Tensor Networks – Variational Ground State Search Algorithm

$$E = \frac{\langle\psi|\, H \,|\psi\rangle}{\langle\psi|\psi\rangle} = \frac{A^\dagger H_{eff} A}{A^\dagger N A}$$

$$\frac{\partial E}{\partial A^\dagger} = 0 = \frac{H_{eff} A}{A^\dagger N A} - \left(\frac{A^\dagger H_{eff} A}{(A^\dagger N A)^2}\right) N A$$

$$H_{eff} A = E A$$





1. Minimise energy w.r.t. an MPS tensor
2. Repeat for other MPS tensors until energy converges

Appendix C

Chiral Anomaly Removing θ at Zero Mass in Continuum

$$Z = \int DA \; D\bar{\psi} \; D\psi \; e^{-S[\psi,\bar{\psi},A,m=0]+i\theta\int d^2x \frac{g}{2\pi} F_{01}}$$

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$$\psi \rightarrow \psi' = e^{i\frac{\theta}{2}\gamma_5}\psi$$

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$$\psi \to \psi' = e^{i\frac{\theta}{2}\gamma_5}\psi$$

$$D\bar{\psi}'\ D\psi' \to J^2 D\bar{\psi}\ D\psi = e^{-i\theta\int d^2x\,\frac{g}{2\pi}F_{01}}D\bar{\psi}\ D\psi$$ [1]

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$$Z[m=0,\theta] \sim Z[m=0,\theta=0]$$

Appendix D

Electric Field Density (EFD) Screening

$$\frac{\mathcal{F}(m,\theta)}{g} = A \cdot \frac{m}{g}\,\sin(\theta) + B \cdot \left(\frac{m}{g}\right)^2 \sin(2\theta) + \mathcal{O}\left(\left(\frac{m}{g}\right)^3\right) \quad [1]$$

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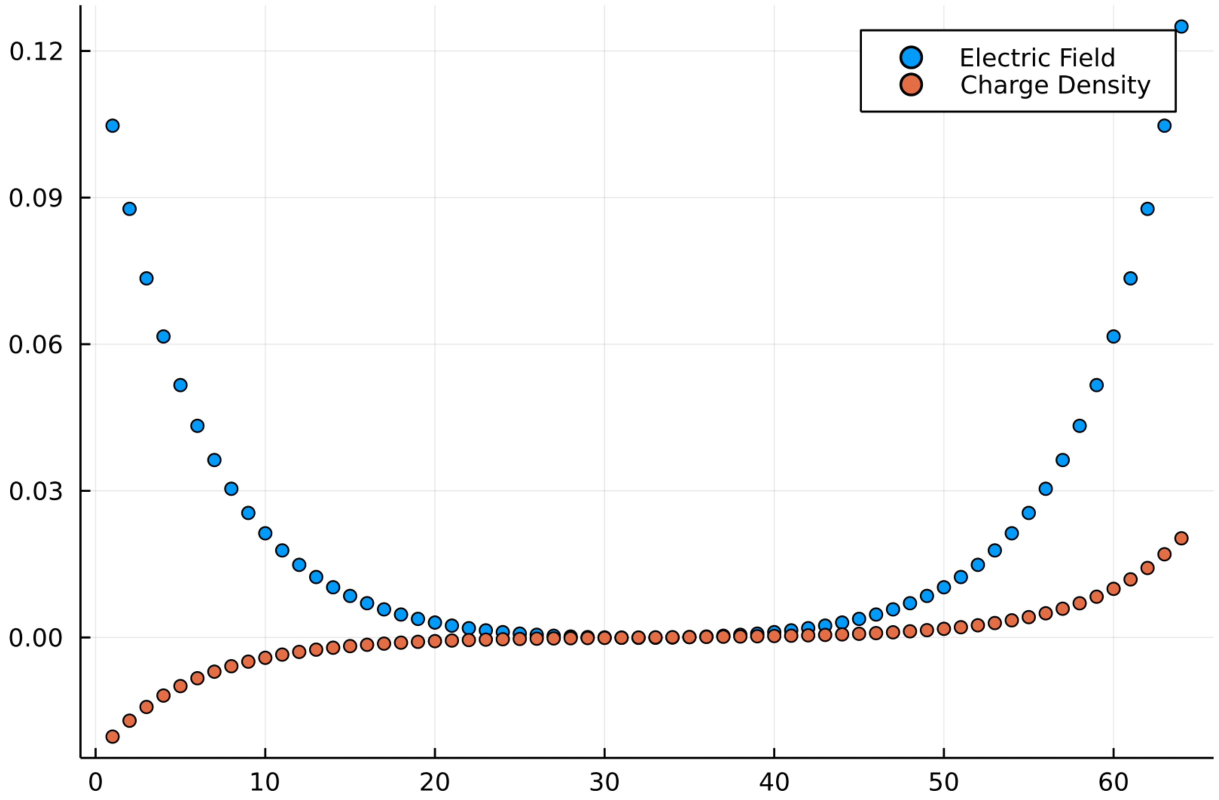

$N = 64$

$x = 10$

$l_0 = 0.125$

$\left(\frac{m}{g}\right)_r \sim 0$