# Master-field simulations of QCD and the exponential clover action

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in collaboration with
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and
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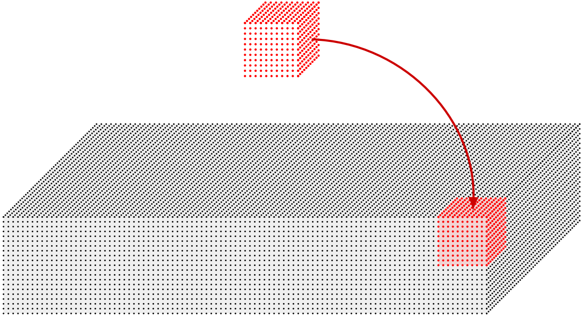
Master fields?

What's that?

Why?

Master fields?
What’s that?
Why?



- *negligible finite-volume effects*
- *absence of topological-freezing problem*
- *access to new kinematic regimes*
- *ideal for position-space methods*
- *better exploitation of exascale machines*
  (large memory footprint $\Rightarrow$ weak-scaling case)
- *new tool to perform different/new calculations*

# The standard lattice QCD approach

### Markov Chain Monte Carlo simulations of QCD

Goal: produce sequence of gauge fields $\{U_i | i = 1, \dots, N_U\}$


$\Rightarrow$ expectation values of physical observables $\mathcal{O}$ from ensemble-average

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U] \, \mathcal{O} \, e^{-\mathcal{S}_{\rm G}[U] - \mathcal{S}_{\rm eff}[U]}$$

$$\approx \frac{1}{N_U} \sum_{i=1}^{N_U} \mathcal{O}[U_i] \, ,$$

with Wilson–Dirac operator $Q$ and

$$e^{-\mathcal{S}_{\rm eff}} \simeq \prod_f \det(Q_f) \, , \qquad f \in \{u, d, s, \dots\}$$

employs **Hybrid Monte-Carlo (HMC) algorithm**

- with importance sampling
- draw conjugate momenta $\pi$ & integrate molecular dynamics (MD) equations
- made exact by (global) Metropolis accept-reject step ($\Delta H = \Delta \mathcal{S}$)
- ergodicity maintained by redrawing the momenta

and **advanced techniques to solve large linear systems**:

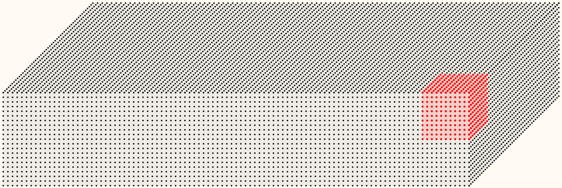
- various (Krylov) solvers
- precondition techniques (eo, det-splitting, …)
- mixed-precision arithmetic
- symplectic integrators w/ multiple time-scales
- architecture dependent optimisations
- …

# The master-field approach[1]

**Master-field lattice**

single master-field replaces classical (Markov chain) ensemble

$$N_V = \frac{V_4^{\mathrm{mf}}}{V_4} = \prod_{i=0}^{3} N_i \simeq 100 - 1000 \approx N_U$$



$\Rightarrow$ expectation values from translation average $\langle\langle \mathcal{O} \rangle\rangle$

$$\langle \mathcal{O}(x) \rangle = \langle\langle \mathcal{O}(x) \rangle\rangle + \mathrm{O}(N_V^{-1/2}) \, ,$$
$$\langle\langle \mathcal{O}(x) \rangle\rangle = \frac{1}{N_V} \sum_z \mathcal{O}(x+z)$$

based on **stochastic locality** due to short-range interaction

- QCD field variables in distant regions fluctuate largely independent
- their distribution is everywhere the same (with periodic bc.)
- translation averages replace ensemble averages provided localisation range of $\mathcal{O} \ll L$ (lattice extent)
- uncertainties estimated using standard methods through correlations in space

Concept successfully applied to SU(3) YM theory.[2]

It isn't straightforward to simulate QCD on very large lattices!

# Critical aspects of lattice QCD simulations

Various choices (strongly) impact simulation cost and reliability of a simulation.

## Discretisation aspects

- gauge action (impacts UV fluctuations)
- fermion action (lattice Dirac operator $D$)
- spectral gap of $D \sim \lambda_{\min}$ (near zero-modes in MD evolution)

## Algorithmic aspects

- update algorithm: Hybrid Monte-Carlo (exploration of phase space)
- integration schemes and length (symplectic integrators)
- numerical precision, e.g. in global sums (Metropolis step) (double precision)
- solver parameters (stability & performance)

## Physical aspects

- coarse $a$ $\rightsquigarrow$ promote large fluctuations of gauge field (roughness of $U$ fields)
- small $m_{\mathrm{ud}}$ $\rightsquigarrow$ result in small eigenvalues $\lambda_{\min}(m_{\mathrm{ud}})$ of lattice Dirac operator
- large $(L/a)^4$ $\rightsquigarrow$ increase risk of exceptional behaviour (e.g. from MD force)

Large potential for algorithmic instabilities and precision issues. $\Rightarrow$ Additional stability measures required.[3]

# Stabilising measures for QCD[1, 3]

include …

- new fermion action / Wilson–Dirac operator $\rightsquigarrow$ +1 page
- new algorithm: stochastic molecular dynamics (SMD) algorithm[4–7] $\rightsquigarrow$ +3 pages
- solver stopping criteria

$$\|D\psi - \eta\|_2 \leq \rho\|\eta\|_2 \ ,$$

$$\|\eta\|_2 = \Big(\sum\nolimits_x \big(\eta(x), \eta(x)\big)\Big)^{1/2} \propto \sqrt{V}$$

✓ $V$-independent uniform norm: $\|\eta\|_\infty = \sup_x \|\eta(x)\|_2$

$\|\eta\|_\infty$ for all forces ($\mathrm{res}_F = 10^{-12} \ldots 10^{-10}$) and some actions ($\mathrm{res}_\phi = 10^{-12}$)

- global Metropolis accept-reject step (numerical precision must increase with $V$)

$$\Delta H \propto \epsilon^p \sqrt{V}$$

✓ quadruple precision in global sums

- well-established techniques

✓ Schwarz Alternating Procedure, local deflation, multi-grid, …
even-odd & mass-preconditioning, multiple time-scales, …

# New Wilson–Dirac operator[3]

with exponential clover term

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$$D = \tfrac{1}{2}\gamma_\mu(\nabla_\mu^* + \nabla_\mu) - a\tfrac{1}{2}\nabla_\mu^*\nabla_\mu + \underline{M_0 + ac_{\rm sw}\tfrac{\rm i}{4}\sigma_{\mu\nu}\hat{F}_{\mu\nu}}$$

Even-odd preconditioning:

$$\hat{D} = D_{\rm ee} - D_{\rm eo}(D_{\rm oo})^{-1}D_{\rm oe}$$

with diagonal part[8] $(M_0 = 4 + m_0)$

$$D_{\rm ee} + D_{\rm oo} = M_0 + c_{\rm sw}\tfrac{\rm i}{4}\sigma_{\mu\nu}\hat{F}_{\mu\nu} \quad \sim \quad \boxed{M_0 \exp\left\{\frac{c_{\rm sw}}{M_0}\frac{\rm i}{4}\sigma_{\mu\nu}\hat{F}_{\mu\nu}\right\}}$$

✗ not protected from arbitrarily small eigenvalues

small mass, rough gauge field, large lattice promote instabilities in $(D_{\rm oo})^{-1}$

✓ Employ exponential mapping

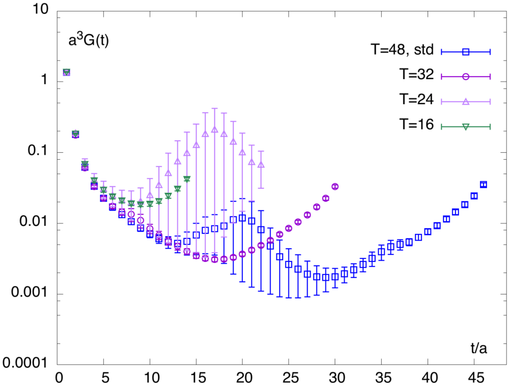
- regulates UV fluctuations
- valid Symanzik expansion/improvement
- guarantees invertibility

# New Wilson–Dirac operator

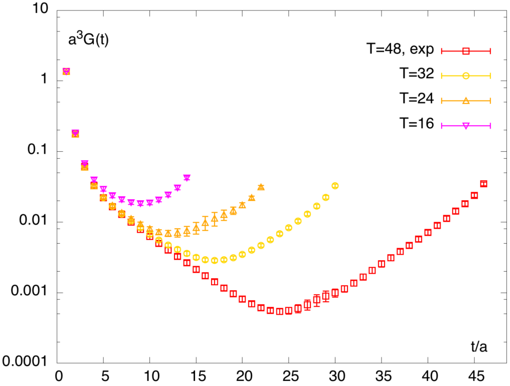
A clean comparison of fermion actions

- Impact best seen in pure gauge theory ($N_\mathrm{f} = 0$, quenched; i.e. same gauge background).
  Ill-defined theory for fermionic observables. $\Rightarrow$ exceptional problems
- Different lattices $L/a \in \{16, 24, 32, 48\}$ and same gluon action ($\beta = 6.0$, $a = 0.094\,\mathrm{fm}$).
- pion correlator $G(t) \propto e^{-m_\pi t}$ at zero momentum, $m_\pi \approx 220\,\mathrm{MeV}$

std. Wilson–Dirac operator




new Wilson–Dirac operator



# Algorithmic improvements for stability

## Stochastic Molecular Dynamics (SMD) algorithm[4–7]

Refresh $\pi(x,\mu)$, $\phi(x)$ by random field rotation

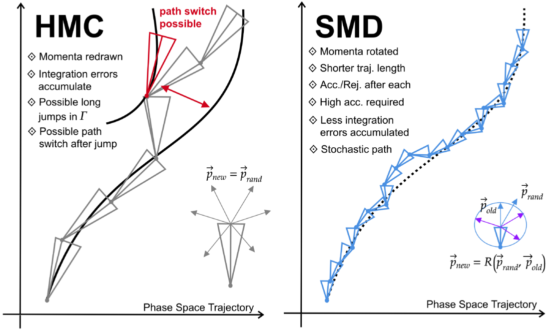
$$\pi \to c_1\pi + c_2 v\,, \qquad c_1 = e^{-\epsilon\gamma}\,,\quad c_1^2 + c_2^2 = 1\,,\qquad v(x,\mu),\eta(x) \in \mathcal{N}(0,1)$$

$$\phi \to c_1\phi + c_2 D^\dagger\eta\,,$$

($\gamma > 0$: friction parameter; $\epsilon$: MD integration time)

+ MD evolution + accept-reject step + repeat. If rejected: $\{\tilde{U},\tilde{\pi},\tilde{\phi}\} \to \{U,-\pi,\tilde{\phi}\}$

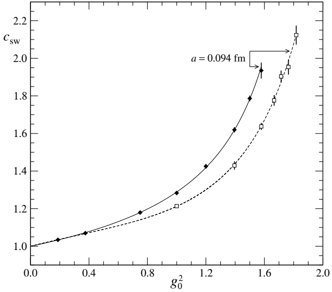
- ergodic[9] for sufficiently small $\epsilon$ (typically $\epsilon < 0.35$ vs. $\tau = 1-2$)
- exact algorithm
- significant reduction of unbounded energy violations $|\Delta H| \gg 1$
- a bit “slower“ than HMC but compensated by shorter autocorrelation times
- smooth changes in $\phi_t, U_t$ improve update of deflation subspace



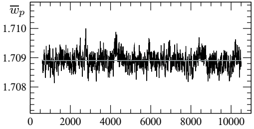
# Comparison to traditional Wilson–Clover action

e.g: $N_\mathrm{f} = 2 + 1$ data of Coordinated Lattice Simulations (CLS) effort[10–12]

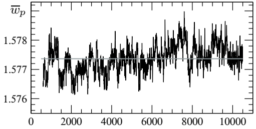
## Non-perturbative improvement:




## plaquette (energy density) with SMD: $a = 0.095\,\mathrm{fm}$




exp. clover $(A_1)$

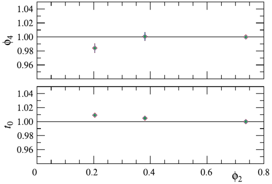



std. clover $(X_1)$

## Chiral trajectory:

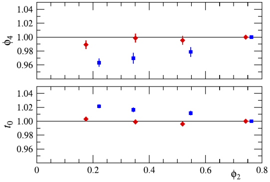
$$t_0 \text{ and } \phi_4 \equiv 8t_0\left(\tfrac{1}{2}m_\pi^2 + m_\mathrm{K}^2\right) = 1.11 \sim \mathrm{Tr}[M_\mathrm{q}]$$

### Stabilised Wilson




$a = 0.095\,\mathrm{fm}$

### Standard Wilson




$a = 0.064, 0.086\,\mathrm{fm}$

### key observation

- smoother fluctuations
- smaller lattice spacing effects


$N_\mathrm{f} = 2 + 1$+all stabilising measures[3]

- M. Cé, M. Bruno, J. Bulava, A. Francis, P. F., J. Green, M. Lüscher, A. Rago, M. Hansen
- $m_\pi = 270\,\mathrm{MeV} = 2m_\pi^{\mathrm{phys}}$
- `openQCD-2.0`, `openQCD-2.4`[13]

# Master fields prefer the target partition function

## Reweighting of observables not available

**QCD simulations necessitate frequency-splitting methods**

- Hasenbusch (mass-)preconditioning for quark doublet $(\mu_n > \ldots > \mu_0)$

$$S_{\mathrm{pf}} = (\phi_0, \frac{1}{D^\dagger D + \mu_n^2}\phi_0) + \sum_{k=1}^{n}(\phi_k, \frac{D^\dagger D + \mu_{n-k+1}^2}{D^\dagger D + \mu_{n-k}^2}\phi_k)$$

requires mass-reweighting if regulator mass $\mu_0 \neq 0$

- rational approximation of strange (or charm) quark determinant is given by

$$\det(D_{\mathrm{s}}) = W_{\mathrm{s}} \det(R^{-1}) \ , \quad R = C \prod_{k=0}^{m-1} \frac{D_{\mathrm{s}}^\dagger D_{\mathrm{s}} + \omega_k^2}{D_{\mathrm{s}}^\dagger D_{\mathrm{s}} + \nu_k^2}$$

: Zolotarev optimal rat. approx.

with reweighting factor $W_{\mathrm{s}} = \det(D_{\mathrm{s}} R)$ to
correct approximation error ($m =$ degree of $[D_{\mathrm{s}}^\dagger D_{\mathrm{s}}]^{-1/2}$)

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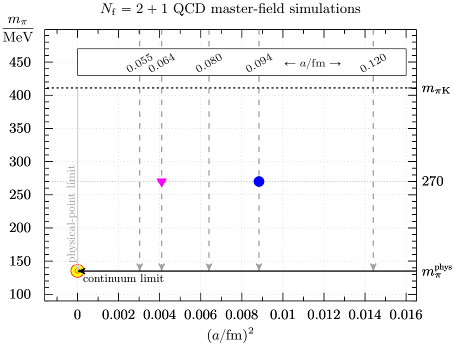
with reweighting factor $W_{\rm s} = \det(D_{\rm s} R)$ to correct approximation error ($m =$degree of $[D_{\rm s}^\dagger D_{\rm s}]^{-1/2}$) $\Rightarrow \checkmark$ if approximation is sufficiently accurate

Complying with strict bound $\frac{\sigma(W_{\rm s})}{\langle W_{\rm s}\rangle} \leq 0.1$ guarantees unbiased results in <u>all</u> observables.

# Summary of $N_f = 2 + 1$ master-field simulations

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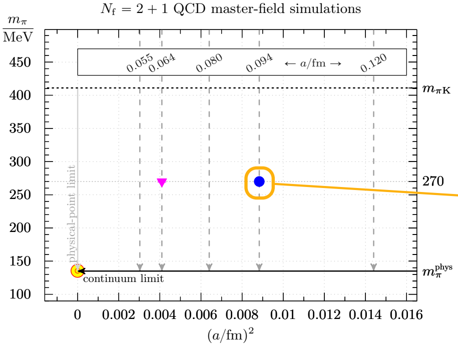


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$V = 96^4$ ($L = 9\,\mathrm{fm}$, $m_\pi L \approx 12.3$)

- $V/V_4 = 3^4 = 81$ ($N_\mathrm{core} = 6144$)
- *Cost:* 3 Mch (thermal.) + 0.2 Mch (add. cfg.)
- *Total memory used:* 1.8 TiB (= 309.1 MiB per core)
- *On disc:* 132 GiB (= 46 GiB $U$ + 61 GiB $\phi$ + 20 GiB $\pi$)

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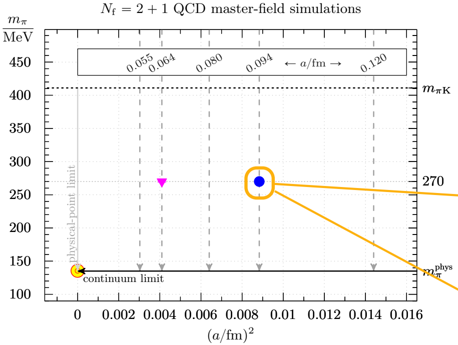


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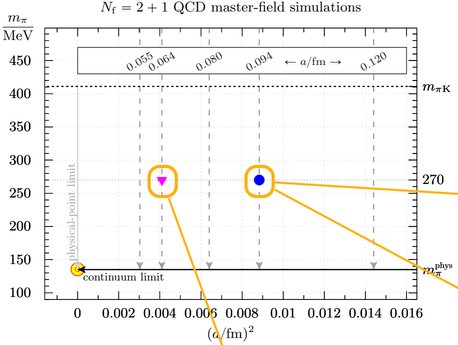


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### $V = 144^4$ ($L = 9.2\,\text{fm}$, $m_\pi L \approx 12.6$)

- $V/V_4 = (144/48)^3 = 81$ ($N_{\text{core}} = 10368$)
- *Cost:* 20 Mch (thermal.) + 13 Mch (per add. cfg.)
- *Total memory used:* 11.1 TiB (= 1.1 GiB per core)
- *On disc:* 642 GiB (= 231 GiB $U$ + 308 GiB $\phi$ + 103 GiB $\pi$)

**Variety of choices:**

time-momentum correlators

$$C(x_0,\mathbf{p}) = \sum_{\mathbf{x}} \exp(-i\mathbf{p}\mathbf{x})C(x,0)$$

have large footprint in space for $\mathbf{p} = \mathbf{0}$
(inexact momentum projection $\rightsquigarrow$ more localized)

$\Rightarrow$ position-space correlators

- single point source (inefficient)
- Dirichlet b.c. on blocks[1] (induce boundary effect)
- random source (useable)
- …

# How to (efficiently) calculate hadronic observables?

**Variety of choices:**

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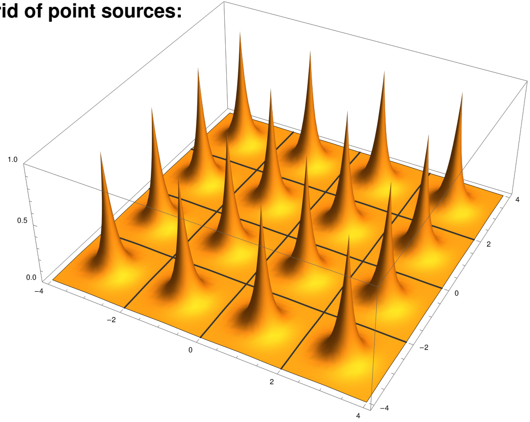
**Grid of point sources:**




2D sketch of exponential decay of „2-pt function“ with $(8a/2a)^2 = 4^2 = 16$ grid source points

Take away message

***employ techniques compatible with MF translation average for <u>single</u> inversion of Dirac op.***

**Hadron propagators**

E.g. meson 2-pt function (like pion propagator):

$$C_{\Gamma\Gamma'}(x) = -\mathrm{Tr}\{\Gamma\gamma_5 D^{-1}(x,0)\gamma_5\Gamma' D^{-1}(x,0)\}\ , \qquad ||D^{-1}(x,0)|| \sim e^{-m|x|/2}$$

with localisation range $1/m$ (not ultra-local)

- Asymptotic form of position-space correlators analytically known when $a = 0$ ($T, L = \infty$). For $|x| \to \infty$:

$$C_{\mathrm{PP}}(x) \to \frac{|c_{\mathrm{P}}|^2}{4\pi^2}\frac{m_{\mathrm{P}}^2}{|x|}K_1(m_{\mathrm{P}}|x|)\ ,$$

$$C_{\mathrm{NN}}(x) \to \frac{|c_{\mathrm{N}}|^2}{4\pi^2}\frac{m_{\mathrm{N}}^2}{|x|}\left[K_1(m_{\mathrm{N}}|x|) + \frac{\not{x}}{|x|}K_2(m_{\mathrm{N}}|x|)\right]$$

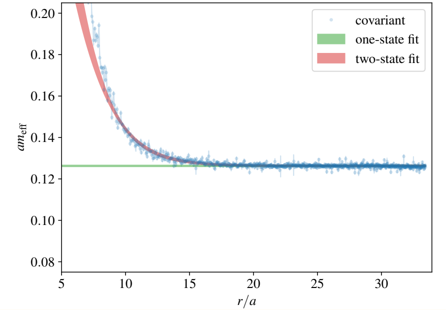
- axis/off-axis directions different cutoff effects
- correlator averaged over equivalent distances $r = |x|$:

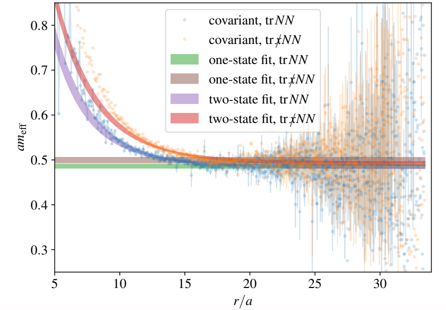
$$\overline{C}(r) = \frac{1}{\mathrm{r}_4((r/a)^2)}\sum\nolimits_{|x|=r} C(x)$$

# Hadronic observables

from position-space correlators & grid-points offset $b = 48a$ ($r_{\max} = 48a/\sqrt{2} \leq 34a$)

## Effective masses of pion and nucleon ($a = 0.94\,\mathrm{fm}$) on $96^4$




## Comments:

- similar statistics (4096 noise src.) and computational effort on $96^4$ ($x_{\mathrm{gs}} = 12$) and $192^4$ ($x_{\mathrm{gs}} = 24$)
- different methods for proper calculations of uncertainties available (bootstrap, $\Gamma$-method)
- using empirical ansatz for excited state effects
- no boundary effects observed
- presented by M. Cè (Aug 11, 11:50)

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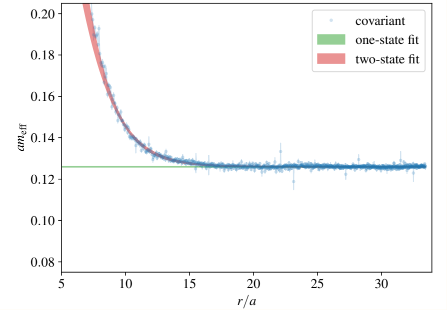


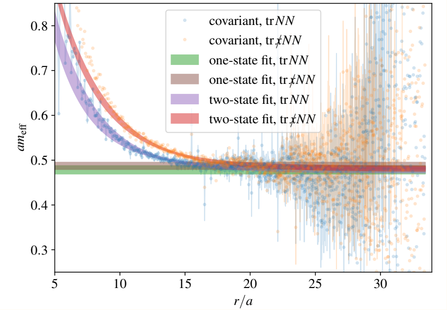


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## $N_{\mathrm{f}} = 2 + 1$ Stabilised Wilson-Fermion simulations

- perform standard-sized lattice simulations
- implement all stabilising measures[14] (Exp-Clover action, SMD, …)
- various lattices $\{a/L, \beta, m_\pi\}$ to complement master-field simulations
- provide ensembles under open science policy
- `https://openlat1.gitlab.io`

# $N_\mathrm{f} = 2 + 1$ Stabilised Wilson-Fermion simulations

**Current OPENLAT team:**


Francesca Cuteri

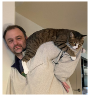

Anthony Francis

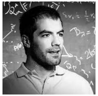

Patrick Fritzsch


Giovanni Pederiva

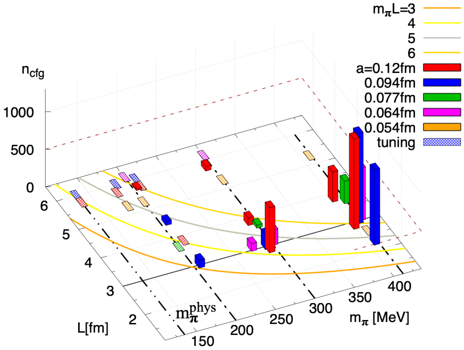

Antonio Rago


Andrea Shindler


André Walker-Loud


Savvas Zafeiropoulos

- more information on policies & status:
  *A. Francis* *(Aug 9, 14:30), Lattice Data session, 461*
  *„OpenLat"*
- first results of spectroscopic calculations:
  *G. Pederiva* *(Aug 9, 16:50), Hadron Spec. & Int., 433*
  *„Benchmark Continuum Limit Results for Spectroscopy with StabWF"*
- *continue to explore the behaviour of StabWF with colleagues from CLS*




- approaching physical points at coarse lattice spacings $a = 0.094, 0.12\,\mathrm{fm}$
- 5 lattice spacings at $SU(3)$-flavour-symmetric point
- no negative eigenvalues of $D_\mathrm{s}$ observed so far

# $N_\mathrm{f} = 2+1$ Stabilised Wilson-Fermions @Lattice22

Growing interest reflected in this year's contributions

**Traditional simulations:**

- *J. Kuhlmann* *(Aug 9, 20:00), Poster session, 302*
  *„On improvement and renormalisation of quark currents with StabWF"*
- *R.F. Basta* *(Aug 9, 20:00), Poster session, 381*
  *„QCD Thermodynamics with StabWF"*
- *F. Joswig* *(Aug 11, 12:10), Hadron Spec. & Int., 189*
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- *J. Green* *(Aug 11, 12:10), Nuclear Physics, 215*
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**Master-field simulations:**

- *A. Francis* *(Aug 8, 14:40), Theoretical Dev., 209*
  *„Translating topological benefits in very cold master-field simulations"*
- *J. Bulava* *(Aug 9, 9:20), Plenary session, 278*
  *„The spectral reconstruction of inclusive rates"*
- *M. Cè* *(Aug 11, 11:50), Hadron Spec. & Int., 356*
  *„Hadronic observables from master-field simulations"*

**Topics covered:**

- Renormalisation & Symanzik improvement
- Thermodynamics
- Hadron spectrum calculations
- Nuclear physics & scattering amplitudes
- Spectral reconstruction

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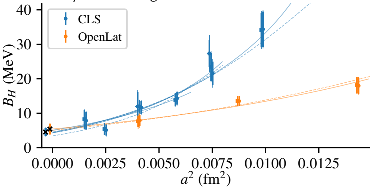
*H* dibaryon: $a \longrightarrow 0$ universality (PRELIMINARY)

BaSc: Baryon Scattering collaboration — combined effort of "Mainz" and sLapHnn




Three exp-clover ensembles with $L \approx 3$ fm.

Second action at SU(3) point: exponentiated clover / stabilized Wilson (OpenLat).
Smaller lattice artifacts than standard clover (CLS).

*The future of master fields & StabWF*

## My personal opinion

- Growing evidence for smaller cutoff effects with StabWF
- Physical point simulations become available
- Increasing user base for StabWF
- Exascale machines may turn out to be particularly suitable for MF simulations
- Master field at *another lattice spacing*, then shift focus *towards physical point* master fields
- Master field simulations may have a political dimension:
  - Much larger computing resources required
  - Probe performance of HPC facilities (landmark sim.)
  - Community-driven effort (within EuroLat, …?)
    - \+ General purpose
    - \+ Open access
    - \+ Computing time for smaller groups?

# Summary

Master-fields require stabilising measures

- modified fermion action (improvement term)
- stochastic Molecular dynamics (SMD) algorithm
- uniform norm & quadruple precision
- multilevel deflation

So far:

- stabilising measures (action, SMD, …) work excellent, especially at coarse lattice spacing $\checkmark$
- $96^4$, $192^4$ $(a = 0.095\,\mathrm{fm})$ and $144^4$ $(a = 0.065\,\mathrm{fm})$ master-field ready for physics applications $\checkmark$
- master-field prefers traget partition function $\checkmark$
- very large volumes like $(18\,\mathrm{fm})^4$ still challenging but doable (or $m_\pi^{\text{phys}}$) $\checkmark$
- position-space correlators $\rightsquigarrow$ hadron masses, decay constants, …

Ongoing:

We just start to uncover new possibilities.

- exploration of physical calculations & benchmarking
- continuum limit scaling behaviour
- master-fields: natural setup to study spectral reconstruction
- complementary large-scale lattice simulations (OPENLAT)

Backup slides

# Exponential clover implementation

Apply **Cayley–Hamilton theorem** for $6 \times 6$ hermitean matrices.

$$\frac{i}{4}\sigma_{\mu\nu}\hat{F}_{\mu\nu}(x) = \begin{pmatrix} A_+(x) & 0 \\ 0 & A_-(x) \end{pmatrix} ,$$

$$\mathrm{tr}\{\mathrm{A}\} = 0 \quad \Rightarrow \quad A^6 = \sum_{k=0}^{4} p_k A^k$$

Any polynomial in $A$ of degree $N \geq 6$ can be reduced to

$$\sum_{k=0}^{5} q_k A^k ,$$

with $A$-dependent coefficients $q_k$, calculated recursively.

Expansion coefficients ($p_k \in \mathbb{R}$)

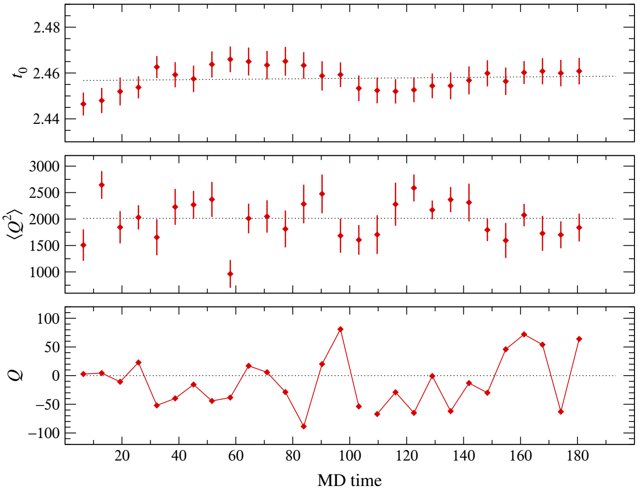
$$\begin{aligned}
p_0 &= \tfrac{1}{6}\mathrm{tr}\{\mathrm{A}^6\} - \tfrac{1}{8}\mathrm{tr}\{\mathrm{A}^4\}\mathrm{tr}\{\mathrm{A}^2\} - \tfrac{1}{18}\mathrm{tr}\{\mathrm{A}^3\}^2 + \tfrac{1}{48}\mathrm{tr}\{\mathrm{A}^2\}^3 \\
p_1 &= \tfrac{1}{5}\mathrm{tr}\{\mathrm{A}^5\} - \tfrac{1}{6}\mathrm{tr}\{\mathrm{A}^3\}\mathrm{tr}\{\mathrm{A}^2\} , \\
p_2 &= \tfrac{1}{4}\mathrm{tr}\{\mathrm{A}^4\} - \tfrac{1}{8}\mathrm{tr}\{\mathrm{A}^2\}^2 , \\
p_3 &= \tfrac{1}{3}\mathrm{tr}\{\mathrm{A}^3\} , \\
p_4 &= \tfrac{1}{2}\mathrm{tr}\{\mathrm{A}^2\} ,
\end{aligned}$$

$\exp(A) = \sum_{k=0}^{N} \frac{A^k}{k!} + r_N(A)$ converges rapidly with bound $||r_N(A)|| \leq \frac{||A||^{N+1}}{(N+1)!}\exp(||A||)$

$\Rightarrow \exp\left(\frac{i}{4}\sigma_{\mu\nu}\hat{F}_{\mu\nu}(x)\right)$ easily obstained to machine precision.

# Monitoring observables (thermalisation)

$96^4$ : $a = 0.095\,\mathrm{fm}$, $m_\pi = 270\,\mathrm{MeV}$, $Lm_\pi = 12.5$ ($L = 9\,\mathrm{fm}$)




Simulations without TM-reweighting:

- no spikes in $\Delta H$
- $\langle e^{-\Delta H} \rangle = 1$ within errors
- acceptance rate 98% or higher
- checked that $\sigma(\hat{D}_s) \in [r_a, r_b]$ of Zolotarev rational approximation
- adapt solver tolerances to exclude statistically relevant effects of numerical inaccuracies
- autocorrelation times: 20-30 MDU

# Master-field simulations

Thermalising $192^4$ ($a = 0.094\,\mathrm{fm}$, $m_\pi = 270\,\mathrm{MeV}$) at LRZ using 768 nodes (36864 cores)

openQCD-2.0.2: multilevel DFL solver (full double prec.)

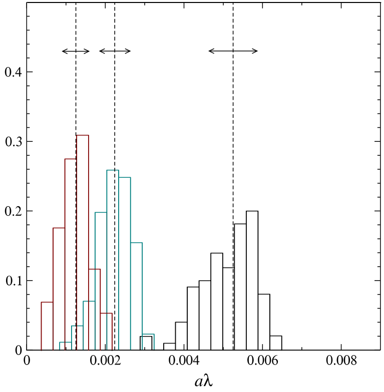
```
SMD parameters:
actions = 0 1 2 3 4 5 6 7 8
npf = 8
mu = 0.0 0.0012 0.012 0.12 1.2
nlv = 2
gamma = 0.3
eps = 0.137
iacc = 1

...

Rational 0:
degree = 12
range = [0.012,8.1]

Level 0:
4th order OMF integrator
Number of steps = 1
Forces = 0

Level 1:
4th order OMF integrator
Number of steps = 2
Forces = 1 2 3 4 5 6 7 8
```

```
Update cycle no 48
dH = -1.4e-02, iac = 1
Average plaquette = 1.708999
Action  1: <status> = 0
Action  2: <status> = 0 [0,0|0,0]
Action  3: <status> = 0 [0,0|0,0]
Action  4: <status> = 0 [0,0|0,0]
Action  5: <status> = 2 [5,2|7,6]
Action  6: <status> = 271
Action  7: <status> = 21 [3,2|5,3]
Action  8: <status> = 22 [3,2|5,3]
Field   1: <status> = 139
Field   2: <status> = 31 [3,2|6,4]
Field   3: <status> = 38 [5,3|8,7]
Field   4: <status> = 33 [5,2|7,6]
Field   5: <status> = 267
Field   6: <status> = 26 [3,2|5,3]
Field   7: <status> = 24 [3,2|5,3]
Force   1: <status> = 91
Force   2: <status> = 22 [3,2|6,4];23 [3,2|5,4]
Force   3: <status> = 28 [5,3|7,6];30 [5,3|7,6]
Force   4: <status> = 29 [5,2|7,6];32 [5,2|7,6]
Force   5: <status> = 28 [5,2|7,5];30 [5,2|7,6]
Force   6: <status> = 303
Force   7: <status> = 22 [3,2|5,3];23 [3,2|5,3]
Force   8: <status> = 23 [3,2|5,3];26 [3,2|5,3]
Modes   0: <status> = 0,0|0,0
Modes   1: <status> = 4,2|5,5 (no of updates = 4)
Acceptance rate = 1.000000
Time per update cycle = 4.34e+03 sec (average = 4.38e+03 sec)
```

# Towards large scale simulations

How does the lowest eigenvalue distribution scale with quark mass?

$a = 0.095\,\mathrm{fm},\ V = 96 \times 32^3$




$m_\pi = 410\,\mathrm{MeV},\ m_\pi L = 6.3$

$m_\pi = 294\,\mathrm{MeV},\ m_\pi L = 4.5$

$m_\pi = 220\,\mathrm{MeV},\ m_\pi L = 3.4$

(historical data missing for detailed comparison)

### Overall behaviour of smallest eigenvalue

- $a\lambda = \min\left\{\mathrm{spec}(D_u^\dagger D_u)^{1/2}\right\}$

$(a\lambda = 0.001 \sim 2\,\mathrm{MeV})$

- median $\mu \propto Zm$
- width $\sigma$ decreases with $m$
- somewhat similar to $N_\mathrm{f} = 2$ case[15] (unimproved Wilson)
- (non-)Gaussian ?
- empirical:[15] $\sigma \simeq a/\sqrt{V}$

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