

Spectator Effects in b -hadron decays on the lattice

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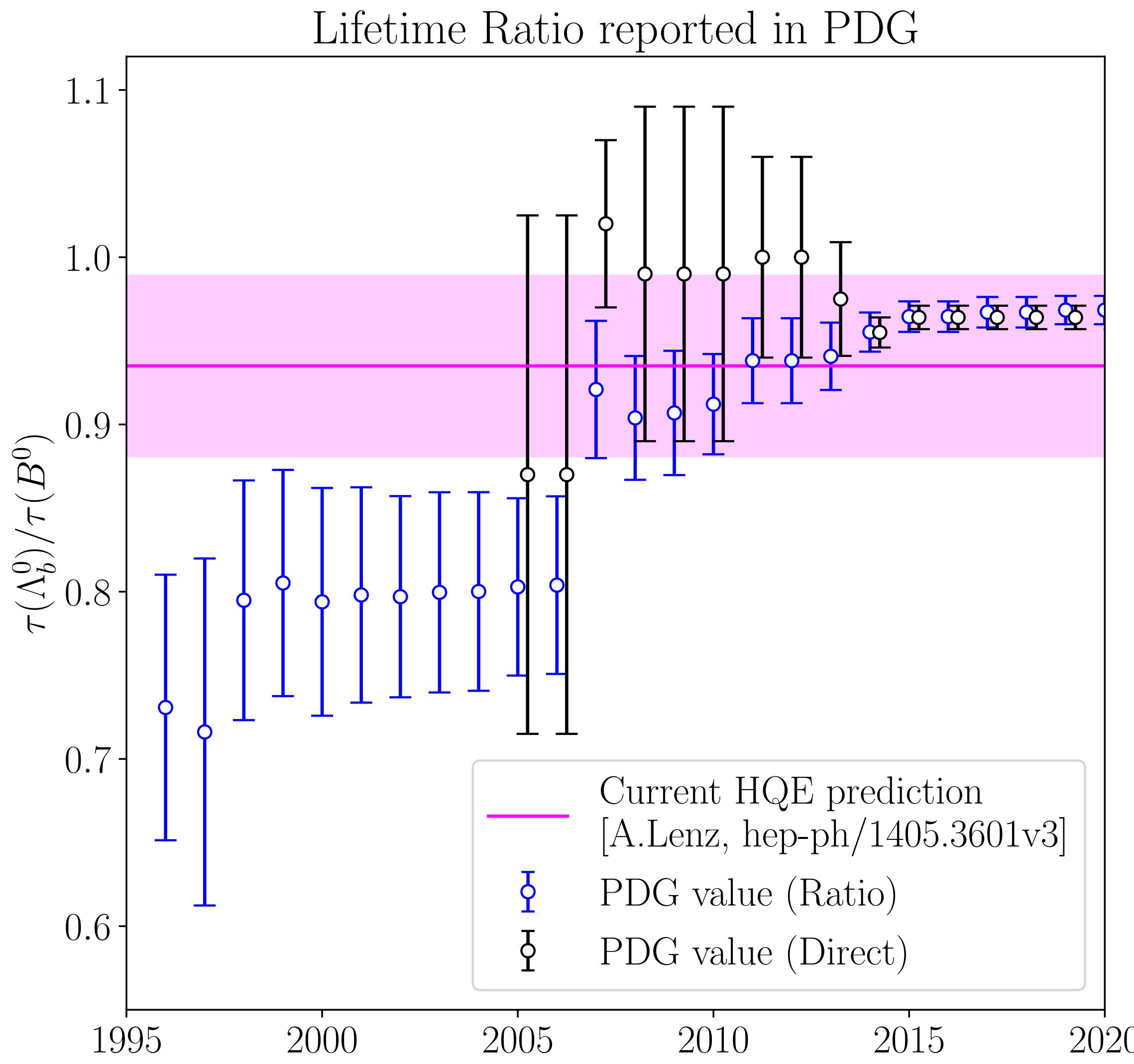
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Lifetimes of b -hadrons

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- Lifetime ratios of b -hadrons can be expanded simultaneously in α_s and $1/m_b$ using the Heavy Quark Expansion (HQE)
$$\frac{\tau(\Lambda_b^0)}{\tau(B^0)} = 1 + \frac{\mu_\pi^2(\Lambda_b^0) - \mu_\pi^2(B^0)}{2m_b^2} + \frac{c_G}{c_3} \frac{\mu_G^2(\Lambda_b^0) - \mu_G^2(B^0)}{2m_b^2} + O(1/m_b^3)$$
- $O(1/m_b^2)$ contributions can be extracted from observed masses of heavy hadrons.
- $O(1/m_b^3)$ corrections are dominated by ‘Spectator Effects’, which are matrix elements of dim-6 $\Delta B = 0$ operators.
- Previous determinations of Spectator matrix elements
 - Sum Rules [M. Kirk et al hep-ph/1711.02100]
 - (quenched) Lattice studies from 2001 [M. Di Pierro et al, hep-lat/9906031].
 - Lattice B -meson MEs [D. Becirevic, hep-ph/0110124]

Heavy Quark Expansion

[Review: A. Lenz, hep-ph/1405.3601]

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$$H_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \left[V_{cb}^* V_{us} (C_1 Q_1 + C_2 Q_2) + \dots V_{cb}^* Q_l^c + \dots \right]$$

$$Q_1 = (\bar{b}_i c_j)_{V-A} (\bar{u}_j s_i)_{V-A}$$

$$Q_l^q = (\bar{b}_i q_i)_{V-A} (\bar{\nu}_l l)_{V-A}$$



Optical Theorem

$$\Gamma(B \rightarrow X) = \frac{1}{2m_B} \langle B | \hat{T} | B \rangle = \frac{1}{2m_B} \langle B | \text{Im } i \int d^4x T \left(H_{\text{eff}}^{\Delta B=1}(x) H_{\text{eff}}^{\Delta B=1}(0) \right) | B \rangle$$



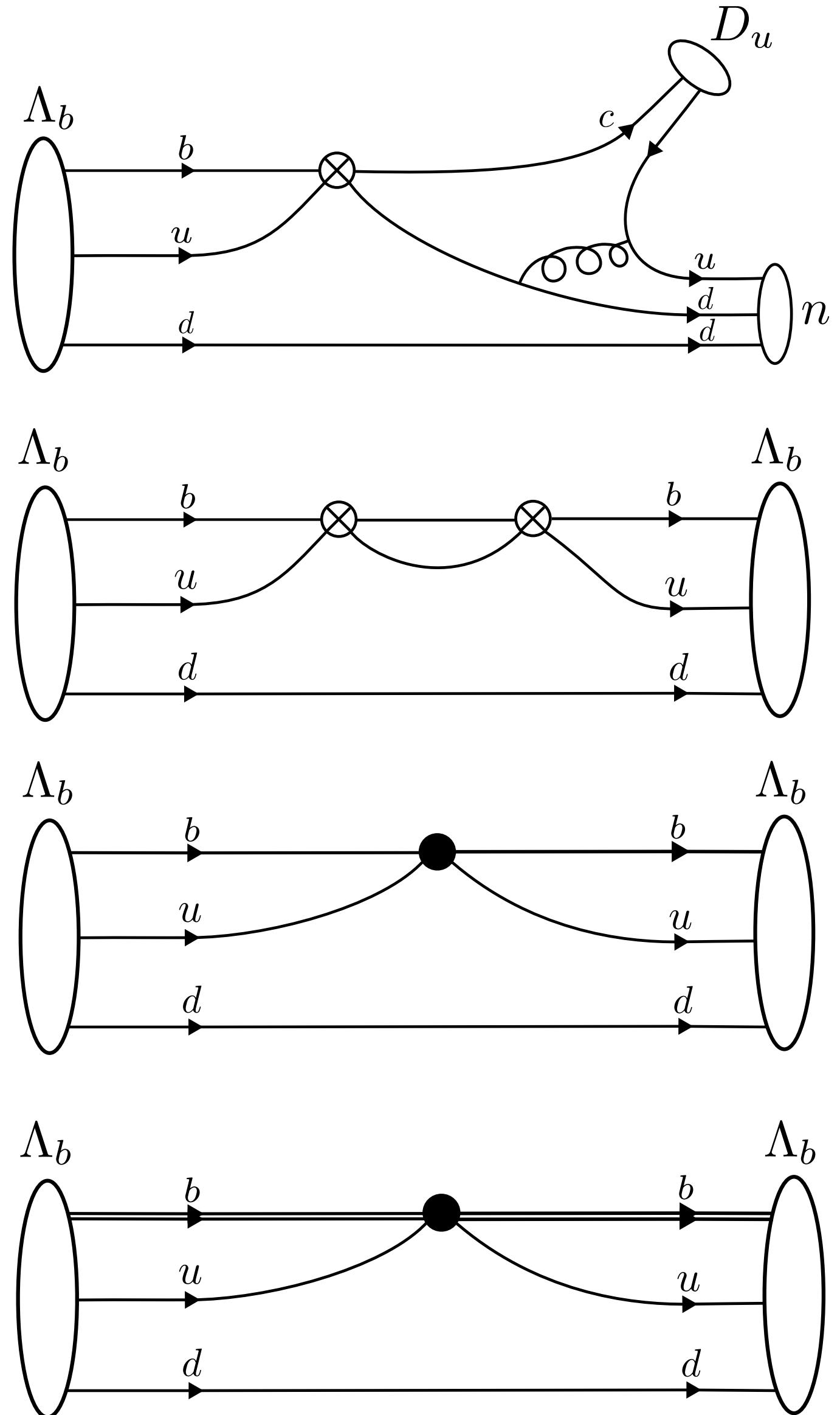
Operator Product Expansion

$$\hat{T} = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} \left[c^{(3)} \bar{b} b + \frac{1}{m_b^2} c^{(5)} g_s \bar{b} \sigma^{\mu\nu} G_{\mu\nu} b + \frac{1}{m_b^3} \sum_k c_k^{(6)} O_k^{(6)} + O(1/m_b^4) \right]$$



Heavy Quark Effective Theory

$$\bar{b} b = \bar{b}_+ b_+ + \bar{b}_+ \frac{D_T^2 + \frac{g}{2} \sigma_{\mu\nu} G^{\mu\nu}}{4m_b^2} b_+ + O\left(\frac{1}{m_b^3}\right)$$



Spectator Effects

[Neubert Sachrajda, hep-ph/9603202]

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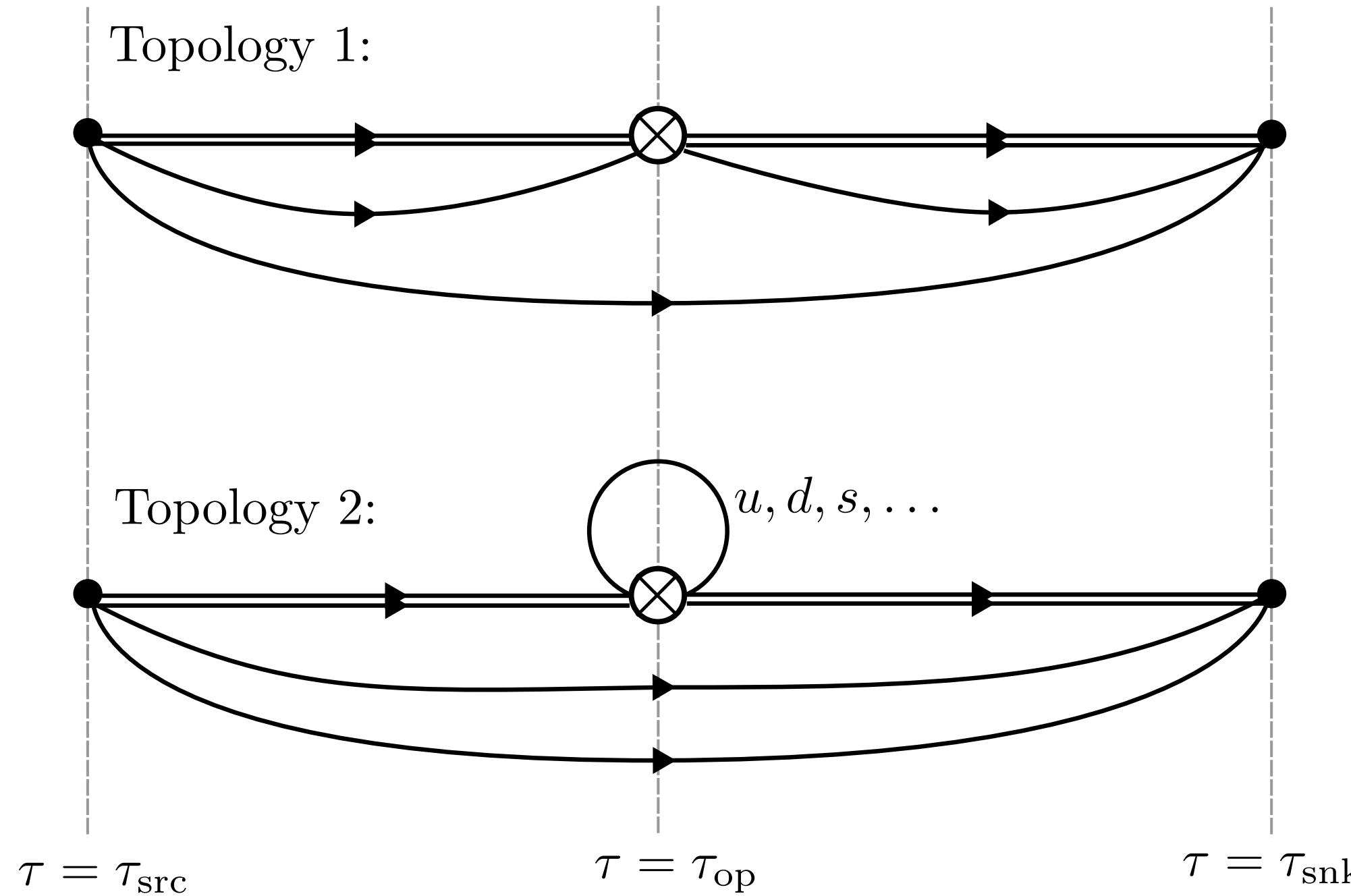
$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3} V_{cb}^2 \left[c_3 \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right) \right) + 2c_5 \left(\frac{\mu_G^2}{m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right) \right) + c_6 \left(\frac{1}{m_b^3} \frac{\langle B | (\bar{b}_+ q)_\Gamma (\bar{q} b_+)_\Gamma | B \rangle}{M_B} + \dots \right) \right]$$

- μ_π^2, μ_G^2 from masses of hadrons, [I.I. Bigi, Th. Mannel, N. Uraltsev, hep-ph/1105.4574]
- Procedure can be modified for inclusive, inclusive semi-leptonic, inclusive non-leptonic decay rates. Differences are encoded in perturbative coefficients c_i , all non-perturbative information encoded in matrix elements.
- Out of the various $\mathcal{O}(1/m_b^3)$ contributions, it turns out the spectator effects have coefficient that is enhanced by $16\pi^2$ due to phase space factors.
- The four-quark operators of interest are:

$$O_1 = (\bar{b}_+ \gamma_\mu P_L q) (\bar{q} \gamma_\mu P_L b_+) \quad O_2 = (\bar{b}_+ P_L q) (\bar{q} P_R b_+) \quad O_3 = (\bar{b}_+ T_a \gamma_\mu P_L q) (\bar{q} T_a \gamma^\mu P_L b_+) \quad O_4 = (\bar{b}_+ T_a P_L q) (\bar{q} T_a P_R b_+)$$

Lattice Strategy

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- Heavy quark is static, all operators lie on a single line.

- Ratios of three to two point functions cancel Z-factors:

$$\langle J(\tau_{\text{snk}})\mathcal{O}(\tau_{\text{op}})J^\dagger(\tau_{\text{src}}) \rangle = \sum_{m,n} Z_m e^{-E_m(\tau_{\text{snk}} - \tau_{\text{op}})} \langle \psi_m | \mathcal{O} | \psi_n \rangle e^{-E_n(\tau_{\text{op}} - \tau_{\text{src}})} Z_n$$

$$R_{\mathcal{O}}(\tau_{\text{src}}, \tau_{\text{op}}, \tau_{\text{snk}}) := \frac{\langle J(\tau_{\text{snk}})\mathcal{O}(\tau_{\text{op}})J^\dagger(\tau_{\text{src}}) \rangle}{\langle J(\tau_{\text{snk}})J^\dagger(\tau_{\text{src}}) \rangle} \xrightarrow{\tau_{\text{snk}} - \tau_{\text{op}}, \tau_{\text{op}} - \tau_{\text{src}} \rightarrow \infty} \langle \psi_0 | \mathcal{O} | \psi_0 \rangle$$

- We only measure Topology 1. Topology 2 does not contribute to lifetime ratios such as $\tau(B^+)/\tau(B_d)$, but does for $\tau(\Lambda_b)/\tau(B_u)$. Formally, to cancel out Topology 2 we are looking at flavor non-singlet versions of our operators [hep-ph/0110375], as is done in previous lattice calculations.

Lattice Details

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- RBC/UKQCD 2+1f Domain Wall Fermion, Iwasaki action configurations were used [hep-lat/1411.7017], matrix elements on $24^3 \times 64$ lattice shown here.

V	1/a (GeV)	$m_{u,d}, m_s$	N
$24^3 \times 64(\times 16)$	1.785(5)	0.005,0.04	228
$32^3 \times 64(\times 16)$	2.383(9)	0.004,0.03	239

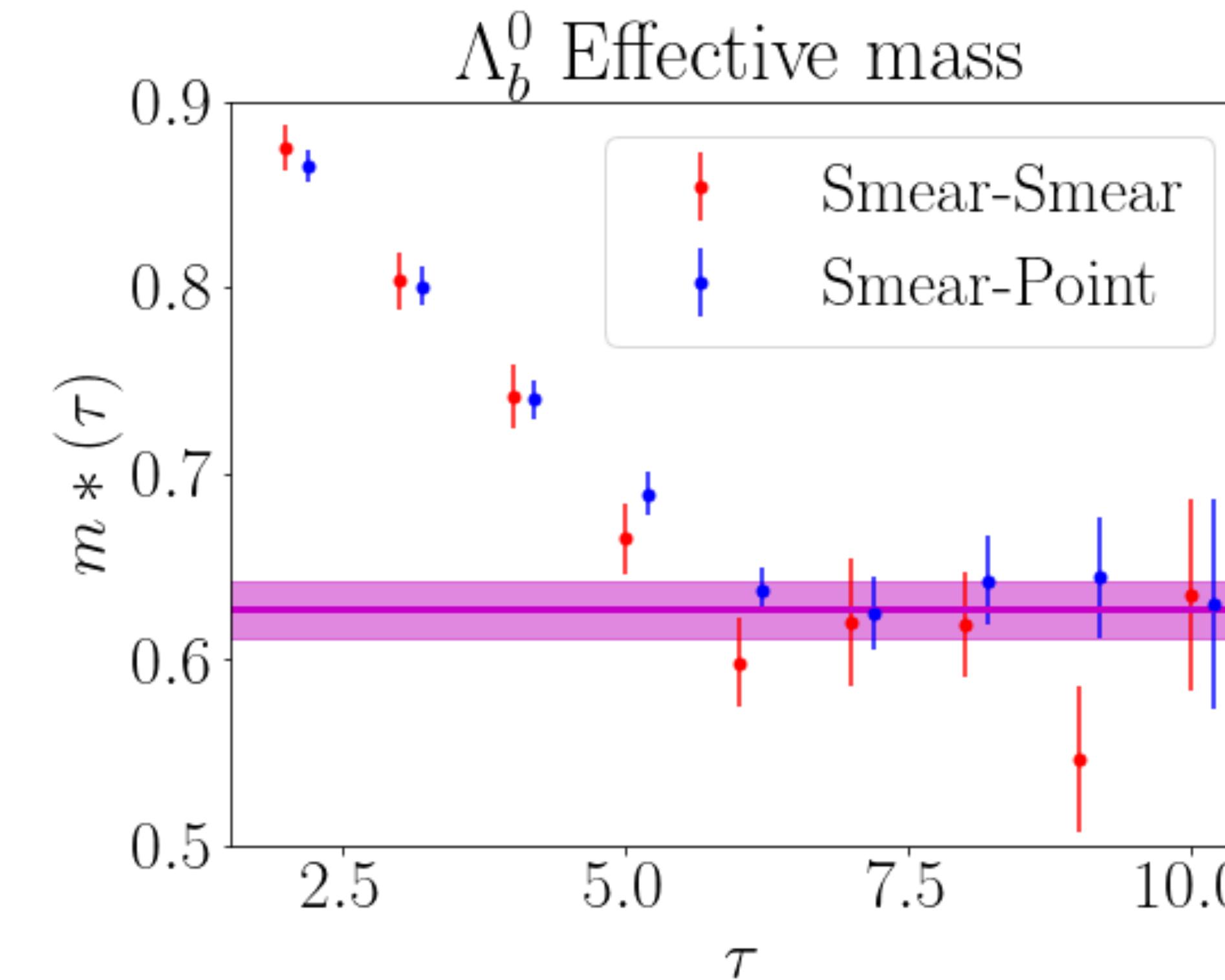
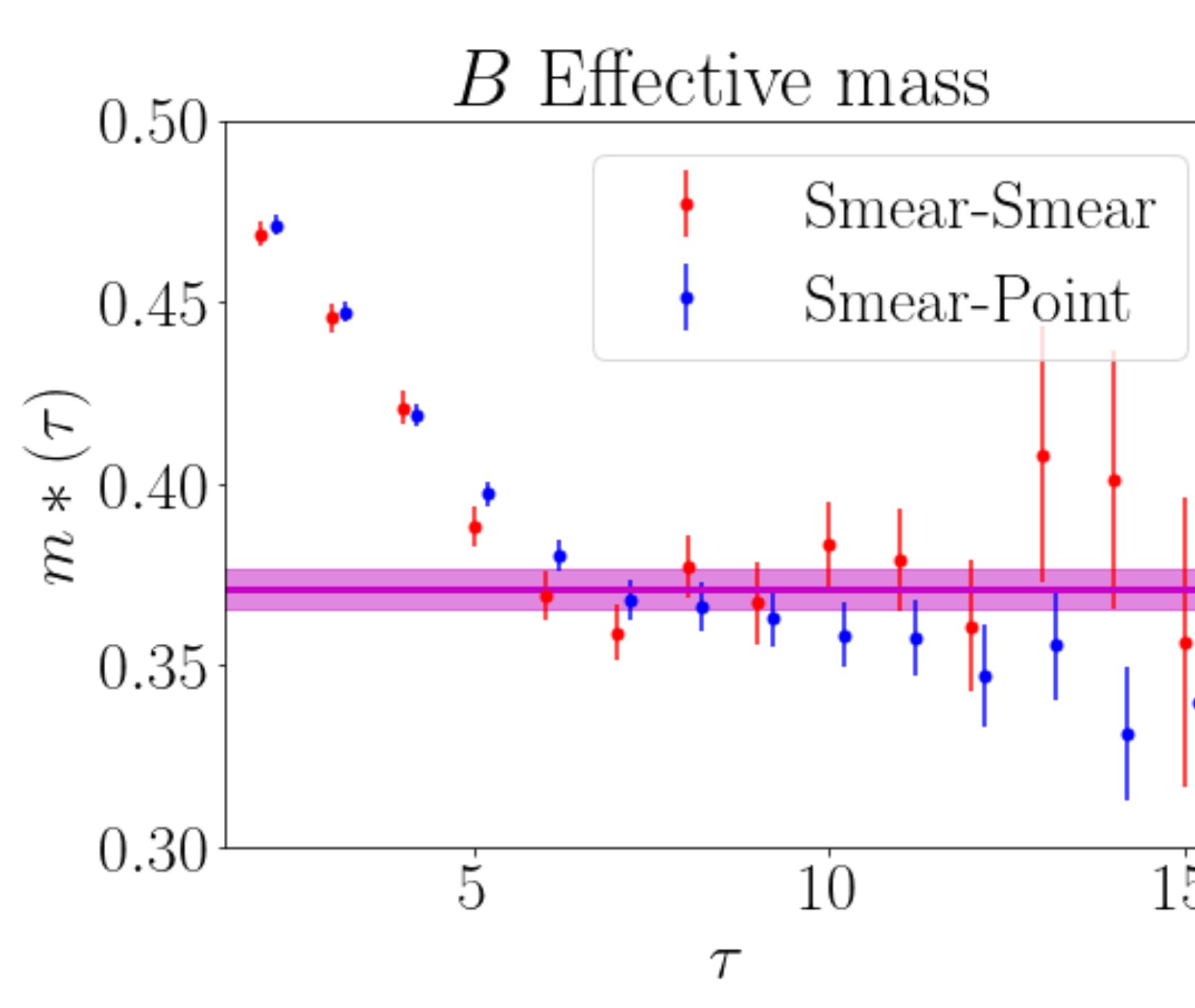
- Wilson Flow was used to improve behaviour of Wilson Lines (static propagator), flow time $\tau_F = 2.0$ used for the plots shown here. Appropriate $a \rightarrow 0$, $\tau_F \rightarrow 0$ extrapolations will need to be taken.
- Coupled two and three-point fits, over 10 different fit ranges. Two-point fits include both smeared-smeared and smeared-point variants. Akaike Information Criterion ($\Delta\chi^2/n_{\text{d.o.f}} > 0.3$), with a cut $\chi^2/n_{\text{d.o.f}} < 1$. Bootstrap to estimate errors.

Effective Mass Plots

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- The two-point function $\langle J_{P/L}(\tau)J_S^\dagger(0) \rangle$ has a gaussian smeared light quark propagator source, and choice of gaussian smeared or point sink. Taking ratios gives us the effective mass:

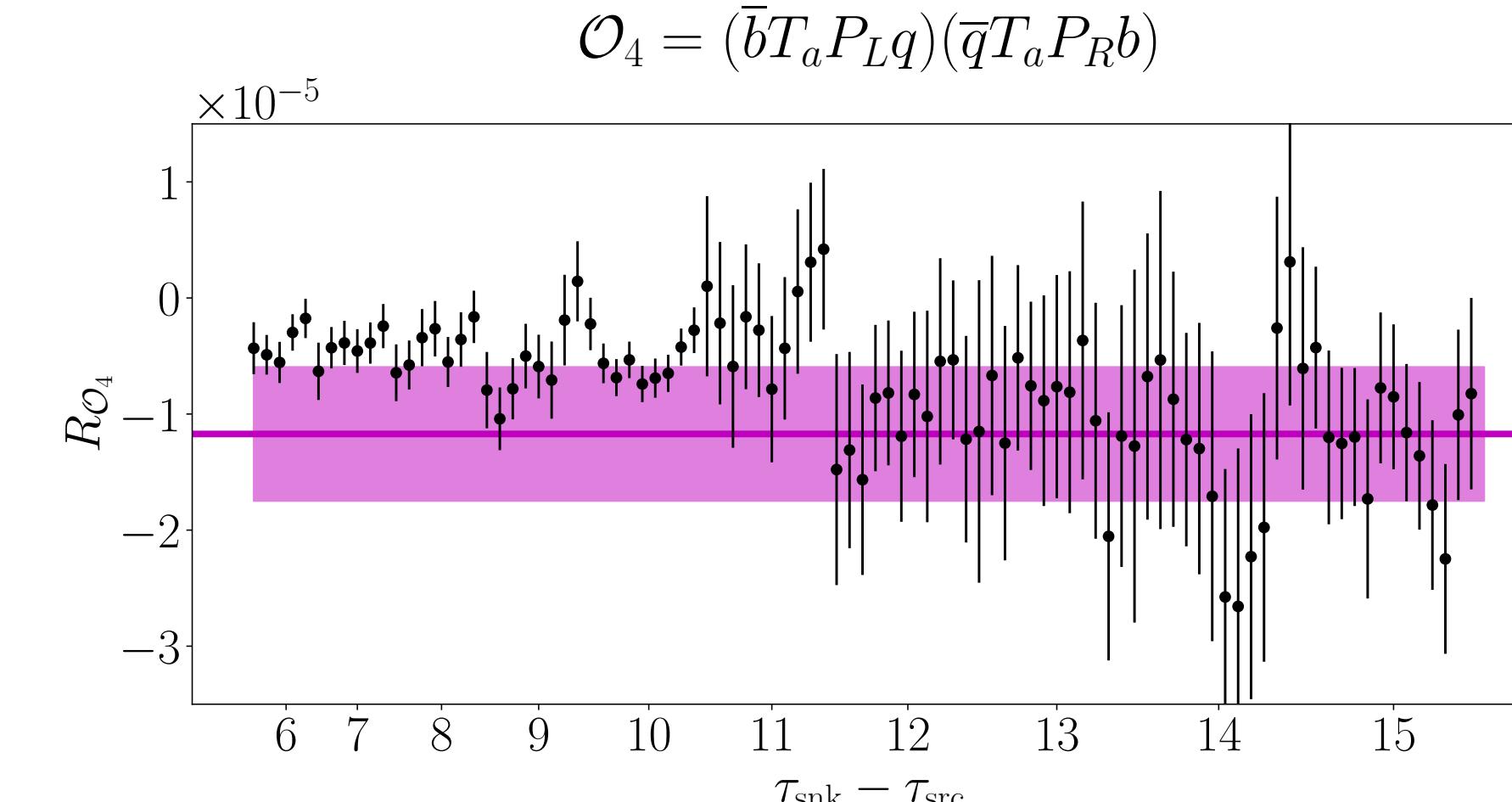
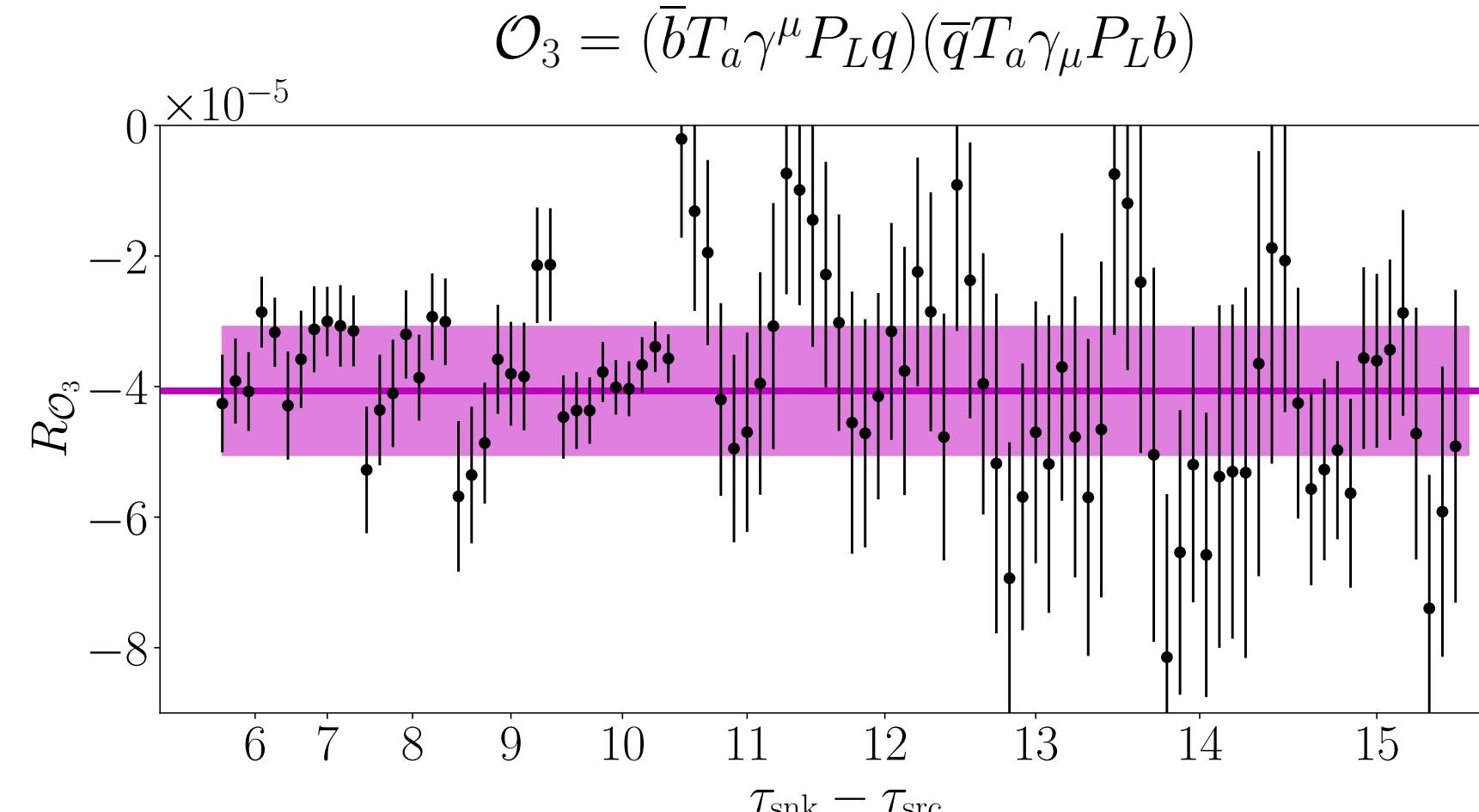
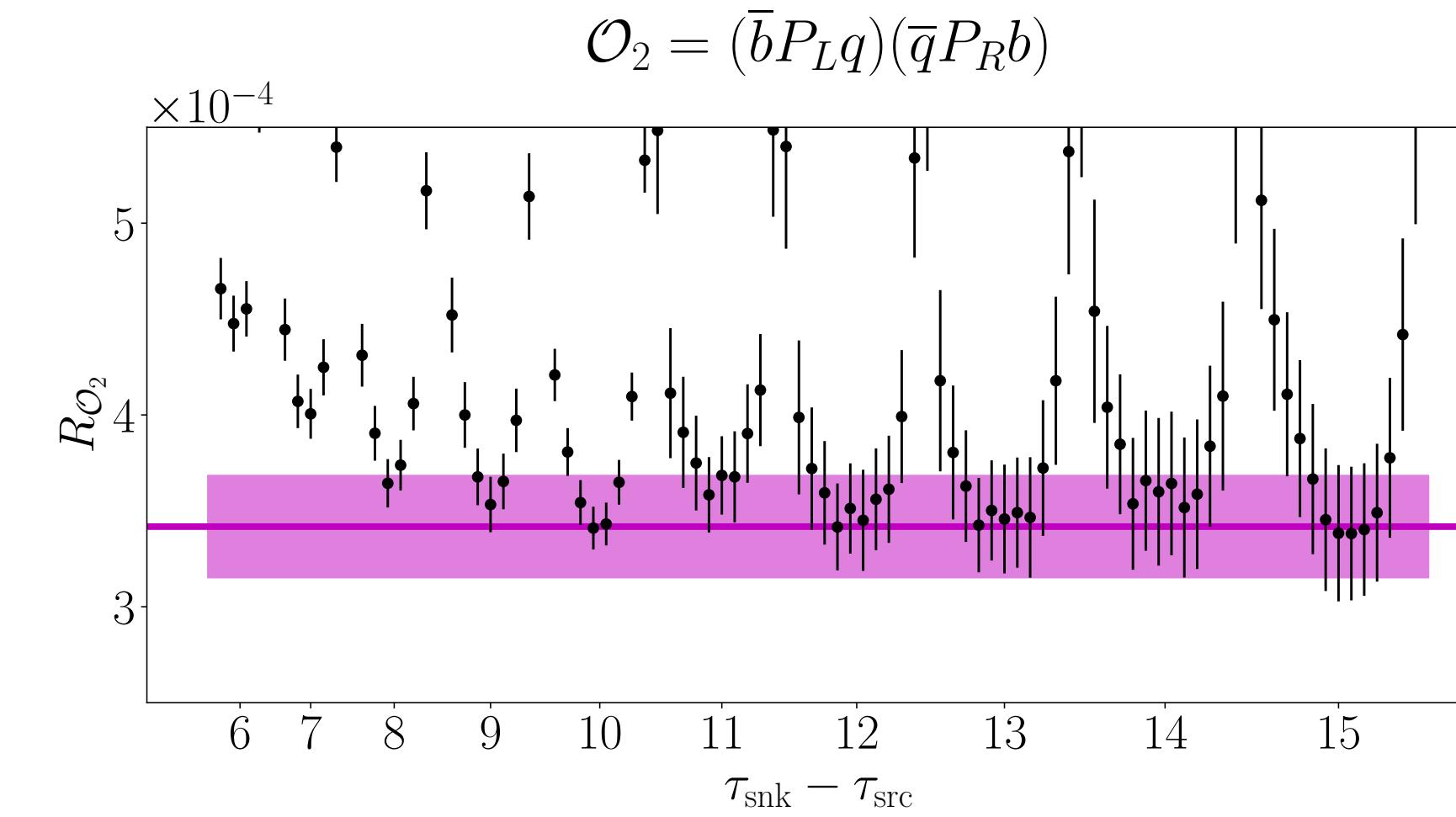
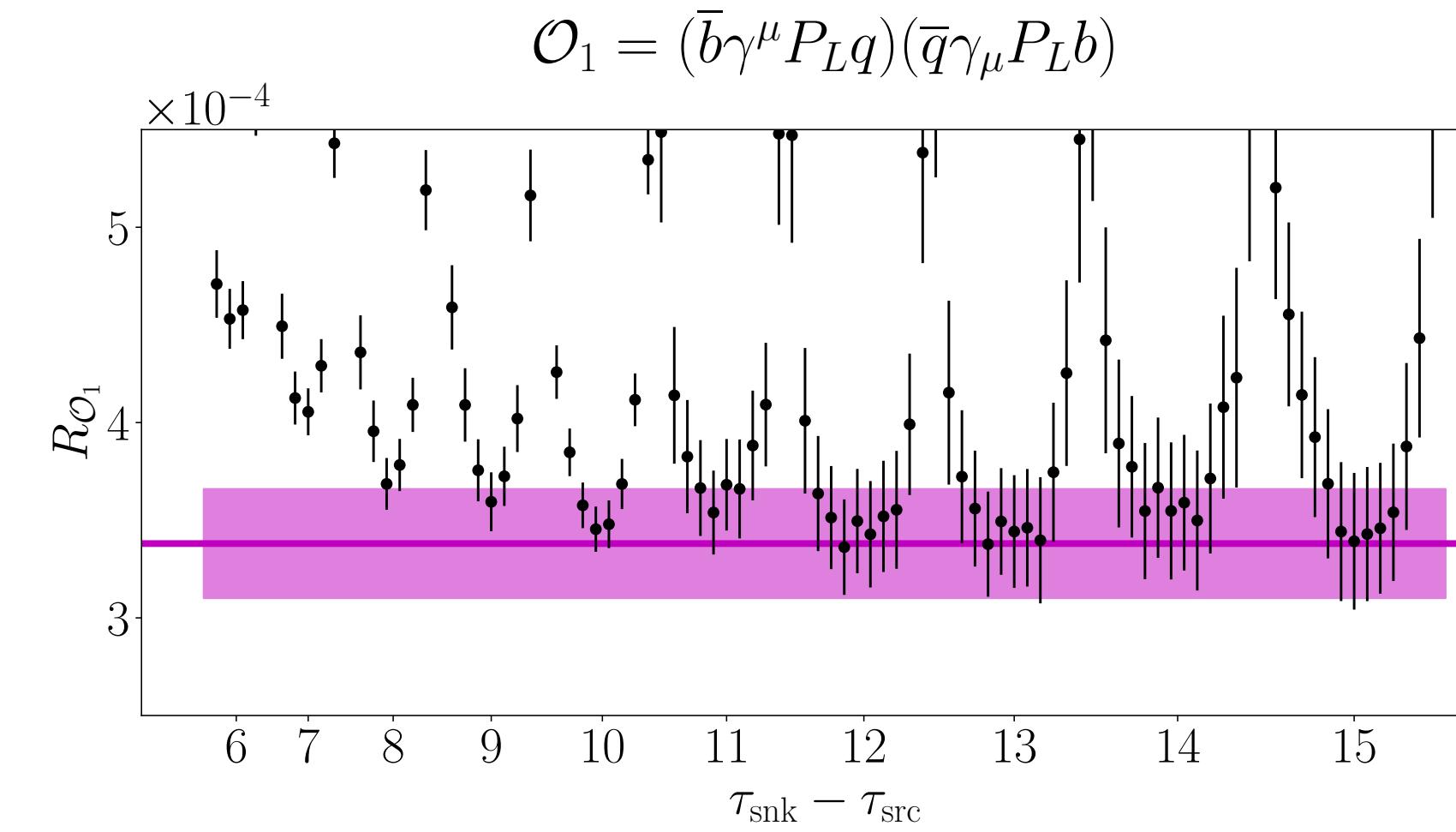
$$m^*(\tau) = \frac{\langle J_{P/S}(\tau+1)J_S^\dagger(0) \rangle}{\langle J_{P/S}(\tau)J_S^\dagger(0) \rangle}$$



Bare B_d Matrix Elements, 24^3

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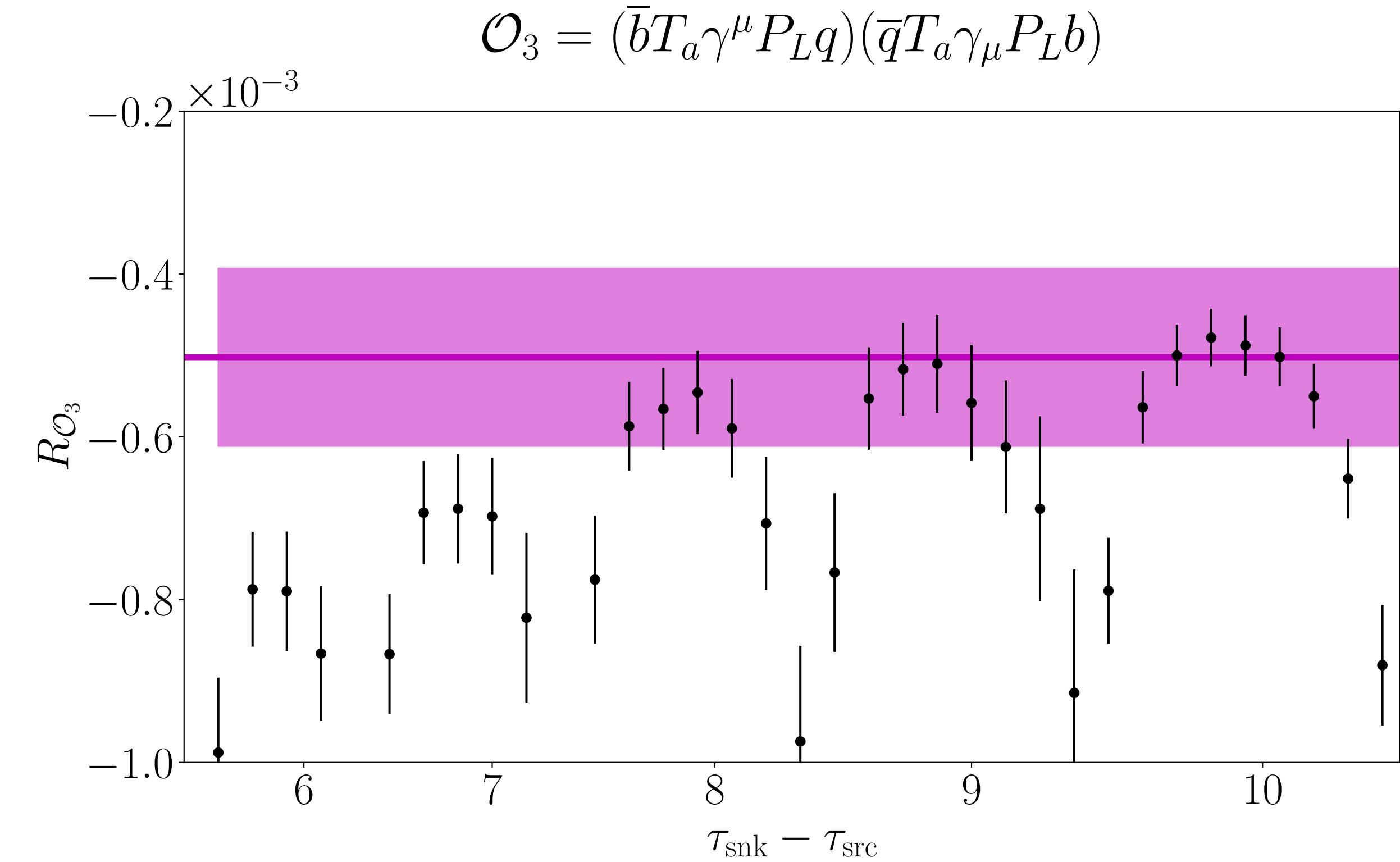
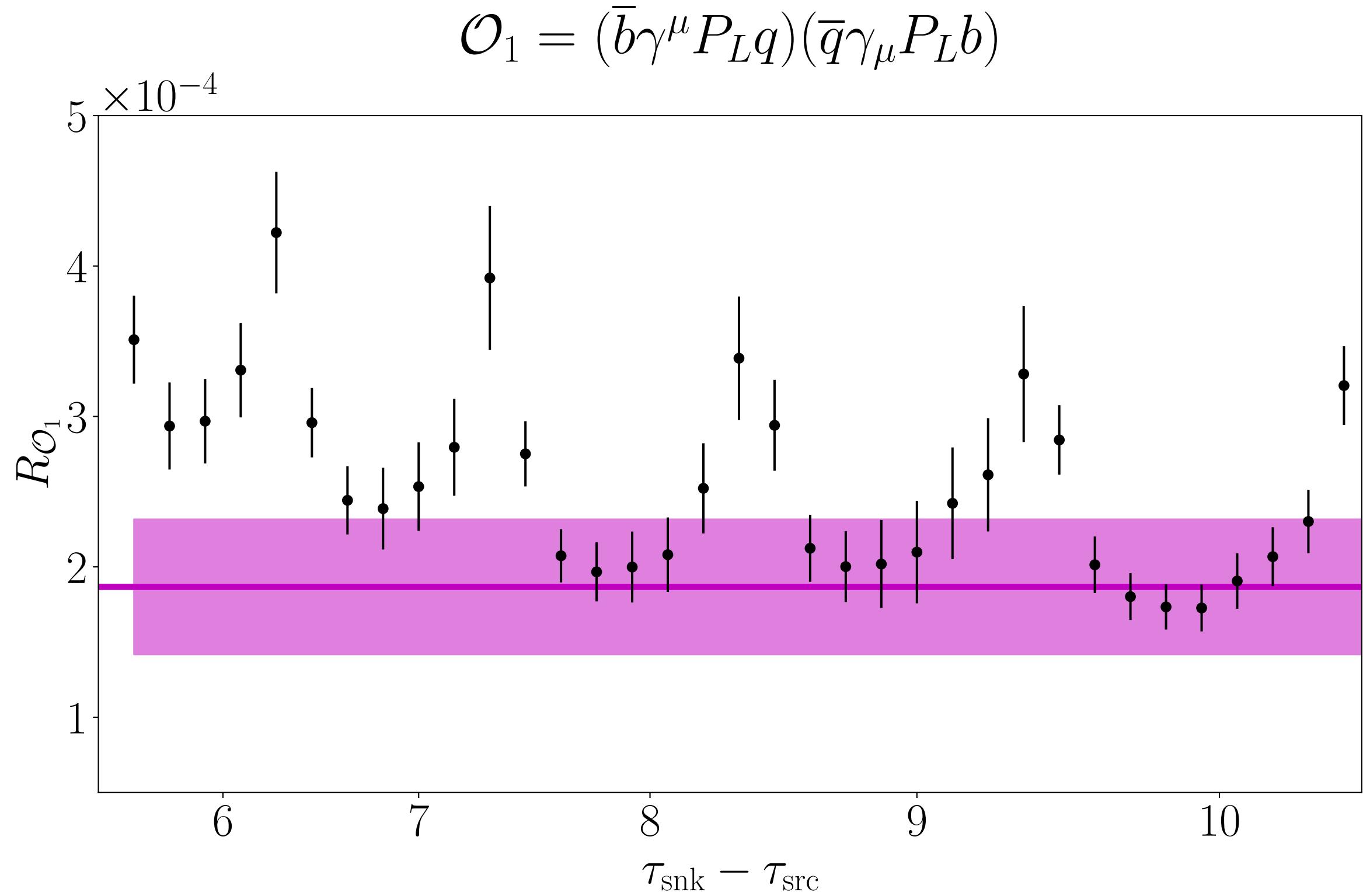
- $R_{\mathcal{O}}(\tau_{\text{snk}}, \tau_{\text{op}}, \tau_{\text{src}}) := \frac{\langle J(\tau_{\text{snk}})\mathcal{O}(\tau_{\text{op}})J^\dagger(\tau_{\text{src}}) \rangle}{\langle J(\tau_{\text{snk}})J^\dagger(\tau_{\text{src}}) \rangle}$ Operators 3 and 4 are suppressed, tree-level values vanish due to color trace.



Bare Λ_b^0 Matrix Elements

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- Heavy quark symmetry cuts down the number of matrix elements in the baryonic case from 4 to 2.
- Only showing $\tau_{\text{snk}} - \tau_{\text{src}} \leq 10$ as the larger separations are very noisy.



Position Space Renormalisation (1)

WORK IN PROGRESS

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- Position space scheme is gauge-invariant and nonperturbative scheme.
 - Bilinear QCD quark operators [V. Gimenez et al. hep-lat/0406019]
 - Heavy-light HQET bilinears [P. Korcyl et al. hep-lat/1512.00069]
- Our proposed position space renormalisation condition for four-quark operators is

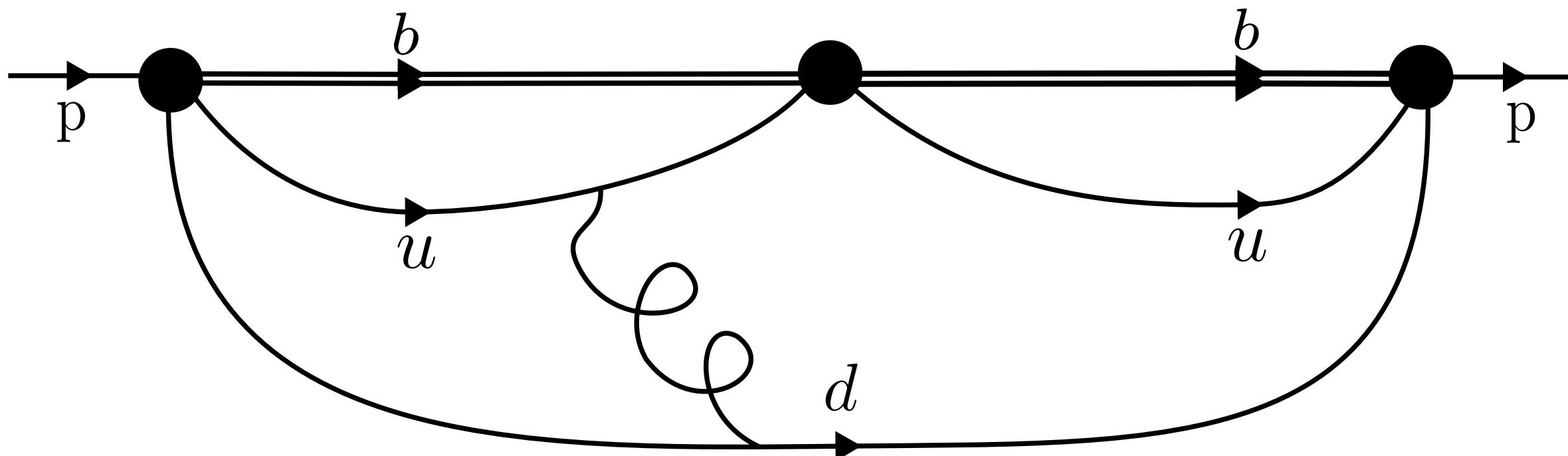
$$Z_{ij} \frac{\langle J_\alpha(x_0) \mathcal{O}_j(\frac{x_0}{2}) J_\alpha^\dagger(0) \rangle}{\langle J_\alpha(x_0) J_\alpha^\dagger(0) \rangle} \Bigg|_{\text{Lattice interacting}} = \frac{\langle J_\alpha(x_0) \mathcal{O}_i(\frac{x_0}{2}) J_\alpha^\dagger(0) \rangle}{\langle J_\alpha(x_0) J_\alpha^\dagger(0) \rangle} \Bigg|_{\text{Lattice tree value}}$$

where x_0 is the renormalisation scale. Taking ratios causes the Z-factors for the currents at either end to cancel.

- To solve for Z , we require that the tree-level values for the different sources J_α are linearly independent vectors over the different operators.

$$J_\alpha = \bar{b}\gamma^5 q, \bar{b}\gamma^i q, \epsilon^{abc} b_a(u_b^T C \gamma_5 d_c), \epsilon^{abc} d_a(b_b^T C \gamma_i u_c)$$

- Connection of X-space scheme to \overline{MS} requires evaluation of position-space three-point functions in Dimensional Regularisation. After Fourier transform, this is equivalent to evaluating p-type Feynman diagrams such as:



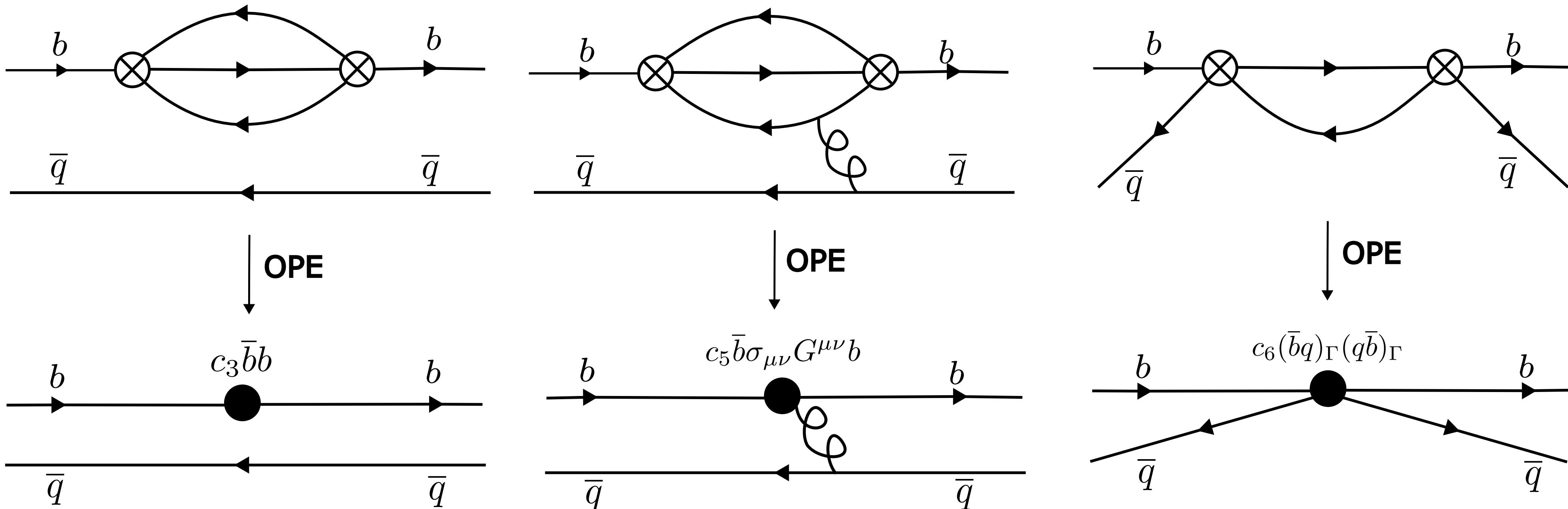
- In the chiral limit, this is made possible by Integration by Parts relations, such as described in [A.G. Grozin, hep-ph/0211328]. This is work in progress!

Thanks!

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Phase Space Factors

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Heavy Quark Expansion (2)

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$$\Gamma(B \rightarrow X) = \frac{1}{2m_B} \langle B | \hat{T} | B \rangle \quad \quad \hat{T} = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} \left[c^{(3)} \bar{b}b + \frac{1}{m_b^2} c^{(5)} g_s \bar{b} \sigma^{\mu\nu} G_{\mu\nu} b + \frac{1}{m_b^3} \sum_k c_k^{(6)} O_k^{(6)} + O(1/m_b^4) \right]$$

- To further simplify matrix elements, use Heavy Quark Effective Theory to expand QCD operators in $1/m_b$. After integrating out the antiquark degrees of freedom, Lagrangian becomes:

$$\mathcal{L}_{\text{HQET}} = \bar{b}_+ (i\nu \cdot D) b_+ + \bar{b}_+ \frac{-D_T^2 - \frac{g}{2} \sigma_{\mu\nu} G^{\mu\nu}}{2m_b} b_+ + O\left(\frac{1}{m_b^2}\right)$$

- And our QCD operators are expanded as:

$$\bar{b}b = \bar{b}_+ b_+ + \bar{b}_+ \frac{D_T^2 + \frac{g}{2} \sigma_{\mu\nu} G^{\mu\nu}}{4m_b^2} b_+ + O\left(\frac{1}{m_b^3}\right) \quad \frac{1}{m_b^2} \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b = \frac{1}{m_b^2} \bar{b}_+ \sigma_{\mu\nu} G^{\mu\nu} b_+ + O\left(\frac{1}{m_b^3}\right)$$

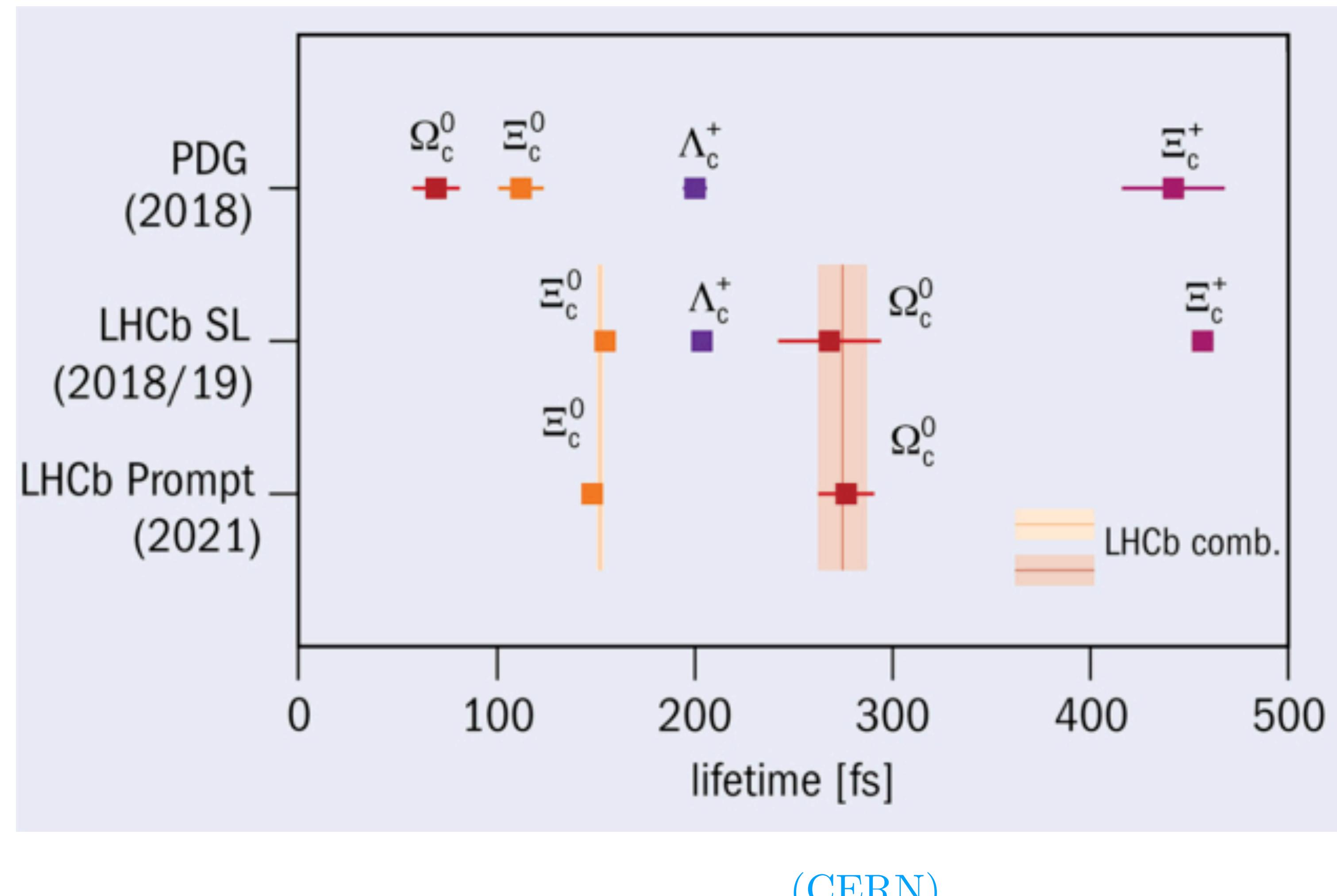
- The operators in red and blue are respectively as the kinetic and chromomagnetic contributions:

$$\mu_\pi^2 = \frac{\langle B | \bar{b}_+ (\overrightarrow{iD})^2 b_+ | B \rangle}{2M_B} + O\left(\frac{1}{m_b}\right) \quad \mu_G^2 = \frac{\langle B | \bar{b}_+ \frac{g_s}{2} \sigma^{\mu\nu} G_{\mu\nu} b_+ | B \rangle}{2M_B} + O\left(\frac{1}{m_b}\right)$$

these can be extracted from the masses of heavy hadrons, as the same terms appear in the HQET-lagrangian.

Possible future work

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Status of Calculations

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Reference	Method	$B_1(m_B)^{\overline{MS}}$	$B_2(m_B)^{\overline{MS}}$	$\epsilon_1(m_B)^{\overline{MS}}$	$\epsilon_2(m_B)^{\overline{MS}}$
arxiv/9709386	QCD Sum Rules	1.01(1)	0.99(1)	-0.08(2)	-0.01(3)
arxiv/9805028	Quenched, Static	1.06(8)	1.01(6)	-0.01(3)	-0.01(2)
arxiv/0110124	Lattice Wilson, Relativistic	1.10(13)(0.2)	0.79(5)(9)	-0.02(2)(0.01)	0.03(1)(0.01)
(this work)	DWF, 2+1f, Static	[to appear]	[to appear]	[to appear]	[to appear]

Strange Multiplets

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