

Estimation of scheme ambiguities in the separation of isospin-breaking effects

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- Motivations and formal problem
- Scheme ambiguities from lattice data
- Outlook and perspectives

Motivations and formal problem

Motivations

- The parameters matching QCD+QED to our world can be **unambiguously determined** by imposing a complete set of experimental hadronic measurement
- The separate determination of isospin-breaking corrections is **prescription dependent**
- **Important phenomenological interest**, for example
 - Comparison of iso-symmetric quantities in theoretical g-2 determinations
 - Radiative corrections to weak decays relatively to QCD decay constants and form factors

Background literature

- ▶ Phenomenology
 - [Gasser & Leutwyler, Phys. Rep. 87(3), pp. 77-169 (1982)]
 - [Gasser, Rusetsky & Scimemi, EPJC 32, pp. 97–114 (2003)]
 - [Gasser & Zarnauskas, PLB 693(2), pp. 122-128 (2010)]
- ▶ Lattice
 - [RM123, Phys. Rev. D 87(11), 114505 (2013)]
 - [BMW, Phys. Rev. Lett. 111(25), 252001 (2013)]
 - [BMW, Science 347 (6229), pp. 1452-1455 (2015)]
 - [QCDSF, JHEP 93 (2016)]
 - [BMW, Phys. Rev. Lett. 117(8), 082001 (2016)]
 - [MILC, Phys. Rev. D 99(3), 034503 (2019)]
 - [RM123-Soton, Phys. Rev. D 100(3), 034514 (2019)]
 - [FLAG, EPJC 80, 113 (2020)]

General problem

- For an observable X one ideally wants an expansion (FLAG notation)

$$X^\phi = \bar{X} + X_\gamma + X_{\text{SU}(2)}$$

strong IB
electromagnetic IB
iso-symmetric

- A complete set of hadron masses defines X^ϕ **unambiguously**
- The separation in 3 contributions requires additional conditions, and are **scheme-dependent**

High-level strategy

- This is quite technical to describe fully, so before anything else...

The key choices in designing a scheme are

- 1) which variables are kept fixed when $\alpha \rightarrow 0$**
 - 2) which variable parametrises $\delta m = m_u - m_d$**
- Both 1) and 2) define the scheme and are sufficient to define the isospin expansion

Linear expansion

- Isospin breaking effects are small.
Up to 1% corrections, **unphysical theories are within a linear correction from the physical point**

$$X_M(M, \alpha) = X^\phi + \frac{\partial X_M}{\partial M} (M - M^\phi) + (\alpha - \alpha^\phi) \frac{\partial X_M}{\partial \alpha}$$

- The space of all possible prescriptions can be explored with the knowledge of the **observable derivatives**
- The variable M can be changed using Jacobians
Requires knowledge of **variable derivatives**

Scheme ambiguities from lattice data

Physical point setup

- Same data than leptonic decay IB corrections
previous talk by M Di Carlo this session 14:00
- Physical point DWF ensemble with $a \simeq 0.12$ fm
- Bare parameters derivatives through operator insertions
- Electro-quenched QED_L
- Universal FV QED_L effects subtracted from masses
- Using best AIC fit results, **statistical errors only**

First step: finding the physical point

- Tilde quantities: lattice units
- Choose a set of known dimensionless ratios ρ
here $\rho = (M_{\pi^+}^2/M_{\Omega^-}^2, M_{K^+}^2/M_{\Omega^-}^2, M_{K^0}^2/M_{\Omega^-}^2)$
- Find physical bare quark masses

$$\tilde{m}_0^\phi = \tilde{m}_0^{\text{sim}} - \left(\frac{\partial \rho}{\partial \tilde{m}_0} \right)^{-1} \left(\rho^{\text{sim}} - \rho^{\text{exp}} + \alpha \frac{\partial \rho}{\partial \alpha} \right)$$

- Predict any observable at the physical point

$$\tilde{X}^\phi = \tilde{X}^{\text{sim}} + \frac{\partial \tilde{X}}{\partial \tilde{m}_0} (\tilde{m}_0^\phi - \tilde{m}_0^{\text{sim}}) + \alpha \frac{\partial \tilde{X}}{\partial \alpha}$$

Scale setting

- Physical point scale setting predicting $a = \tilde{M}_{\Omega^-}^\phi / M_{\Omega^-}^{\text{exp}}$
- For this ensemble $a^{-1} = 1739(9)$ GeV
- We will keep that fixed across theories
- More ambiguities depending on that choice, e.g.
 - use dimensionless ratios as variables
 - re-determine lattice spacing(s) at unphysical point(s)
- Close to no impact in the present exercise, needs to be studied more, with continuum limit

Will be discussed in N Tantalo plenary talk Saturday 11:45

Consistency check: quark masses

- Exercise: find $\overline{\text{MS}}$ physical quark masses at $\mu = 2 \text{ GeV}$
- Using renormalisation constants from RBC-UKQCD and 100% error on undetermined QED corrections

$$m_{ud} = 3.33(2) \text{ MeV}$$

$$m_s = 92.7(5) \text{ MeV}$$

$$m_u/m_d = 0.457(4)$$

this analysis

$$m_{ud} = 3.38(4) \text{ MeV}$$

$$m_s = 92.2(1.0) \text{ MeV}$$

$$m_u/m_d = 0.485(19)$$

[FLAG 2021 $N_f = 2 + 1$]

- This is just a check, not a new result**
Systematics and continuum limit needed

Second step: apply scheme

- Choose a variable set M
- If M is not known experimentally, predict M^ϕ
Choose prescription for \hat{M}, \bar{M}
- Derivatives in M can be computed using the Jacobian
$$\frac{\partial X_M}{\partial(M, \alpha)} = \frac{\partial X}{\partial(m_0, \alpha)} \left[\frac{\partial(M, \alpha)}{\partial(m_0, \alpha)} \right]^{-1}$$
- Compute IB corrections, for example QED corrections

$$X_\gamma = \frac{\partial X_M}{\partial M} (M^\phi - \hat{M}) + \alpha \frac{\partial X}{\partial \alpha}$$

Quark mass scheme

- Prescription: take physical renormalised quark masses

$$m^\phi = (m_{ud}^\phi, m_s^\phi, m_u^\phi - m_d^\phi)$$

- Then with $\alpha \rightarrow 0$

$$\text{pure QCD} \quad \hat{m} = (m_{ud}^\phi, m_s^\phi, m_u^\phi - m_d^\phi)$$

$$\text{iso-symmetric QCD} \quad \bar{m} = (m_{ud}^\phi, m_s^\phi, 0)$$

- Generally implicit scheme for EFT calculations
- Introduced in lattice calculations by RM123
as “GRS scheme” for electro-quenched theories

[RM123, Phys. Rev. D 87(11), 114505 (2013)]

BMW 2013 scheme

[BMW, Phys. Rev. Lett. 111(25), 252001 (2013)] [BMW, Phys. Rev. Lett. 117(8), 082001 (2016)]

- Connected $\bar{q}q$ meson masses as a proxy for quark masses

$$M_{\bar{q}q}^2 = 2B_0 m_q + \text{NLO}$$

[Bijnens & Danielsson, Phys. Rev. D 75(1), 014505 (2007)]

- Variable set

$$M_{ud}^2 = \frac{1}{2}(M_{\bar{u}u}^2 + M_{\bar{d}d}^2) = 2B_0 m_{ud} + \text{NLO}$$

$$\Delta M^2 = (M_{\bar{u}u}^2 - M_{\bar{d}d}^2) = 2B_0(m_u - m_d) + \text{NLO}$$

$$2M_{K_\chi}^2 = M_{K^+}^2 + M_{K^0}^2 - M_{\pi^+}^2 = 2B_0 m_s + \text{NLO}$$

- Scheme defined by

$$\hat{M} = (M_{ud}^{2,\phi}, \Delta M^{2,\phi}, 2M_{K_\chi}^{2,\phi}) \quad \text{pure QCD}$$

$$\bar{M} = (M_{ud}^{2,\phi}, 0, 2M_{K_\chi}^{2,\phi}) \quad \text{iso-symmetric QCD}$$

BMW 2013 scheme

- M_{ud}^2 and ΔM^2 are unphysical and need to be determined at the physical point, we found

$$M_{ud}^2 = 18251(15) \text{ MeV}^2$$

$$\Delta M^2 = -13127(104) \text{ MeV}^2$$

- Scheme slightly modified in BMW 2022 g-2 calculation
- They obtained

$$\Delta M^2 = -13170(320)(270) \text{ MeV}^2$$

Mainz scheme

[Mainz, arXiv:2203.08676]

- Identical to BMW 2013 up to the substitution

$$\Delta M^2 \mapsto \Delta_8^2 = M_{K^+}^2 - M_{K^0}^2 - M_{\pi^+}^2 + M_{\pi^0}^2$$

- $2\Delta_8^2$ and ΔM^2 are both equal to $2B_0(m_u - m_d)$ at LO
- Δ_8^2 is known experimentally, but potentially receives large corrections at NLO (Dashen theorem violations)

$$(\Delta M^2)_{\text{LO}} = -13459(756) \text{ MeV}^2$$

[FLAG 2021]

[RBC-UKQCD, PRD 93(7), 074505 (2016)]



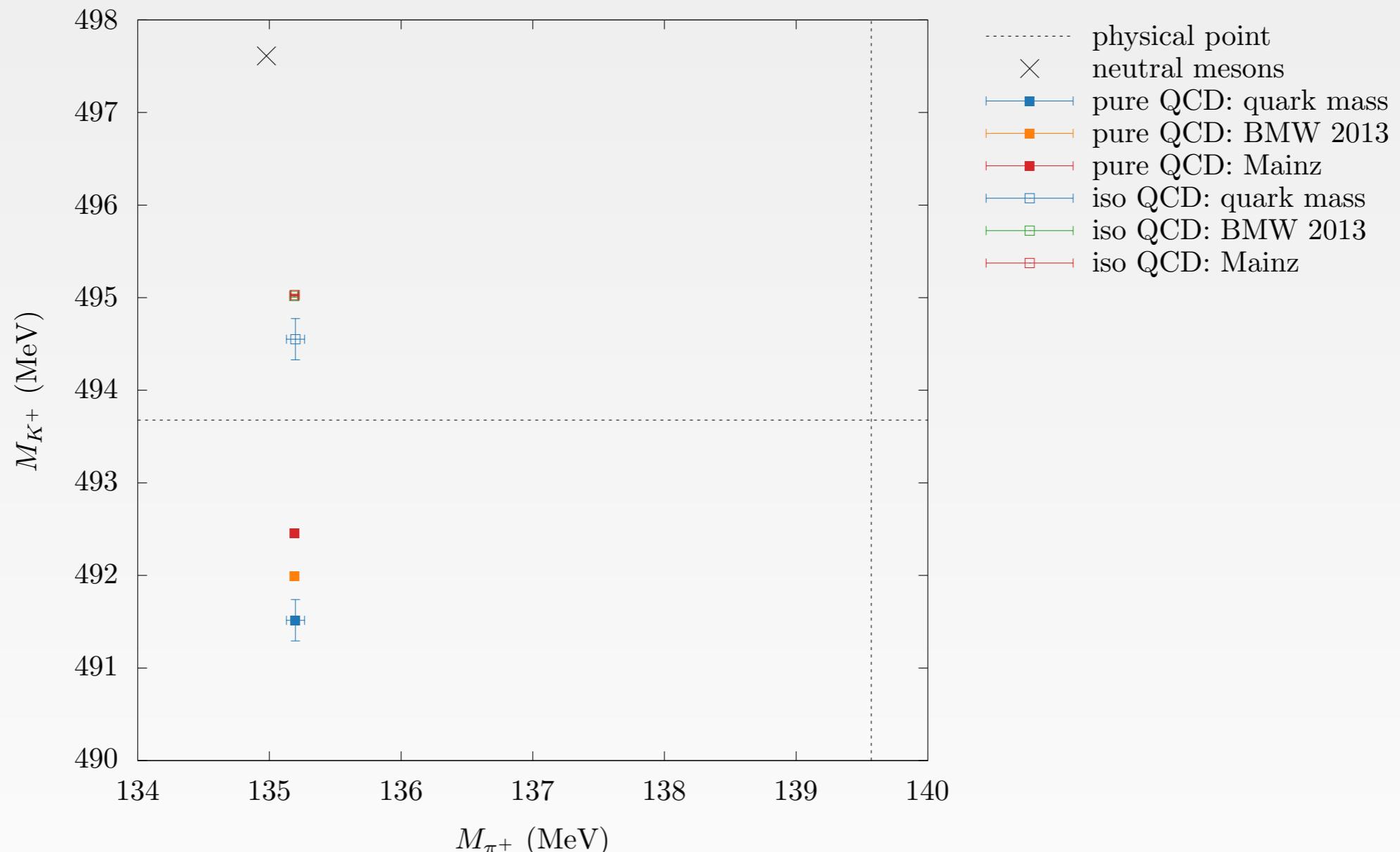
$$\Delta M^2 = -13127(104) \text{ MeV}^2$$

this analysis

$$2\Delta_8^2 = -10322(41) \text{ MeV}^2$$

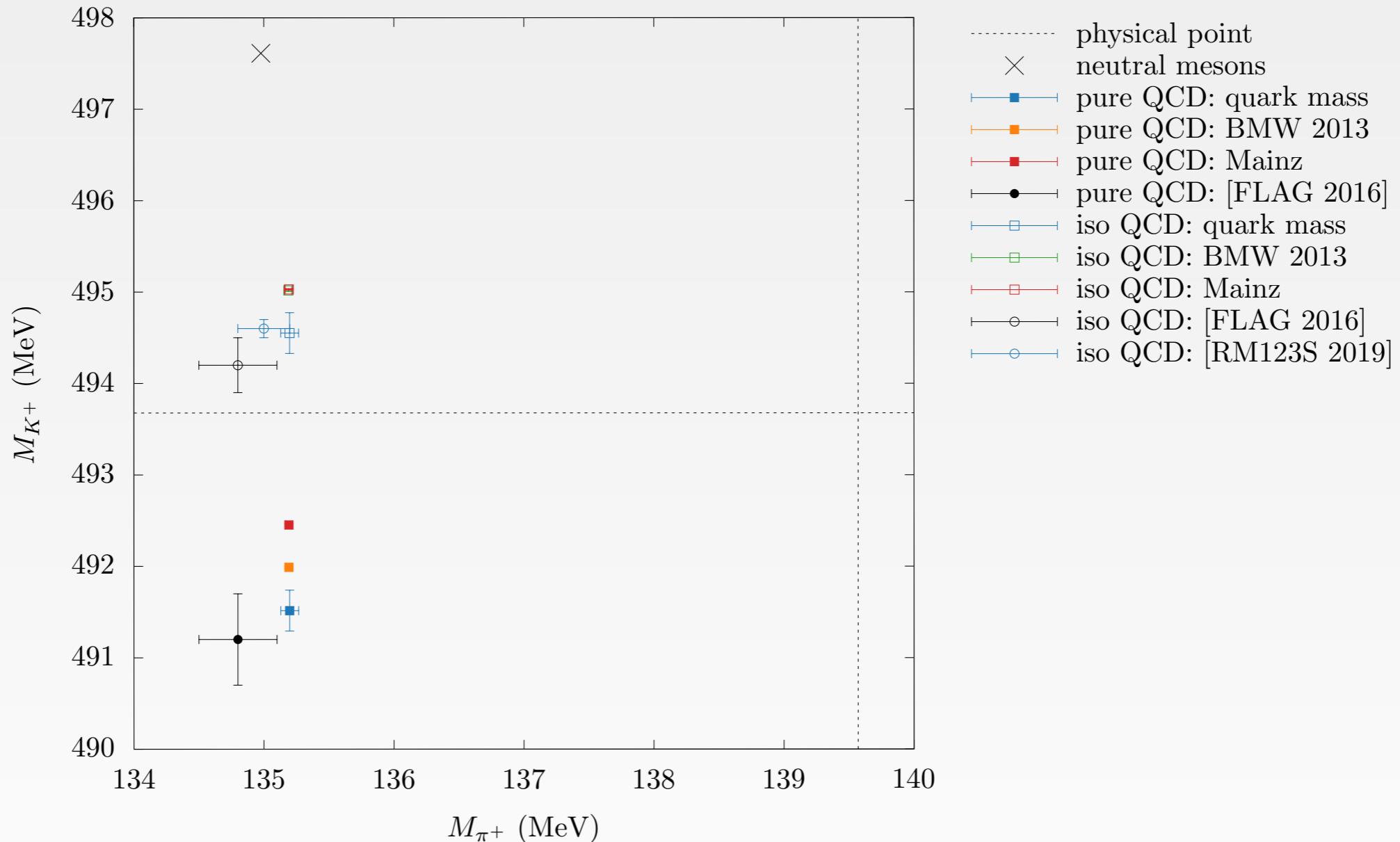
[PDG 2022]

Pion/kaon plane landscape



Open symbols: iso QCD / Full symbols: pure QCD

Pion/kaon plane landscape

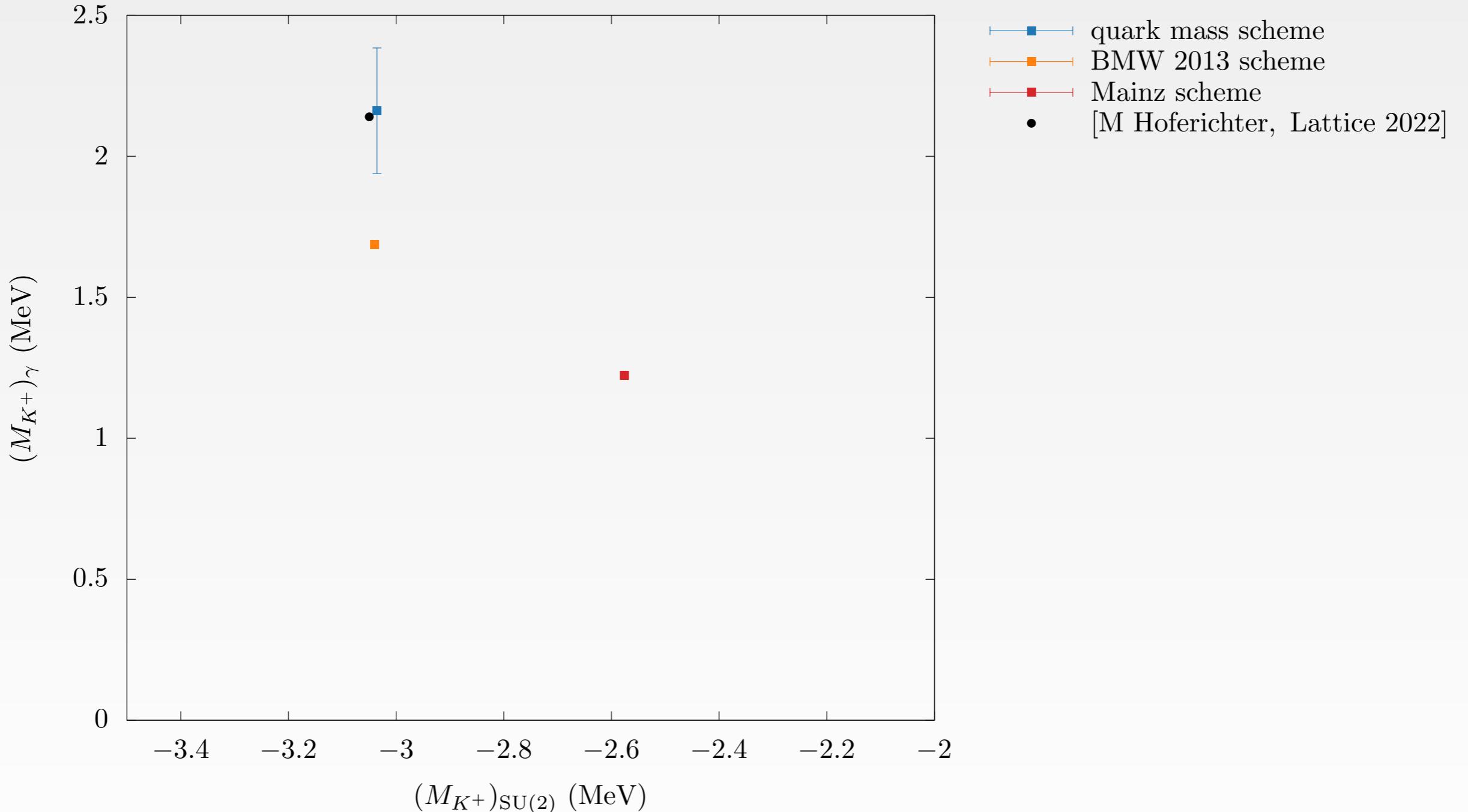


Open symbols: iso QCD / Full symbols: pure QCD

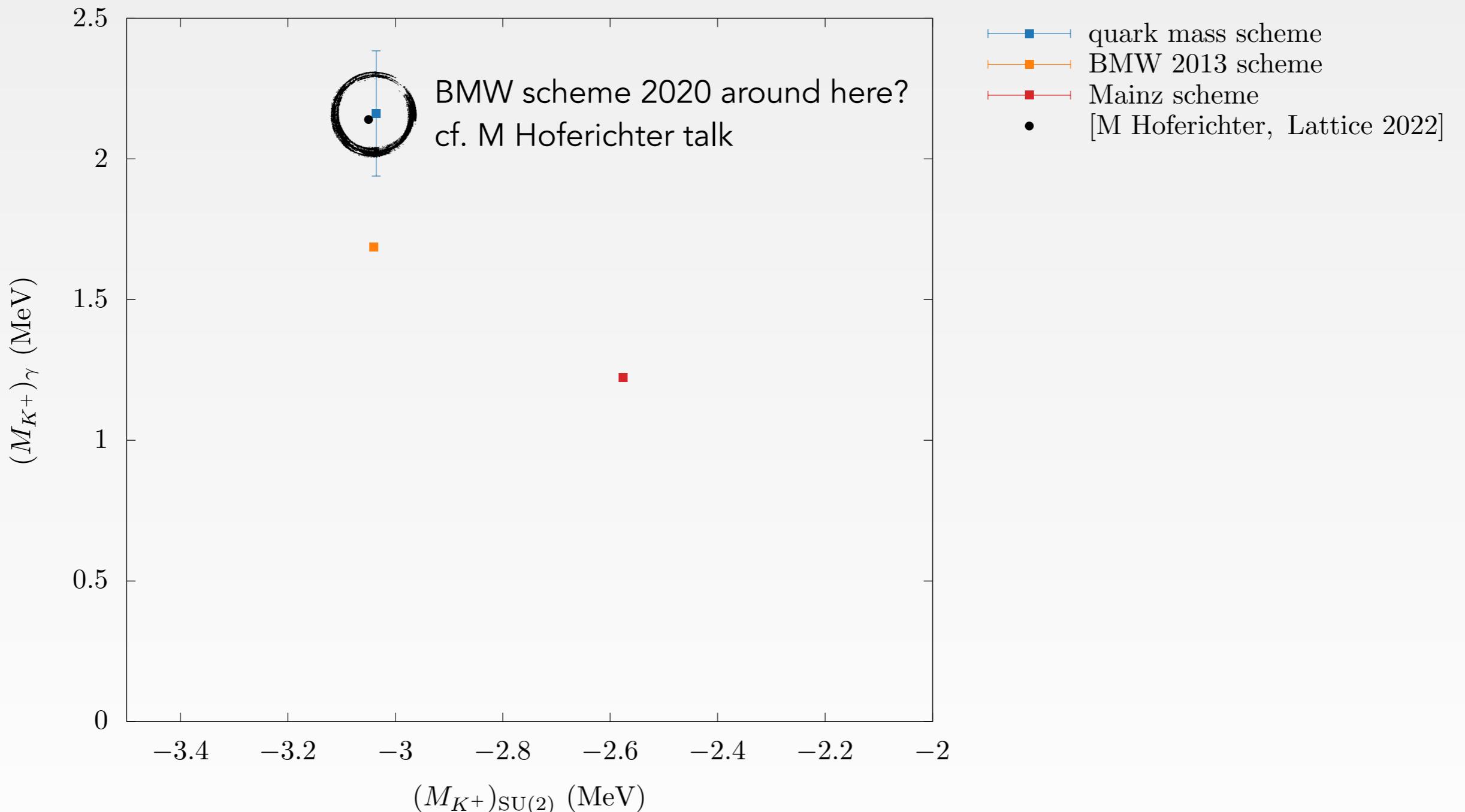
[RM123S 2019]: equivalent to quark mass scheme (electro-quenched GRS)

[FLAG 2016]: equivalent to quark mass scheme (pheno estimate)

Charged kaon decomposition



Charged kaon decomposition



Outlook and perspectives

Outlook

- Isospin-breaking corrections with non-zero isospin limit are **scheme-dependent**
- Because of the small size of isospin corrections, all possible prescriptions are **within a linear correction from each other**
- Conversion between schemes is achievable if projects **publish derivatives of observables and variables** in bare parameters
- Pheno schemes generally based on quark masses, lattice schemes tend to prefer meson masses as variables

Perspectives

- Study continuum limit and scale-setting ambiguities
- Study scheme ambiguities for leptonic decays width corrections
- Provide scheme coordinates with systematic errors
- Implement QED corrections to quark mass renormalisation constants
- Study scheme ambiguities formally using EFT