Towards precision lattice determination of semileptonic $D \to \pi\ell\nu$, $D \to K\ell\nu$ and $D_s \to K\ell\nu$ decay form factors

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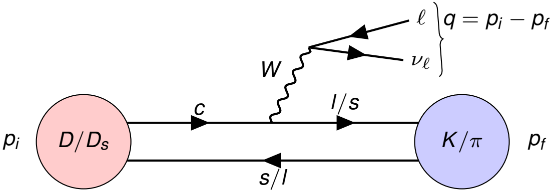
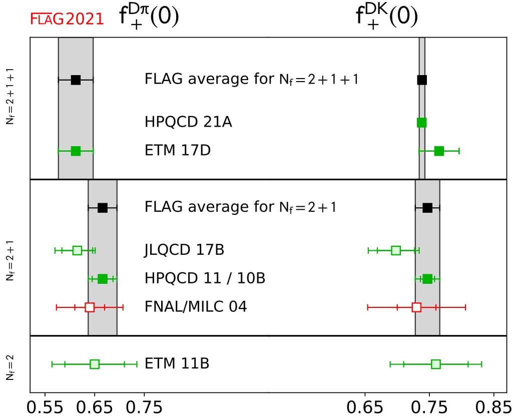
Outline

- Exclusive semileptonic decays $D/D_s \to K/\pi\ell\nu$
- Form factors over the entire physical q^2 range
- Combined with experiment, yields $|V_{cs}|$ and $|V_{cd}|$
- These quantities ultimately constrain BSM physics

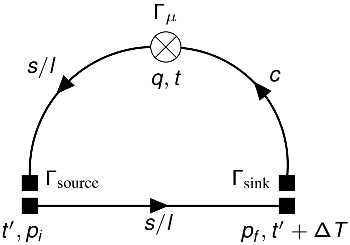


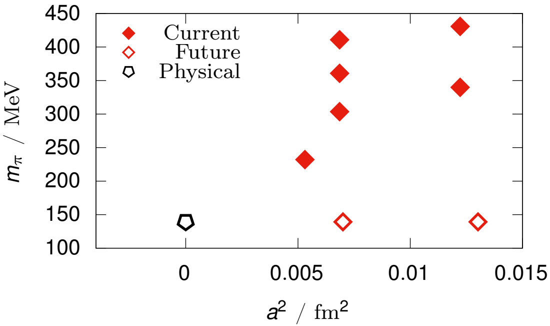
Using domain wall fermions



FLAG 2021 [1]

- All domain wall fermions
- Automatic $\mathcal{O}(a)$ improved
- RBC/UKQCD 2+1 flavour ensembles
- Stout-smeared Möbius charm





Our setup

- Tuned charm at rest
- Full kinematic range
- Sequential propagator: spectator then charm
- Multiple operators
  - $\mathbb{Z}_2$ [2] stochastic average **point**-like sources
  - Coulomb gauge-fixed **wall**-sources

Semileptonic form factor definitions i/ii

Experiment measures $f_+\left(q^2\right)$ and $f_0\left(q^2\right)$

$$\langle K|\mathcal{V}^\mu|D_s\rangle = f_+\left(q^2\right)\left(p^\mu_{D_s} + p^\mu_K - \frac{m^2_{D_s} - m^2_K}{q^2}\, q^\mu\right) + f_0\left(q^2\right)\frac{m^2_{D_s} - m^2_K}{q^2}\, q^\mu\,.$$

q^μ is momentum transfer to the lepton pair

$$q^\mu = p^\mu_{D_s} - p^\mu_K = \left(E_{D_s} - E_{P_L}, \mathbf{p}_{D_s} - \mathbf{p}_K\right)\,.$$

In our setup, the heavy meson is at rest ($\mathbf{p}_{D_s} = 0$):

$$q^2 = m^2_{D_s} + m^2_K - 2m_{D_s}E_K\,.$$

Energies extracted from fits to two point functions.

Semileptonic form factor definitions ii/ii

On the lattice we parameterise $f_{||}$ and $f_{\perp}$

$$f_{||}(E_K) = \frac{Z_V \langle K|V_4|D_s\rangle}{\sqrt{2m_{D_s}}}$$

$$f_{\perp}(E_K) = \frac{Z_V \langle K|V_i|D_s\rangle}{\sqrt{2m_{D_s}}} \frac{1}{p_{K,i}} .$$

We relate lattice to experiment

$$f_+\left(q^2\right) = \frac{1}{\sqrt{2m_{D_s}}} \left(f_{||}(E_K) + (m_{D_s} - E_K) f_{\perp}(E_K)\right)$$

$$f_0\left(q^2\right) = \frac{\sqrt{2m_{D_s}}}{m_{D_s}^2 - m_K^2} \left((m_{D_s} - E_K) f_{||}(E_K) + \left(E_K^2 - m_K^2\right) f_{\perp}(E_K)\right) .$$

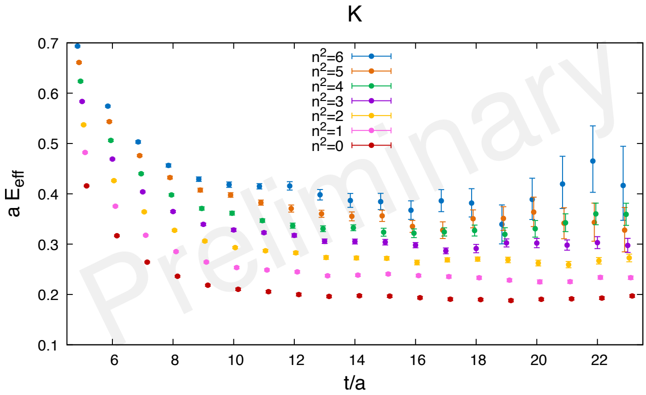
Matrix elements from ratios of 3pt and 2pt functions ($C^{(3)}$ and $C^{(2)}$).

Z_V from charge conservation.

- Access to momenta **p** (for integers **n**)

$$\mathbf{p} = \frac{2\pi}{L}\mathbf{n} \quad .$$

- Point- and wall-source 2-point data
  - Simultaneously fitted
  - Scan many initial and final fit times, t_i / t_f
  - Check results under control varying t_i / t_f
- Choice per meson/momentum/ensemble
  - Masses
  - Energies
  - Overlap coefficients $A^*_{f,j} = \langle j|\hat{O}^\dagger_f|\Omega\rangle$



Extracting Z_V

Charge conservation, $f_+^{PP}(0) = 1$ and rest frame $p_i + p_f = (2E_P, \mathbf{0}) \implies$

$$Z_V = \frac{2E_P}{\langle P(\mathbf{0}) | V_4(\mathbf{0}) | P(\mathbf{0}) \rangle} = \frac{C_P(t_f - t_i, \mathbf{0})}{C^{(4)}_{PP\mathrm{bare}}(t_i, t, t_f, \mathbf{0}, \mathbf{0})} .$$

Extract Z_V as ratio of two-point over three-point correlators

Simplification (smeared charm with unsmeared strange/light action):

$$Z_V = \sqrt{Z_{V,\mathrm{action\ 1}} Z_{V,\mathrm{action\ 2}}}$$

Longer term: fully non-perturbative RI/SMOM determination of mixed action Z_V[3] [4].

Ratios used to examine three-point data

Many ratios could be used to extract the matrix elements of interest. We choose

$$R^\mu = A_i A_f^* Z_V \frac{C^\mu_{P_i P_f}(t_i, t, t_f, \mathbf{p}_i, \mathbf{p}_f)}{C_{P_i}(t - t_i, \mathbf{p}_i)\, C_{P_f}(t_f - t, \mathbf{p}_f)}$$

$$\simeq \cancel{A_i A_f^*} Z_V \frac{\cancel{A_i^* A_f}}{\cancel{|A_i|^2 |A_f|^2}} \langle P_f(\mathbf{p}_f) | V^\mu \left(\mathbf{q}^2\right) | P_i(\mathbf{p}_i) \rangle \frac{\cancel{e^{-E_i(t-t_i)} e^{-E_f(t_f-t)}}}{\cancel{e^{-E_i(t-t_i)} e^{-E_f(t_f-t)}}}.$$

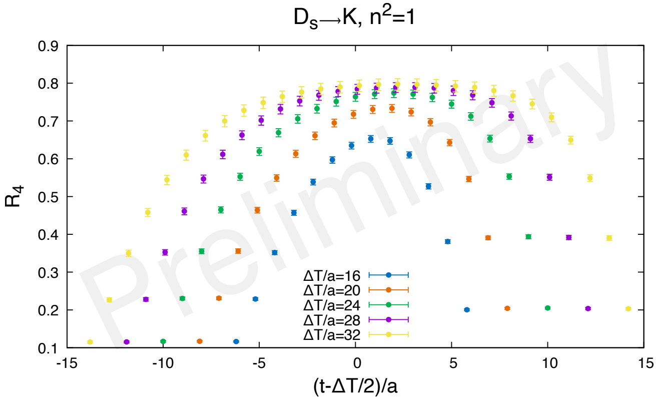
Exponentials cancel (where the ground-state dominates) yielding the renormalised matrix element

$$R^\mu = Z_V \langle P_f(\mathbf{p}_f) | V^\mu \left(\mathbf{q}^2\right) | P_i(\mathbf{p}_i) \rangle \ .$$

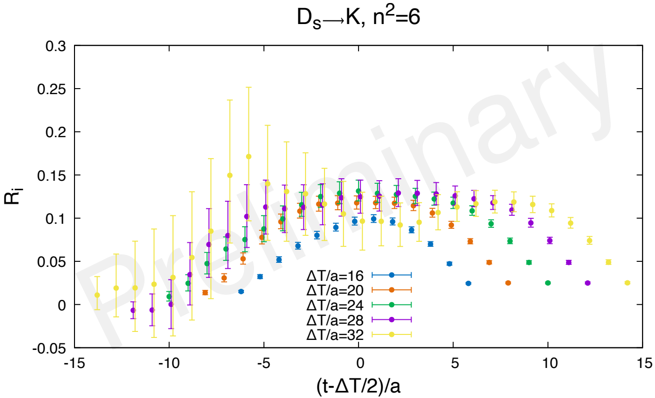
Ratio constructed from bare correlators, using Z_V and $A^*_{f,j} = \langle j | \hat{O}_f^\dagger | \Omega \rangle$ extracted from fits.

Three-point functions – matrix elements

- Form ratios to extract matrix elements
  - Excited-state contamination low ΔT
  - Higher ΔT compatible
- We observe error growth at higher ΔT
- Plan to simultaneously fit multiple ΔT
  - for reduced error
- For this talk we fit a single ΔT

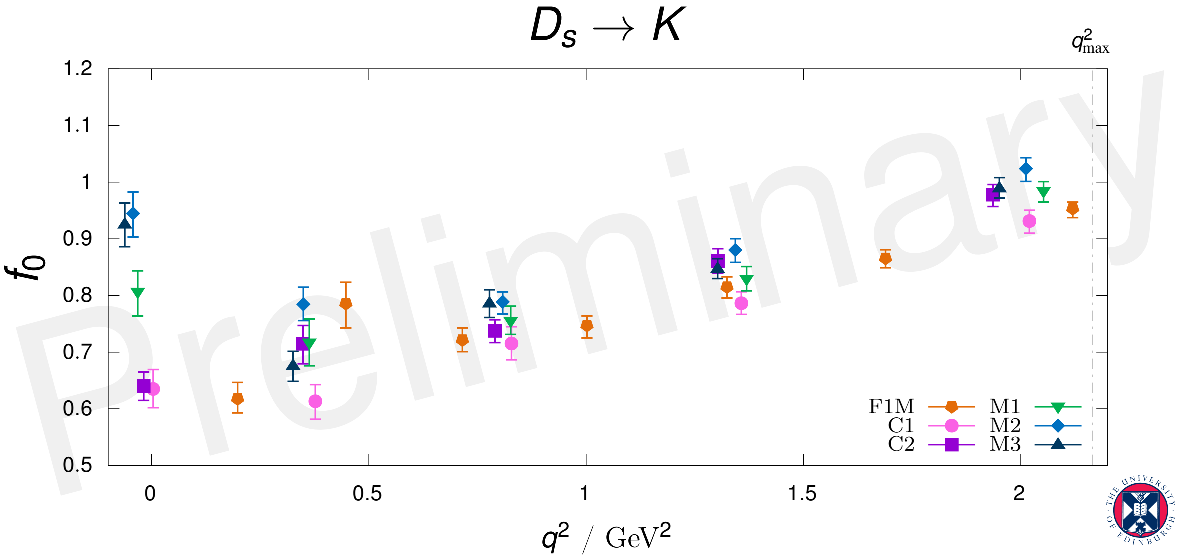


Good signal over the physically allowed kinematic range



- Higher momenta noiser
- Data usable
- For this talk
  - Manually select plateau
  - Fit to a constant
  - Statistical error only

We see a signal over the entire physical q^2 range on all ensembles


$D_s \to K$
q^2_{max}
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
f_0
0
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q^2 / GeV^2
F1M
C1
C2
M1
M2
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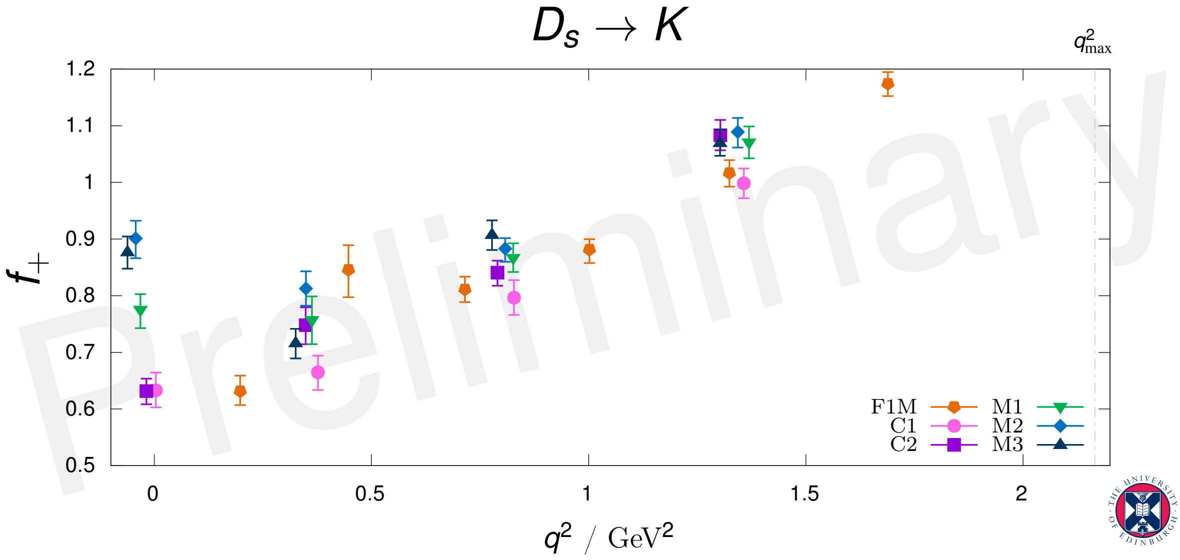
Preliminary



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Next steps

Analysis and fitting strategies

- Model excited-state behaviour
- Simultaneous fits to multiple ΔT
- Include other operators (wall source)
- Fully non-perturbative RI/SMOM determination Z_V[3] [4]
- Parameterise pion mass dependence, lattice artifacts and q^2-dependence over whole kinematic range
- HMχPT and global fit
- Possible dispersive bounds analysis
- Compare alternative ratios to extract form factors

This is the focus

Data production

- Short term: increase statistics on existing ensembles for better signal
- Longer term: further non-physical and physical m_π ensembles

Summary and outlook

Achieved to date

- 6 ensembles, three lattice spacings and several pion masses down to 230 MeV
- Usable data across the full q^2 range on all ensembles
- First preview of q^2-dependence of form factors
- Work in progress
  - Have not (yet) performed complete analysis
  - Do not (yet) quote results on form factors

Outlook

- Detailed analysis and global fit
- Treat discretisation and chiral extrapolation effects
- Fully non-perturbative renormalisation

This work used the DiRAC Extreme Scaling HPC Service (https://www.dirac.ac.uk)

Data produced using Grid [5] and Hadrons [6]

References I

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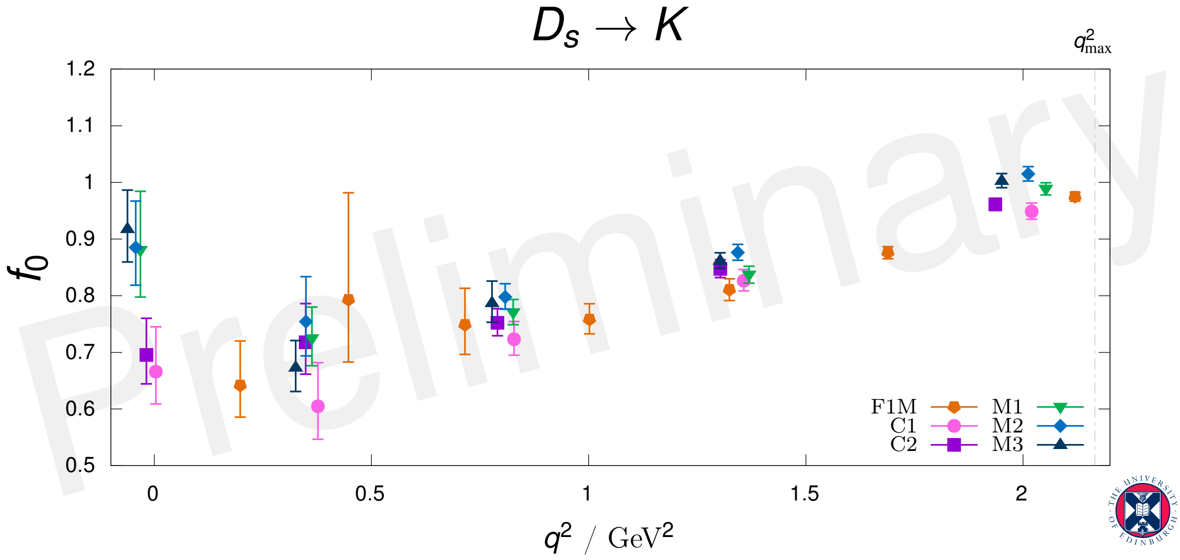
Ensemble properties

ID	L/a	T/a	a^{-1} / GeV	m_π / MeV	$m_\pi L$	Configs
C1	24	64	1.8	340	4.6	160
C2	24	64	1.8	430	5.8	128
M1	32	64	2.4	300	4.1	128
M2	32	64	2.4	360	4.8	128
M3	32	64	2.4	410	5.5	120
F1M	48	96	2.7	230	4.1	72

ΔT in form factor plots approximately

	a^{-1} / GeV	$\Delta T/a$	ΔT / fm
C	1.8	20	2.2
M	2.4	28	2.3
F	2.7	32	2.3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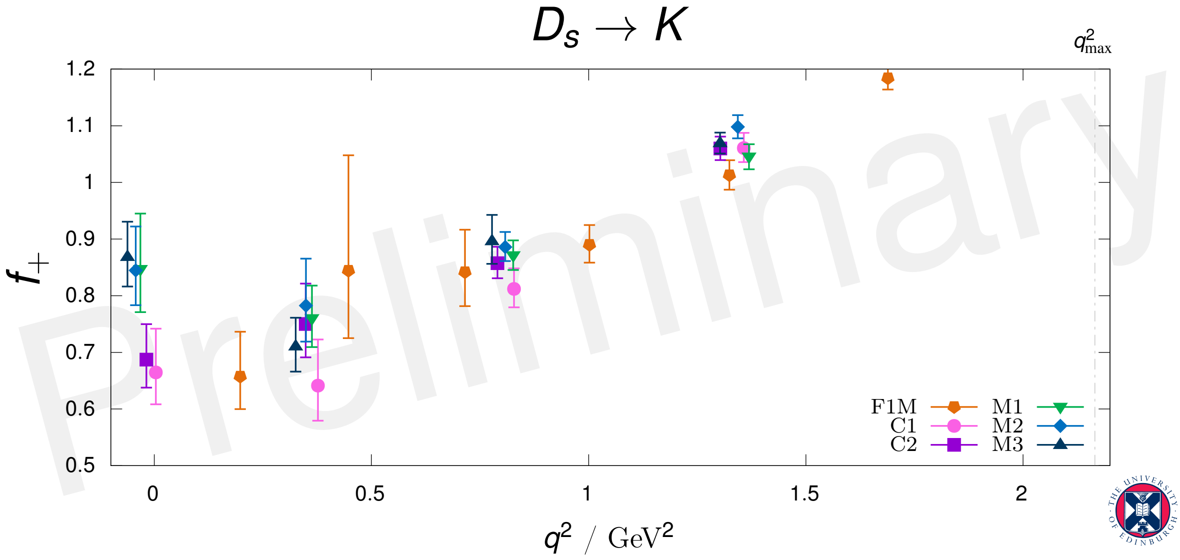
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M2
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Preliminary

$$R_1^{\mu}\left(t_i, t, t_f, \mathbf{p}_i, \mathbf{p}_f\right) = 2\sqrt{E_i E_f}\sqrt{\frac{C^{\mu}_{P_i P_f}\left(t_i, t, t_f, \mathbf{p}_i, \mathbf{p}_f\right) C^{\mu}_{P_f P_i}\left(t_i, t, t_f, \mathbf{p}_f, \mathbf{p}_i\right)}{\tilde{C}_{P_i}\left(t_f - t_i, \mathbf{p}_i\right)\tilde{C}_{P_f}\left(t_f - t_i, \mathbf{p}_f\right)}} \tag{1}$$

i.e.

$$R_1 = \cancel{\sqrt{4E_i E_f}}\sqrt{\frac{Z_V \frac{Z_i^* Z_f}{\cancel{4E_i E_f}} Z_V \frac{Z_f^* Z_i}{4E_f E_i}}{\frac{|Z_i|^2}{2E_i}\frac{|Z_f|^2}{2E_f}}}\left\langle P_f\left(\mathbf{p}_f\right)\right| V_{\mu}\left(\mathbf{q}^2\right)\left|P_i\left(\mathbf{p}_i\right)\right\rangle \tag{2}$$

$$\sqrt{\frac{\mathrm{e}^{-E_i(t-t_i)}\,\mathrm{e}^{-E_f(t_f-t)}\,\mathrm{e}^{-E_f(t-t_i)}\,\mathrm{e}^{-E_i(t_f-t)}}{\mathrm{e}^{-E_i(t_f-t_i)}\,\mathrm{e}^{-E_f(t_f-t_i)}}} \tag{3}$$

$$= Z_V \left\langle P_f\left(\mathbf{p}_f\right)\right| V_{\mu}\left(\mathbf{q}^2\right)\left|P_i\left(\mathbf{p}_i\right)\right\rangle \sqrt{\frac{\mathrm{e}^{-E_f(t_f-t_i)}\,\mathrm{e}^{-E_i(t_f-t_i)}}{\mathrm{e}^{-E_i(t_f-t_i)}\,\mathrm{e}^{-E_f(t_f-t_i)}}} \tag{4}$$

i.e. R_1 yields the renormalised matrix element when built with renormalised 3-pt correlators

$$R_1 = Z_V \left\langle P_f\left(\mathbf{p}_f\right)\right| V_{\mu}\left(\mathbf{q}\right)\left|P_i\left(\mathbf{p}_i\right)\right\rangle \tag{5}$$

More usefully, we can construct the ratio from bare correlators and multiply by Z_V at the end