Estimating Excited-States Contamination of $B \to \pi$ Form Factors Using Heavy Meson Chiral Perturbation Theory

Alexander Broll

in collaboration with

Oliver Bär and Rainer Sommer

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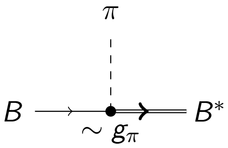
Heavy Meson Chiral Perturbation Theory

- **Low energy** effective theory for the interaction of **heavy** (D, B) and **light** (π, K) mesons



- **Chiral symmetry**: $SU(N_f)_L \times SU(N_f)_R$

- **Static Limit**: Heavy Quark Spin Symmetry, broken by $1/m_B$ corrections

$$H = \frac{1 - \mathrm{i}\not{v}}{2}(\mathrm{i}\gamma_\mu B^*_\mu + \mathrm{i}\gamma_5 B)$$

Interpolating Fields

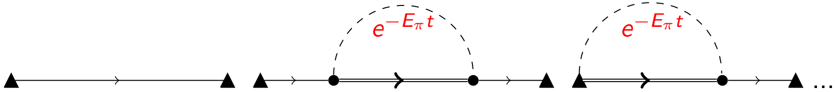
- HMChPT analogues of $J_\Gamma = \bar{q}_f \Gamma Q$ needed, **heavy-light currents**
- Order by order in **chiral expansion** and $\mathbf{1/m_B}$
- Form dictated by: χ sym., heavy quark spin sym.
- **One** low energy constant (LEC) at leading order ($\hat{f} = f_B \sqrt{m_B}$), **two** at NLO ($\beta_1\,,\beta_2$) in the chiral expansion

$$\mathcal{B} = \hat{f}\left(B\left(1 - \frac{1}{f^2}\pi^2\right) + \frac{\mathrm{i}\beta_1}{f} B_k^* \partial_k \pi + \frac{\beta_2}{f^2} B \partial_4 \pi \pi + \mathcal{O}(p^2, \pi^3)\right)$$

- Expectation: $\beta_i = \mathcal{O}\left(\Lambda_\chi^{-1}\right)$
- Smeared Fields: $\tilde{\beta}_i$

Excited States

- Consider B meson 2-point function



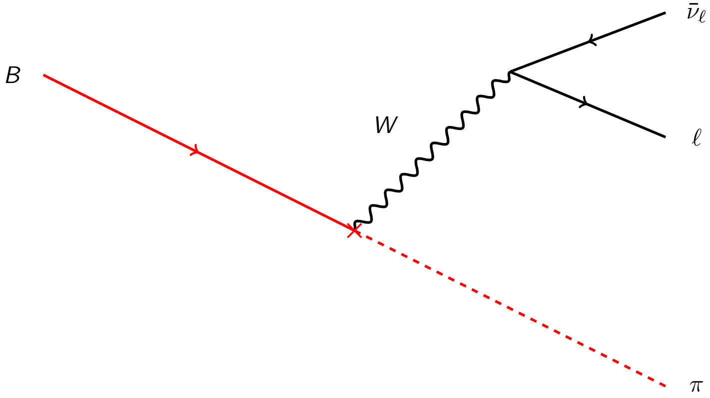
- Result can be written as

$$C(t) = C^B(t) + C^{B^*\pi}(t) + ... = C^B(t)\left(1 + \frac{C^{B^*\pi}(t) + ...}{C^B(t)}\right)$$
$$= C^B(t)\left(1 + \boldsymbol{\delta C(t)}\right)$$

- Results reliable for $t \gtrsim 1.3\,\mathrm{fm}$

Semileptonic Decays of B Mesons

$$B \to \pi \ell \bar{\nu}_\ell$$



Semileptonic Decays of B Mesons

- Parametrisation in Heavy Quark Effective Theory, rest frame of B

$$(2m_B)^{-\frac{1}{2}} \langle \pi(p)|V_4|B(0)\rangle = h_{\|}(E_\pi)$$
$$(2m_B)^{-\frac{1}{2}} \langle \pi(p)|V_k|B(0)\rangle = p_k h_{\perp}(E_\pi)$$

- Important for determination of $|V_{ub}|$
- $h_\perp$ dominant

Extraction of Matrix Elements

- Correlator

$$C_{3,\mu}(t,t_v,\vec{p}) = \int \mathrm{d}^3\vec{x}\,\mathrm{d}^3\vec{z}\,e^{-\mathrm{i}\vec{p}(\vec{x}-\vec{z})}\left\langle \Pi^+(t,\vec{x})V_\mu^-(t_v,\vec{z})\bar{\mathcal{B}}^{0\dagger}(0,\vec{0})\right\rangle$$

- "Effective Form Factor"

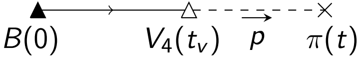
$$h_\mu(t,t_v,\vec{p}) = \frac{2\sqrt{E_\pi}\,C_{3,\mu}(t,t_v,\vec{p})}{\sqrt{C_2^B(2t_v)C_2^\Pi(2t-2t_v)}} = \left\langle \pi^+ \middle| V_\mu^- \middle| \bar{B}^0 \right\rangle + \ldots$$

- Excited states

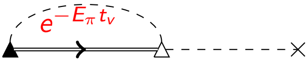
$$\delta h_\mu(t,t_v,\vec{p}) = \delta C_{3,\mu}(t,t_v,\vec{p}) - \frac{1}{2}\delta C_2^B(2t_v)$$

$h_\parallel \propto \langle \pi | V_4 | B \rangle$

- The leading order diagram:



- Example for a $B^*\pi$ excited state:



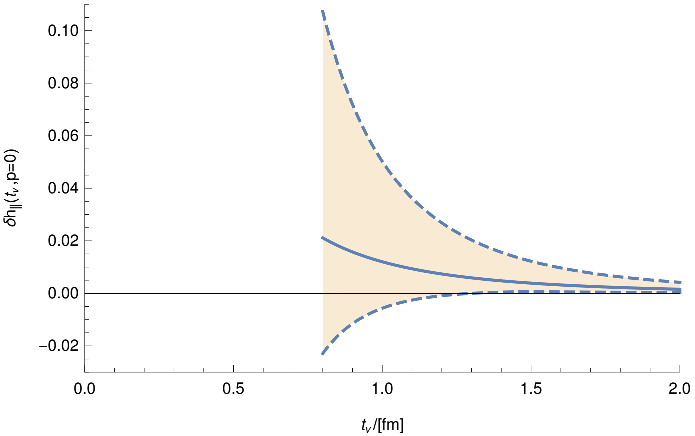
- General form of the $B^*\pi$ excited states:

$$\delta C_{3,4} = \sum_{\vec{l}} \frac{1}{f^2 L^3 E_\pi(\vec{l})} c(\vec{l}, \vec{p}, \beta_i) e^{-E_\pi(\vec{l}) t_v}$$

- No contributions from the NLO Lagrangian
- NLO LECs: estimate $\beta_i \in [-\Lambda_\chi^{-1}, \Lambda_\chi^{-1}]$

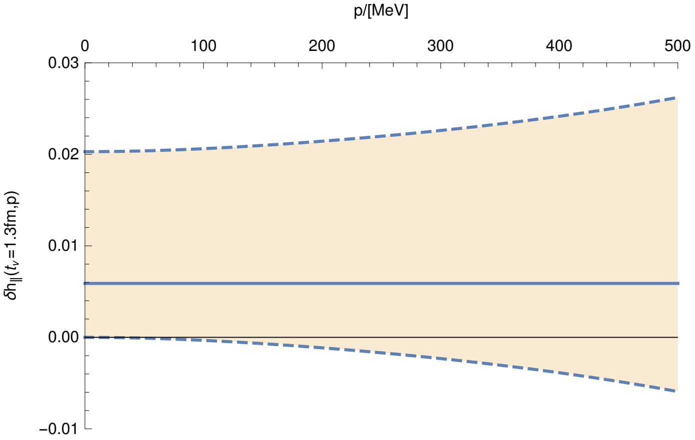
$h_\parallel$

Excited states as a function of **time** t_v for $|\vec{p}| = 0$



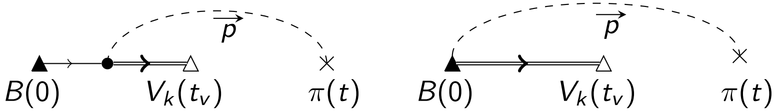
$h_{\parallel}$

Excited states as a function of **pion momentum** $|\vec{p}|$ for $t_v = 1.3\,\mathrm{fm}$



$h_\perp \propto \langle \pi | V_k | B \rangle$

- Excited states which are not "volume suppressed"

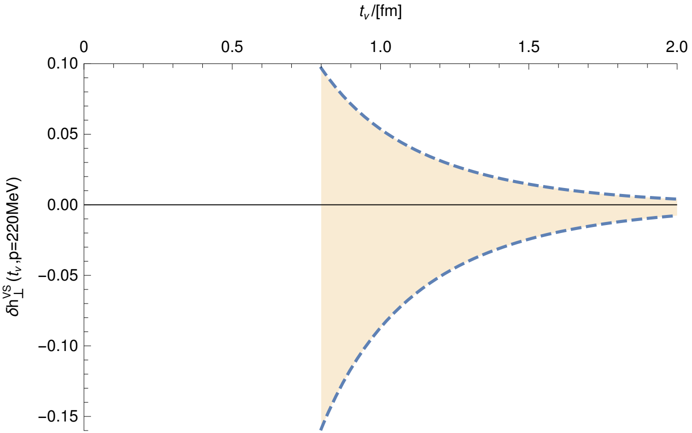


$$\delta C_{3,k} = -\frac{1+\tilde{\beta}_1 E_\pi(\vec{p})/g}{1-\beta_1 E_\pi(\vec{p})/g} e^{-E_\pi(\vec{p})t_v} + \sum_{\vec{l}} \frac{1}{f^2 L^3 E_\pi(\vec{l})} d(\vec{l},\vec{p},\beta_i,\gamma) e^{-E_\pi(\vec{l})t_v}$$

- Dominant contribution to excited states

$h_\perp$

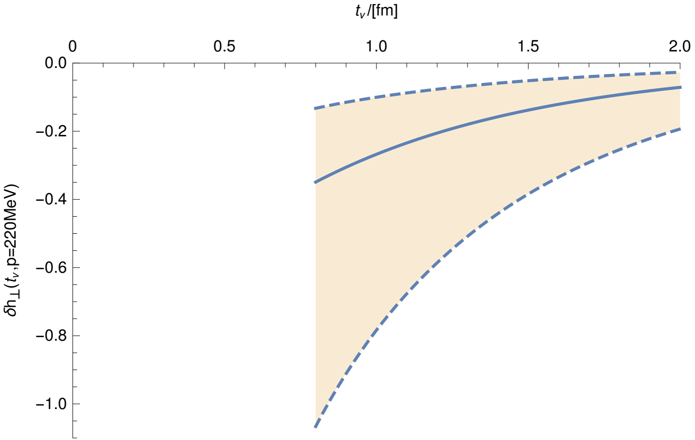
Volume suppressed contributions are small...



$h_\perp$

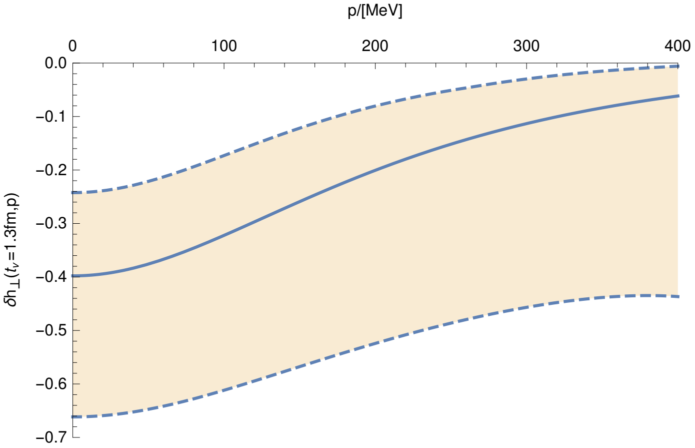
... compared to the full result

Excited states as a function of **time** t_v for $|\vec{p}| = 220\,\mathrm{MeV}$



$h_\perp$

Excited states as a function of **pion momentum** $|\vec{p}|$ for $t_v = 1.3\,\mathrm{fm}$



$$\delta h_\perp = -\frac{1 + \tilde{\beta}_1 E_\pi(\vec{p})/g}{1 - \beta_1 E_\pi(\vec{p})/g} e^{-E_\pi(\vec{p}) t_v} + \dots$$

How to determine β_2?

- Consider the correlator:

$$C_{hl}(t, t_A, \vec{q}) = \int \mathrm{d}^3\vec{x}\,\mathrm{d}^3\vec{z}\,e^{\mathrm{i}\vec{q}\vec{x}} \left\langle A_4^{ll,+}(\vec{x}, t) A_k^{hl,-}(\vec{z}, t_A) \bar{\mathcal{B}}_k^{0*\dagger}(\vec{0}, 0) \right\rangle$$

- For $t > t_A > 0$, define the ratio:

$$R = \frac{C_{hl}}{C_2^B} \sim (1 - \beta_2 E_\pi(\vec{q}))\, e^{-E_\pi(\vec{q})(t - t_A)}$$

Conclusions

- First analytic results for the $B^*\pi$ excited states of $B \to \pi$ vector form factors in the static limit
- Of the three unknown LECs, one can in principle determine β_1 and β_2 (interpolating fields) from 3-point functions, γ (NLO Lagrangian/light-light axial current) requires presumably a 4-point function
- Nonetheless, our estimates imply that the excited states for $\mathbf{h}_{\parallel}$ are of the order of a **few percent**, whereas for $\mathbf{h}_{\perp}$ we find **considerable deviations**, at leading order $-e^{-E_\pi(\vec{p})t_v}$ stemming from the diagram