

# A comparison of Wilson and twisted mass valence quarks for charmed semileptonic form factors

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# Overview



## Introduction

- Collaboration(s)
- Common ensembles
- The twisted project
- The Wilson step-scaling project
- Common contraction strategy

## Checks on some first results

- The dispersion relation
- Vector Ward identity

## Extracting matrix elements

- Combined fit
- Model average
- Time correlations
- Model averaging results: an example
- VWI and Form Factor decomposition
- Results for  
 $D(p_D = 0) \rightarrow \pi(p_\pi = 0)$
- A quick look at the leptomics

# Collaboration(s)

Both projects within the context of ALPHA, CLS

## Twisted mass

JF, Andrea Bussone, Alessandro Conigli,  
Gregorio Herdoíza, Carlos Pena, Jose Ángel  
Romero, Alejandro Saez, Javier Ugarrio

### A. Conigli 15:40 in SM parameters

[1711.06017] [1812.01474] [1812.05458]  
[1903.00286] [1911.02412] [2112.00666]  
[2112.03047] [PoS LATTICE2021 (2022) 258]

## Wilson step-scaling

JF, Alessandro Conigli, Patrick Fritzsch,  
Antoine Gérardin, Jochen Heitger, Gregorio  
Herdoíza, Carlos Pena, Hubert Simma,  
Rainer Sommer



# Common ensembles

CLS  $N_f = 2 + 1$  configurations

- > non-perturbative  $O(a)$  improvement ( $c_{sw}$  kept everywhere)
- > OpenBC: Lowers the barrier between topological sectors  
Finer ensembles are accessible with good sampling

id	$\beta$	$a^{-1}$ [fm]	$N_s$	$N_t$	$m_\pi$ [MeV]	$m_K$ [MeV]	$m_\pi L$	$a\mu_c$
H101	3.40	0.086	32	96	420	420	5.8	0.22
H400	3.46	0.075	32	96	420	420	5.2	0.21
N202	3.55	0.064	48	128	420	420	6.5	0.18
N300	3.70	0.050	48	128	420	420	5.1	0.14



# The twisted project

## Goal

Get rid of  $O(am_c)$  terms

## Implementation

- > Partial quenching, only the mass parameters are changed
- > Tuned to maximal twist for each ensemble
- > Meson masses matched to unitary theory

Mostly limited to charm for now.

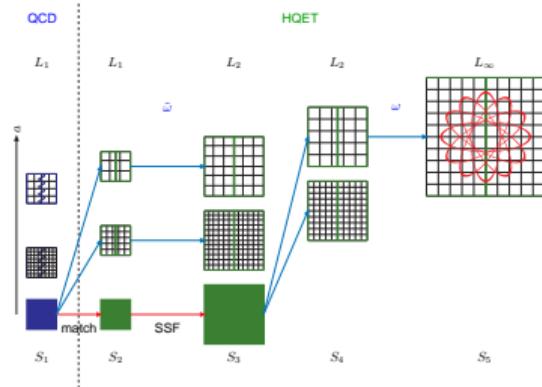
## Theoretical benefits

- > Automatic  $O(a)$  improvement (no  $am_h$ , residual sea effects)
- > Renormalisation factors already known
- > Renormalisation simplified
- > Tuning to maximal twist easier *after* gauge generation



# The Wilson step-scaling project

- > Ratios are formed before continuum limit
- > Both HQET and relativistic
- > Charm sector up to static limit
- > Focused on B
- > Needs many improvement parameters already part of the ALPHA program

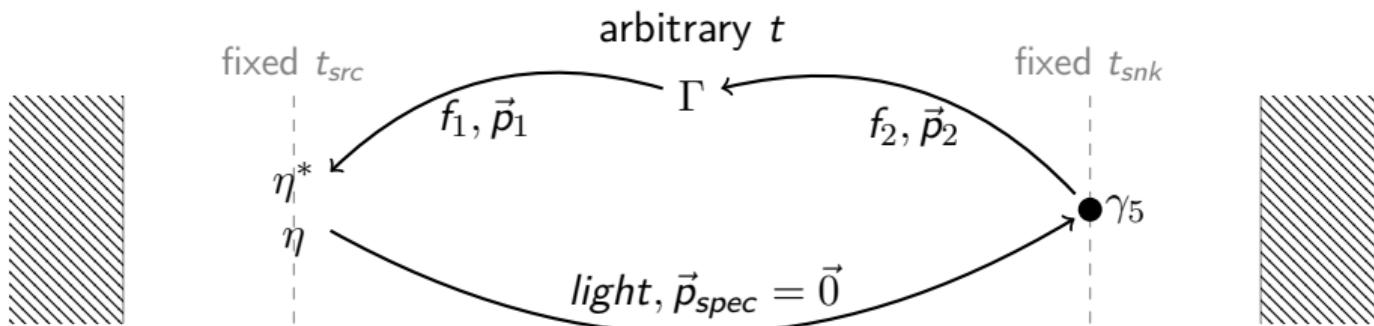


**Here:** only the relativistic part of the  $L_\infty$  computation!



# Common contraction strategy

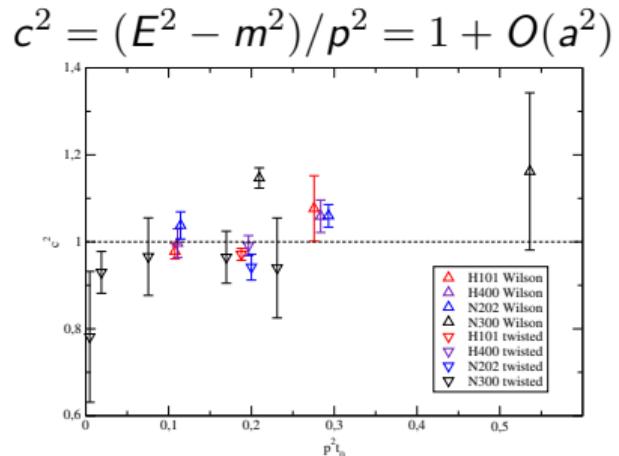
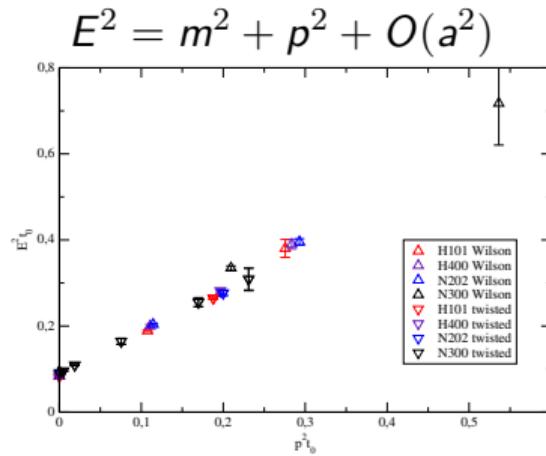
- > One-hand trick and sequentials
- > A few choices of  $t_{src}$  and  $t_{snk}$
- > Spectator always light
- > Flavours ( $f_1, f_2$ ) and momenta ( $\vec{p}_1, \vec{p}_2$ ) depend on project



# Checks on some first results

# The dispersion relation

Both discretisations compatible with continuum dispersion relation on pion



# Vector Ward identity

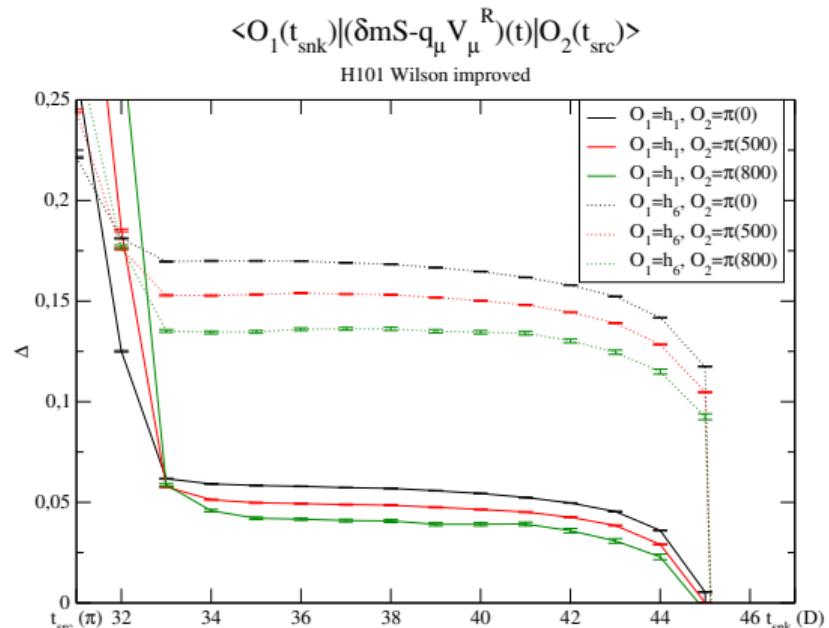
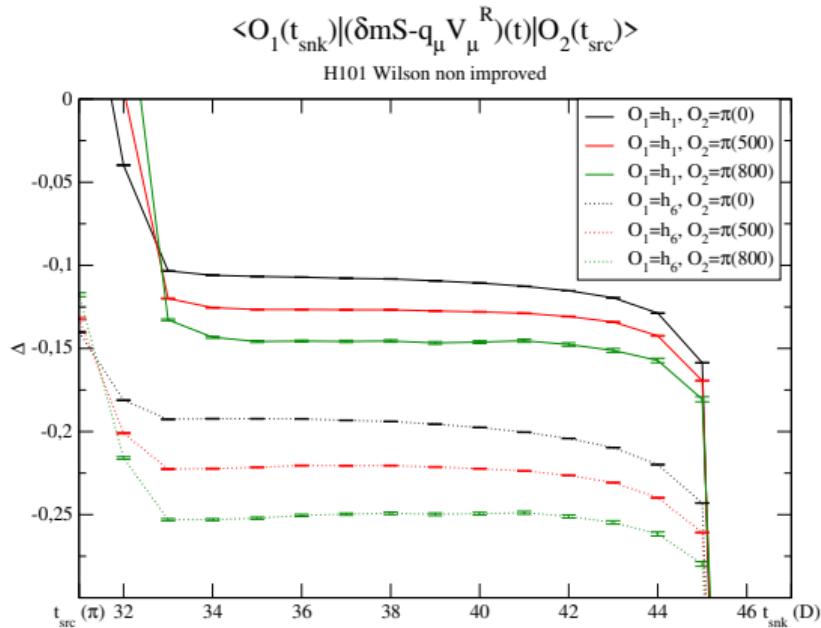
Define, similarly for both discretisations:

$$\Delta(t) \equiv \delta m C_S^{3pt}(t) - Z_V^{\text{imp}} \left( \tilde{\partial}_t C_{V_0}^{3pt}(t) + \tilde{q}_i C_{V_i}^{3pt}(t) \right) = 0 + O(a^2)$$

- > Can be checked to very high precision
- > Combination  $\delta m C_S^{3pt}$  doesn't need renormalisation (bare mass)
- >  $\delta m C_S^{3pt}$  already  $O(a)$ -improved [[hep-lat/0511014](#)]
- >  $c_V$  contribution cancels out algebraically in VWI
- >  $b_V$  important in the Wilson case:  $Z_V^{\text{imp}} = Z_V(1 + \textcolor{red}{b_V am_q} + \bar{b}_V \text{Tr}[aM])$



# Vector Ward identity(2)

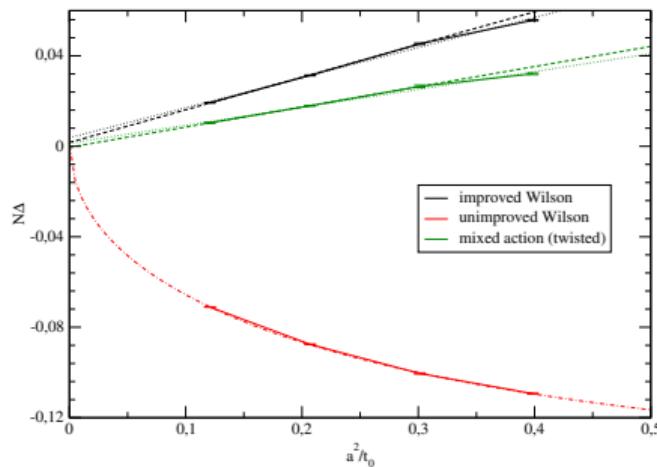


# Vector Ward identity(3)

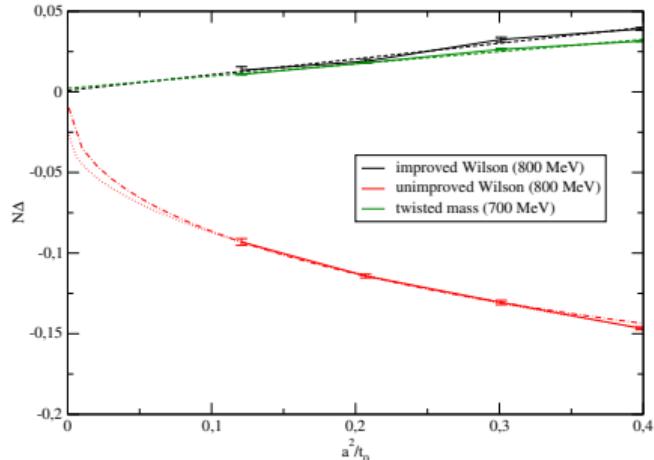
Keep middle time slice and normalise with

$$1/N^2 = \left( \delta m C_S^{3pt}(t) \right)^2 + \left( Z_V^{\text{imp}} \tilde{\partial}_t C_{V_0}^{3pt}(t) \right)^2 + \sum_i \left( Z_V^{\text{imp}} \tilde{q}_i C_{V_i}^{3pt}(t) \right)^2$$

Relative Ward identity violation at  $p_D=p_\pi=0$



Relative Ward identity violation for D to  $\pi$  at large momentum



# Extracting matrix elements



# Combined fit

One needs a way to reliably estimate excited state contaminations  
⇒ combined multi-exponential fit:

$$\begin{aligned} C_{\Gamma}^{3pt}(t, t_{snk}) &= \sum_{i=0}^{N_{\pi,max}} \sum_{j=0}^{N_{D,max}} p_{ij} e^{-E_i^{\pi}(t-t_{src})} e^{-E_j^D(t-t_{snk})} \\ C_{\pi}(t) &= \sum_{i=0}^{N_{\pi,max}} \alpha_i e^{-E_i^{\pi}(t-t_{src})} \\ C_D(t) &= \sum_{j=0}^{N_{D,max}} \beta_j e^{-E_j^D(t-t_{snk})} \end{aligned}$$

Typical case:  $N_{\pi,max} = N_{D,max} = 2$       ⇒ 12 parameters



# Model average

Many hyper-parameters:  $N_{\pi,\max}, N_{D,\max}, t_{\Gamma,\min}, t_{\Gamma,\max}, t_{\pi,\min}, t_{D,\max}, t_{D,\min}, t_{D,\max}$   
We need a way to *reliably* set them as well

## pseudo-Bayesian Model Average

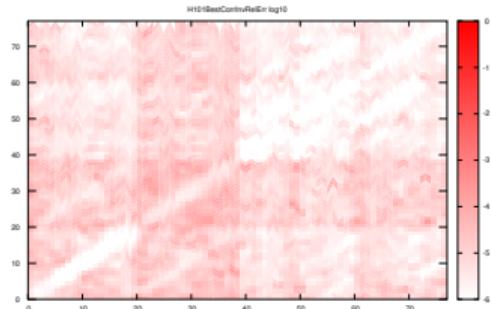
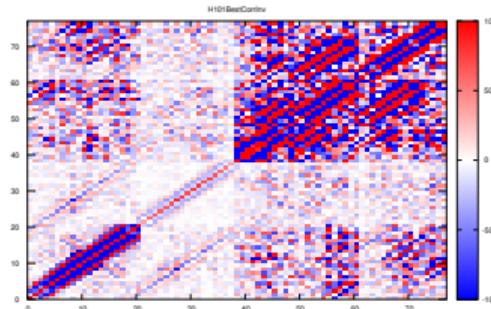
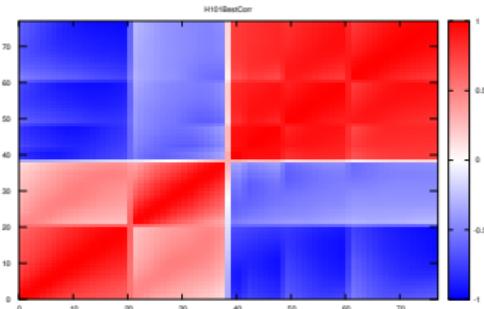
- >  $AIC = \chi^2 + 2k$  (cuts are parameters), as in [2008.01069]
- > We loop over all values of the hyper-parameters  
( $O(10k)$  models, times 2000 bootstraps, but models with negligible weight dropped)
- > bootstrap implementation:  $P(X_b = x_b^i) \propto e^{-AIC/2}$
- > Model selection adds a contribution to the bootstrap error

⇒ only works if we can build a decent  $\chi^2$ , can we?

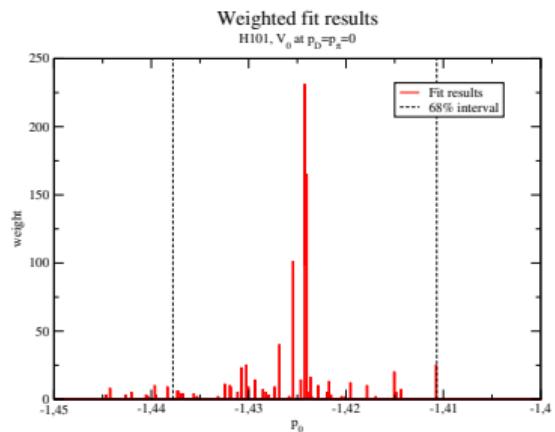
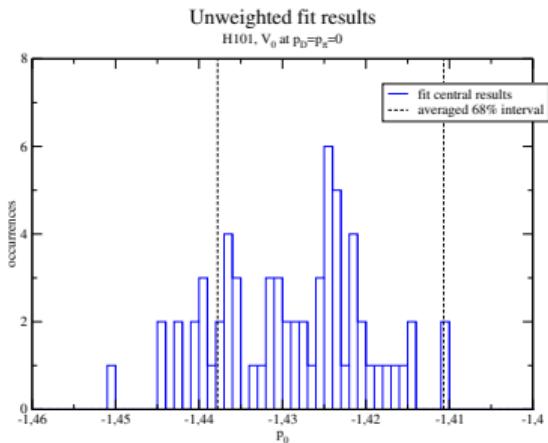
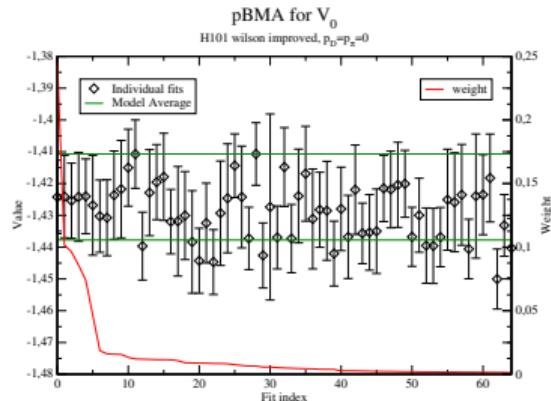


# Time correlations

Data is very strongly correlated, but good signal to build a *correlated  $\chi^2$*



# Model averaging results: an example



# VWI and Form Factor decomposition

The WI on 3pt functions can be promoted to a WI on matrix elements:

$$\Delta_{WI} = \delta m \langle S \rangle - \tilde{q}_\mu \langle \hat{V}_\mu \rangle$$

which is already encoded in the usual FF decomposition:

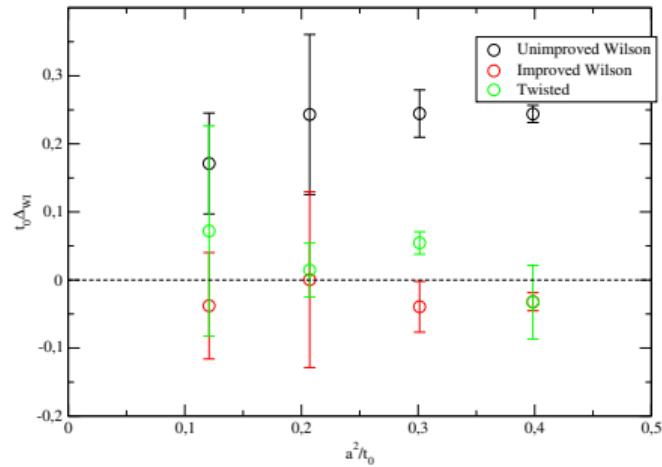
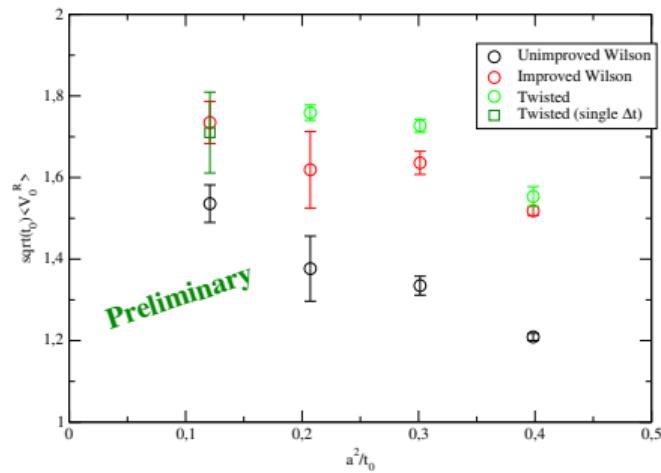
$$\begin{aligned}\delta m \langle S \rangle &= (M_D^2 - M_P^2) f_0(q^2) \\ \langle \hat{V}_\mu \rangle &= \left[ P_\mu - q_\mu \frac{M_D^2 - M_P^2}{q^2} \right] f_+(q^2) + q_\mu \frac{M_D^2 - M_P^2}{q^2} f_0(q^2)\end{aligned}$$



# Results for $D(p_D = 0) \rightarrow \pi(p_\pi = 0)$

Particular kinematics which is exactly common to both projects

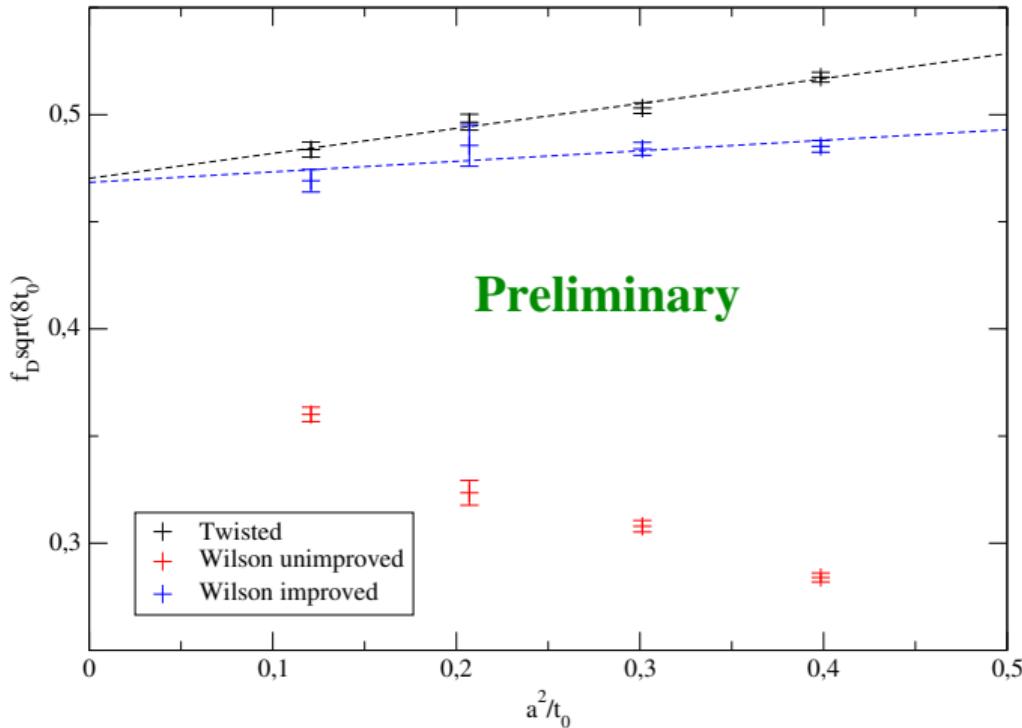
Here WI essentially checks whether FF from  $\langle S \rangle$  is compatible with FF from  $\langle V_0 \rangle$



# A quick look at the leptomics



# A quick look at the leptronics (From A.Conigli)



# Conclusion

- > We presented **two studies** of heavy-light decays with Wilson and Wtm
- > We showed how a **Ward identity** can be used to assess scaling violations to **high precision**

The scaling of this quantity is consistent with expectations

- > We presented a set of techniques to **extract matrix elements**
- > The two discretisations **agree on finest ensemble**, but full statistics not yet available
- > A fifth ensemble at 0.039 fm will soon allow this to be tested further.



# Thank you!

## Contact

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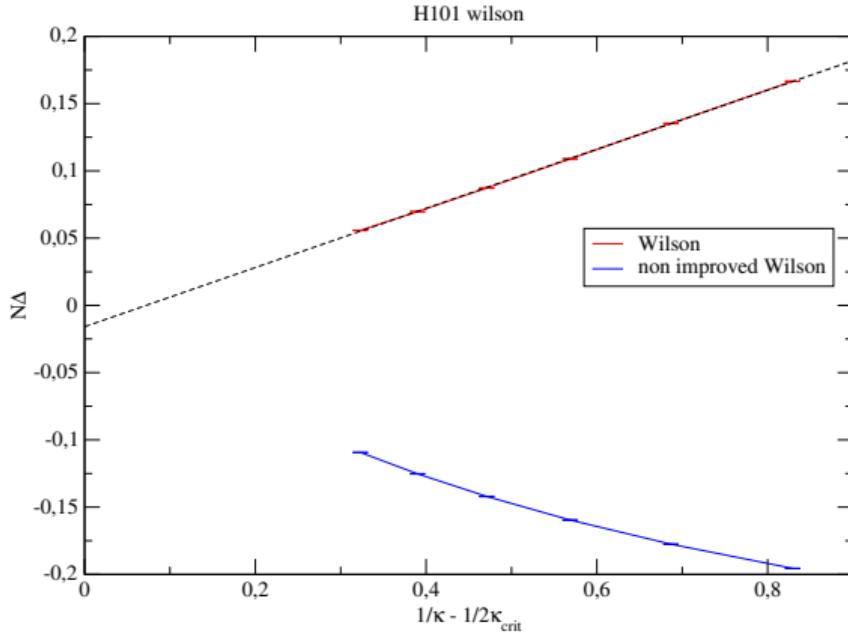


# Appendix



# Heavier masses

## Mass-dependence of Vector Ward Identity violation



Looks like large  $O([a\Lambda_{QCD}][am_h])$  effect

# Full time correlations

