

Leading isospin breaking effects in $\Gamma(K_{\mu 2})/\Gamma(\pi_{\mu 2})$ with close to physical chiral fermions

Matteo Di Carlo

10th August 2022



THE UNIVERSITY
of EDINBURGH



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Outline of the talk

- QED corrections to leptonic decays on the lattice
- The RBC/UKQCD way to $\delta R_{K\pi}$
- First (preliminary) lattice results with chiral and close-to-physical fermions
- Comparison with published calculation by RM123+Soton collaboration
- Final comments on $|V_{us}/V_{ud}|$

The goal: testing the Standard Model

Indirect searches of new physics using CKM matrix unitarity constraints

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

in the Standard Model:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$



The goal: testing the Standard Model

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$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Matrix elements can be extracted e.g. from **leptonic** and **semileptonic** decays of mesons

$$\underbrace{\frac{\Gamma[K \rightarrow l\nu_l(\gamma)]}{\Gamma[\pi \rightarrow l\nu_l(\gamma)]}}_{\text{experiments}} \propto \underbrace{\left| \frac{V_{us}}{V_{ud}} \right|^2}_{\text{experiments}} \underbrace{\left(\frac{f_K}{f_\pi} \right)^2}_{\text{QCD}} \quad \underbrace{\Gamma[K \rightarrow \pi l\nu_l(\gamma)]}_{\text{experiments}} \propto \underbrace{|V_{us}|^2}_{\text{experiments}} \underbrace{|f_+^{K\pi}(0)|^2}_{\text{QCD}}$$



FLAG 2021
Flavour Lattice Averaging Group

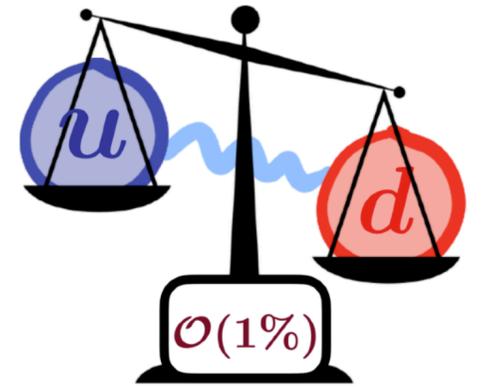
f_K/f_π and $f_+^{K\pi}(0)$ determined from lattice QCD with sub percent precision!

Isospin-breaking effects on the lattice

Current level of precision requires the inclusion of isospin breaking (IB) corrections

- strong effects $[m_u - m_d]_{\text{QCD}} \neq 0$
- electromagnetic effects $\alpha \neq 0$

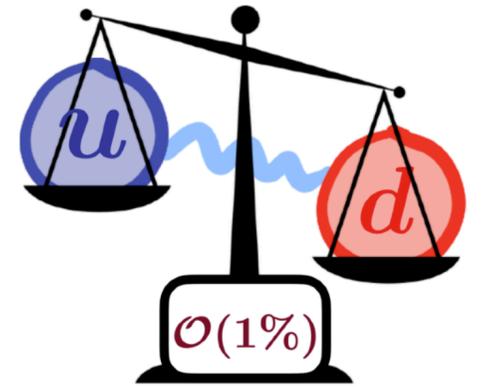
Different ways to include them on the lattice...



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Different ways to include them on the lattice...

In this calculation:

■ RM123 perturbative approach

$$\begin{aligned} \langle \mathcal{O} \rangle &= \int \mathcal{D}\Phi \mathcal{O} e^{-S_{\text{iso}} - \Delta S} \\ &= \langle \mathcal{O} \rangle_{\text{iso}} + \langle \Delta S \mathcal{O} \rangle_{\text{iso}} + \dots \end{aligned}$$



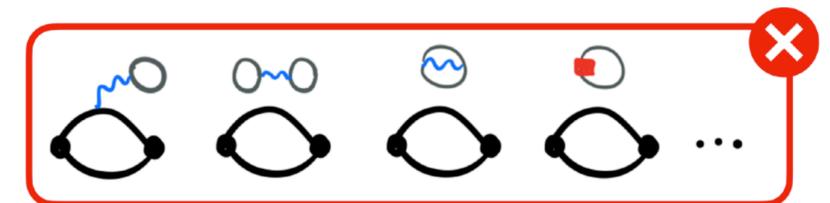
■ QED_L photon prescription

$$\Delta_{\mu\nu}^{\gamma}(x) = \frac{1}{V} \sum_{k_0} \sum_{\mathbf{k} \neq 0} \Delta_{\mu\nu}^{\gamma}(k) e^{ik \cdot x}$$

& power-like finite volume effects

■ "electro-quenched" QED

neutral sea quarks



Decay rate at $\mathcal{O}(\alpha)$

N. Carrasco et al., PRD 91 (2015)
V. Lubicz et al., PRD 95 (2017)
D. Giusti et al., PRL 120 (2018)
MDC et al., PRD 100 (2019)

The RM123+Soton (original) recipe

$$\Gamma(P_{\ell 2}) = \lim_{L \rightarrow \infty} \left\{ \text{on the lattice} \right\} + \lim_{m_\gamma \rightarrow 0} \left\{ \text{in perturbation theory} \right\}$$

The equation shows the decomposition of the decay rate $\Gamma(P_{\ell 2})$ into two parts. The first part is a limit as $L \rightarrow \infty$ of a difference between two lattice diagrams. The second part is a limit as $m_\gamma \rightarrow 0$ of a sum of two perturbation theory diagrams. The lattice diagrams show a fermion loop (black) with a photon (blue wavy) and a lepton (green) line. The perturbation theory diagrams show a tree-level process with a photon (blue wavy) and a lepton (green) line.

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on the lattice in perturbation theory

Possible extensions:

- improving finite volume scaling (see next talk by N.Hermansson-Truedsson)
- including structure-dependent real photon emission

MDC et al., PRD 105 (2022)

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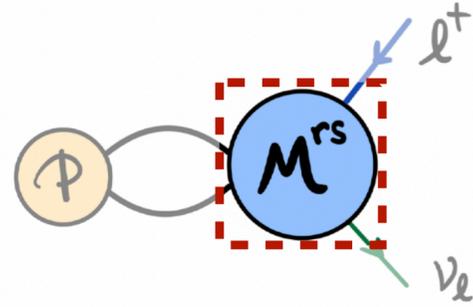
Virtual decay rate (on the lattice)

$$\Gamma(P_{\ell 2}) = \Gamma_P^{\text{tree}} (1 + \delta R_P) \quad \xrightarrow{\text{PDG convention}} \quad \Gamma_P^{\text{tree}} = \frac{G_F^2}{8\pi} m_\ell^2 \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2 m_P [f_{P,0}]^2 \quad \xrightarrow{\quad} \quad \delta R_P = 2 \left(\frac{\delta \mathcal{A}_P}{\mathcal{A}_{P,0}} - \frac{\delta m_P}{m_{P,0}} + \frac{\delta \mathcal{Z}}{\mathcal{Z}_0} \right)$$

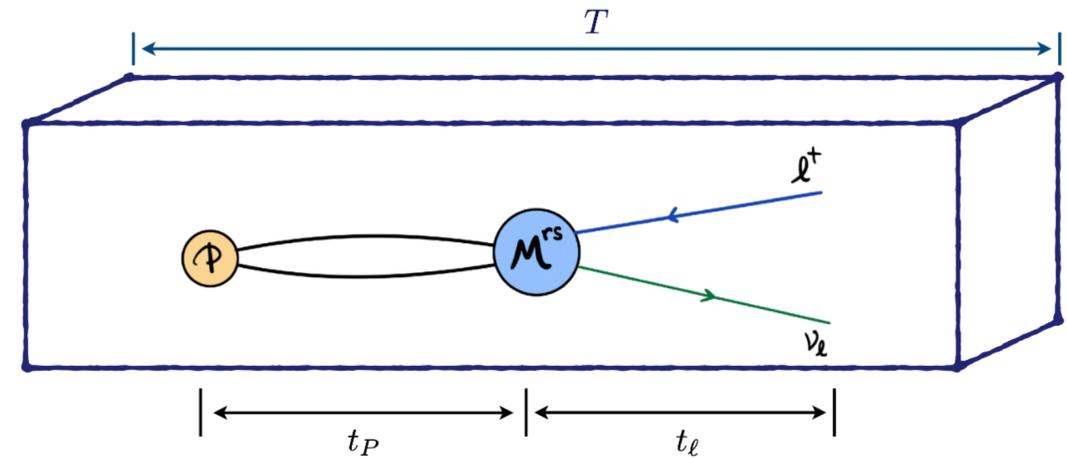
Our target:
$$\delta R_{K\pi} = 2 \left(\frac{\delta \mathcal{A}_K}{\mathcal{A}_{K,0}} - \frac{\delta m_K}{m_{K,0}} \right) - 2 \left(\frac{\delta \mathcal{A}_\pi}{\mathcal{A}_{\pi,0}} - \frac{\delta m_\pi}{m_{\pi,0}} \right)$$

From correlators to matrix elements

Our goal:

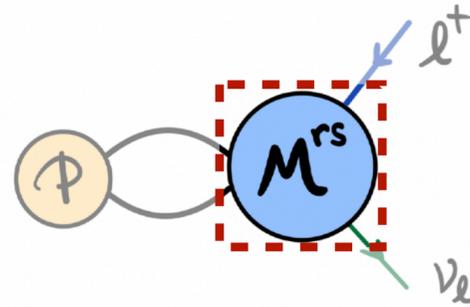


How we realise it:

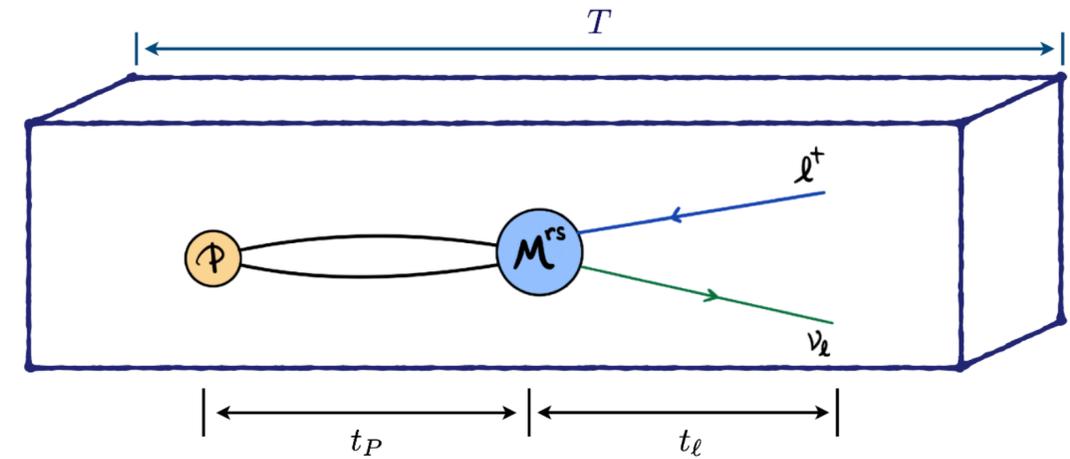


From correlators to matrix elements

Our goal:



How we realise it:



Tree-level decay amplitude: $|\mathcal{M}_0(\mathbf{p}_\ell)|^2 = |\mathcal{A}_{P,0}|^2 |\mathcal{L}_0(\mathbf{p}_\ell)|^2$ $\mathcal{A}_{P,0} = \langle 0|A^0|P\rangle_0 = im_{P,0} [f_{P,0}]$

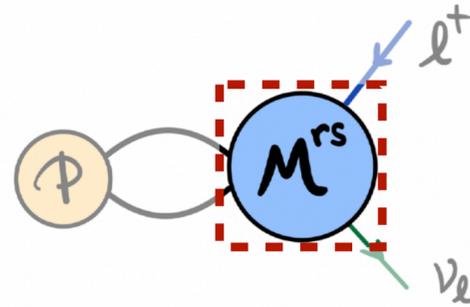
$$\text{Diagram } \phi_0 \text{ loop } A^0 = \langle 0|A^0(0)\phi^\dagger(-t)|0\rangle \rightarrow \frac{Z_{P,0}\mathcal{A}_{P,0}}{2m_{P,0}} e^{-m_{P,0}t}$$

$$Z_{P,0} = \langle P, \mathbf{p} = \mathbf{0}|\phi^\dagger|0\rangle_0$$

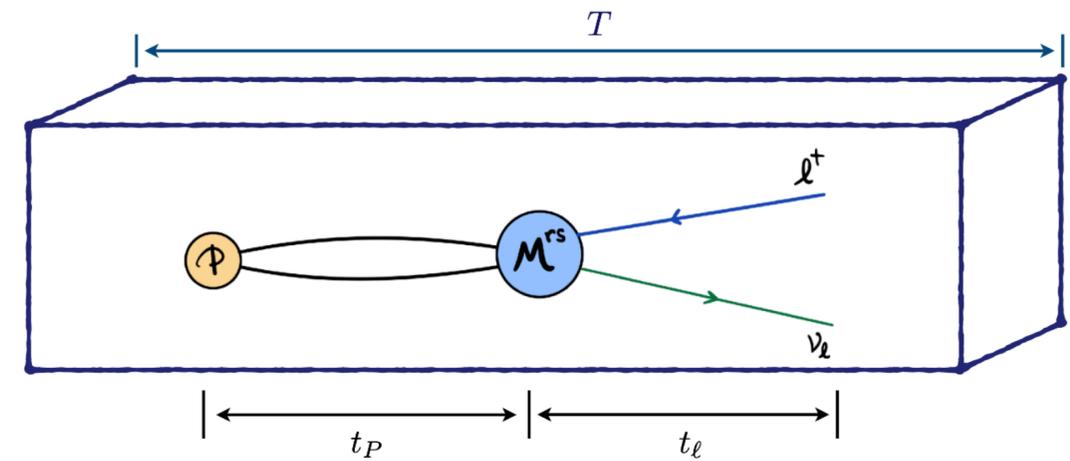
$$\text{Diagram } \phi_0 \text{ loop } \phi_0 = \langle 0|\phi(0)\phi^\dagger(-t)|0\rangle \rightarrow \frac{Z_{P,0}^2}{2m_{P,0}} e^{-m_{P,0}t}$$

From correlators to matrix elements

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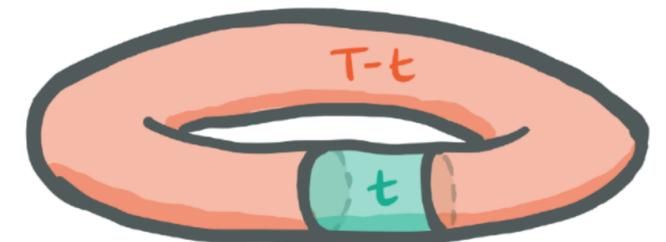


Tree-level decay amplitude: $|\mathcal{M}_0(\mathbf{p}_\ell)|^2 = |\mathcal{A}_{P,0}|^2 |\mathcal{L}_0(\mathbf{p}_\ell)|^2$ $\mathcal{A}_{P,0} = \langle 0|A^0|P\rangle_0 = im_{P,0} [f_{P,0}]$

$$\phi_0 \text{ loop with } A^0 = \langle 0|A^0(0)\phi^\dagger(-t)|0\rangle_T \rightarrow \frac{Z_{P,0}\mathcal{A}_{P,0}}{2m_{P,0}} \left\{ e^{-m_{P,0}t} - e^{-m_{P,0}(T-t)} \right\}$$

$$Z_{P,0} = \langle P, \mathbf{p} = \mathbf{0}|\phi^\dagger|0\rangle_0$$

$$\phi_0 \text{ loop with } \phi_0 = \langle 0|\phi(0)\phi^\dagger(-t)|0\rangle_T \rightarrow \frac{Z_{P,0}^2}{2m_{P,0}} \left\{ e^{-m_{P,0}t} + e^{-m_{P,0}(T-t)} \right\}$$



IB corrections to the decay amplitude

$$|\mathcal{M}(\mathbf{p}_\ell)|^2 = |\mathcal{M}_0(\mathbf{p}_\ell)|^2 + \delta_{\text{fact}}|\mathcal{M}(\mathbf{p}_\ell)|^2 + \delta_{\text{non-fact}}|\mathcal{M}(\mathbf{p}_\ell)|^2$$

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factorisable

only quarks involved

$$\frac{\text{Diagram with } \phi_0 \text{ and } A^0 \text{ connected by a wavy line}}{\text{Diagram with } \phi_0 \text{ and } A^0 \text{ connected by a straight line}} \rightarrow \frac{\delta_{\text{fact}} \mathcal{A}_P}{\mathcal{A}_{P,0}} + \frac{\delta Z_P}{Z_{P,0}} - \frac{\delta m_P}{m_{P,0}} f_{\text{PA}}(t, T)$$

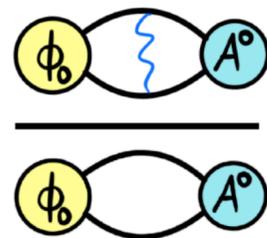
$$f_{\text{PA}}(t, T) = 1 + m_{P,0} \left\{ \frac{T}{2} - \left(t - \frac{T}{2} \right) \coth \left[m_{P,0} \left(t - \frac{T}{2} \right) \right] \right\} \approx 1 + m_{P,0} t$$

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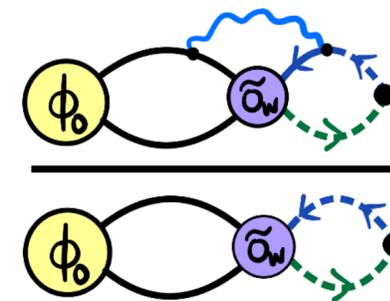


$$\frac{\text{Diagram 1}}{\text{Diagram 2}} \rightarrow \frac{\delta_{\text{fact}} \mathcal{A}_P}{\mathcal{A}_{P,0}} + \frac{\delta Z_P}{Z_{P,0}} - \frac{\delta m_P}{m_{P,0}} f_{\text{PA}}(t, T)$$

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non-factorisable

photon exchanged between quarks and lepton



$$\frac{\text{Diagram 1}}{\text{Diagram 2}} \rightarrow \frac{\delta_{\text{non-fact}} \mathcal{A}_P}{\mathcal{A}_{P,0}} f_{\text{P}\ell}(t, T)$$

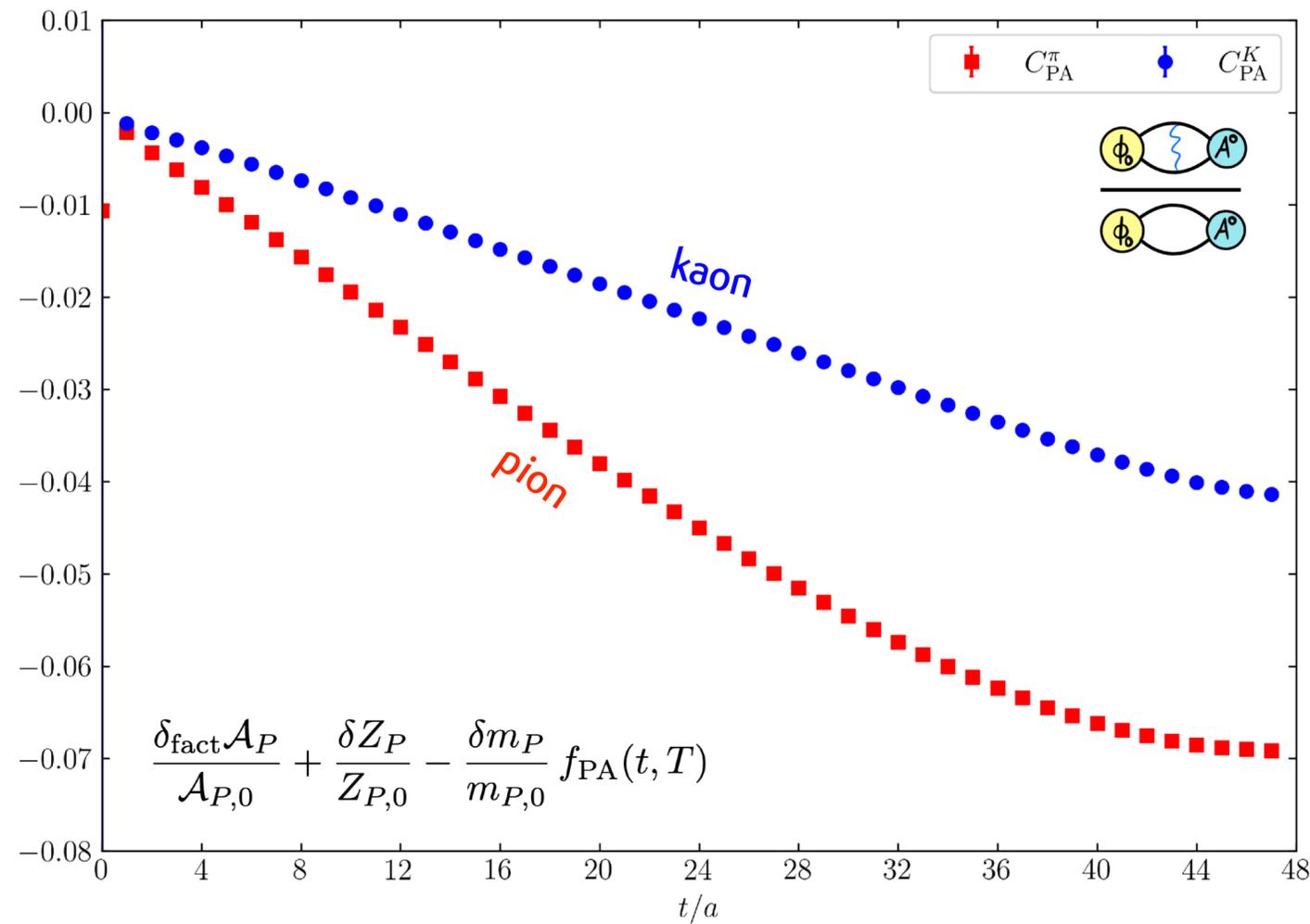
$$f_{\text{P}\ell}(t, T) = \frac{1}{2} \left\{ (1 + \kappa_\ell) - (1 - \kappa_\ell) \coth \left[m_{P,0} \left(t - \frac{T}{2} \right) \right] \right\} \approx 1$$

$$\text{---}\leftarrow\text{---} = \sum_r u_\nu^r(-\mathbf{p}_\ell) \bar{u}_\nu^r(-\mathbf{p}_\ell) \quad \text{---}\rightarrow\text{---} = \sum_s v_\ell^s(\mathbf{p}_\ell) \bar{v}_\ell^s(\mathbf{p}_\ell)$$

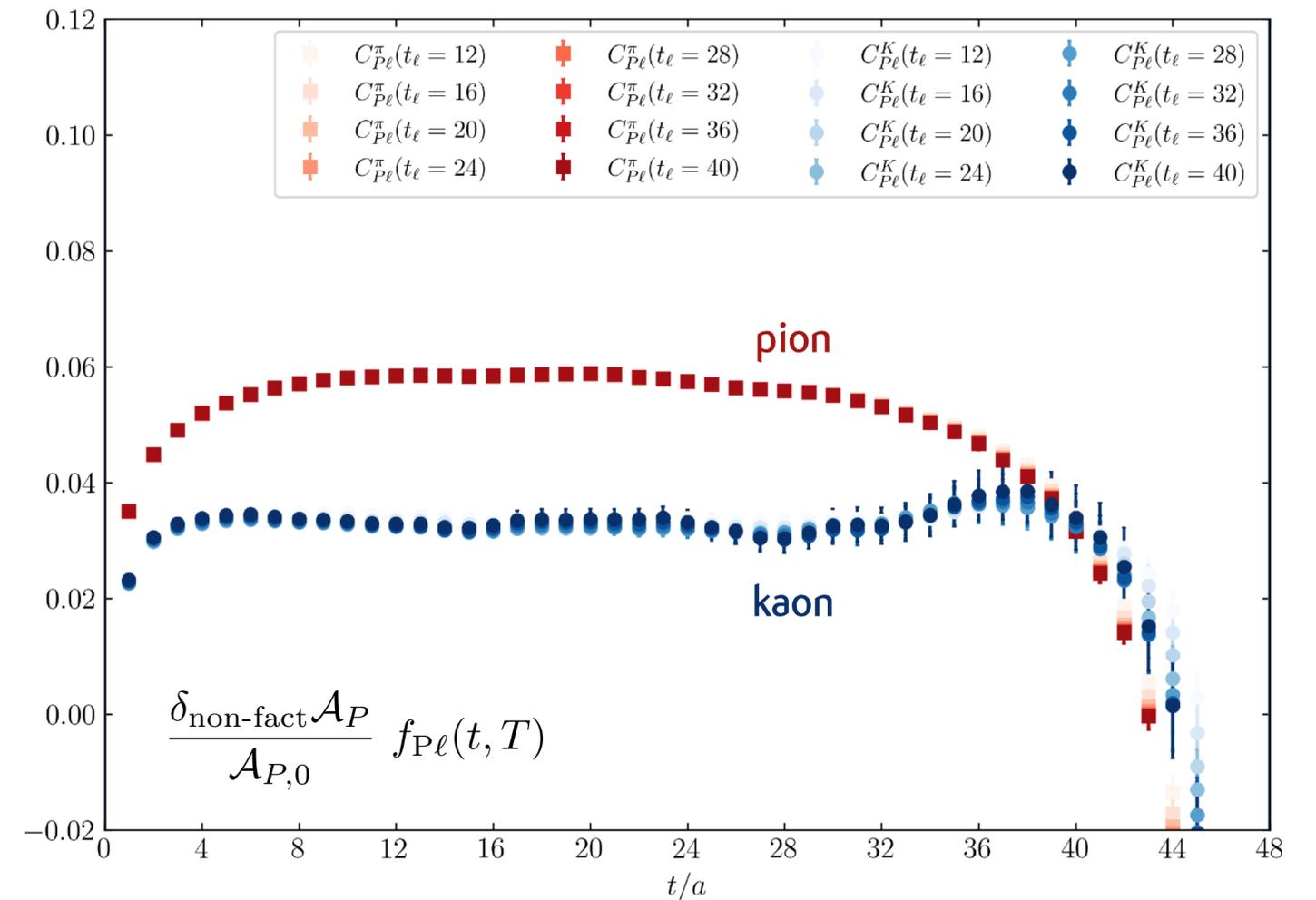
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factorisable

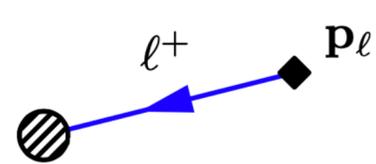


non-factorisable



Non-factorisable QED corrections

The lepton in a finite volume

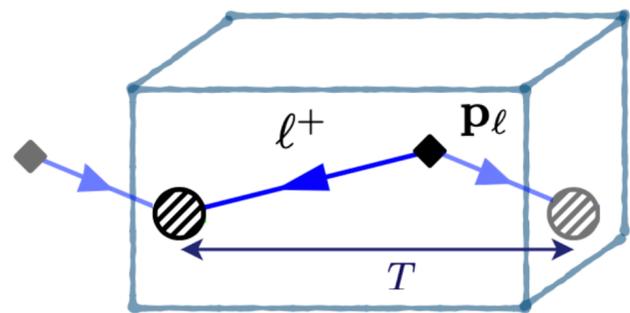


A Feynman diagram showing a lepton line. It starts with a shaded circle on the left, followed by a blue arrow pointing to the right, and ends with a black diamond on the right. The label l^+ is placed above the arrow, and the label \mathbf{p}_ℓ is placed to the right of the diamond.

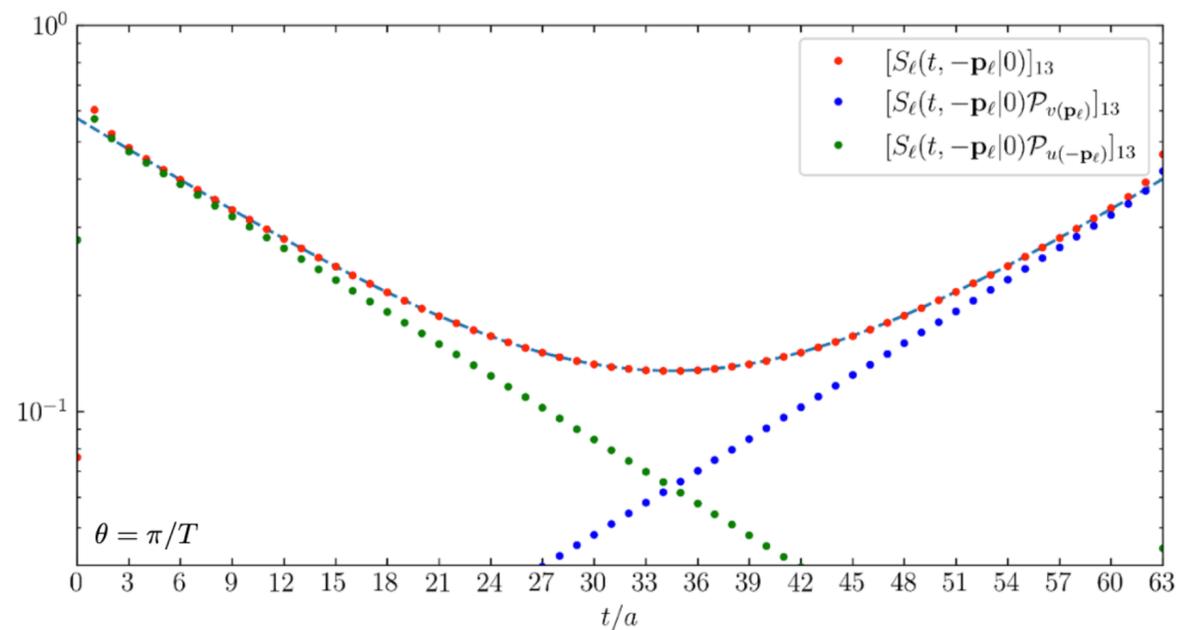
$$= S(0|t, \mathbf{p}_\ell) = \sum_r \left\{ -e^{-tE_\ell} \frac{v_r(\mathbf{p}_\ell) \bar{v}_r(\mathbf{p}_\ell)}{2\Omega_\ell} \right\}$$

Non-factorisable QED corrections

The lepton in a finite volume



$$S(0|t, \mathbf{p}_\ell) = \sum_r \left\{ -e^{-tE_\ell} \frac{v_r(\mathbf{p}_\ell)\bar{v}_r(\mathbf{p}_\ell)}{2\Omega_\ell} + e^{i\theta T} e^{-(T-t)E_\ell} \frac{u_r(-\mathbf{p}_\ell)\bar{u}_r(-\mathbf{p}_\ell)}{2\Omega_\ell} \right\} \times \frac{1}{1 - e^{-TE_\ell} e^{i\theta T}}$$



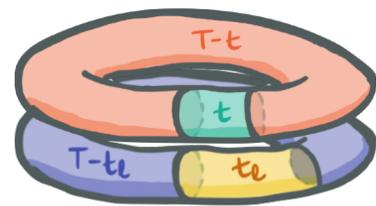
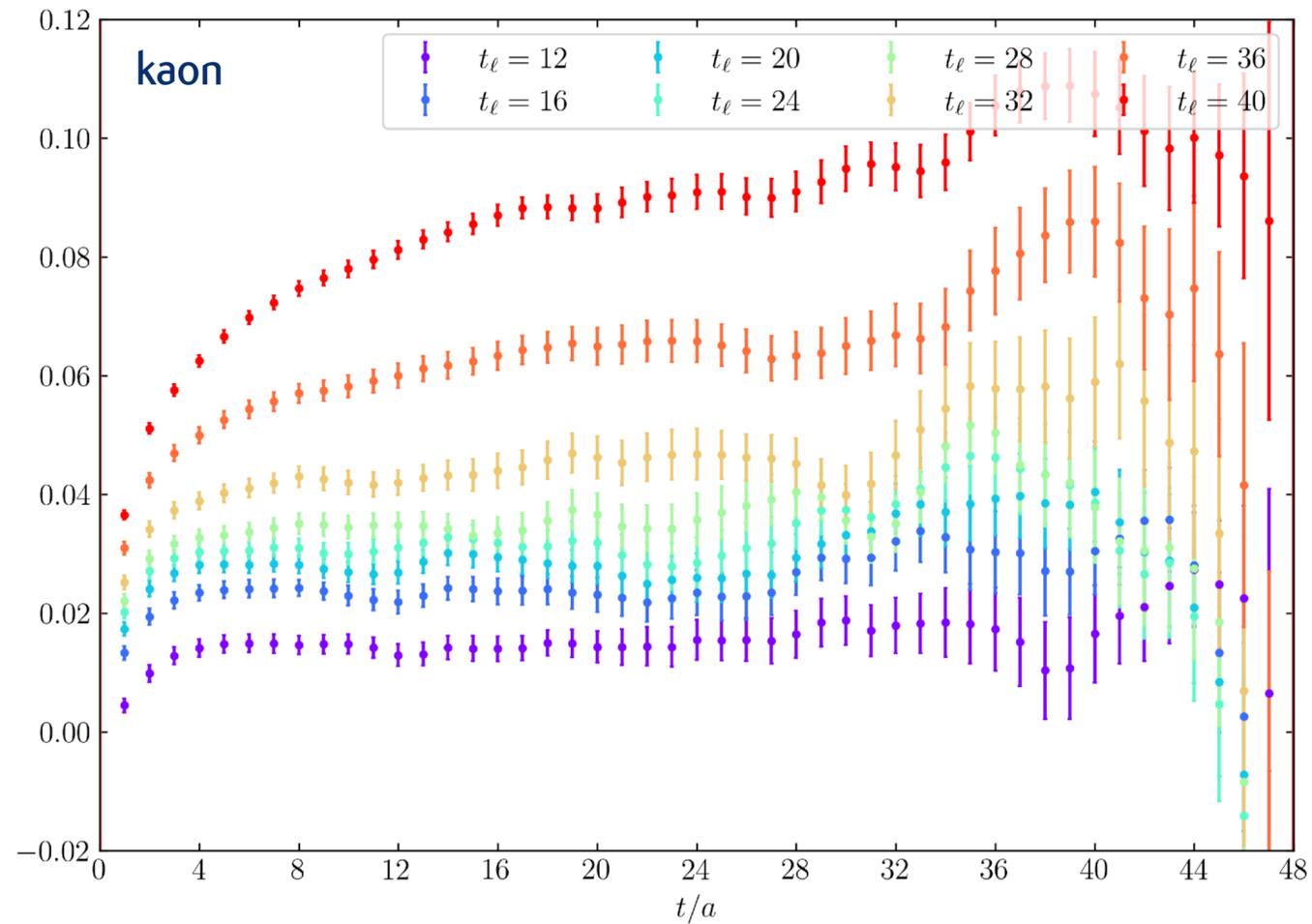
We can select specific components using projectors:

$$\begin{aligned} \left[\begin{array}{c} \bullet \\ \bullet \end{array} \right] \cdot \mathcal{P}_{v(\mathbf{p}_\ell)} &= \left[\begin{array}{c} \bullet \\ \bullet \end{array} \right] \\ \left[\begin{array}{c} \bullet \\ \bullet \end{array} \right] \cdot \mathcal{P}_{u(-\mathbf{p}_\ell)} &= \left[\begin{array}{c} \bullet \\ \bullet \end{array} \right] \end{aligned}$$

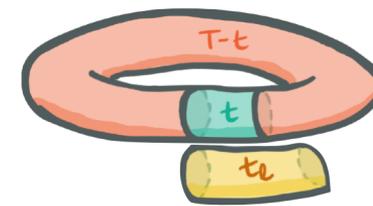
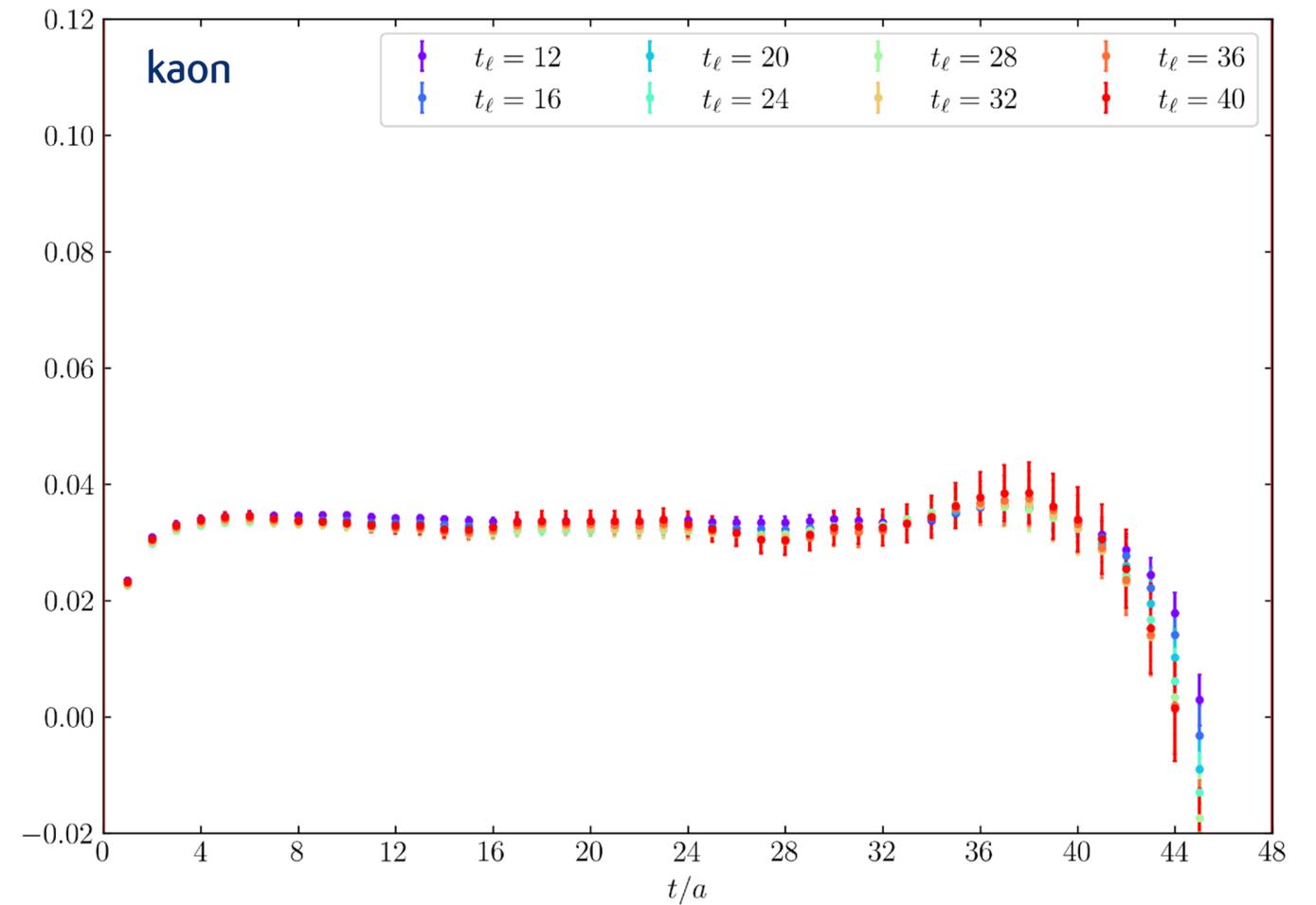
$$\begin{aligned} \mathcal{P}_{v(\mathbf{p}_\ell)} &= \{u_t(-\mathbf{p}_\ell)\bar{u}_t(-\mathbf{p}_\ell) + v_s(\mathbf{p}_\ell)\bar{v}_s(\mathbf{p}_\ell)\}^{-1} [v_r(\mathbf{p}_\ell)\bar{v}_r(\mathbf{p}_\ell)] \\ \mathcal{P}_{u(-\mathbf{p}_\ell)} &= \{u_t(-\mathbf{p}_\ell)\bar{u}_t(-\mathbf{p}_\ell) + v_s(\mathbf{p}_\ell)\bar{v}_s(\mathbf{p}_\ell)\}^{-1} [u_r(-\mathbf{p}_\ell)\bar{u}_r(-\mathbf{p}_\ell)] \end{aligned}$$

Non-factorisable QED corrections

$$\frac{\text{Diagram with } \phi_0 \text{ and } \tilde{\sigma}_w \text{ and a photon loop}}{\text{Diagram with } \phi_0 \text{ and } \tilde{\sigma}_w} \rightarrow \frac{\delta_{\text{non-fact}} \mathcal{A}_P}{\mathcal{A}_{P,0}} f_{P\ell}(t, T)$$



without projection



with projection

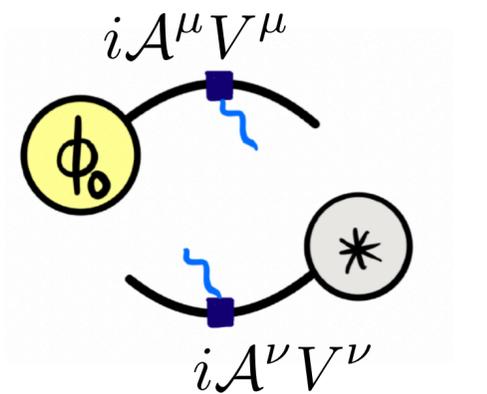
Numerical implementation of correlators



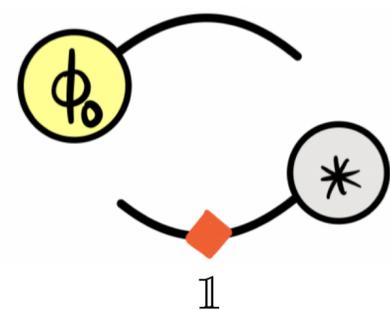
Hadrons

<https://github.com/paboyle/Grid>

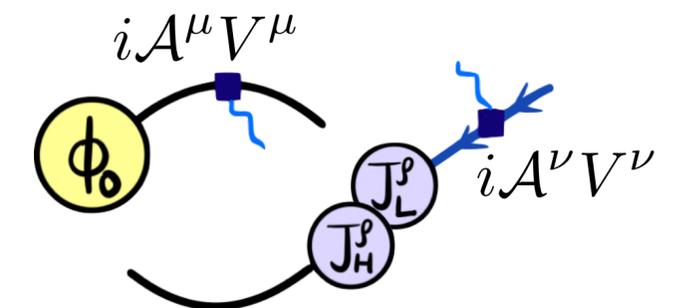
<https://github.com/aportelli/Hadrons>



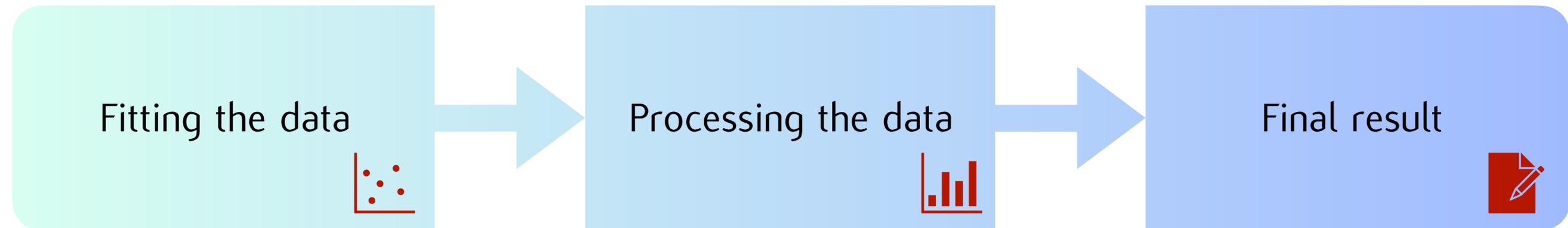
$$* = \{ A^0, \phi_0 \}$$



- Correlators created using sequential propagators
- $N_T = 96$ Coulomb gauge-fixed wall sources ϕ_0 per configuration
- Muon momentum $\mathbf{p}_\ell \propto \{1, 1, 1\}$ fixed by energy conservation & injected via twisted boundary conditions
- Muon propagator evaluated for various source-sink separations
- Photon fields sampled from Gaussian distribution (QED_L)
- Electromagnetic current: renormalised local vector current



Extracting results from data



- Simultaneous correlated fit of correlators
- Best AIC fit range selection through genetic algorithm



- Assemble fit results to get $\delta R_{K\pi}$
- AIC-based model averaging [BMW, Science 347 \(2015\)](#), [W.Jay & E.T.Neil, PRD 103 \(2008\)](#)



- Determine $\delta R_{K\pi}$ as median of distribution
- Estimate associated statistical and systematic uncertainty

A general comparison of the calculations

	RBC/UKQCD	RM123+Soton
physical masses	✓ physical point simulations	extrapolation needed
chiral symmetry	✓ at finite lattice spacing	recovered in the continuum
fermionic action	Domain Wall	Twisted Mass
continuum limit	single lattice spacing	✓ continuum limit (3)
infinite volume limit	single volume	✓ multiple volumes
QED prescription	QED _L	QED _L
sea effects	electro-quenching	electro-quenching
(*) IB scheme	BMW ^[a]	GRS ^[b]

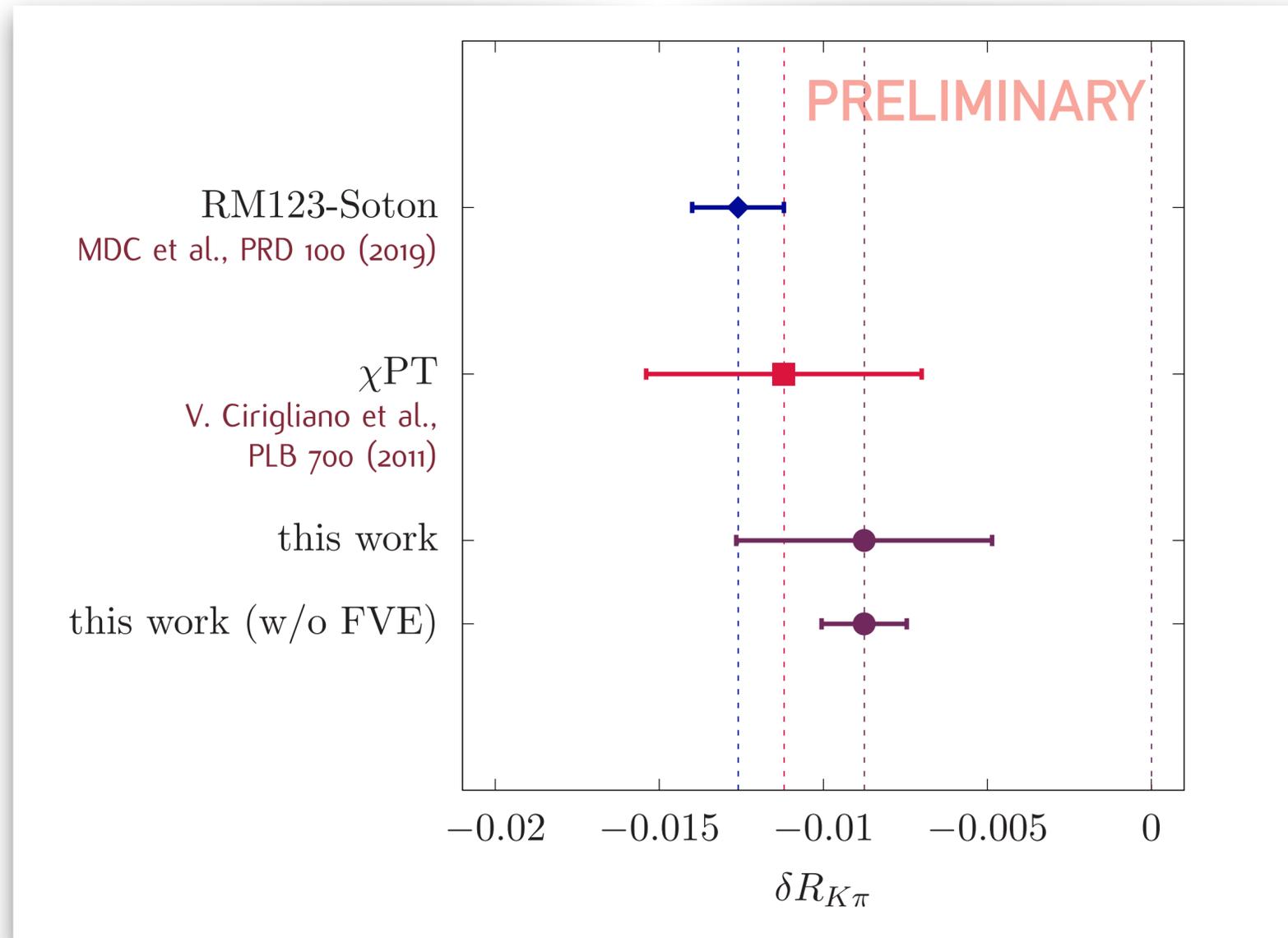
[a] BMW, PRL 111 (2013); BMW, PRL 117 (2016)

[b] Gasser, Rusetsky & Scimemi, EPJC 32 (2003); RM123, PRD 87 (2013)

(*) see also talks by A.Portelli at 14.40
& N.Tantalo on Saturday 11.45

Results for $\delta R_{K\pi}$

Results for $\delta R_{K\pi}$



$$\delta R_{K\pi} = -0.0088 (39)$$

total	(39)
total (w/o FVE)	(13)
statistical	(3)
FVE	(37)
fit	(11)
QED quenching	(5)
discretisation	(5)

PRELIMINARY

RM123-Soton: $\delta R_{K\pi} = -0.0126 (14)$

χ PT: $\delta R_{K\pi} = -0.0112 (21)$

Our main systematic uncertainty

Finite volume effects

V. Lubicz et al., PRD 95 (2017)

N. Tantalo et al., [1612.00199]

MDC et al., PRD 105 (2022)

$$\Gamma_0(L) = \Gamma_0^{\text{tree}} \left\{ 1 + 2 \frac{\alpha}{4\pi} Y(L) \right\}$$

$$Y(L) = Y_{\log}(L) + Y_0 + \frac{1}{m_P L} Y_1 + \frac{1}{(m_P L)^2} Y_2 + \frac{1}{(m_P L)^3} Y_3$$

Our main systematic uncertainty

Finite volume effects

V. Lubicz et al., PRD 95 (2017)

N. Tantalo et al., [1612.00199]

MDC et al., PRD 105 (2022)

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$Y_{K\pi}^{(1)}(L/a = 48) \approx -3.96$ **57%**

$Y_{K\pi}^{(2)}(L/a = 48) \approx -6.20$

$Y_{K\pi}^{(3),\text{pt}}(L/a = 48) \approx -2.83$ **-54%**
pointlike approximation

see next talk by N.Hermansson-Truedsson for more details

Significant correction from **pointlike $1/L^3$** !

Central value: $1/L^2$ subtracted result

Systematic error: conservative ~50% error



FV scaling should be carefully studied!

Prospects for $|V_{us}/V_{ud}|$

A speculative exercise on the error budget

$$\left| \frac{V_{us}}{V_{ud}} \right|^2 = \left[\frac{\Gamma(K\ell 2)}{\Gamma(\pi\ell 2)} \frac{M_{K^+}^3}{M_{\pi^+}^3} \frac{(M_{K^+}^2 - M_{\mu^+}^2)^2}{(M_{\pi^+}^2 - M_{\mu^+}^2)^2} \right]_{\text{exp}} \cdot \left[\frac{f_{K,0}}{f_{\pi,0}} \right]^2 (1 + \delta R_{K\pi})$$

- Let us use $\delta R_{K\pi} = -0.0088 (13)$ (assume FV issue solved)

	$[f_{K,0}/f_{\pi,0}]$	$ V_{us}/V_{ud} $
FLAG 2+1 best [a]	1.1945 (45)	$0.23127 (28)_{\text{exp}} (14)_{\delta R} (87)_{f_P}$
FLAG 2+1 average	1.1930 (33)	$0.23155 (28)_{\text{exp}} (14)_{\delta R} (65)_{f_P}$
FLAG 2+1+1 best [b]	1.1980 (15)	$0.23059 (28)_{\text{exp}} (14)_{\delta R} (28)_{f_P}$
FLAG 2+1+1 average	1.1966 (18)	$0.23086 (28)_{\text{exp}} (14)_{\delta R} (35)_{f_P}$

[\[a\]](#) RBC/UKQCD14 481 [\[b\]](#) FNAL/MILC 17

- From RM123+Soton calculation $\delta R_{K\pi} = -0.0126 (14)$

	$[f_{K,0}/f_{\pi,0}]$	$ V_{us}/V_{ud} $
FLAG 2+1+1 average	1.1966 (18)	$0.23131 (28)_{\text{exp}} (17)_{\delta R} (35)_{f_P}$

- ▶ the uncertainty on $[f_{K,0}/f_{\pi,0}]$ dominates in the error budget
- ▶ if improved, precision from lattice starts being competitive with the experimental one

Conclusions

- **New results** for radiative virtual correction $\delta R_{K\pi}$ from lattice calculation with Domain Wall fermions at the physical point (**soon on arXiv!**)
- **Finite volume effects** have to be carefully studied, including order $1/L^3$ (looking forward to seeing results with different QED prescriptions: QED_C , QED_m , QED_∞)
- including IB effects is necessary to claim **precision**, but we should not forget about improving the **iso-QCD part**
- With small further improvement in lattice calculations, we're close to be **competitive** with precision from experimental inputs on $|V_{us}/V_{ud}|$

Thank you