Direct CP violation in $K \to \pi\pi$ decay at the physical point with periodic BC

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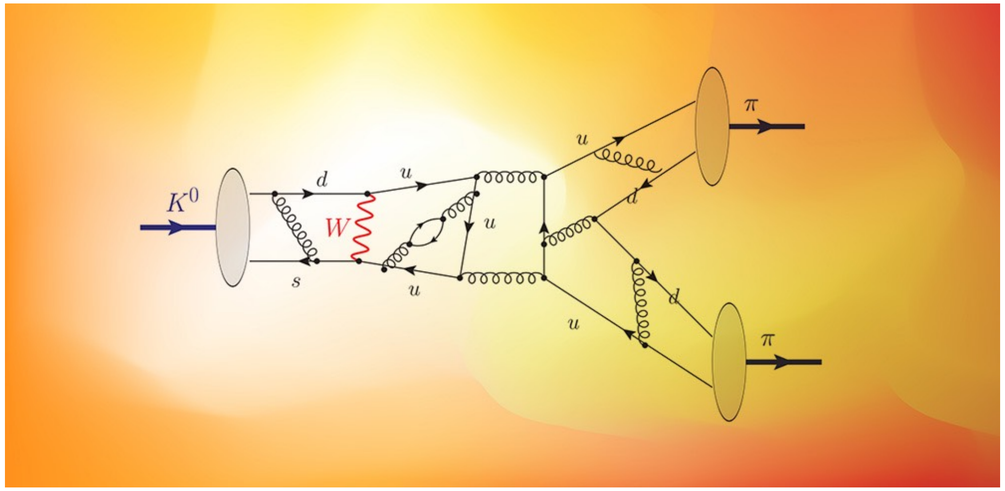
K → ππ & Direct CPV

$$|K_L\rangle = \underbrace{|K_2\rangle}_{\text{CP odd}} + \varepsilon \underbrace{|K_1\rangle}_{\text{CP even}}$$

$|K_2\rangle \xrightarrow{\varepsilon'}$ (direct CPV) $|\pi\pi\rangle$

$|K_1\rangle \xrightarrow{\varepsilon}$ (indirect CPV) $|\pi\pi\rangle$

$|\pi\pi\rangle$ CP even



- ε′ vs ε
  - Re $(\varepsilon'/\varepsilon)_{exp}$ = 16.6(2.3) x 10^{-4} (KTeV, NA48)
  - Explained by SM?
- Key to understanding the nature of matter/anti-matter asymmetry

G-parity BC calculation performed

- $E_{\pi\pi} = 2m_\pi \approx 280$ MeV state in Euclidean correlators forbidden
- Useful to extract $E_{\pi\pi} = m_K$ state as the ground $\pi\pi$ state

- **PRD 102,054509 (2020)**
- **C. Kelly just talked about continuation**

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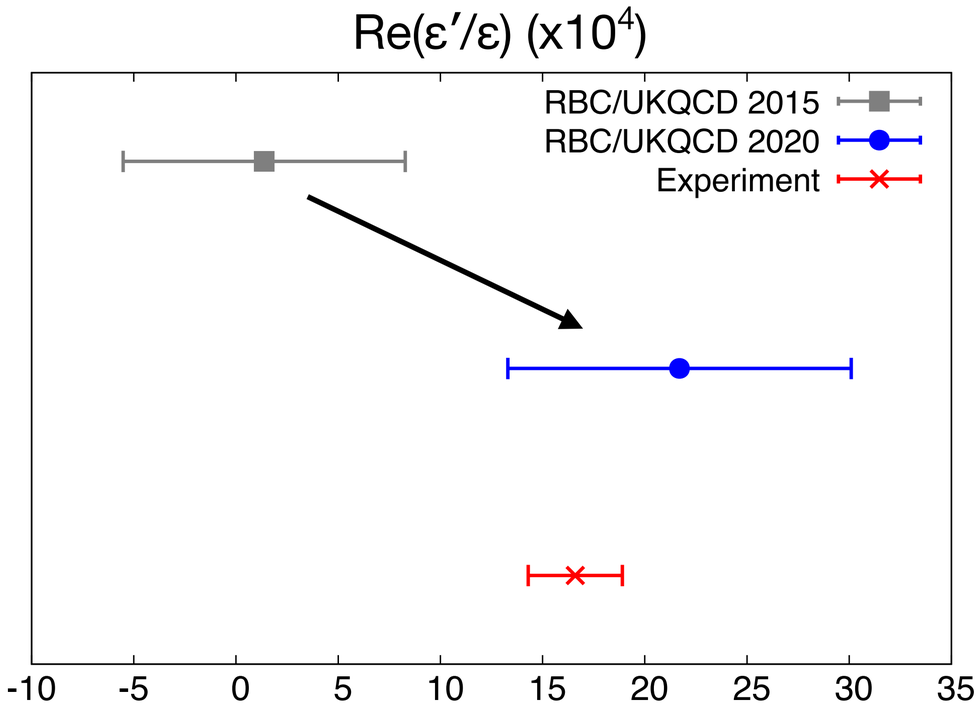

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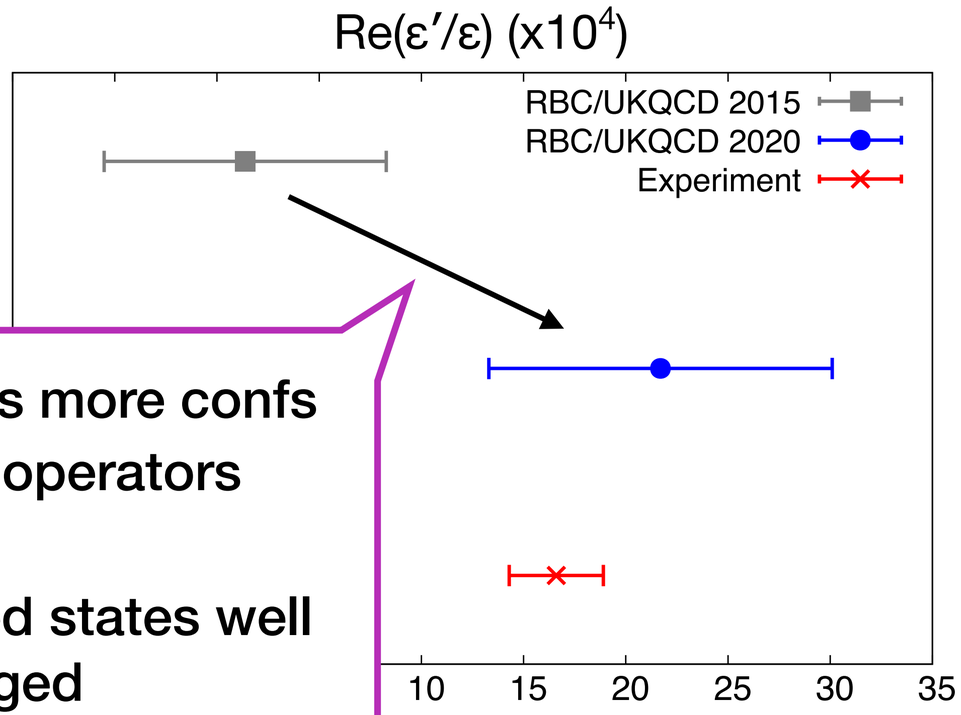

- 3+ times more confs
- # of $\pi\pi$ operators
  - 1 → 3
  - excited states well managed
- Step scaling in NPR

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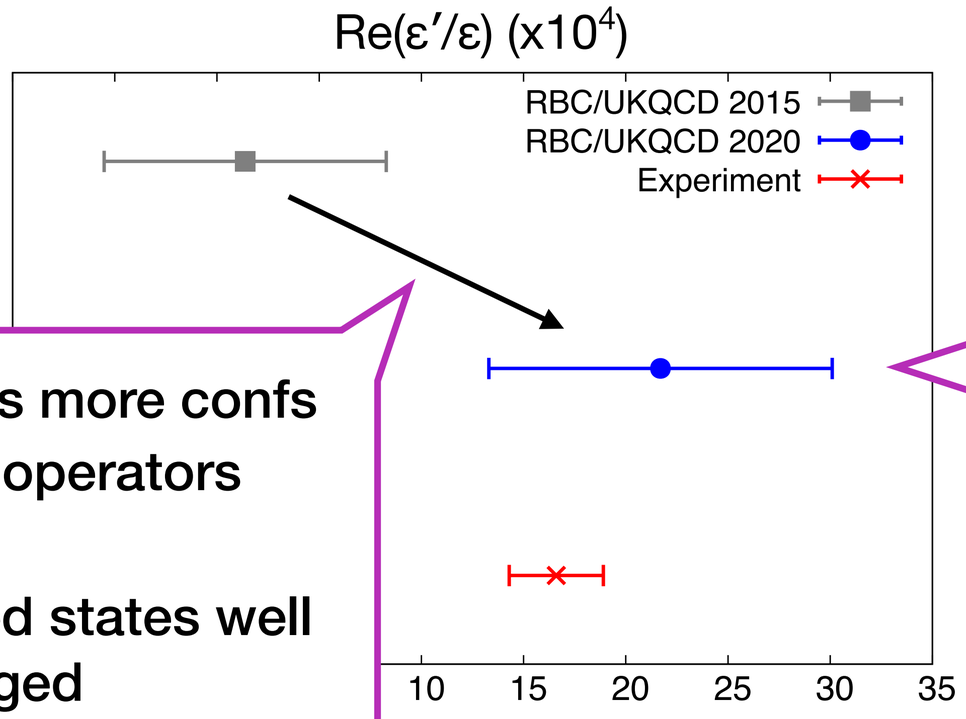

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$$21.7(2.6)_{\text{stat}}(6.2)_{\text{sys}}(5.0)_{\text{EM/IB}} \times 10^{-4}$$

- Independent calculations desired b/c of
  - Phenomenological importance of ε′
  - Complexity of calculation
- Systematic errors
  - Isospin breaking effects
  - Finite lattice spacing
  - Wilson coefficients

Why periodic BC?

- Already have lattice ensembles with physical pion mass
  - a^{-1} = 1 GeV, 24^3 x 64 & a^{-1} = 1.4 GeV, 32^3x64 & …
  - Continuum limit easier

- Hope to introduce QED/IB effects near future
  - Complicated with G-parity BC
  - Maybe possible with periodic BC

- Presence of $E_{\pi\pi} = 2m_\pi$ state challenging
  - S/N ratio of $E_{\pi\pi} = m_K$ state should be the same as in G-parity BC: $\sim e^{-(m_K - 2m_\pi)t}$
  - interesting to see if it's possible to extract signal of excited states

What to calculate

ππ phase shifts

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\epsilon}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]\right\} \qquad (\omega = \mathrm{Re}\,A_2/\mathrm{Re}\,A_0)$$

Lellouch-Lüscher finite volume correction

Renormalization matrix

$$A_I = F\frac{G}{\sqrt{2}}V_{us}^{*}V_{ud}\sum_{i,j}\underbrace{[z_i(\mu)+\tau y_i(\mu)}_{\text{Wilson coefs. pQCD}}\underbrace{Z_{ij}(\mu)}_{\text{LQCD (+pQCD)}}\underbrace{\langle(\pi\pi)_I|Q_j^{lat}|K\rangle}_{\text{LQCD}}$$

- δ_I, F being determined via π-π scattering & Lüscher formalism
- $A_I = \langle(\pi\pi)_I|\,H_W\,|K\rangle$ from 3pt correlation functions
- I = 0 challenging — disconnected diagrams, power divergences

Exploring excited-state signals

- ππ energies in PBC
  - ≈ $2m_\pi$ for ground st.
  - Need excited-state signals to extract on-shell kinematics of K → ππ

Picture in non-interacting 2-pion system with CoM frame

	$\vec{p}$	$E = 2\sqrt{\lvert\vec{p}\rvert^2 + m_\pi^2}$
ground st.	(0,0,0)	$2m_\pi$
1st excited st.	2π/L x (1,0,0)	could be ≈ m_K
2nd excited st.	2π/L x (1,1,0)	

- Variational method useful
  - Solving GEVP (Generalized Eigenvalue Problem)

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0)$$

$C(t)$: N x N correlator matrix

$$C_{ab}(t) = \langle O_a(t)O_b(0)^\dagger\rangle$$

  - $O'_n = \sum_a v^*_{n,a}O_a$ couples with only n-th, N+1-th & higher states
  - $\lambda_n(t,t_0) = e^{-E_n(t-t_0)}$

Matrix elements

- For extraction of ground-state ME

$$M^{\mathrm{eff}}(t_2, t_1) = C^{(3)}(t_2, t_1) \left[\frac{e^{E^{\pi\pi} t_2} e^{E^K t_1}}{C^{\pi\pi}(t_2) C^K(t_1)} \right]^{1/2} \xrightarrow{\text{large } t_1 \ \& \ t_2} M$$

- Excited ππ state needed for on-shell kinematics with PBC

$$\boxed{M_n^{\mathrm{eff}}(t_2, t_1) = C_n^{(3)}(t_2, t_1) \left[\frac{e^{E_n^{\pi\pi} t_2} e^{E^K t_1}}{C_n^{\pi\pi}(t_2) C^K(t_1)} \right]^{1/2}} \xrightarrow{\text{large } t_1 \ \& \ t_2} M_n$$

$C_n^{\pi\pi}$: 2-pt function of ππ operators diagonalized by GEVP

$C_n^{(3)}$: K → ππ 3-pt function with ππ operator used in $C_n^{\pi\pi}$

Lattice setup

- RBC/UKQCD's 2+1-flavor MDWF ensembles at physical pion & kaon masses
  - $24^3 \times 64$, $a^{-1} = 1.0$ GeV, 258 confs
  - $32^3 \times 64$, $a^{-1} = 1.4$ GeV, 107 confs

- Chiral symmetry of DWF → strong constraints on operator mixings
  - with lower-dimensional operators
  - among different representations WRT chiral symmetry (8,1), (8,8) & (27,1)

Measurement strategy

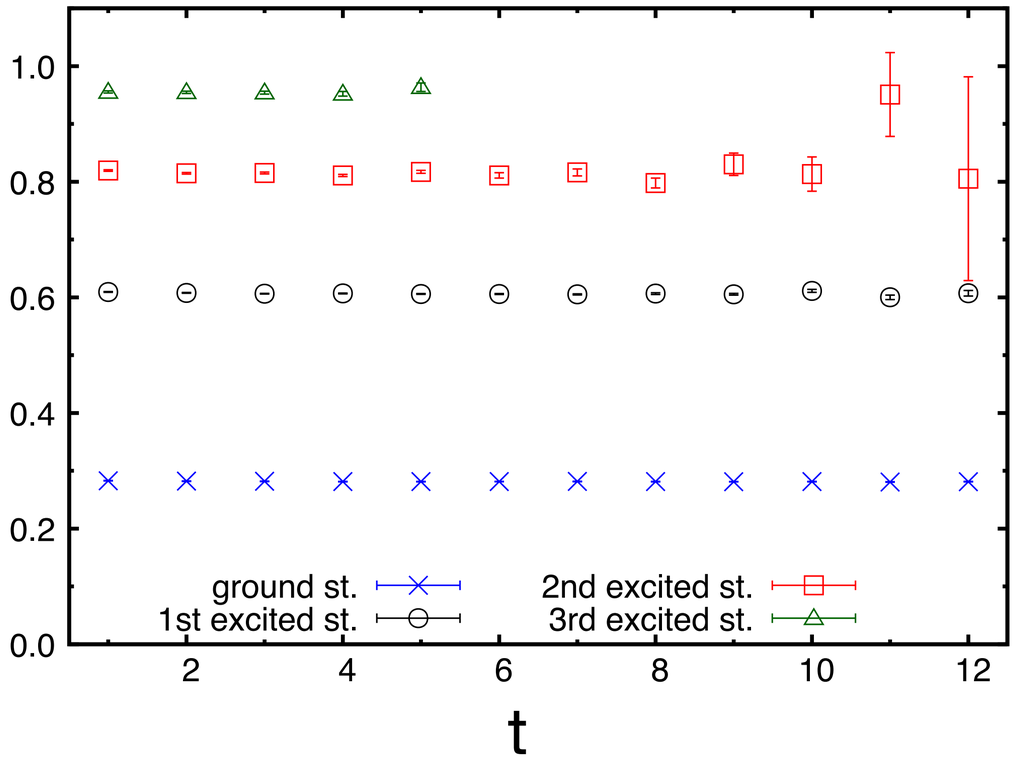
- All-to-all quark propagators
  - 2,000 low modes for light quarks (no low mode for strange)
  - high-mode part: spin, color and time dilutions => 768 inversions
- 28 (5 independent) interpolation ππ operators
  - $\pi_{p=(0,0,0)}\pi_{p=(0,0,0)}$, $\pi_{p=(0,0,1)}\pi_{p=(0,0,-1)}$, $\pi_{p=(0,1,1)}\pi_{p=(0,-1,-1)}$, $\pi_{p=(1,1,1)}\pi_{p=(-1,-1,-1)}$ & σ
  - to control effects from various states

ππ states

- Effective ππ energies from 2pt functions
- Used GEVP with some improvements

Paper on this topic in preparation

$aE_{\pi\pi}(I=2)$



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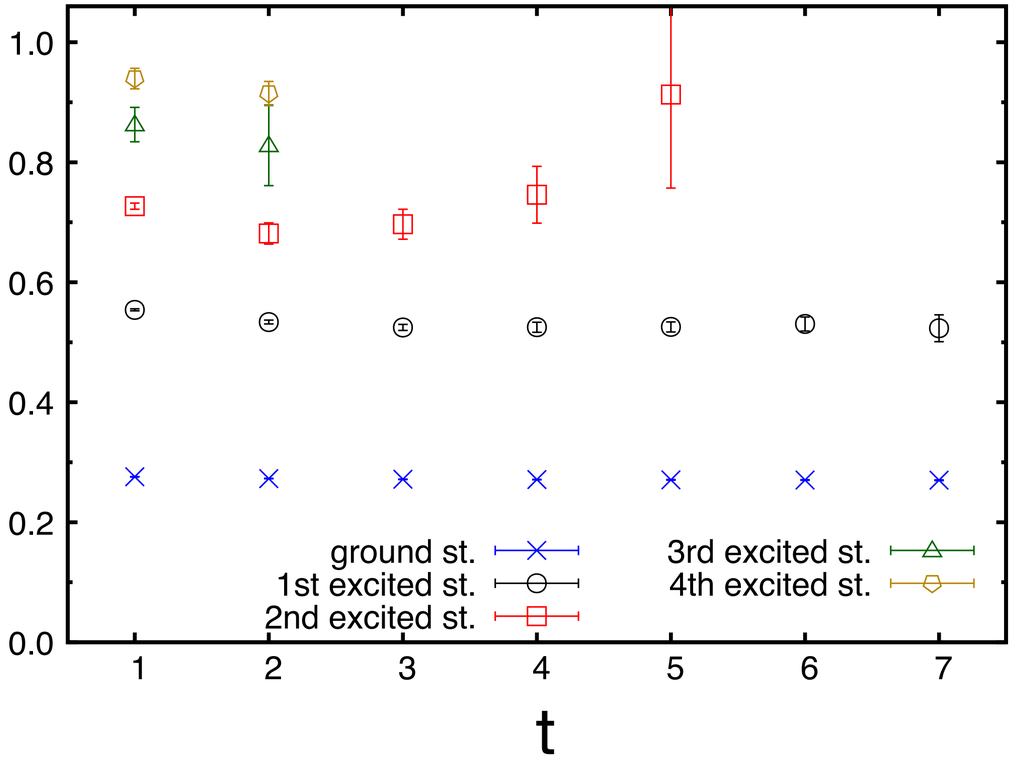

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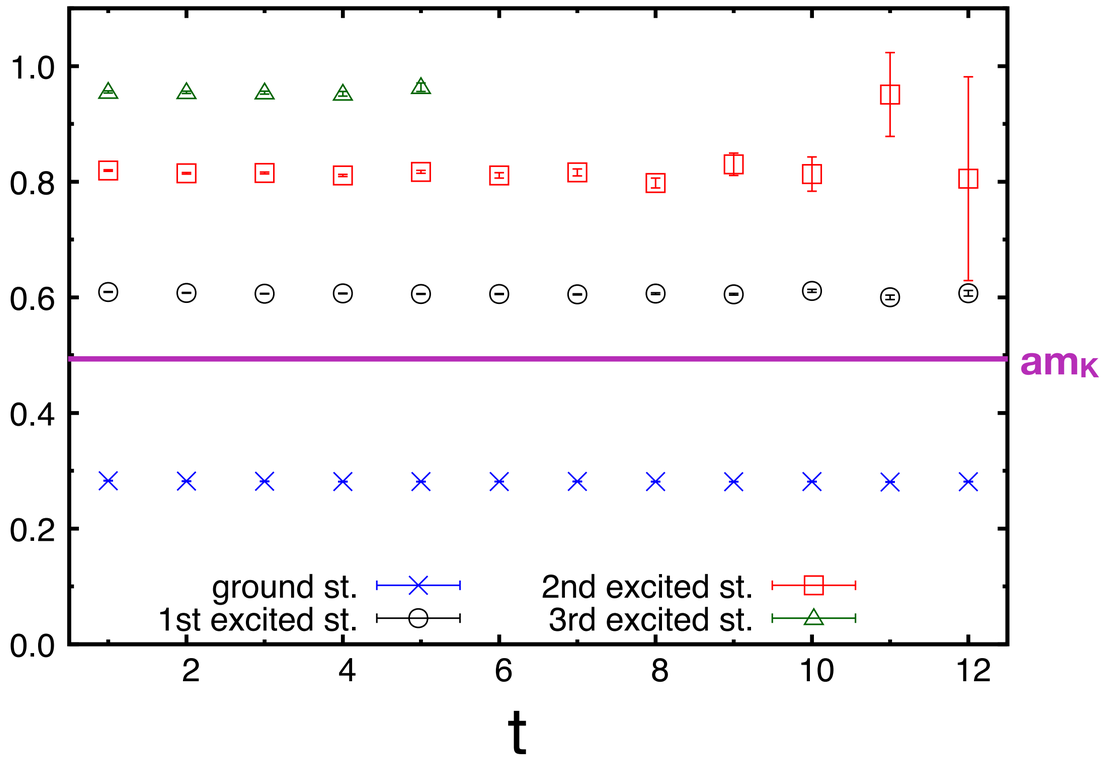

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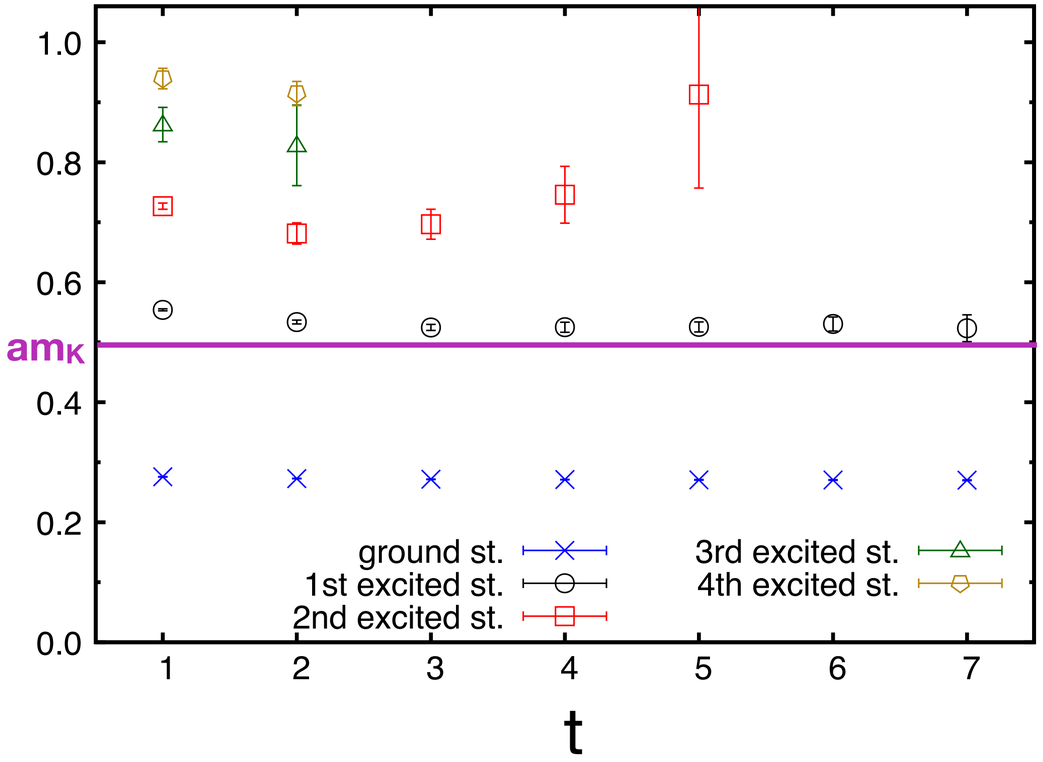

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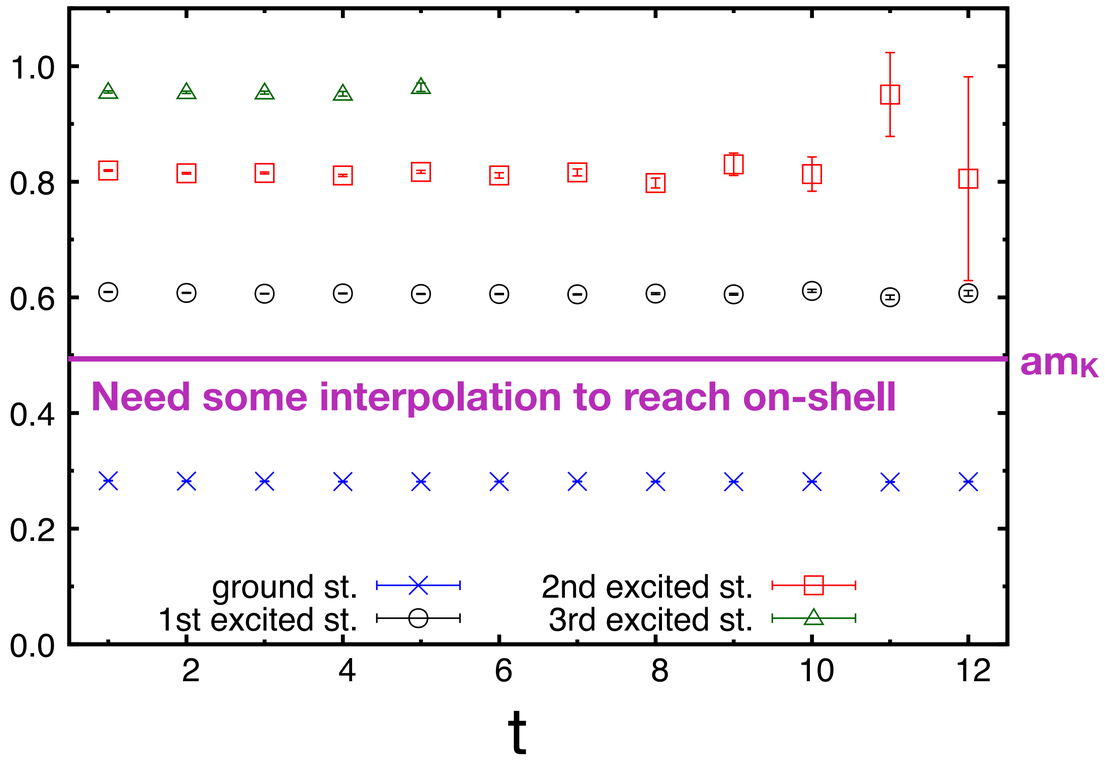

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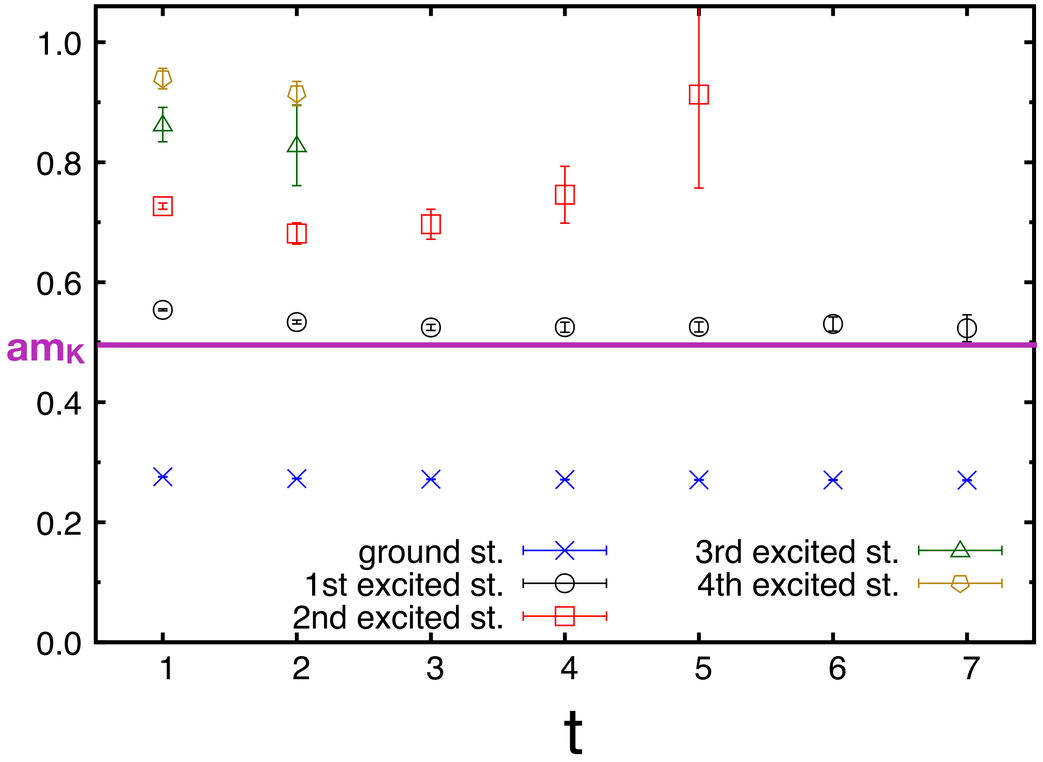
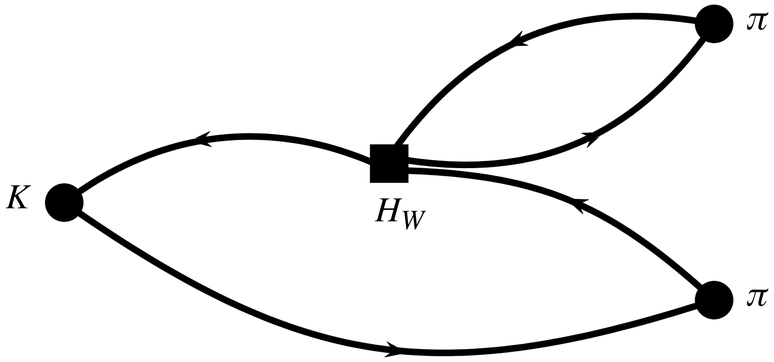
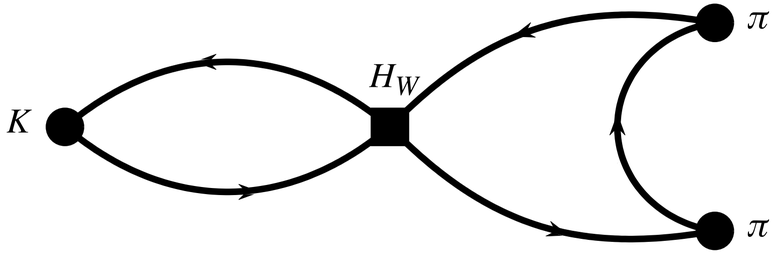


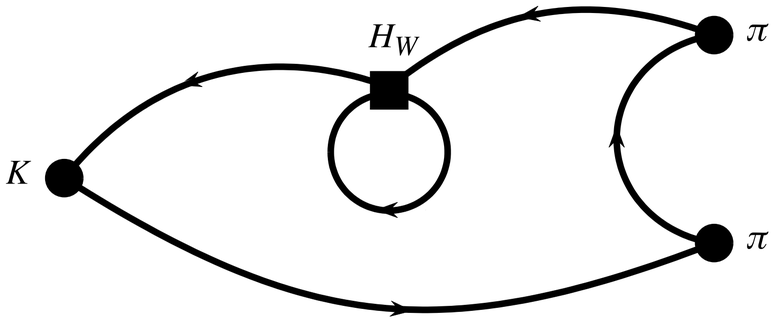
Diagrams for K → ππ 3pt functions



type 1



type 2



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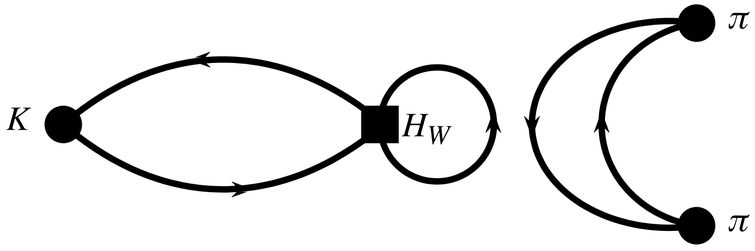
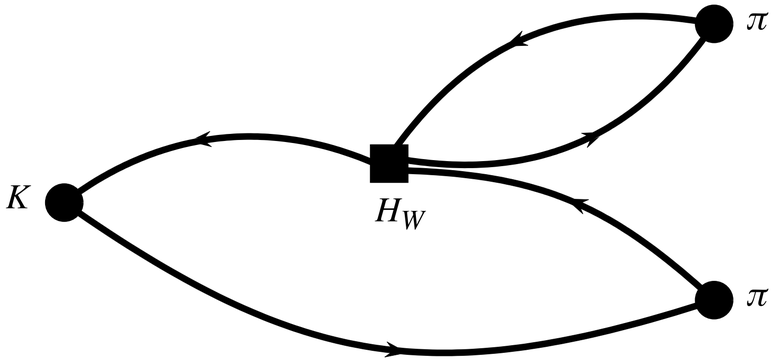


type 4

Diagrams for K → ππ 3pt functions

Main



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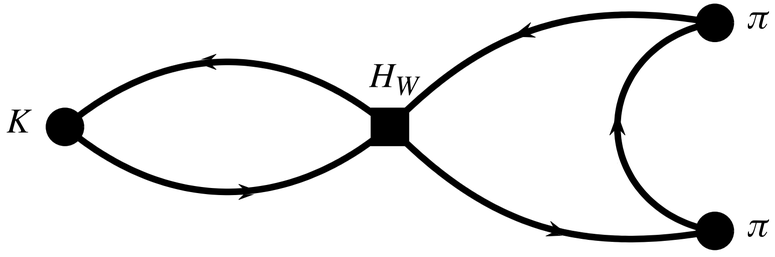

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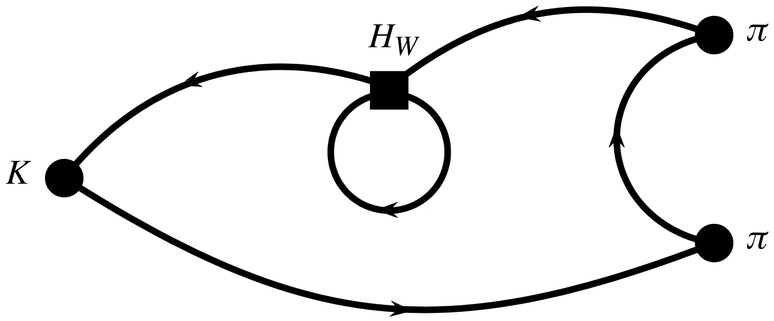

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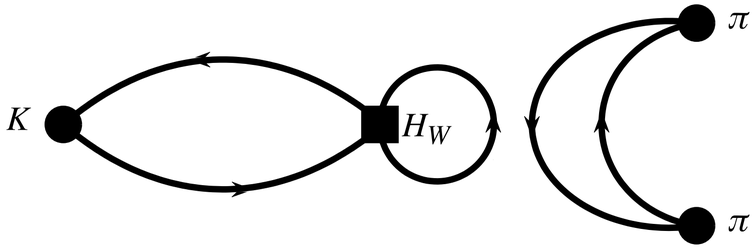
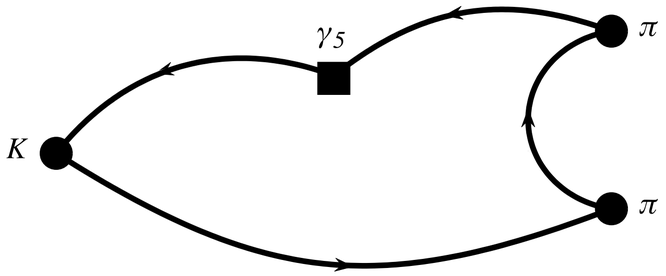
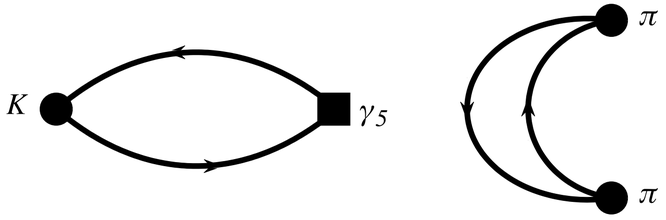


type 4

for subtraction of a^{-2} divergence



type 3'

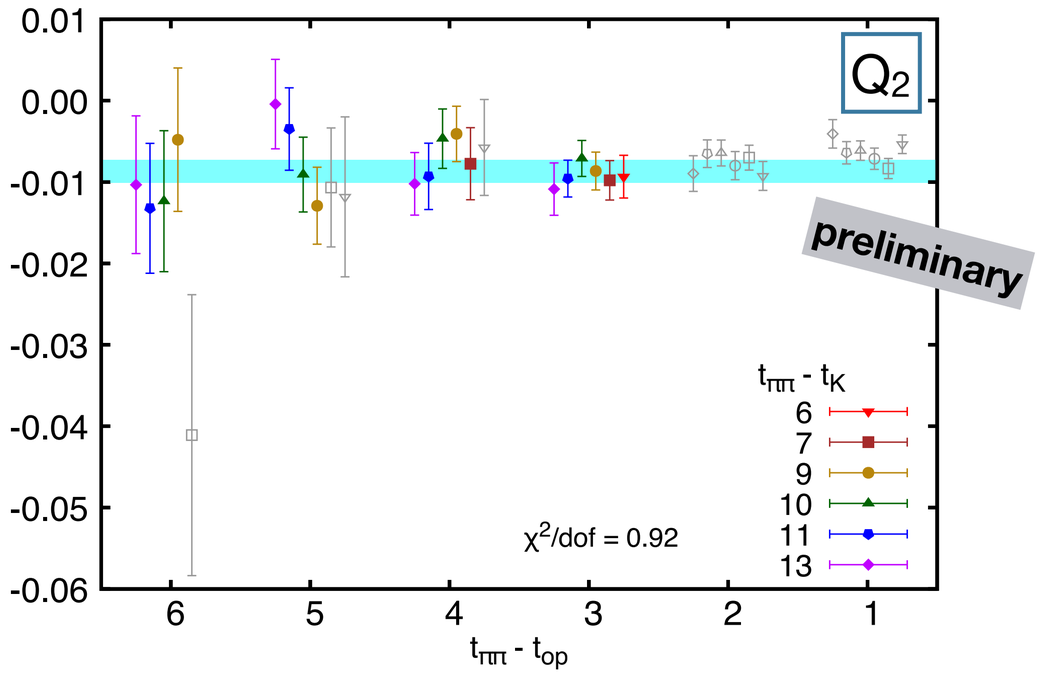


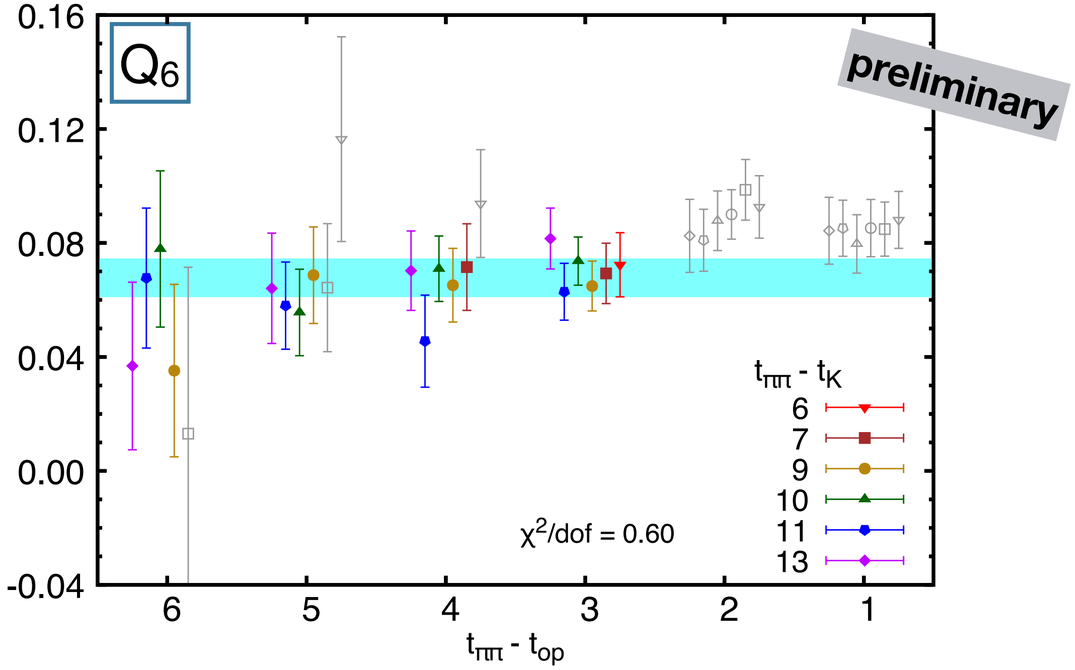
type 4'

Effective matrix elements ($\Delta I = 1/2$)

- Plateau appears
- : An example of correlated fit result with

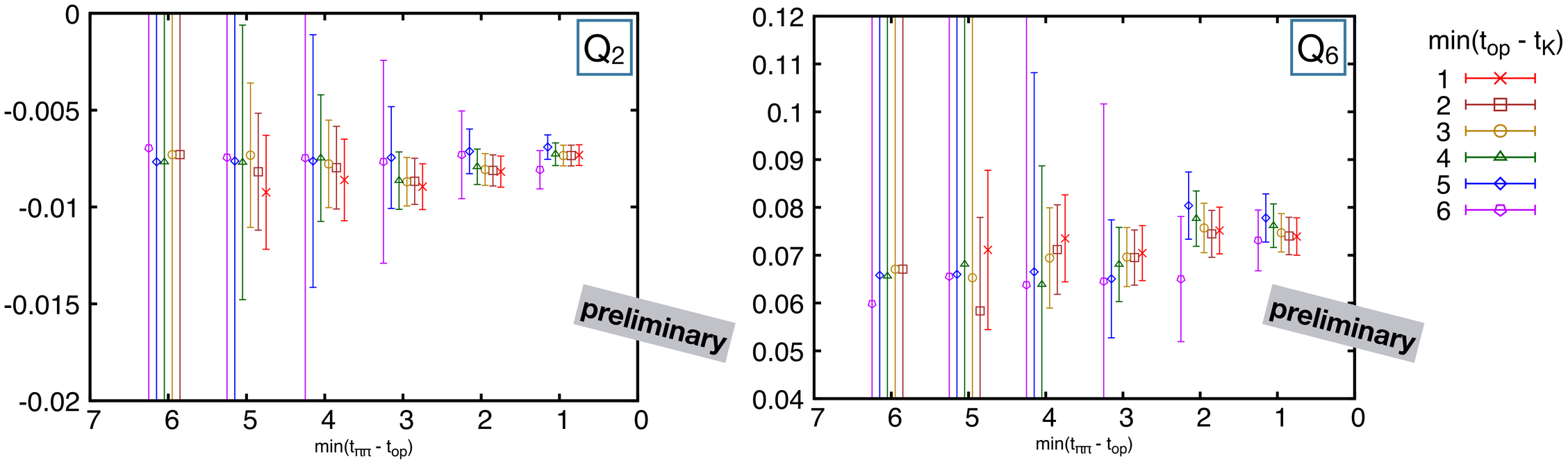
  $t_{op}-t_K \geq 3$ && $t_{\pi\pi}-t_{op} \geq 3$ (colored filled data points)





Fit results (ΔI = 1/2)

- **Fit range-dependence**

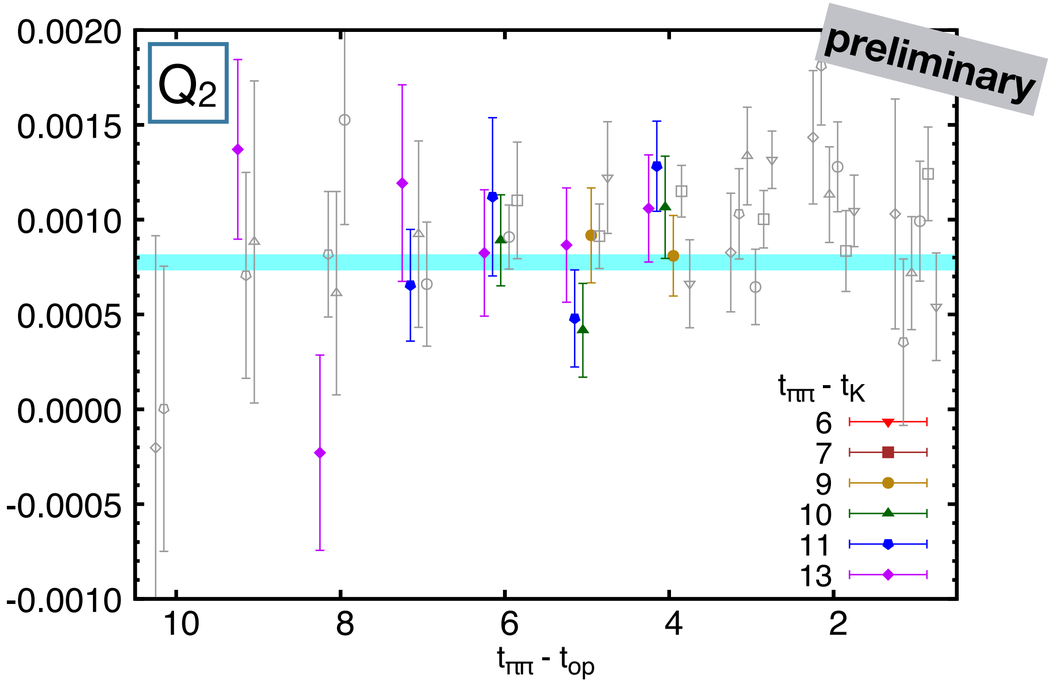


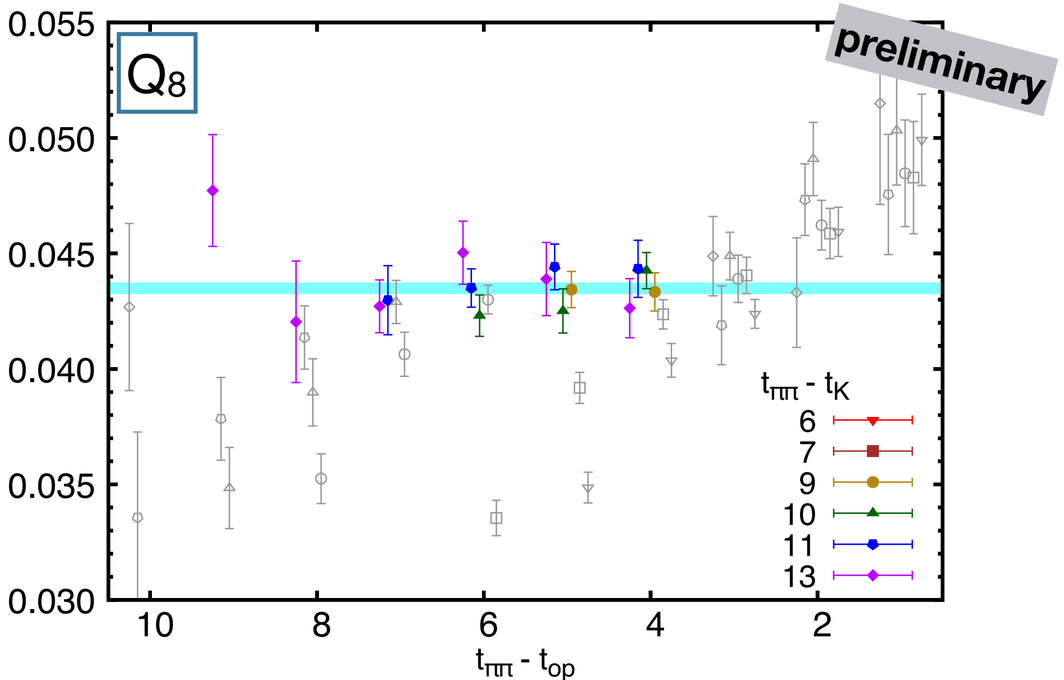
- No significant dependence on fit range for $\min(t_{op}-t_K)$, $\min(t_{\pi\pi}-t_{op}) \geq 3$

Effective matrix elements ($\Delta I = 3/2$)

- Statistically higher precision → excited-state contamination more visible for Q_8
- : An example of correlated fit result with

  $t_{op}-t_K \geq 4$ && $t_{\pi\pi}-t_{op} \geq 4$ (colored filled data points)





Precision performance

	32^3 G-parity BC (previous work)	24^3 Periodic BC	32^3 Periodic BC (w/o AMA correction)
# of configurations	741	258	107
$\Delta I = 1/2$ ME via Q_2^{lat}	10%	14%	14%
$\Delta I = 1/2$ ME via Q_6^{lat}	6.5%	8.9%	11%
Re A_0	11%	13%	14%

Error %

Preliminary

- Good precision performance of PBC (ME with excited-state ππ) compared to G-parity BC calculation (ME with ground-state ππ)

Summary

- Purpose
  - New independent calculation of $K \to \pi\pi$ decays
  - to get measure of direct CPV
  - Periodic-BC study gives prospect of introducing QED/IB effects

- On-shell final state = 1st excited state
  - Interesting challenge to extract signal of excited state
  - GEVP works
  - Good precision seen

- First results being finalized
  - 24^3 paper expected to be released in a few month