

Progress towards an improved lattice calculation of Standard Model direct CP-violation in kaon decays

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Motivation

- Likely explanation for matter/antimatter asymmetry in Universe, baryogenesis, requires violation of CP.
- Amount of CPV in Standard Model appears too low to describe measured M/AM asymmetry: tantalizing hint of new physics.
- Direct CPV first observed in late 90s at CERN (NA31/NA48) and Fermilab (KTeV) in $K^0 \rightarrow \pi\pi$:

$$\eta_{00} = \frac{A(K_L \rightarrow \pi^0\pi^0)}{A(K_S \rightarrow \pi^0\pi^0)}, \quad \eta_{+-} = \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)}.$$

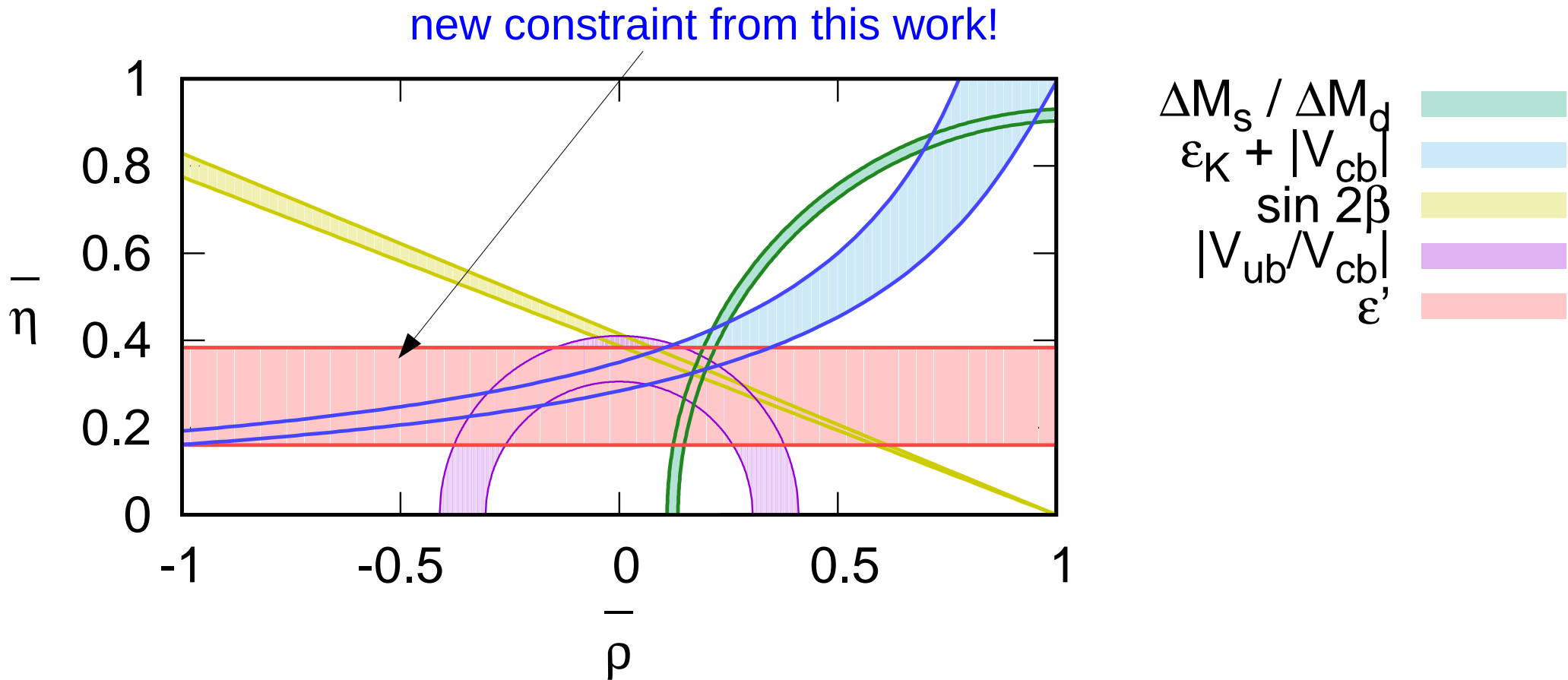
$$\text{Re}(\epsilon'/\epsilon) \approx \frac{1}{6} \left(1 - \left| \frac{\eta_{00}}{\eta_{\pm}} \right|^2 \right) = 16.6(2.3) \times 10^{-4} \quad (\text{experiment})$$

measure of direct CPV

measure of indirect CPV

- **Small size of ϵ' makes it particularly sensitive to new direct-CPV introduced by many BSM models.**
- Looking for deviations from experiment may help shed light on origin of M/AM asymmetry.

- A Standard Model prediction of ε' also provides a new horizontal band constraint on CKM matrix in ρ - η plane:



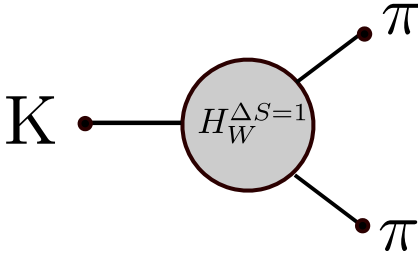
- While underlying weak process occurs at high energies $\sim M_W = 80$ GeV, $K \rightarrow \pi\pi$ decays receive large corrections from low-energy hadronic physics $O(\Lambda_{\text{QCD}}) \sim 250$ MeV.
- Lattice QCD is the only known *ab initio*, **systematically improvable** technique for studying non-perturbative QCD.

Overview of calculation

$$\begin{aligned}
 A(K^0 \rightarrow \pi^+\pi^-) &= \sqrt{\frac{2}{3}}A_0e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2e^{i\delta_2}, \\
 A(K^0 \rightarrow \pi^0\pi^0) &= \sqrt{\frac{2}{3}}A_0e^{i\delta_0} - 2\sqrt{\frac{1}{3}}A_2e^{i\delta_2}.
 \end{aligned}
 \rightarrow \epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}} \left(\frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right)$$

$\omega = \text{Re}A_2/\text{Re}A_0$
 $\pi\pi$ phase shifts
I=2 decay I=0 decay

- Hadronic energy scale $\ll M_W$ – use weak effective theory (3 flavors)



LL finite-volume correction

$$A^I = F \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{i=1}^{10} \sum_{j=1}^7 \left[(z_i(\mu) + \tau y_i(\mu)) Z_{ij}^{\text{lat} \rightarrow \overline{\text{MS}}} M_j^{I, \text{lat}} \right]$$

perturbative Wilson coeffs. $M_j^{I, \text{lat}} = \langle (\pi\pi)_I | Q_j | K \rangle$ (lattice)

renormalization matrix (mixing)
 Use RI-SMOM
 convert to MSbar
 perturbatively

10 effective four-quark operators

$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} = 0.0014606 + \mathbf{0.00060408i}$$

Imaginary part solely responsible for CPV
(everything else is pure-real)

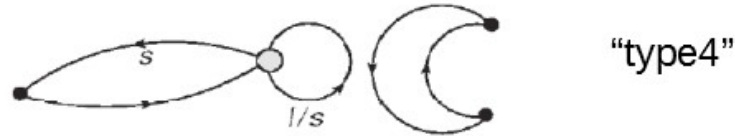
$I=2$ calculation

[Phys.Rev. D91 (2015) no.7, 074502]

- A_2 can be measured very precisely using “standard” lattice techniques.
- Most recent result (2015):
 - Computed with large, $\sim (5.5 \text{ fm})^3$ volumes
 - Physical quark masses
 - Two lattice spacings (2.36 GeV and 1.73 GeV) \rightarrow **Continuum limit taken.**
- $<1\%$ statistical error!
- 10% and 12% total errors on $\text{Re}(A_2)$ and $\text{Im}(A_2)$ resp.
- Dominant sys. errors due to truncation of PT series in computation of renormalization and Wilson coefficients.

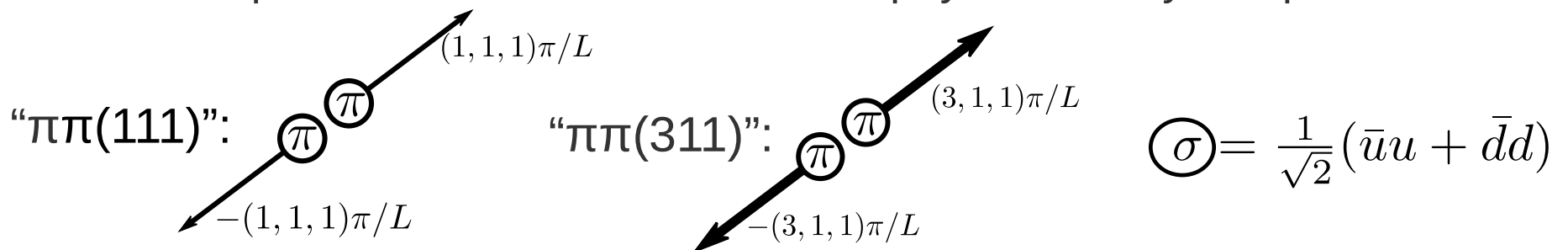
I=0 Calculation

- A_0 is more difficult than A_2 , primarily because I=0 $\pi\pi$ state has *vacuum quantum numbers*.
- “Disconnected diagrams” dominate statistical noise



2020 calculation [[arXiv:2004.09440](https://arxiv.org/abs/2004.09440)]

- Physical quark masses on single, coarse lattice ($a^{-1} = 1.38$ GeV) but with large $(4.6 \text{ fm})^3$ physical volume to control FV errors.
- G-parity boundary conditions remove dominant unphysical contribution from stationary $\pi\pi$ state.
- $3\times \pi\pi$ operators allow clean isolation of physical decay component.



- Achieved **O(10%) statistical precision** on both $\text{Re}(A_0)$ and $\text{Im}(A_0)$!
- **O(20%) systematic errors**

Current result for ϵ'

$$\begin{aligned} \operatorname{Re} \left(\frac{\epsilon'}{\epsilon} \right) &= \operatorname{Re} \left\{ \frac{i\omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2}\epsilon} \left[\frac{\operatorname{Im}A_2}{\operatorname{Re}A_2} - \frac{\operatorname{Im}A_0}{\operatorname{Re}A_0} \right] \right\} \\ &= 0.00217(26)(62)(50) \end{aligned}$$

stat sys IB + EM

Consistent with experimental result:

$$\operatorname{Re}(\epsilon'/\epsilon)_{\text{expt}} = 0.00166(23)$$

- In order to match precision of experiment we must focus on addressing the systematic errors.
- Primary systematic errors:
 - Wilson coefficients: 12%
 - Isospin breaking + electromagnetic effects: 23%
 - Finite lattice spacing: 12%

Wilson coefficients

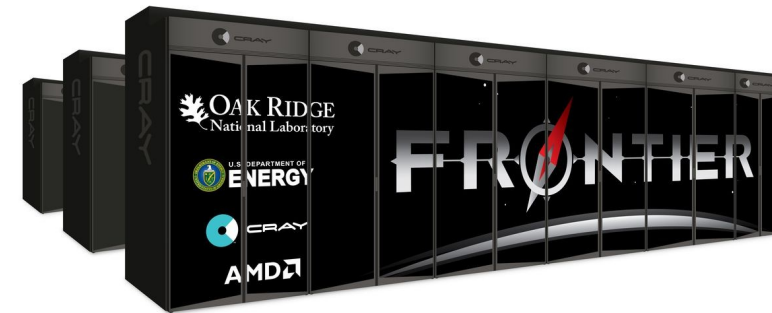
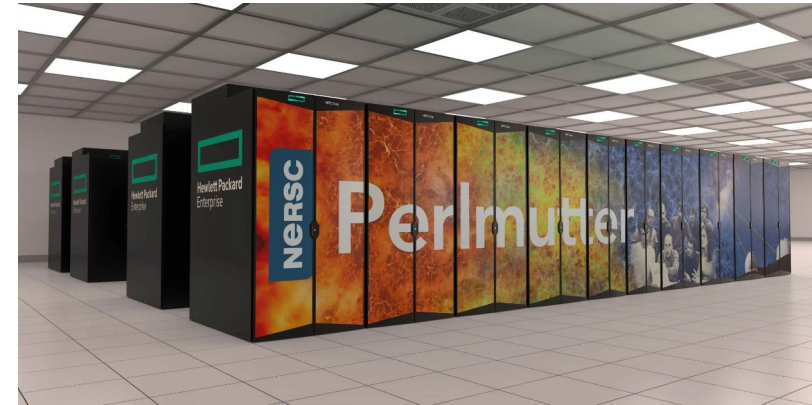
- Perturbative Wilson coefficients incorporate high-energy physics and running down to 3-flavor theory.
- Currently computed in $\overline{\text{MS}}$ scheme to NLO [Buchalla *et al.* *Rev. Mod. Phys.* 68, 1125]
- Matching to renormalized lattice calculation is performed at high energy (4 GeV)
- However PT is still used internally to cross the charm threshold at $m_c=1.3$ GeV \longrightarrow significant systematic error $\sim 12\%$
- Progress towards a complete NNLO calculation is underway which can be expected to significantly improve this error. [Cerda-Sevilla *et al.* *Acta Phys.Polon.B* 4 (2018) 1087-1096]
- We are also investigating a direct non-perturbative calculation of the $4f \rightarrow 3f$ matching
 - Directly compare 4f and 3f matrix elements on a 3f background gauge field
 - Position space technique reduces mixing with irrelevant operators
 - Preliminary demonstration on $16^3 \times 32$, $a^{-1}=1.78$ GeV DWF ensemble shows promising potential in approach [M. Tomii, *PoS LATTICE2019* (2020)]
- In longer term a direct 4f calculation will largely eliminate this error but requirement for high statistics, large volume and fine lattice spacing make this presently unfeasible.

Isospin breaking + EM effects

- Simulation does not include isospin breaking or EM effects.
- Typically these effects are $O(1\%)$
- However $\epsilon' \propto \text{Re}(A_2)/\text{Re}(A_0) \approx 1/22.45$
small due to “ $\Delta I=1/2$ rule”, a non-perturbative QCD effect.
- Thus relative of EM+IB on A_2 and hence ϵ' expected $O(20\%)$.
- Current best determination uses NLO χ PT and $1/N_c$ expansion, predicts 23% correction → separate sys err. [Cirigliano *et al*, JHEP 02 (2020) 032]
- Developing approaches to measuring using lattice QCD. Challenging:
 - Need to reconcile long-distance nature of QED with the local interaction assumptions of the Luscher FV formalism
 - The mixing of final state two-pions by isospin breaking
 - Soft-photon emission introducing additional final states
- Promising start in this direction: A complete demonstration of calculation of (dominant) Coulomb correction to $\pi^+\pi^+$ scattering [Christ *et al*. PRD 106 (2022) 1, 014508]
- Additional challenges remain including computing transverse radiation contribution.

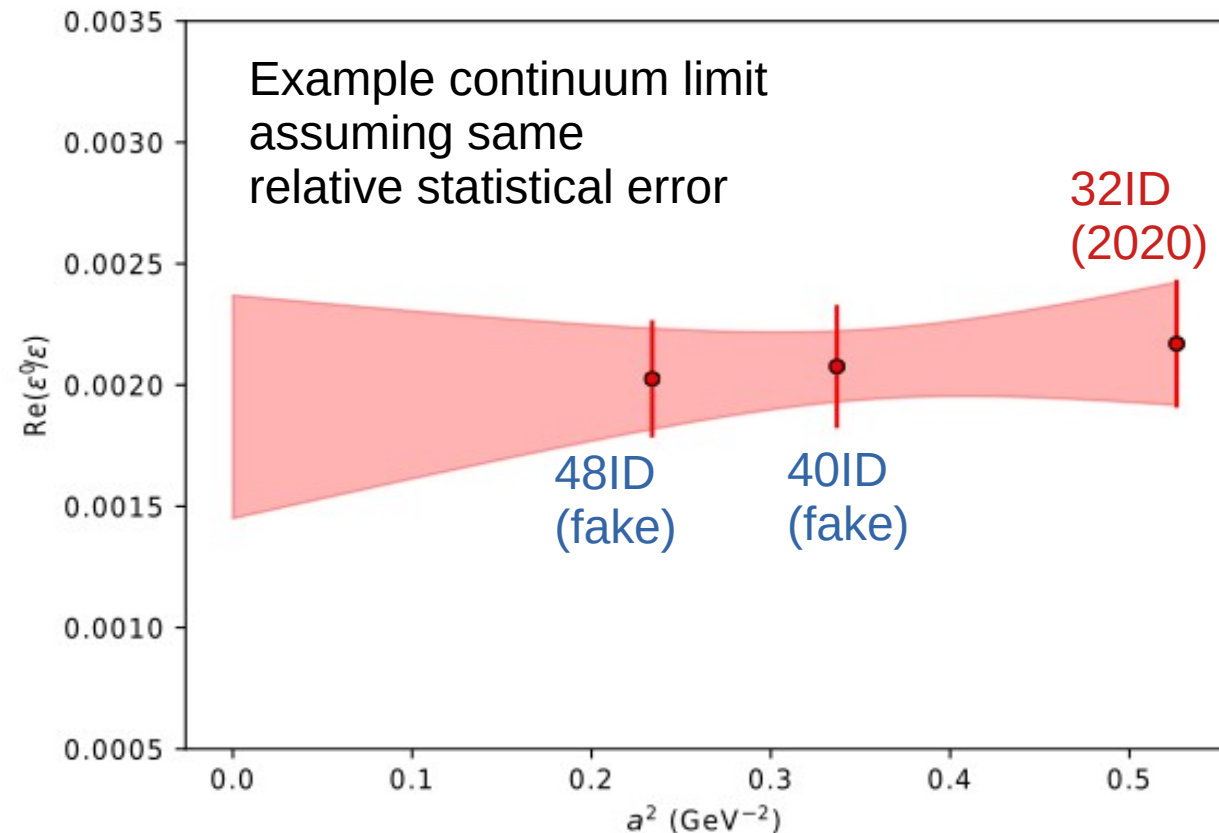
Discretization errors

- Primary pure-lattice error ($\sim 12\%$) and “easiest” to address
- Currently estimated using scaling of $l=2$ ops. but may be significant “error on the error”.
- Exploit new exascale and pre-exascale hardware to perform **continuum limit**.
- Extensive effort in porting measurement code to Intel, NVidia and AMD GPUs almost complete.
- G-parity BCs requires us to also generate new lattices.



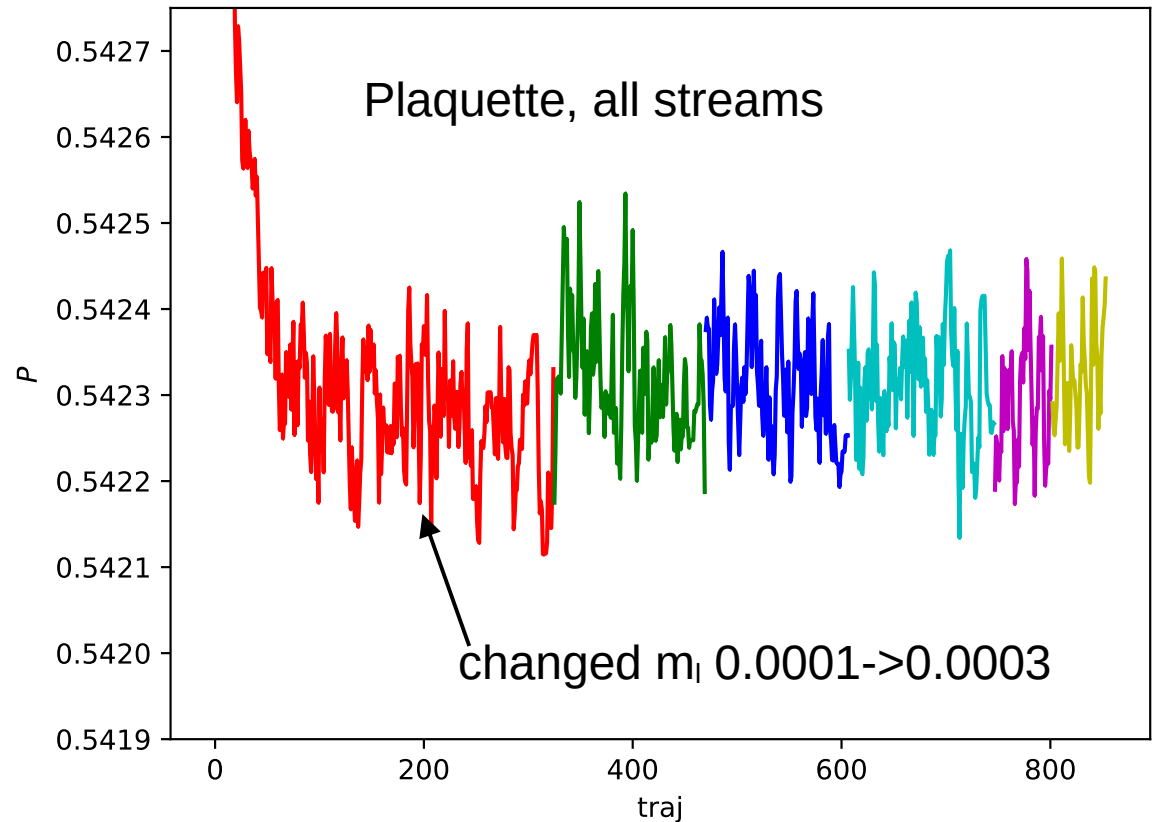
Continuum extrapolation lever-arm

- Utilizing and expanding on HMC capacities of Grid framework to run efficiently on GPUs
- Two new lattices:
 - **40ID**: $40^3 \times 64 \times 12$ DWF+ID
 $a^{-1} = 1.723$ GeV
 - **48ID**: $48^3 \times 64 \times 12$ DWF+ID
 $a^{-1} = 2.068$ GeV
- Physical pion masses
- GPBC in 3 directions
- Same physical volume $(4.6 \text{ fm})^3 \rightarrow$
 $\pi\pi$ energy remains the same as before and **interaction remains physical**.



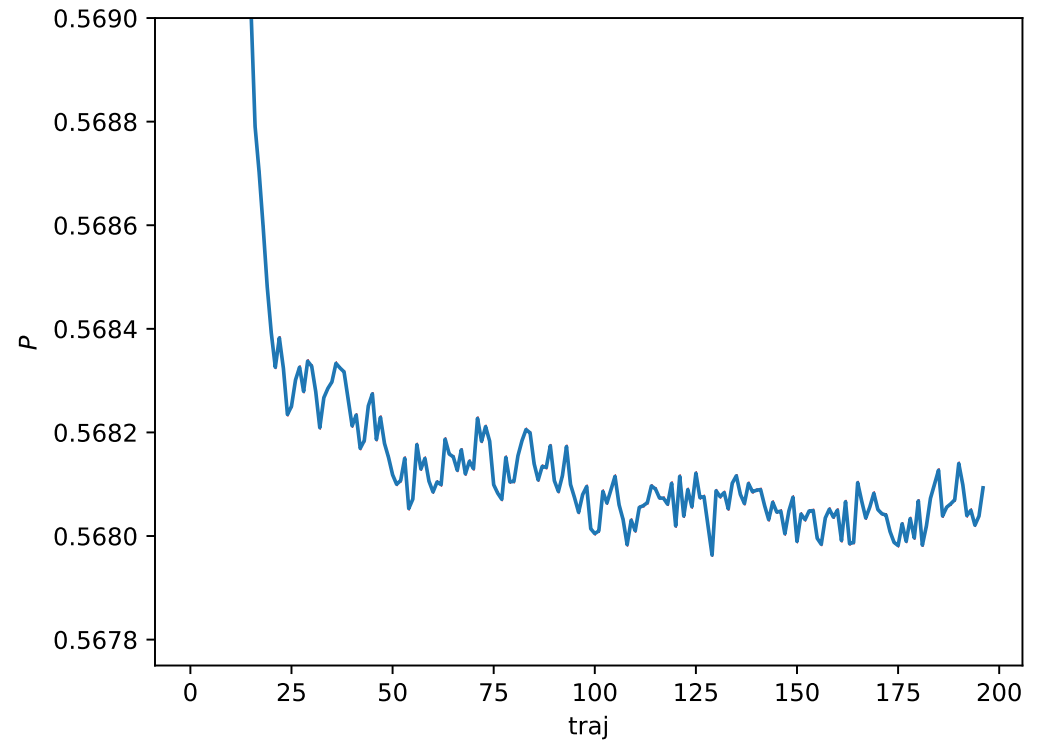
401D status

- Tuning and running performed on Perlmutter machine.
- Thermalized with 1 initial stream.
- 5 additional streams started from thermalized configs.
- Job time ~6hrs on 32 nodes (128 NVidia A-100 GPUs)
- Severely hampered by Slingshot 10 network
 - Expect significant improvements with new phase 2 (Slingshot 11) network.



48ID status

- Tuning progress hampered by wall-clock time limits on Perlmutter and weak network
- Expect significant improvement with Slingshot 11 network



$K \rightarrow \pi\pi$ *without G-parity*

- Independent calculation of ϵ' using multiple operators to extract on-shell matrix elements as excited-state contributions in a periodic lattice is well under way.
 - Avoid complications of using G-parity BCs
 - Uses existing MDWF+I ensembles with physical pion masses
 - 2 lattice spacings allowing continuum limit

[See Masaaki Tomii's talk - next!]

Conclusions

- Result for ε' consistent with experimental value but total error is still $\sim 3.6x$ that of experiment.
- ε' remains a promising avenue to search for new physics, but greater precision is required.
- RBC & UKQCD are working to improve all 3 primary systematic errors:
 - Attempt to address EM+IB errors through lattice calculation (hard!).
 - Investigating direct lattice calculation of 3f-4f matching in Wilson coefficients.
 - (Potential for NNLO calculation of EM+IB in near future may reduce urgency.)
 - Addressing discretization error by introducing two finer lattices
 - Independent calculation with different systematics using periodic BCs.