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Calculation of Hyperon Transition Form Factors from the Feynman-Hellman method

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QCDSF-UKQCD-CSSM Collaboration

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SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

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CKM matrix

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97401(11) & 0.22650(48) & 0.00361(11) \\ 0.22636(48) & 0.97320(11) & 0.04053(83) \\ 0.00854(23) & 0.03978(82) & 0.99917(04) \end{bmatrix}$$

PDG Review of particle physics 2020

- SM requires unitarity: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
- $|V_{us}|$ can be constrained by
  - (Semi-) leptonic Kaon decays
  - Semi-leptonic hyperon decays
- Transition matrix element for semi-leptonic hyperon decay:

$$T = \frac{G_F}{\sqrt{2}} V_{us} \left[\langle B' | \bar{u} \gamma_\mu \gamma^5 s | B \rangle - \langle B' | \bar{u} \gamma_\mu s | B \rangle \right] \bar{l} \gamma^\mu (1 - \gamma^5) \nu_l$$

From experimental decay widths

Axial-vector

Vector

- Lattice QCD determinations can improve on phenomenological form factor values

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Form Factors

$$\langle B'|\bar{u}\gamma_\mu s|B\rangle = \gamma_\mu f_1^{BB'}(Q^2) + \sigma_{\mu\nu}q_\nu \frac{f_2^{BB'}(Q^2)}{M_B + M_{B'}} - iq_\mu \frac{f_3^{BB'}(Q^2)}{M_B + M_{B'}}$$

- Last term is zero for B=B' due to the Conserved Vector Current (CVC)
- Mainly interested in $f_1(0)$ for constraining V_{us}.
- Usually calculated by using three-point correlation functions
  - Need a lot of configurations to keep signal-to-noise ratio high and achieve ground state dominance
- Feynman- Hellmann approach offers an alternative which relies only on two-point correlation functions

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$\Sigma^- \to n$ Transition

- Consider the action: $S = S_g + \int_x (\bar{u}, \bar{s}) \begin{pmatrix} D_u & -\lambda\mathcal{T} \\ -\lambda\mathcal{T}' & D_s \end{pmatrix} \begin{pmatrix} u \\ s \end{pmatrix} + \int_x \bar{d}\, D_d\, d$

  where $\mathcal{T}(x, y; \vec{q}) = \gamma e^{i\vec{q}\cdot\vec{x}} \delta_{x,y}$

- Construct a matrix of correlation functions:

$$C_{\lambda B'B} = \begin{pmatrix} C_{\lambda\Sigma\Sigma} & C_{\lambda\Sigma N} \\ C_{\lambda N\Sigma} & C_{\lambda NN} \end{pmatrix}_{B'B}$$

- This can be diagonalised and the energies related to the matrix element

$$\Delta E_{\lambda\Sigma N} = \sqrt{(E_N(\vec{q}) - M_\Sigma)^2 + 4\lambda^2 \left| \langle N(\vec{q})|\bar{u}\gamma_4 s|\Sigma(\vec{0})\rangle \right|^2}$$

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$\Sigma^- \to n$ Transition

$$C_{\lambda B'B} = \begin{pmatrix} C_{\lambda\Sigma\Sigma} & C_{\lambda\Sigma N} \\ C_{\lambda N\Sigma} & C_{\lambda NN} \end{pmatrix}_{B'B}$$

- Each correlator is built from a Green's function:

$$\begin{pmatrix} G_{uu} & G_{us} \\ G_{su} & G_{ss} \end{pmatrix} = \begin{pmatrix} (\mathcal{M}^{-1})_{uu} & (\mathcal{M}^{-1})_{us} \\ (\mathcal{M}^{-1})_{su} & (\mathcal{M}^{-1})_{ss} \end{pmatrix}$$

- Where:

$$\begin{aligned} G^{(uu)} &= (1 - \lambda^2 D_u^{-1} \mathcal{T} D_s^{-1} \gamma_5 \mathcal{T}^\dagger \gamma_5)^{-1} D_u^{-1} \\ G^{(ss)} &= (1 - \lambda^2 D_s^{-1} \gamma_5 \mathcal{T}^\dagger \gamma_5 D_u^{-1} \mathcal{T})^{-1} D_s^{-1} \end{aligned}$$

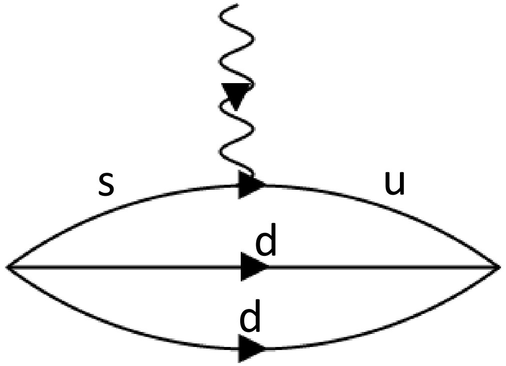
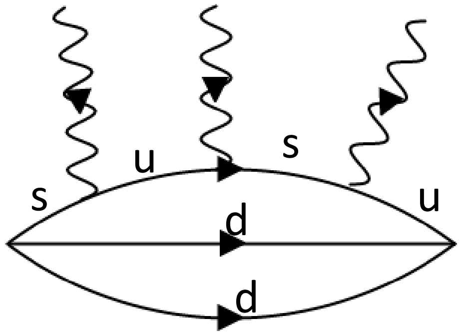
→ Problem: inversion within another inversion

$$\begin{aligned} G^{(us)} &= \lambda D_u^{-1} \mathcal{T} G^{(ss)}\,, \\ G^{(su)} &= \lambda D_s^{-1} \gamma_5 \mathcal{T}^\dagger \gamma_5 G^{(uu)} \end{aligned}$$

Iterative Green's functions

- We can expand the Green's functions for small lambda
- Will give the exact result as n goes to infinity.
- We will consider up to order $\mathcal{O}(\lambda^4)$
- This allows changing the value of lambda after the inversions!

$$
\begin{aligned}
G^{(uu)}_{2n+2} &= D_u^{-1} + \lambda^2 D_u^{-1}\mathcal{T} D_s^{-1}\gamma_5\mathcal{T}^\dagger\gamma_5 G^{(uu)}_{2n}\,, \\
G^{(ss)}_{2n+2} &= D_s^{-1} + \lambda^2 D_s^{-1}\gamma_5\mathcal{T}^\dagger\gamma D_u^{-1}\mathcal{T} G^{(ss)}_{2n}\,,
\end{aligned}
$$

$G_3^{(us)}$ = λ  + λ^3 

Kinematics

- Choose the Sigma to be at rest and change the momentum of the nucleon
- Choose the operator to be the vector current γ_4
- For hyperons with quasi-degenerate energies the shift due to a perturbation in the action lambda is

$$\Delta E_{\lambda\Sigma N} = \sqrt{\left(E_N(\vec{q}) - M_\Sigma\right)^2 + 4\lambda^2 \left| \langle N(\vec{q})|\bar{u}\gamma_4 s|\Sigma(\vec{0})\rangle \right|^2}$$

- When the energy of the nucleon equals the mass of the Sigma, this will be linear in lambda.
  - How far from the degenerate energy point can we make this work?
    - At Q^2=0?

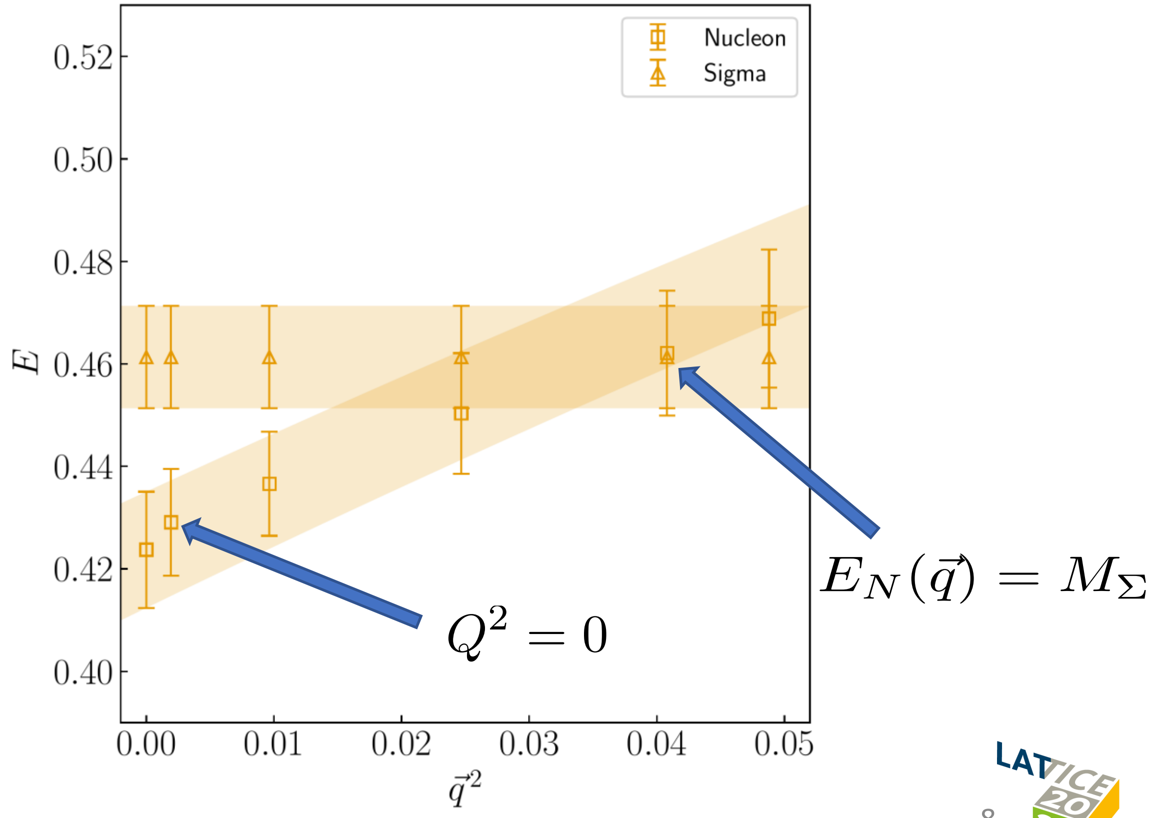
(Partially) Twisted Boundary Conditions

$$Q^2 = -(M_\Sigma - E_N(\vec{q}))^2 + \vec{q}^2$$

- Momentum on the lattice is quantised
  - how do we get to the energy-degenerate point?
- Twisted boundary conditions add a complex phase to the boundary conditions
  - Gives lattice correlators any momentum

$$q(\vec{x} + N_s\vec{e}_i, t) = e^{i\theta_i} q(\vec{x}, t)$$

- Using twisted boundary conditions we can tune the nucleon energy to be very close to the mass of the Sigma baryon.
- Can also be used to get form factors at exactly $Q^2=0$



Generalized EigenValue Problem (GEVP)

- Diagonalise the matrix

$$C_{\lambda B'B} = \begin{pmatrix} C_{\lambda\Sigma\Sigma} & C_{\lambda\Sigma N} \\ C_{\lambda N\Sigma} & C_{\lambda NN} \end{pmatrix}_{B'B}$$

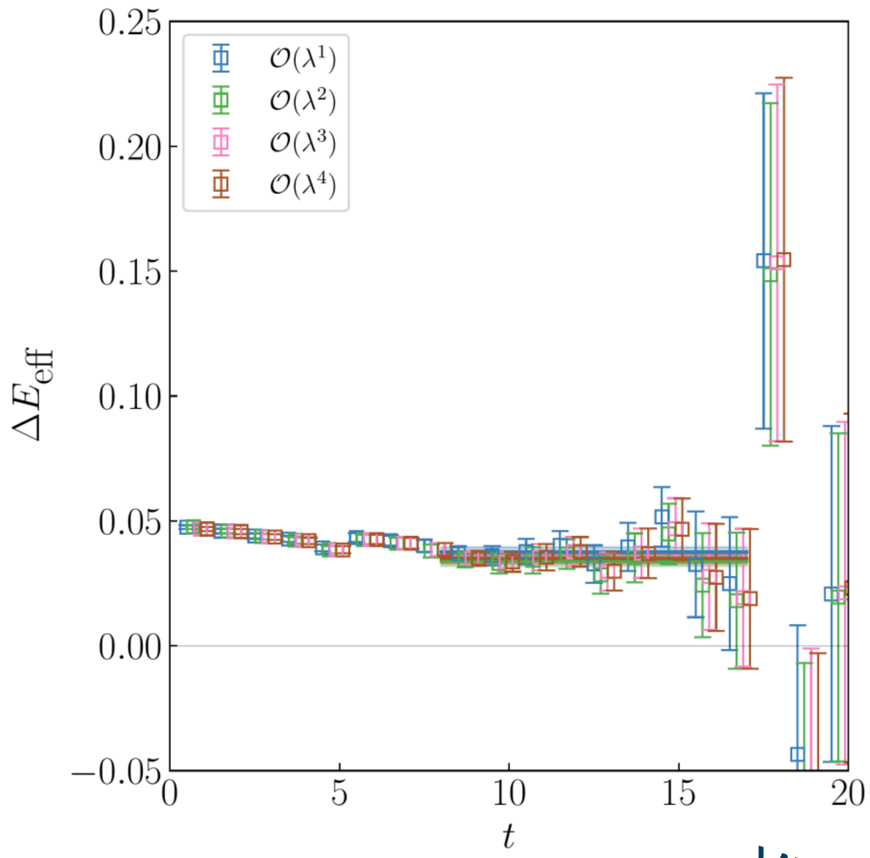
  - Gives two eigenvectors and eigenvalues
  - Eigenvalues related to the energy

- Use the eigenvectors to project out two correlation functions:

$$C_\lambda^{(i)}(t) = v^{(i)\dagger} C_\lambda(t) u^{(i)}\,, \quad i = \pm$$

- Take the ratio of the two correlators and fit to the energy shift ΔE.

$$R_\lambda(t;\vec{0},\vec{q}) = \frac{C_\lambda^{(-)}(t;\vec{0},\vec{q})}{C_\lambda^{(+)}(t;\vec{0},\vec{q})} \overset{t\gg 0}{\propto} e^{-\Delta E_\lambda(\vec{0},\vec{q})t}$$



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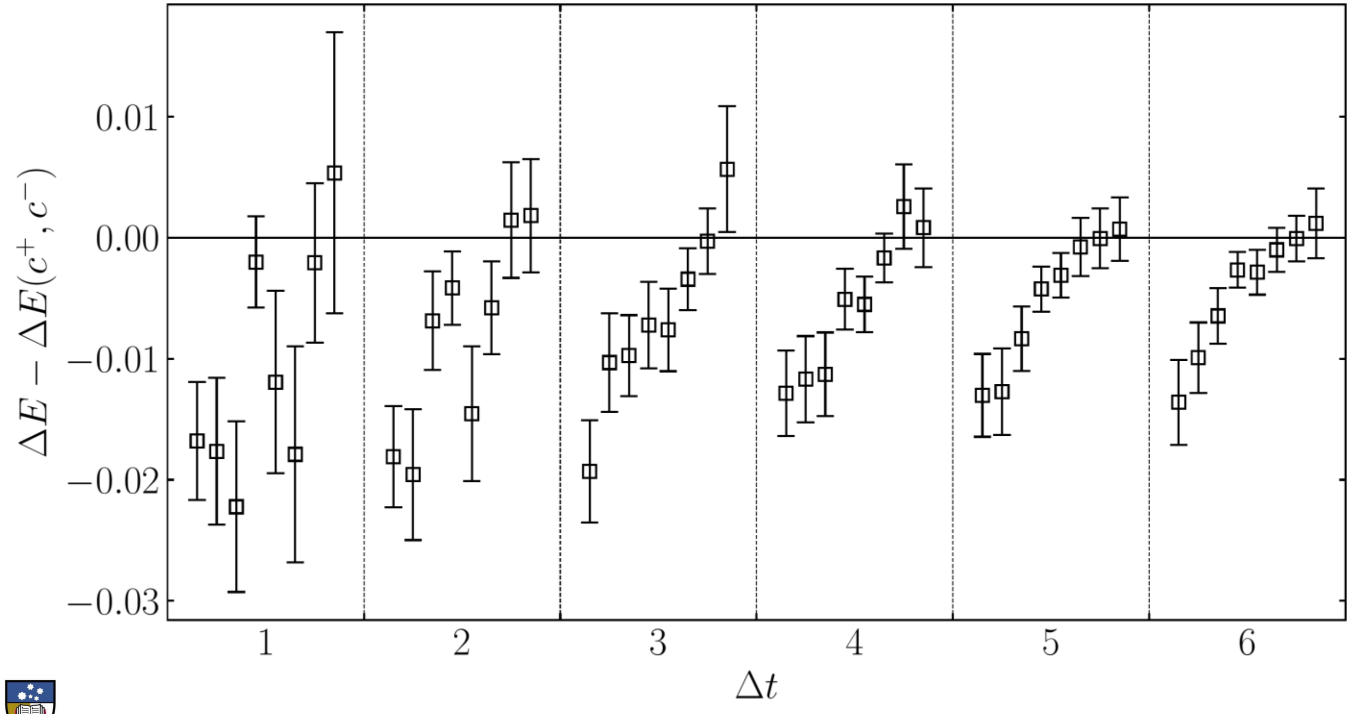
GEVP Stability

GEVP depends on two parameters (t_0 & Δt):

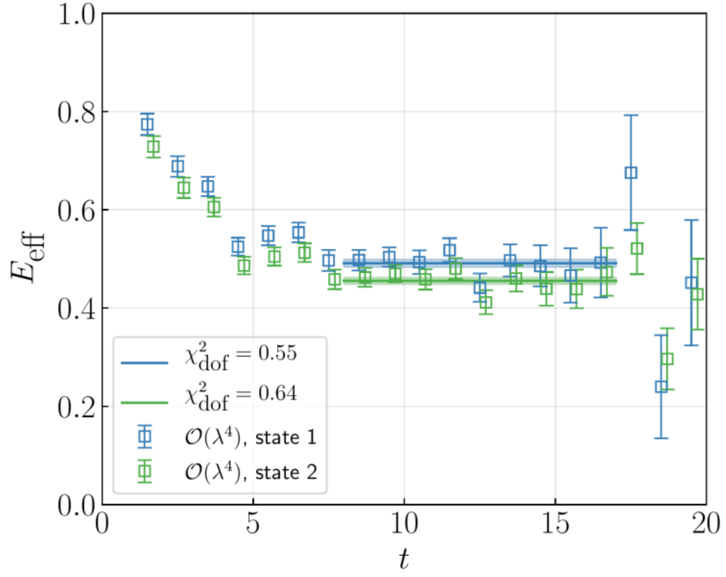
$$C_\lambda^{-1}(t_0)C_\lambda(t_0+\Delta t)e^{(i)} = c^{(i)}e^{(i)}$$

Stable under GEVP parameters?

- Do the GEVP for many value of t_0 and Δt
- Calculate the value of $\Delta E(c^+,c^-)$ from the eigenvalues
- Compare with ΔE from the fit to the ratio of correlators
- For each Δt we show results from t_0=1-8



Ground state saturated:



Result from GEVP are stable in range $\Delta t \geq 4$ and $t_0 \geq 6$

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Lattice details

- 32^3x64 lattice size
- Lattice spacing a=0.074fm
- N_f = 2 + 1, O(a)-improved clover Wilson fermions
- Up and down quark are degenerate
- O(500) configurations used for each choice of momentum

run #	θ_2/π	$\vec{q}^2$	E_N	$M_\Sigma - E_N$	$Q^2[\mathrm{GeV}^2]$
1	0.0	0.0	0.424(11)	0.0366(33)	-0.0095
2	0.448	0.0019	0.429(10)	0.0351(35)	0.0048
3	1	0.0096	0.437(10)	0.0301(42)	0.0620
4	1.6	0.0247	0.450(12)	0.0182(57)	0.1732
5	2.06	0.0408	0.462(12)	0.0030(69)	0.2901
6	2.25	0.0488	0.469(13)	-0.0037(78)	0.3472

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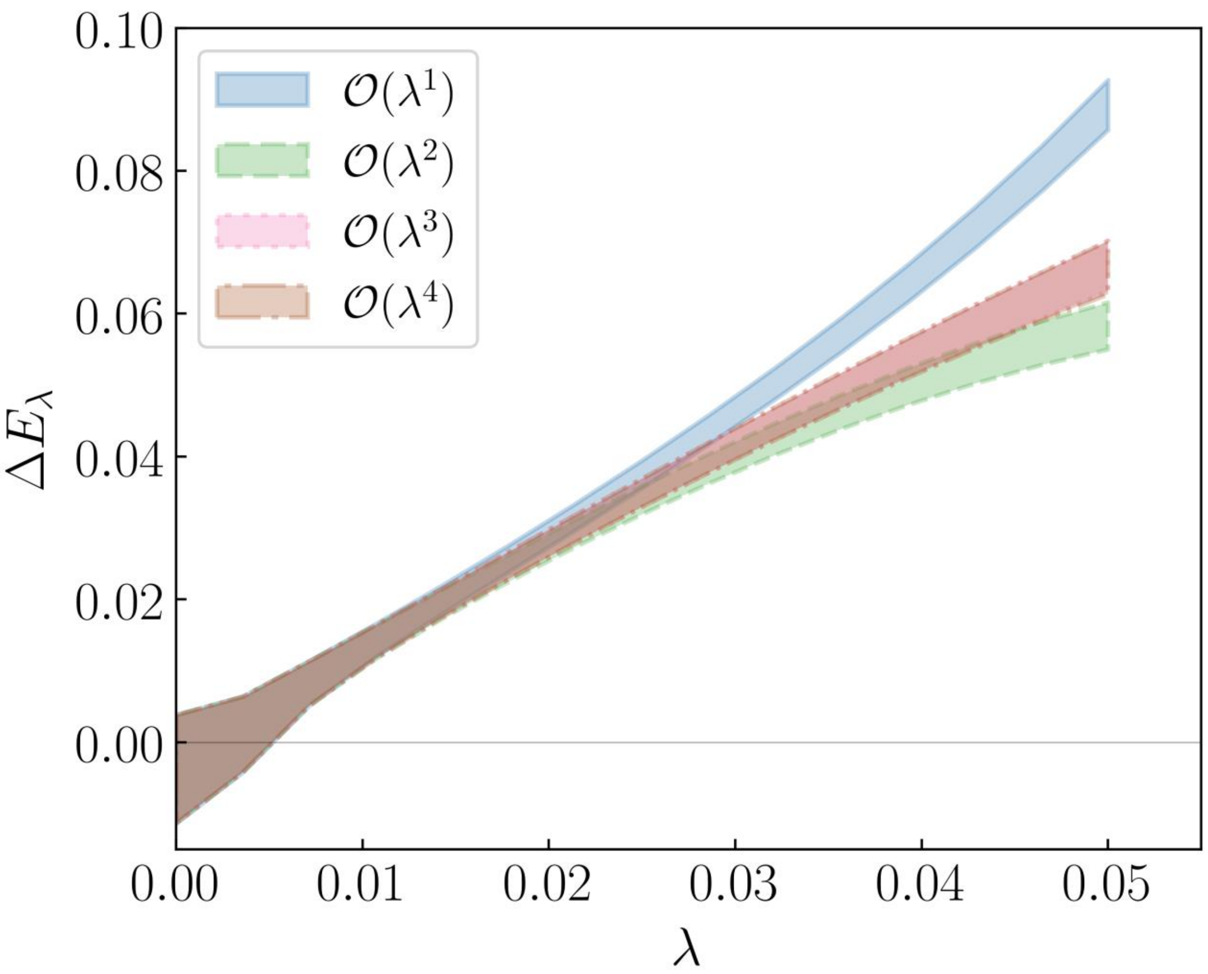
ΔE as a function of λ

$$R_\lambda(t;\vec{0},\vec{q}) = \frac{C_\lambda^{(-)}(t;\vec{0},\vec{q})}{C_\lambda^{(+)}(t;\vec{0},\vec{q})} \overset{t\gg 0}{\propto} e^{-\Delta E_\lambda(\vec{0},\vec{q})t}$$

- Iterative Method: higher orders in lambda increase the range over which our approximation holds
- We want to fit in the region where the dependence is linear
- Choose the region where the two highest order results agree
  - The expansion in λ holds here



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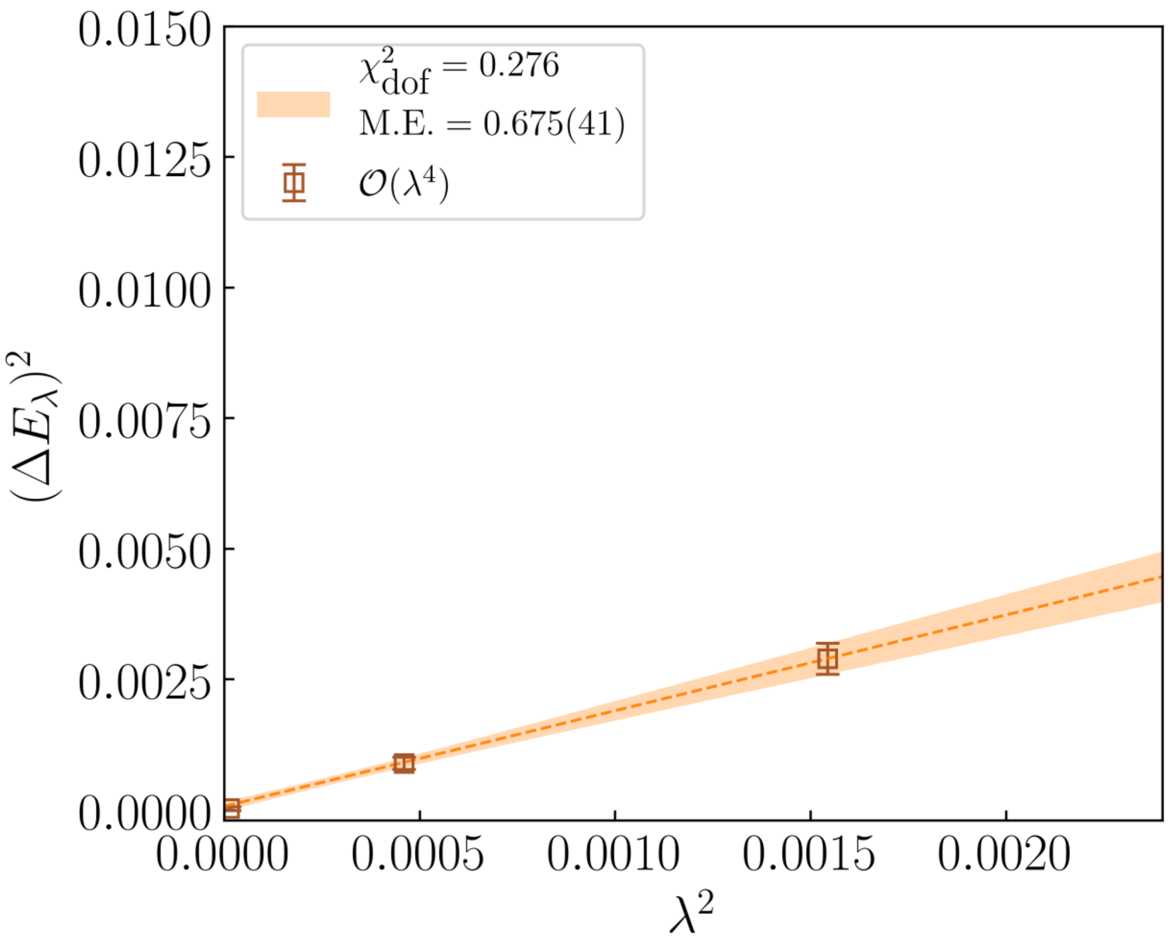
Matrix element fit

$$\Delta E_{\lambda\Sigma N} = \sqrt{(E_N(\vec{q}) - M_\Sigma)^2 + 4\lambda^2 \left| \langle N(\vec{q}) | \bar{u}\gamma_4 s | \Sigma(\vec{0}) \rangle \right|^2}$$

- Fit to data up to $\mathcal{O}(\lambda^4)$
- Values of ΔE for all lambda are very correlated
- We fit to the square of ΔE to ensure the values are positive
- We use three fit points which span the region where the dependence on lambda is linear



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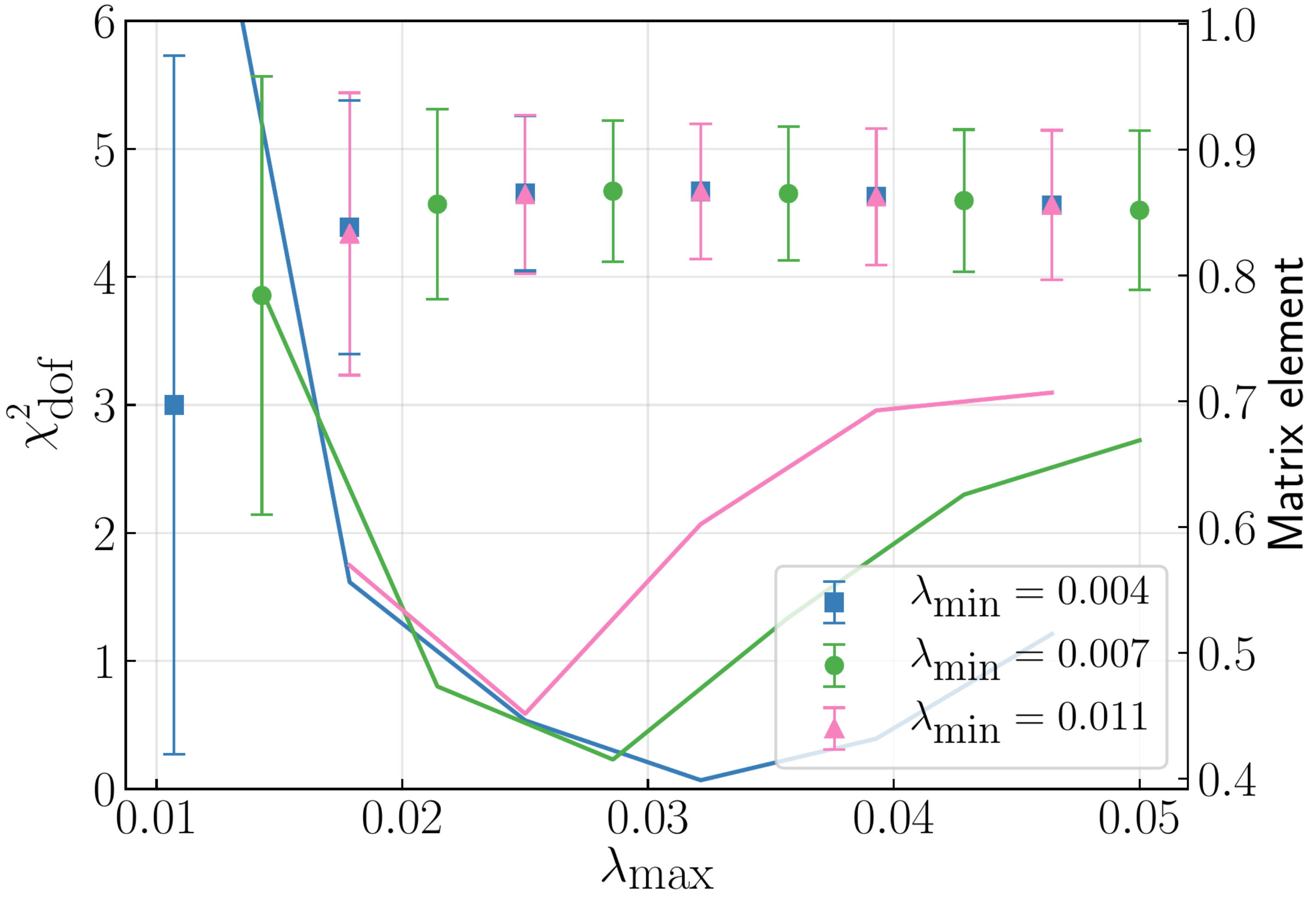
Fit Stability

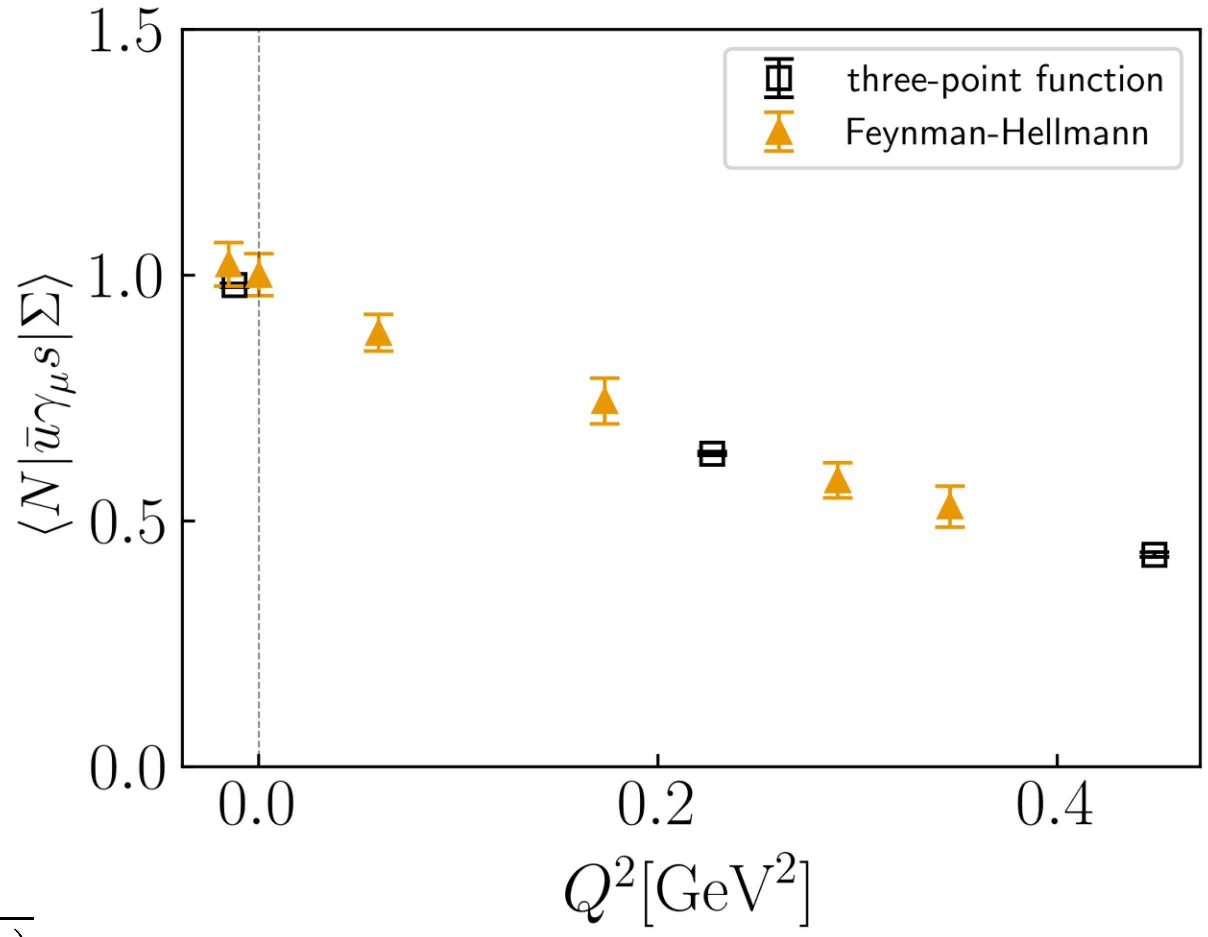
We fit over multiple ranges of lambda using three points for each fit

The results stabilise when the fit extends up to lambda>0.03



Matrix element results

Three-point function results are preliminary, using one source-sink separation and using higher statistics



$$\left\langle N(\vec{q})\middle|\bar{u}\gamma_4 s\middle|\Sigma(\vec{0})\right\rangle = \sqrt{2M_\Sigma(E_N(\vec{q})+M_N)}$$
$$\left(f_1^{\Sigma N}(Q^{f2}) + \frac{E_N(\vec{q})-M_N}{M_N+M_\Sigma}f_2^{\Sigma N}(Q^2) + \frac{E_N(\vec{q})-M_\Sigma}{M_N+M_\Sigma}f_3^{\Sigma N}(Q^2)\right)$$

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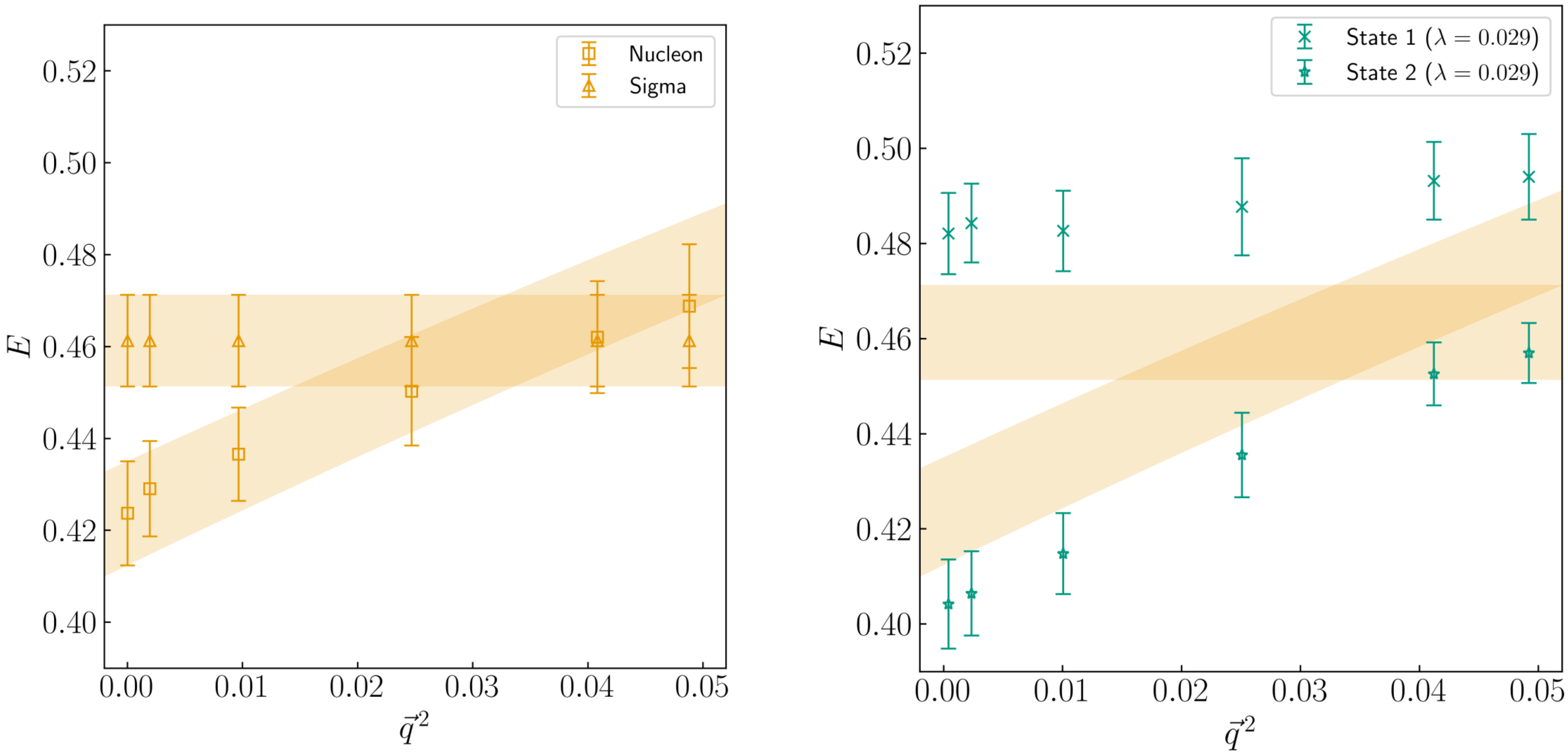
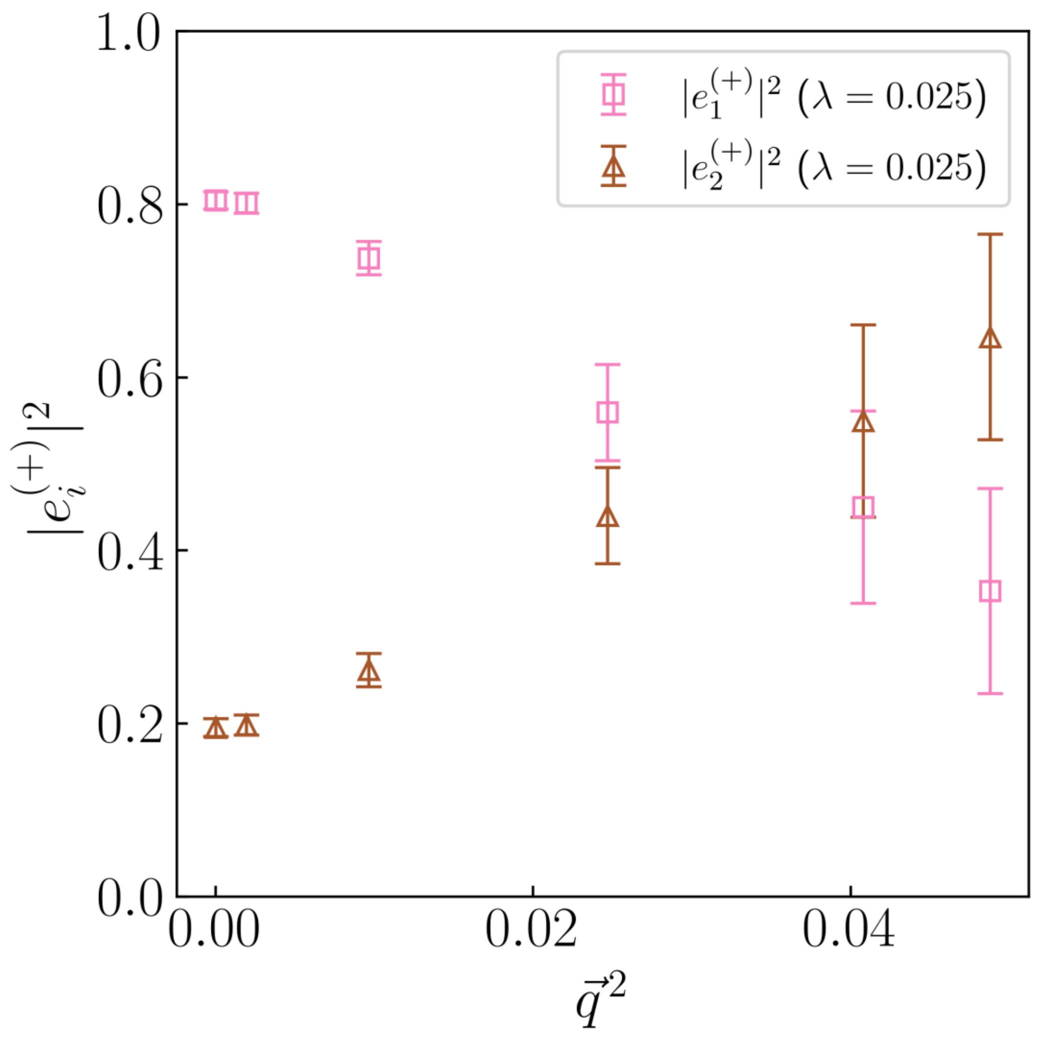
Avoided level crossing



Eigenvectors

- As in Roger's presentation, the eigenvectors show the mixing between the states once the interaction is turned on

$$\vec{e}^{(\pm)} = \begin{pmatrix} e_1^{(\pm)} \\ e_2^{(\pm)} \end{pmatrix}$$



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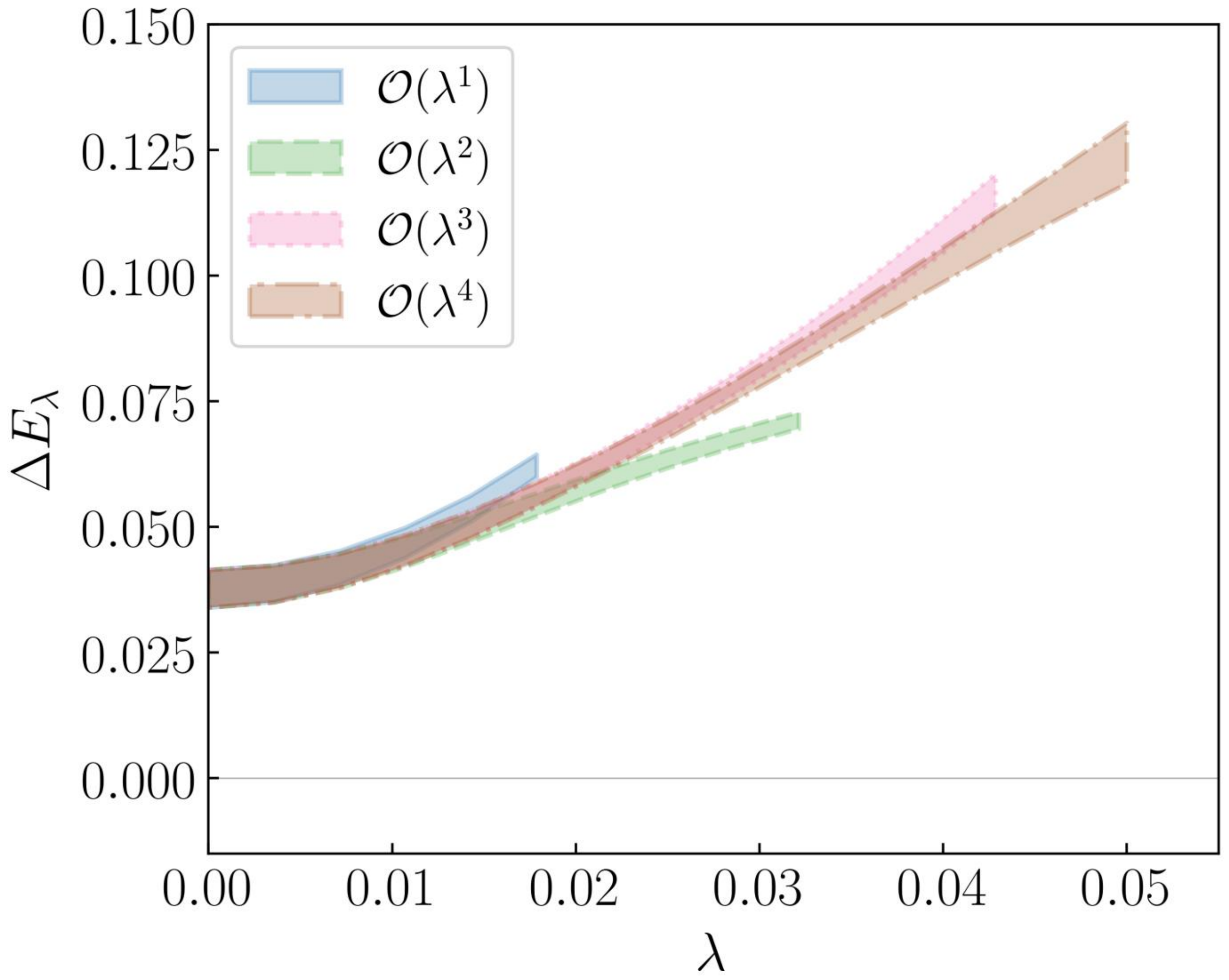
Conclusion

- The Feynman-Hellman method can be used to calculate hyperon transition form factors
- Only requires one Euclidean time parameter to be optimised to extract the ground state.
- Using multiple different operators will allow for the extraction of separate form facors
- Method should be tested on lattices with larger splittings between the light and strange quarks

Extra: Q^2=0 results

- Expansion in λ breaks down earlier
- Choose smaller λ values for the fit
- Still possible to fit and get a good result
- Higher orders could be required when the quark mass splitting is increased on other lattice ensembles.



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