Structure-dependent form factors in radiative leptonic decays with Domain Wall fermions

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OUTLINE

- Motivations
- Leptonic decays of pseudoscalar mesons

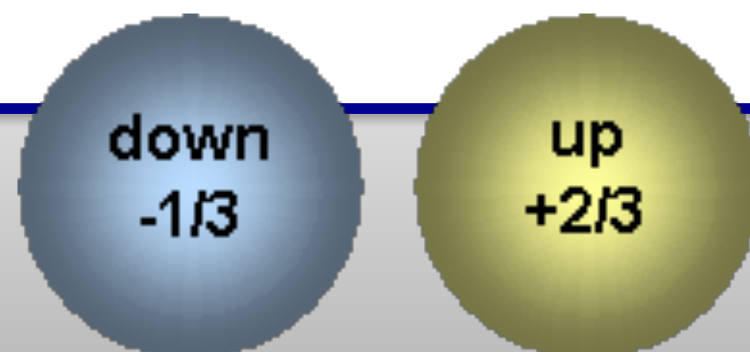
  $H \to \ell \nu_\ell \gamma$
- Outlook

In collaboration with

C. Kane, C. Lehner, S. Meinel and A. Soni

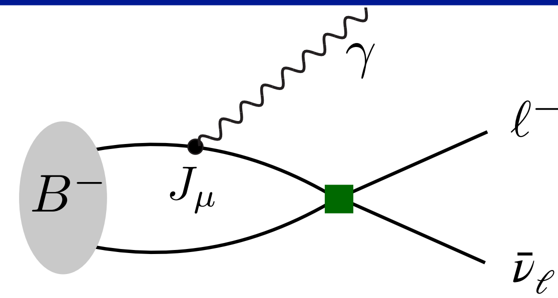
(for early results: arXiv:1907.00279, arXiv:2110.13196)

Phenomenological motivations



Radiative corrections to leptonic B-meson decays

$B^- \to \ell^- \bar{\nu}_\ell \gamma$



- The emission of a real hard photon removes the $(m_\ell / M_B)^2$ helicity suppression
- This is the simplest process that probes (for large E_γ) the first inverse moment of the B-meson LCDA

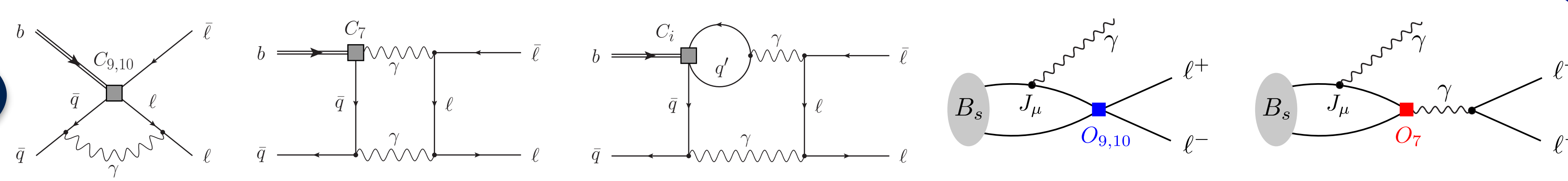
$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega} \Phi_{B+}(\omega, \mu)$$

λ_B is an important input in QCD-factorization predictions for non-leptonic B decays but is poorly known

M. Beneke, V. M. Braun, Y. Ji, Y.-B. Wei, 2018

- Belle 2018: $\mathscr{B}(B^- \to \ell^- \bar{\nu}_\ell \gamma, E_\gamma > 1 \text{ GeV}) < 3.0 \cdot 10^{-6}$ $\longrightarrow$ $\lambda_B > 0.24$ GeV
- QCD sum rules in HQET: $\lambda_B(1 \text{ GeV}) = 0.46\,(11)$ GeV

$B_q \to \ell^+ \ell^- (\gamma)$



- Enhancement of the virtual corrections by a factor M_B / Λ_{QCD} and by large logarithms

M. Beneke, C. Bobeth, R. Szafron, 2019

- The real photon emission process is a clean probe of NP: sensitiveness to C_9, C_{10}, C_7

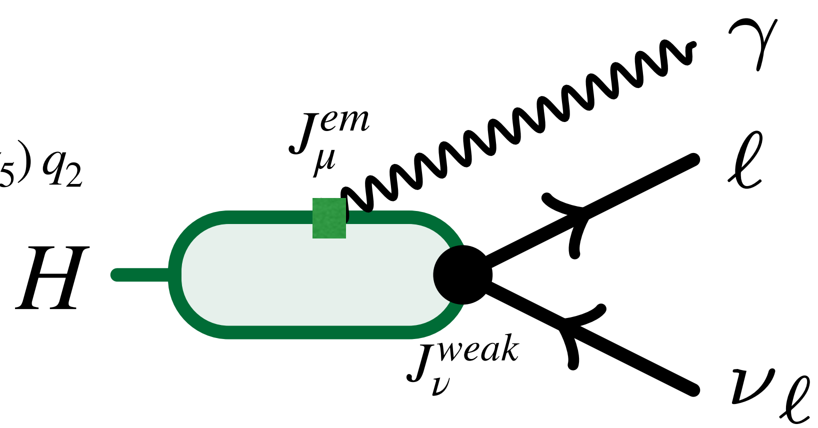
Lattice calculation of

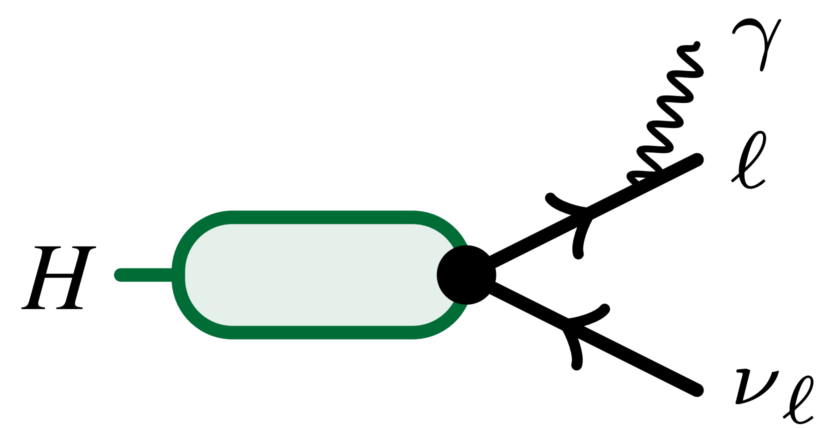
$$H \to \ell \nu_\ell \gamma$$

Hadronic tensor and form factors

$$J_\mu^{em} = \sum_q Q_q \bar{q}\gamma_\mu q$$

$$J_\nu^{weak} = \bar{q}_1\gamma_\nu(1-\gamma_5)\, q_2$$





$$T_{\mu\nu} = -i\int d^4x\, e^{ip_\gamma\cdot x}\langle 0|\mathbf{T}\left(J_\mu^{em}(x)\,J_\nu^{weak}(0)\right)|H(\vec{p}_H)\rangle \qquad (p_H = m_H v)$$

$$= \varepsilon_{\mu\nu\tau\rho}p_\gamma^\tau v^\rho F_V + i\left[-g_{\mu\nu}(p_\gamma\cdot v) + v_\mu(p_\gamma)_\nu\right]F_A - i\frac{v_\mu v_\nu}{p_\gamma\cdot v}m_H f_H + (p_\gamma)_\mu - \text{terms}$$

$$F_A = F_{A,SD} + (-Q_\ell f_H/E_\gamma^{(0)}), \qquad E_\gamma^{(0)} = p_\gamma\cdot v$$

Goal: Calculate $F_V, F_{A,SD}$ as a function of $E_\gamma^{(0)}$

$$\phi_H^\dagger = -\bar{q}_2\gamma_5 q_1$$

$$C_{3,\mu\nu}(t_{em}, t_H) = \int d^3x \int d^3y\; e^{-i\vec{\mathbf{p}}_\gamma\cdot\vec{\mathbf{x}}}e^{i\vec{\mathbf{p}}_H\cdot\vec{\mathbf{y}}}\langle J_\mu^{em}(t_{em},\vec{\mathbf{x}})J_\nu^{weak}(0)\phi_H^\dagger(t_H,\vec{\mathbf{y}})\rangle$$

- safe analytic continuation from Minkowsky to Euclidean spacetime, because of the absence of intermediate states lighter than the pseudoscalar meson

C. Kane *et al.*, arXiv:1907.00279, RM123 & Soton Coll., arXiv:2006.05358

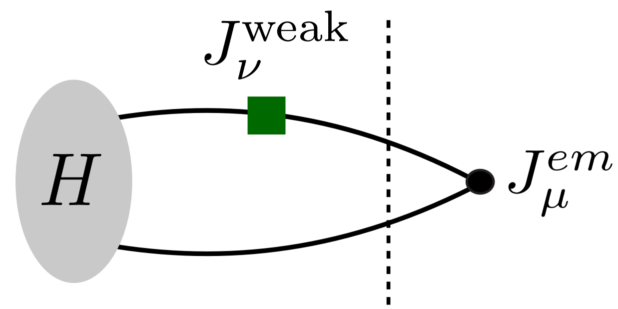
Euclidean correlation function

$$C_{3,\mu\nu}(t_{em}, t_H) = \int d^3x \int d^3y\ e^{-i\vec{\mathbf{p}}_\gamma\cdot\vec{\mathbf{x}}} e^{i\vec{\mathbf{p}}_H\cdot\vec{\mathbf{y}}} \langle J_\mu^{\rm em}(t_{em},\vec{\mathbf{x}}) J_\nu^{\rm weak}(0) \phi_H^\dagger(t_H,\vec{\mathbf{y}})\rangle$$

$$I^{<}_{\mu\nu}(T, t_H) = \int_{-T}^{0} dt_{em} e^{E_\gamma t_{em}} C_{3,\mu\nu}(t_{em}, t_H)$$

$$I^{>}_{\mu\nu}(T, t_H) = \int_{0}^{T} dt_{em} e^{E_\gamma t_{em}} C_{3,\mu\nu}(t_{em}, t_H)$$

Time ordering: $t_{em} > 0$



$$T^{>}_{\mu\nu} = -\sum_n \frac{\langle 0| J_\mu^{em}(0) |n(\vec{\mathbf{p}}_\gamma)\rangle \langle n(\vec{\mathbf{p}}_\gamma)| J_\nu^{weak}(0) |H(\vec{\mathbf{p}}_H)\rangle}{2E_{n,\vec{\mathbf{p}}_\gamma}(E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma})}$$

$$I^{>}_{\mu\nu}(t_H, T) = \int_0^T dt_{em}\ e^{E_\gamma t_{em}} C_{\mu\nu}(t_{em}, t_H)$$

$$= -\sum_m e^{E_m t_H} \frac{\langle m(\vec{\mathbf{p}}_H)| \phi_H^\dagger(0) |0\rangle}{2E_{m,\vec{\mathbf{p}}_H}}$$

$$\times \sum_n \frac{\langle 0| J_\mu^{em}(0) |n(\vec{\mathbf{p}}_\gamma)\rangle \langle n(\vec{\mathbf{p}}_\gamma)| J_\nu^{weak}(0) |m(\vec{\mathbf{p}}_H)\rangle}{2E_{n,\vec{\mathbf{p}}_\gamma}(E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma})} \left[1 - e^{(E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma})T}\right]$$

$t_H \to -\infty$ to achieve ground state saturation

$T \to \infty$ to remove unwanted exponentials that come with intermediate states

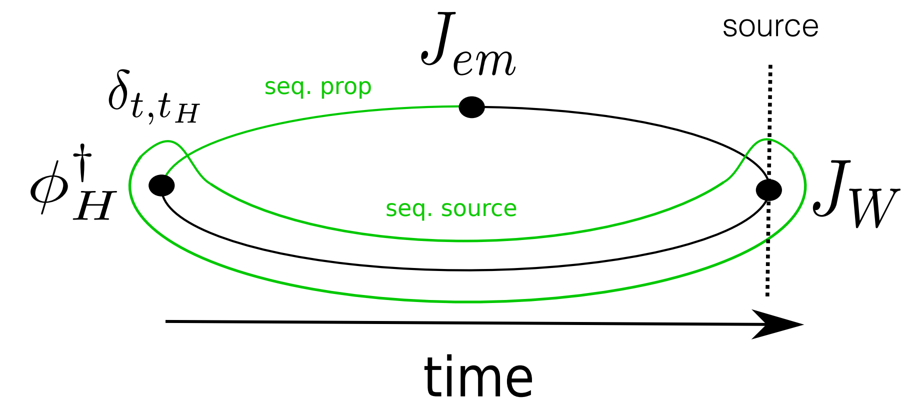
Calculating $I_{\mu\nu}(T, t_H)$

$$T_{\mu\nu} = \lim_{T\to\infty} \lim_{t_H\to-\infty} \frac{-2E_H e^{-E_H t_H}}{\langle H(\vec{\mathbf{p}}_H)|\,\phi_H^\dagger\,|0\rangle} \underbrace{\int_{-T}^{T} dt_{em}\; e^{E_\gamma t_{em}} C_{3,\mu\nu}(t_{em}, t_H)}_{I_{\mu\nu}(T,t_H)}$$

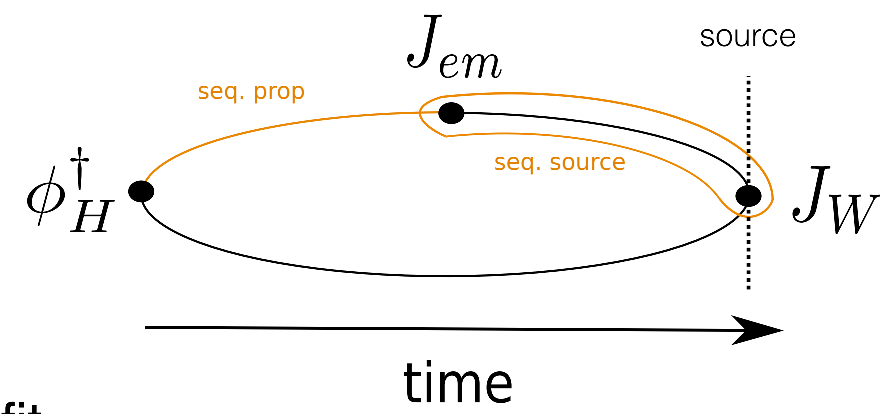
Two methods to calculate $I_{\mu\nu}(T, t_H)$:

1: 3d (timeslice) sequential propagator through $\phi_H^\dagger$ $\to$ calculate $C_{3,\mu\nu}(t_{em}, t_H)$ on lattice, fixed t_H get all t_{em} for free

arXiv:1907.00279 & arXiv:2110.13196



2: 4d sequential propagator through J_μ^{em} $\to$ calculate $I_{\mu\nu}(T, t_H)$ on lattice, fixed T get all t_H for free

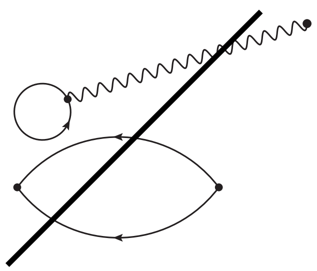


RM123 & Soton Coll., arXiv:2006.05358: Set $T = N_T/2$ and fit to constant in t_H where data has plateaued

- For a comparison of 3d *vs* 4d methods see arXiv:2110.13196

Simulation details

- $N_f = 2+1$ DWF, RBC/UKQCD ensemble $M_\pi = 340\,(1)$ MeV, $a \simeq 0.11$ fm, charm valence quarks $\longrightarrow$ Möbius DW with "stout" smearing
- 25 configurations, AMA with 16 sloppy and 1 exact samples per config
- Disconnected diagrams are neglected

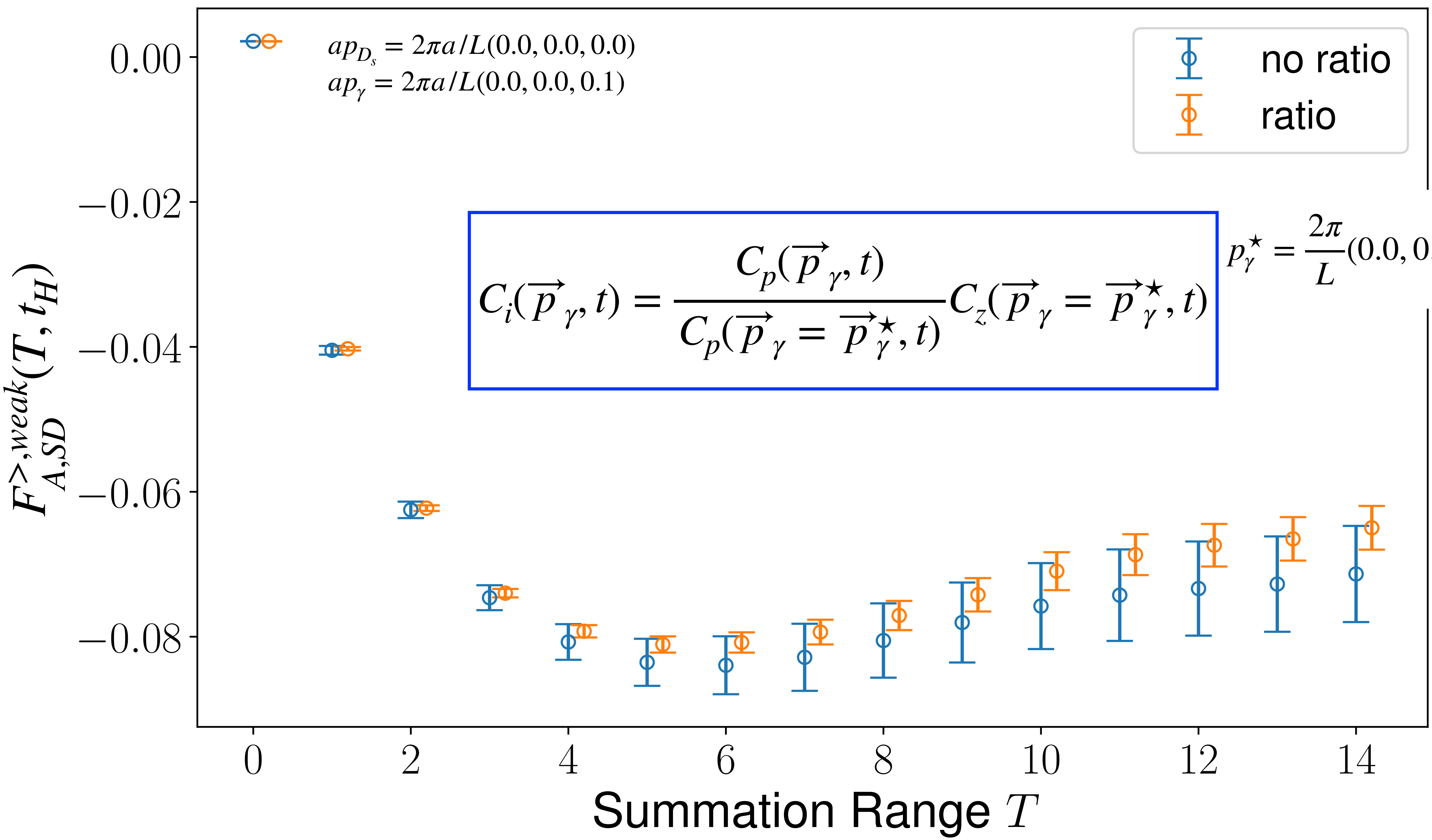


- $\mathbb{Z}_2$ random wall sources & randomly placed point sources
- Local electromagnetic current + mostly non-perturbative RCs
- Two datasets: $J^{weak}(0)$ or $J^{em}(0)$
- For point sources use translational invariance to fix em/weak operator at $\mathbf{0}$

→ use a "sine-cardinal reconstruction" to generate data for arbitrary photon momenta (only exp. small FVEs are introduced)

$$C_{3,\mu\nu} = \int d^3x\, d^3y\, e^{-i\vec{p}_\gamma\cdot\vec{x}} \langle J^{em}_\mu(t_{em}, \vec{x}) J^{weak}_\nu(0) \phi^\dagger_H(t_H, \vec{y})\rangle \qquad \vec{p}_H = 0, \text{several } \vec{p}_\gamma$$

Improved form factors estimators

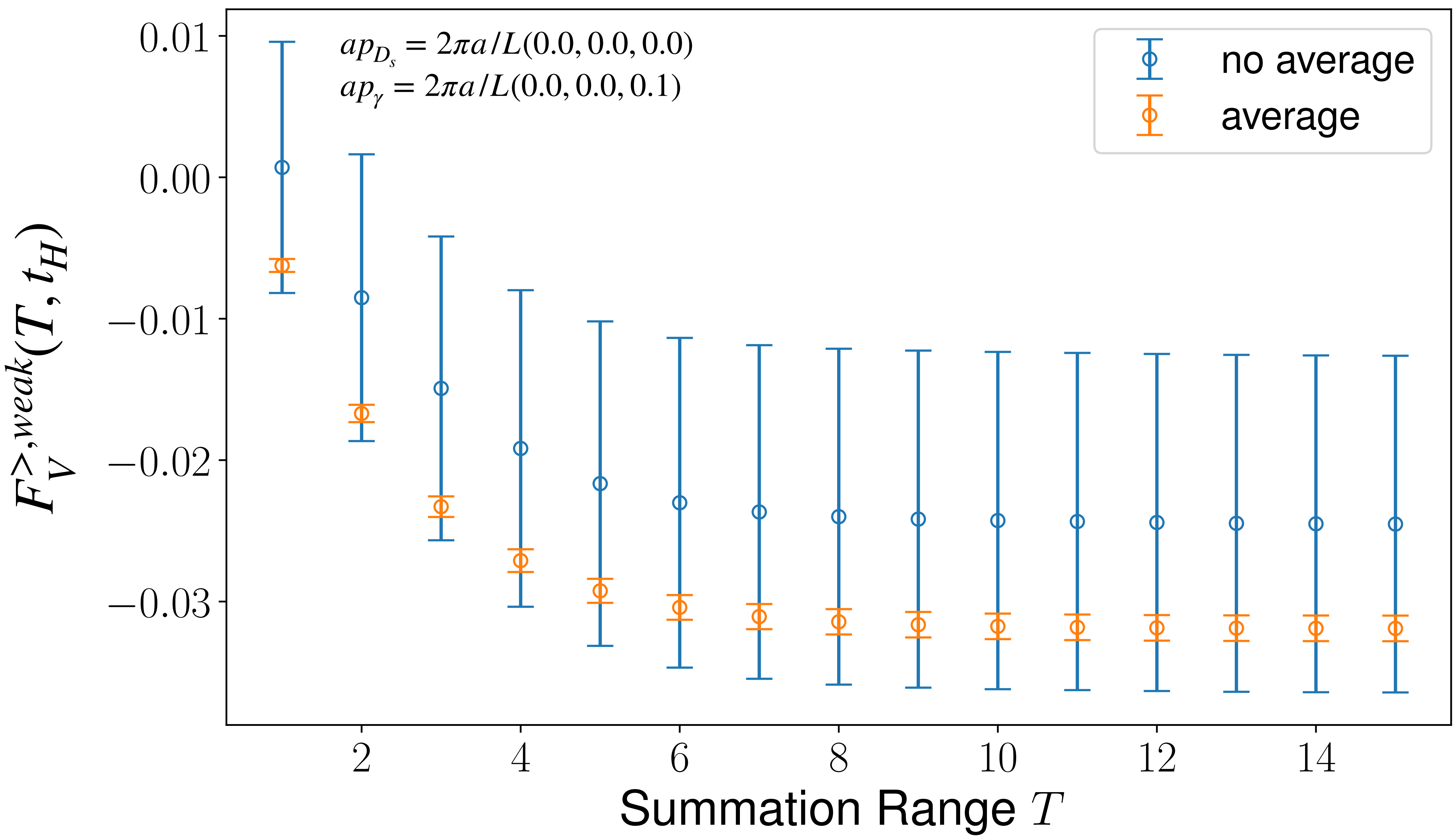


$$C_i(\vec{p}_\gamma, t) = \frac{C_p(\vec{p}_\gamma, t)}{C_p(\vec{p}_\gamma = \vec{p}^{\,\star}_\gamma, t)} C_z(\vec{p}_\gamma = \vec{p}^{\,\star}_\gamma, t)$$

$$p^\star_\gamma = \frac{2\pi}{L}(0.0, 0.0, 0.0)$$

Improved form factors estimators [2]

$\pm \overrightarrow{p}_\gamma$ average



Fit form: 3d method

Include terms to fit

(1) unwanted exponential from first intermediate state
(2) first excited state

Fit form factors F_V and $F_{A,SD}$ directly instead of $I_{\mu\nu}$

$t_H < t_{em} < 0 \quad t_H < 0 < t_W$

$$F_{<}^{weak}(t_H, T) = F^{<} + B_F^{<}\left(1 + B_{F,exc}^{<} \overbrace{e^{\Delta E(T+t_H)}}\right) \overbrace{e^{-(E_\gamma - E_H + E^{<})T}} + C_F^{<} \overbrace{e^{\Delta E t_H}}$$

$$F_{>}^{em}(t_H, T) = F^{<} + B_F^{<}\left[1 + B_{F,exc}^{<} \frac{E_\gamma + E^{<} - (\Delta E + E_H)}{E_\gamma + E^{<} - E_H} e^{\Delta E t_H}\right] e^{-(E_\gamma - E_H + E^{<})T} + \tilde{C}_F^{<} e^{\Delta E t_H}$$

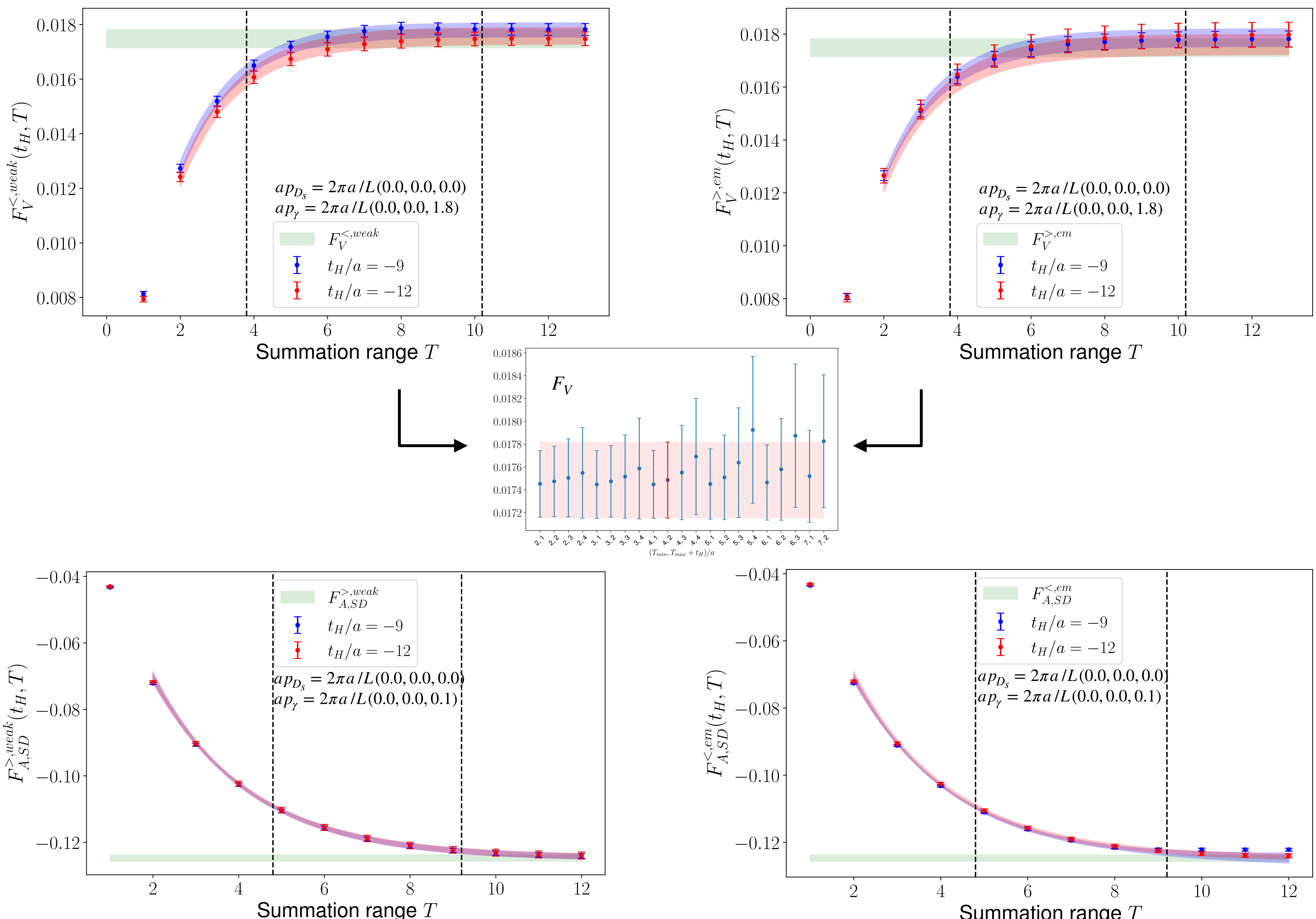
$t_H < 0 < t_{em} \quad t_H < t_W < 0$

$$F_{>}^{weak}(t_H, T) = F^{>} + B_F^{>}\left(1 + B_{F,exc}^{>} e^{\Delta E t_H}\right) e^{(E_\gamma - E^{>})T} + C_F^{>} e^{\Delta E t_H}$$

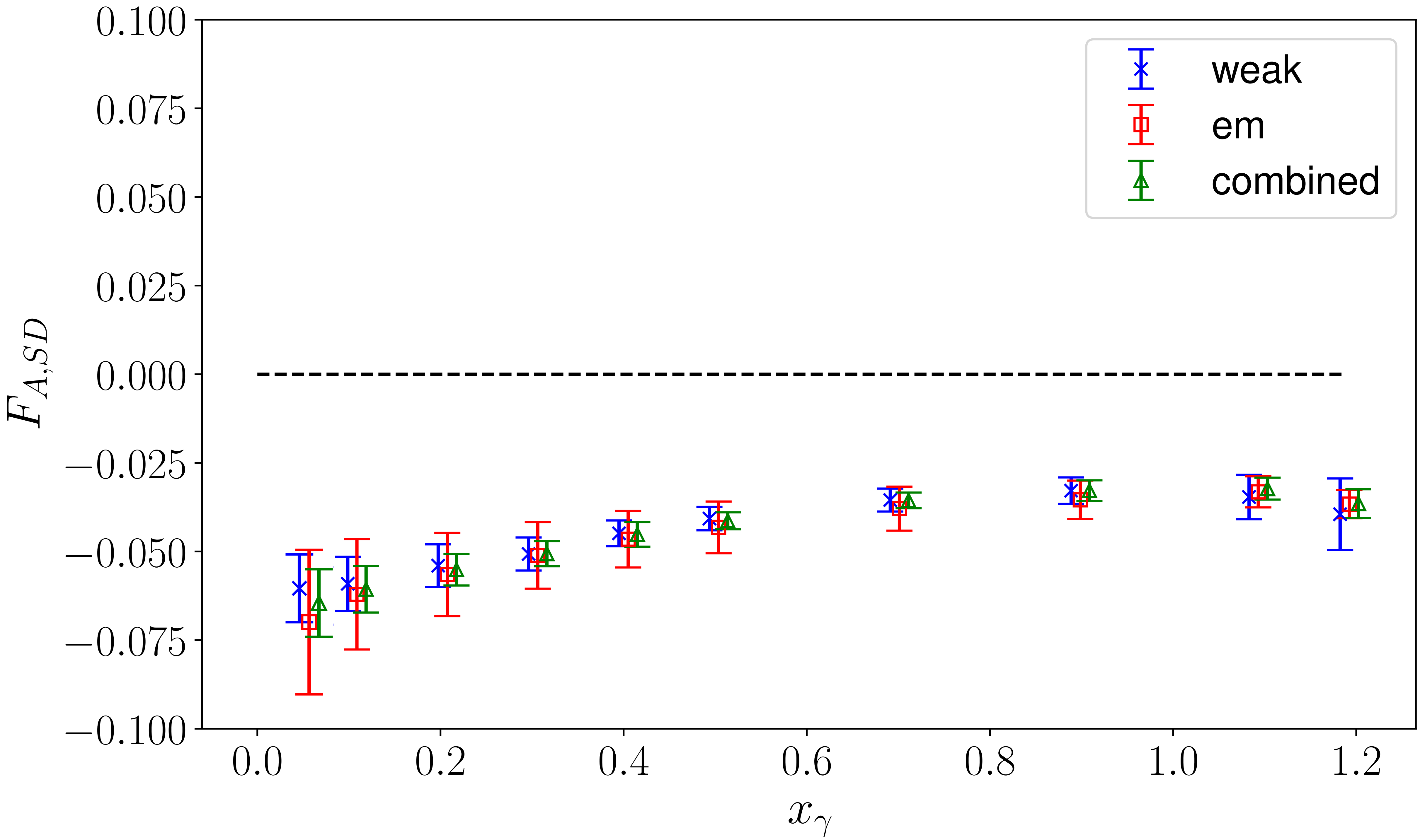
$$F_{<}^{em}(t_H, T) = F^{>} + B_F^{>}\left[1 + B_{F,exc}^{>} \frac{E_\gamma - E^{>}}{E_\gamma - E^{>} + \Delta E} e^{\Delta E(T+t_H)}\right] e^{(E_\gamma - E^{>})T} + \tilde{C}_F^{>} e^{\Delta E t_H}$$

Only have two values of t_H, fitting multiple exponentials not possible
$\rightarrow$ Determine ΔE from the pseudoscalar two-point correlation function
$\rightarrow$ use result as Gaussian prior in form factor fits

$D_s \to \ell\nu_\ell\gamma$: 3d method



$D_s \to \ell\nu_\ell\gamma$: 3d method

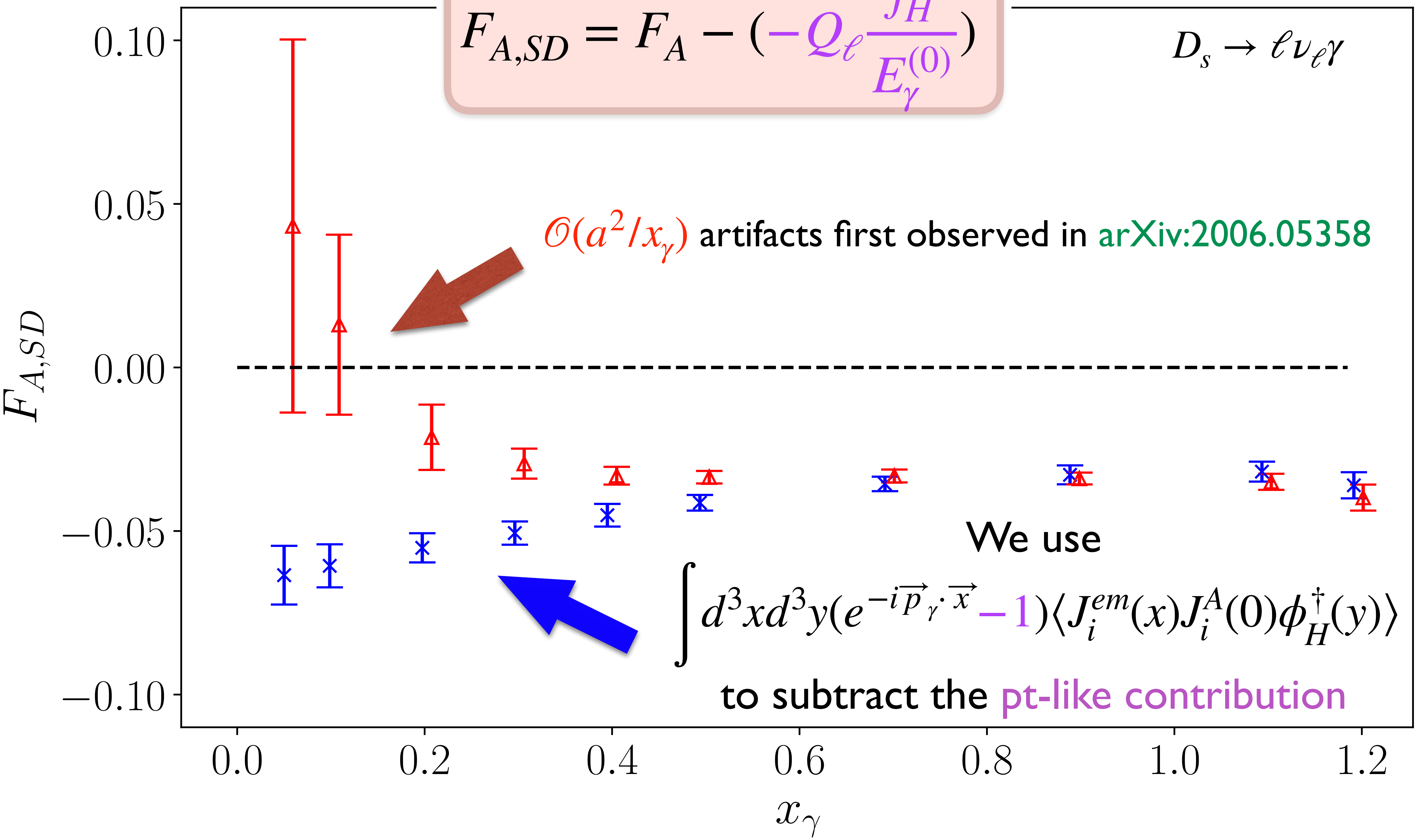


$$x_\gamma = \frac{2p_H \cdot p_\gamma}{m_H^2} \xrightarrow{\vec{p}_H = \mathbf{0}} x_\gamma = \frac{2E_\gamma^{(0)}}{m_H}$$

$$0 \le x_\gamma \le 1 - \frac{m_\ell^2}{m_H^2}$$

NP subtraction of IR-divergent discretization effects

$$F_{A,SD} = F_A - \left(-Q_\ell \frac{f_H}{E_\gamma^{(0)}}\right)$$



$\mathcal{O}(a^2/x_\gamma)$ artifacts first observed in arXiv:2006.05358

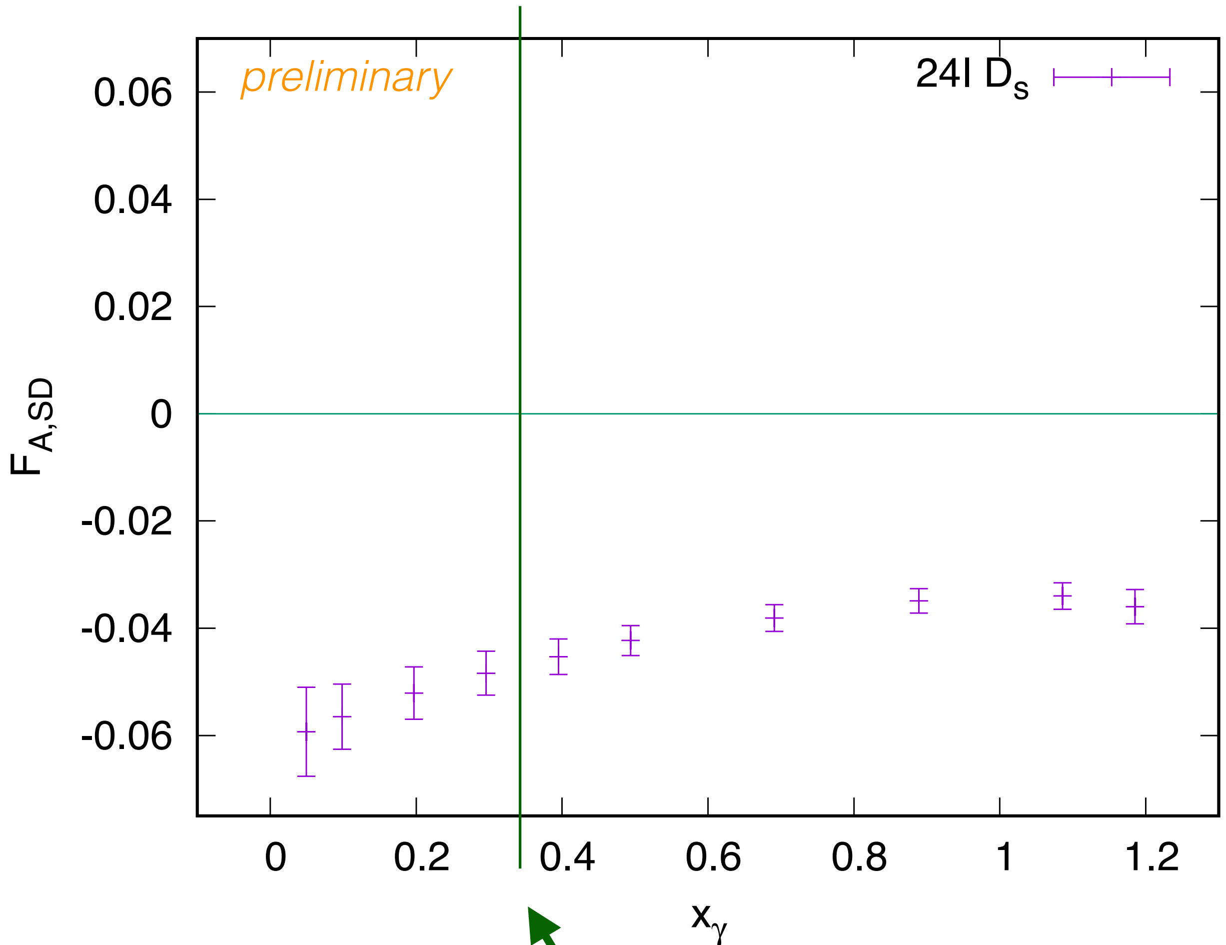
We use

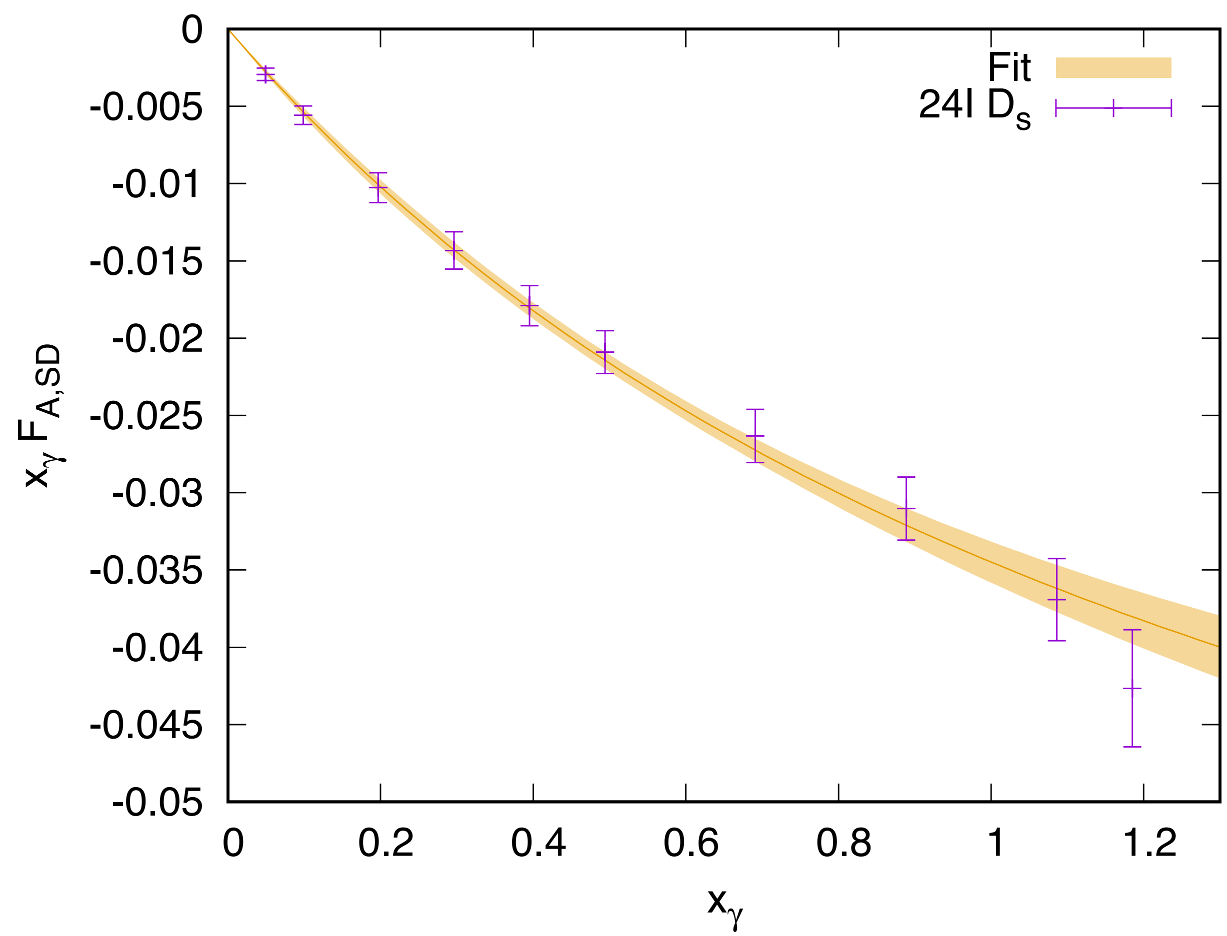
$$\int d^3x d^3y (e^{-i\vec{p}_\gamma \cdot \vec{x}} - 1) \langle J_i^{em}(x) J_i^A(0) \phi_H^\dagger(y) \rangle$$

to subtract the pt-like contribution

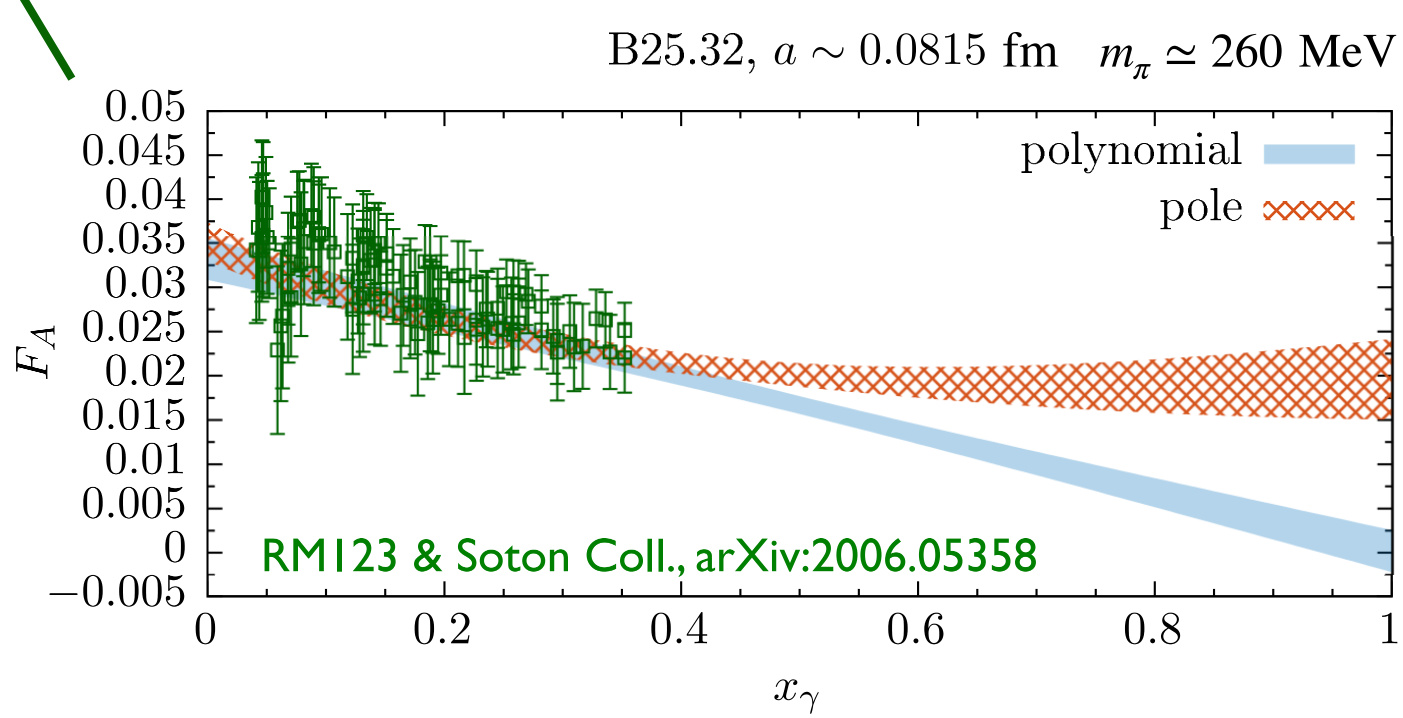
Blue data: improved subtraction of pt-like contribution

$D_s \to \ell \nu_\ell \gamma$: results (3d method)

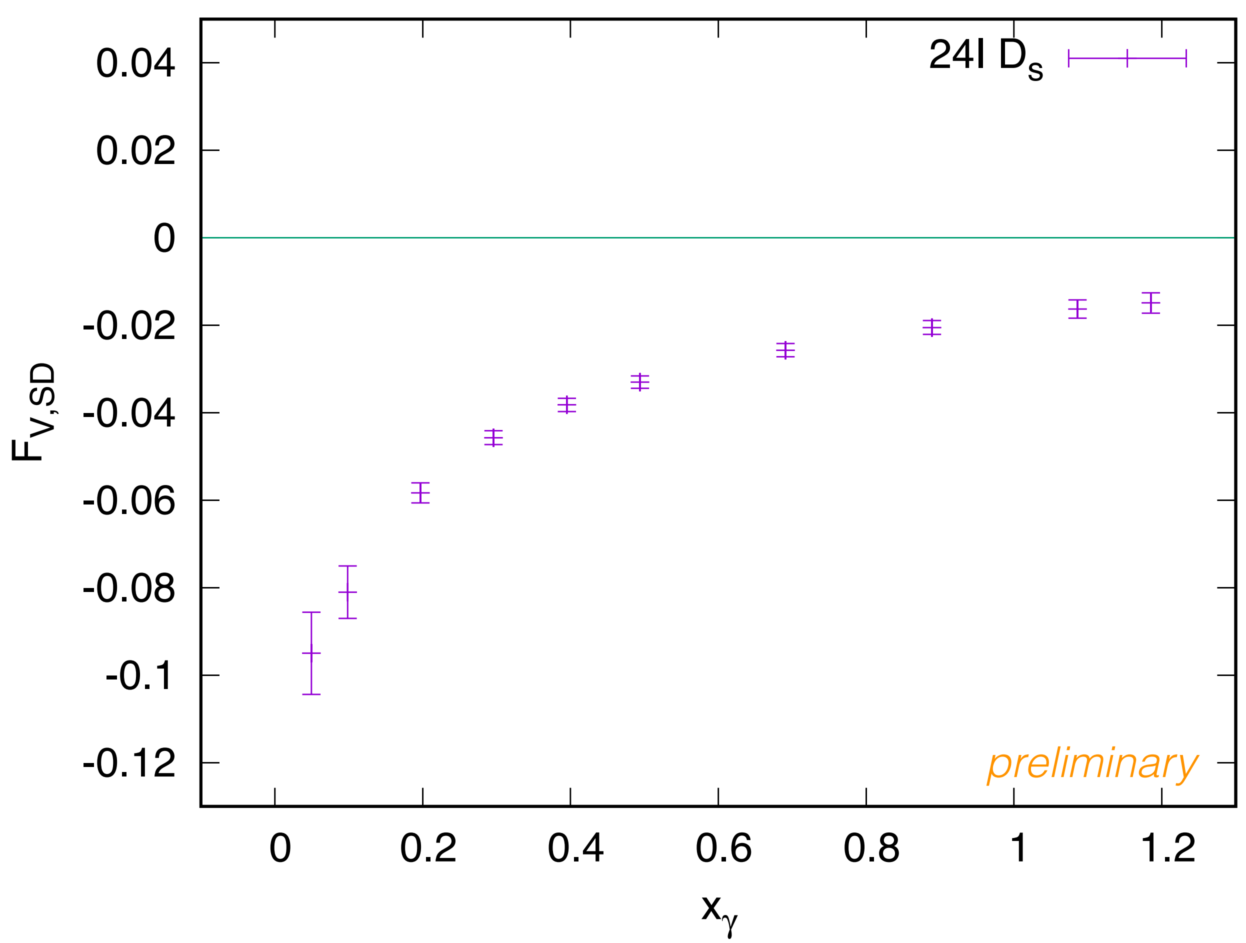


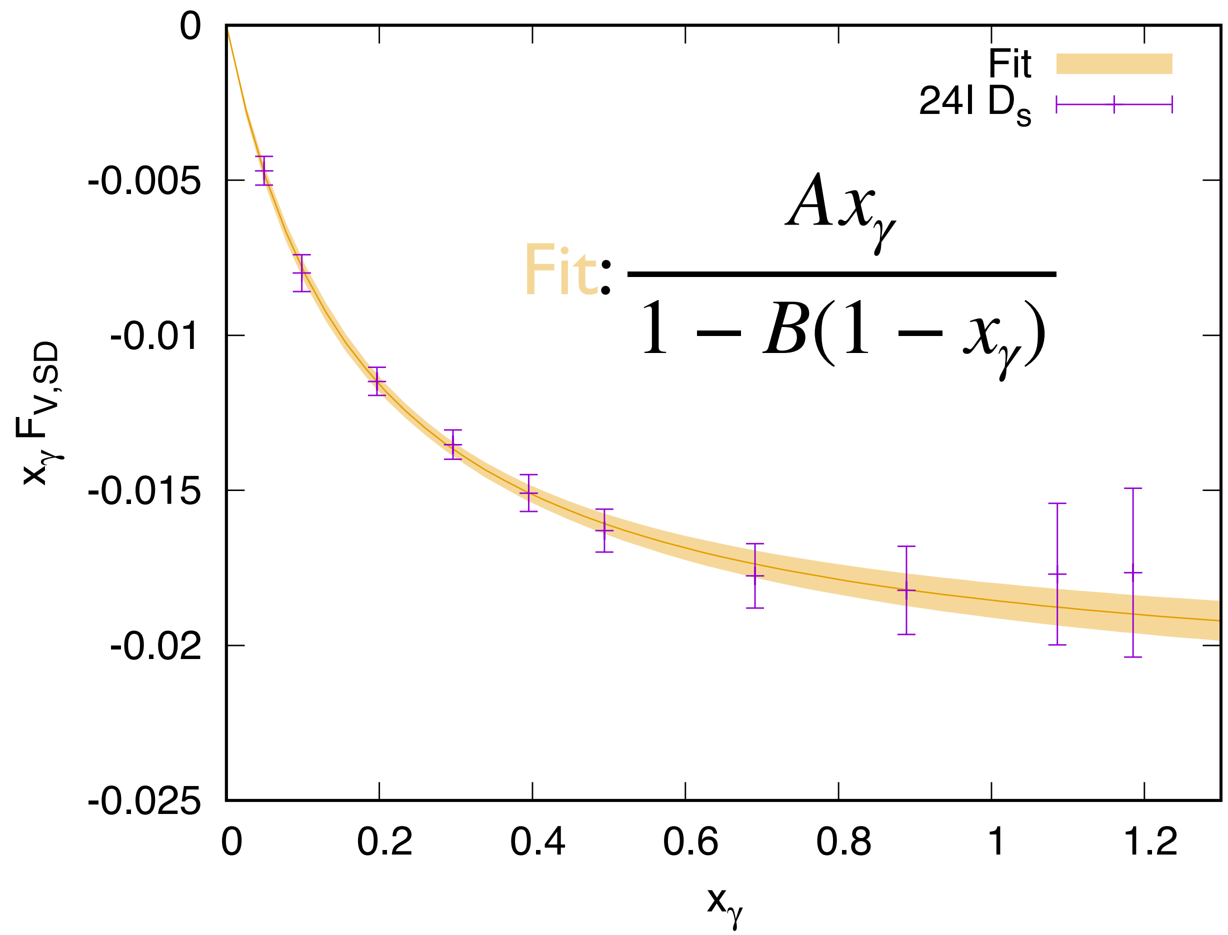


sign: different FFs parameterization



$D_s \to \ell\nu_\ell\gamma$: results (3d method) [2]



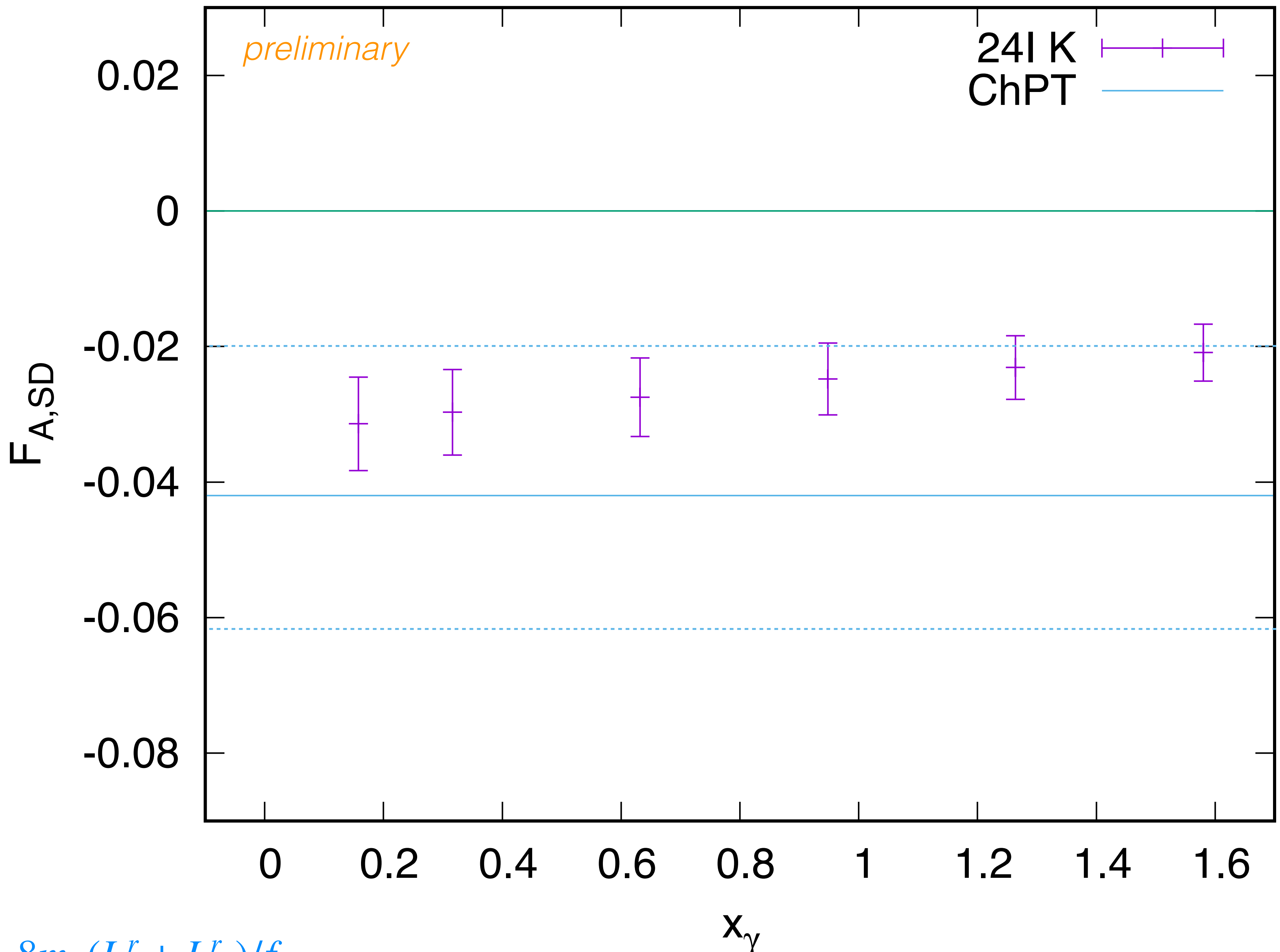


- $D_s^+ \to e^+\nu\gamma$: $\mathcal{B}(E_\gamma > 10\ \text{MeV}) < 1.3 \times 10^{-4}$ SM: $\mathcal{O}(10^{-4})$

  [BESIII Collaboration, arXiv:1902.03351]

- Fit Ansatz inspired by the phenomenological analysis of arXiv:0907.1845

$K \to \ell\nu_\ell\gamma$: results



ChPT : $8m_K(L_9^r + L_{10}^r)/f_K$

J. Bijnens *et al.*, 1993

Conclusions and future perspectives

- The form factors for real emissions are accessible from Euclidean correlators

- We compared analysis methods using 3d and 4d data. 3d method results in smallest statistical uncertainties. A method paper will appear very soon

- With moderate statistics we are able to provide rather precise, first-principles results for the form factors in the full kinematical (photon-energy) range

- Lattice calculations of radiative leptonic heavy-meson decays at high photon energy could provide useful information to better understand the internal structure of hadrons

- The analysis on a variety of ensembles with $m_\pi \simeq m_\pi^{phys}$ is in progress to reach the continuum limit. To extend the study to B-meson decays we will take advantage of new RBC/UKQCD ensembles at $a^{-1} \approx (3.5, 4.5)$ GeV

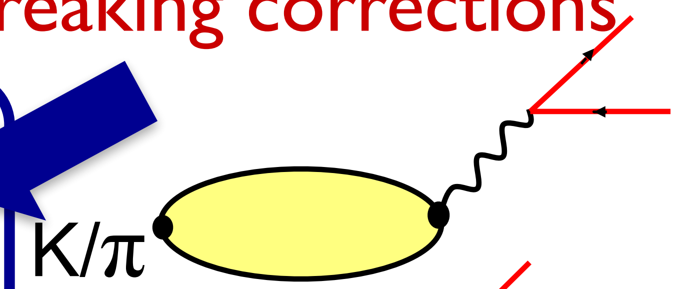
	48I	64I	96I
$L^3 \cdot T/a^4$	$48^3 \cdot 96$	$64^3 \cdot 128$	$96^3 \cdot 192$
β	2.13	2.25	2.31
am_l	0.00078	0.000678	0.0054
am_h	0.0362	0.02661	0.02132
α	2.0	2.0	2.0
a^{-1} (GeV)	1.730(4)	2.359(7)	≈ 2.8
a (fm)	0.1141(3)	0.0837(3)	≈ 0.071
L (fm)	5.476(12)	5.354(16)	≈ 6.8
L_s/a	24	12	12
m_π (MeV)	139.2(4)	139.2(5)	≈ 135
$m_\pi L$	3.863(6)	3.778(8)	≈ 4.7
$N_{\rm conf}$	120	160	20

Supplementary slides

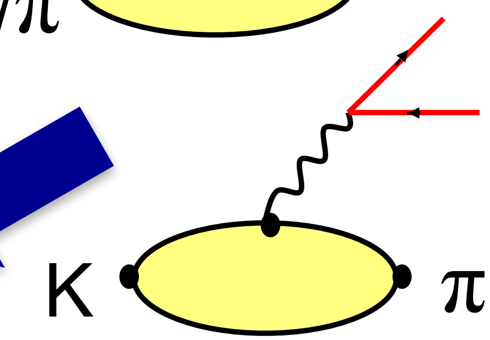
Electromagnetic and isospin-breaking effects

Given the present exper. and theor. (LQCD) accuracy, an important source of uncertainty are long distance electromagnetic and SU(2)-breaking corrections

$$\frac{\Gamma\left(K^+\to \ell^+\nu_\ell(\gamma)\right)}{\Gamma\left(\pi^+\to \ell^+\nu_\ell(\gamma)\right)} = \left(\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi}\right)^2 \frac{M_{K^+}\left(1-m_\ell^2/M_{K^+}^2\right)^2}{M_{\pi^+}\left(1-m_\ell^2/M_{\pi^+}^2\right)^2}\left(1+\delta_{EM}+\delta_{SU(2)}\right)$$



$$\Gamma\left(K^{+,0}\to\pi^{0,-}\ell^+\nu_\ell(\gamma)\right) = \frac{G_F^2 M_{K^{+,0}}^5}{192\pi^3} C_{K^{+,0}}^2 \left|V_{us} f_+^{K^0\pi^-}(0)\right|^2 I_{K\ell}^{(0)} S_{EW}\left(1+\delta_{EM}^{K^{+,0}\ell}+\delta_{SU(2)}^{K^{+,0}\pi}\right)$$



For $\Gamma_{Kl2}/\Gamma_{\pi l2}$

At leading order in ChPT both δ_{EM} and $\delta_{SU(2)}$ can be expressed in terms of physical quantities (e.m. pion mass splitting, f_K/f_π, …)

- $\delta_{EM} = -0.0069(17)$ 25% of error due to higher orders ➡ 0.2% on $\Gamma_{Kl2}/\Gamma_{\pi l2}$

  M.Knecht *et al.*, 2000; V.Cirigliano and H.Neufeld, 2011

- $$\delta_{SU(2)} = \left(\frac{f_{K^+}/f_{\pi^+}}{f_K/f_\pi}\right)^2 - 1 = -0.0044(12)$$ 25% of error due to higher orders ➡ 0.1% on $\Gamma_{Kl2}/\Gamma_{\pi l2}$

  J.Gasser and H.Leutwyler, 1985; V.Cirigliano and H.Neufeld, 2011

ChPT is not applicable to D and B decays

Real photon emission amplitude

By setting $p_\gamma^2 = 0$, at fixed meson mass, the form factors depend on $p_H \cdot p_\gamma$ only. Moreover, by choosing a *physical* basis for the polarization vectors, *i.e.* $\epsilon_r(\mathbf{p}_\gamma) \cdot p_\gamma = 0$, one has

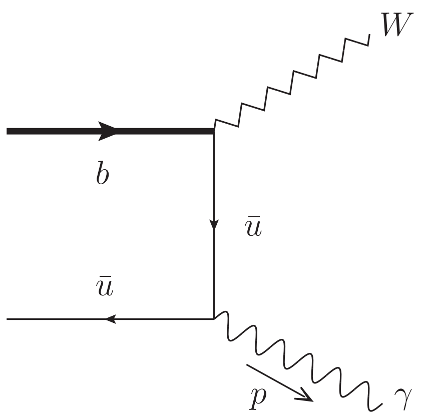
$$\epsilon_\mu^r(\mathbf{p}_\gamma)\, T^{\mu\nu}(p_\gamma, p_H) = \epsilon_\mu^r(\mathbf{p}_\gamma) \left\{ \varepsilon^{\mu\nu\tau\rho}(p_\gamma)_\tau v_\rho F_V + i\left[-g^{\mu\nu}(p_\gamma \cdot v) + v^\mu p_\gamma^\nu\right] F_A - i\frac{v^\mu v^\nu}{p_\gamma \cdot v} m_H f_H \right\}$$

In the case of off-shell photons ($p_\gamma^2 \neq 0$) $\longrightarrow$ $\Gamma[H \to \ell \nu_\ell \ell^+ \ell^-]$ expressed in terms of 4 form factors

For large photon energies and in the B-meson rest frame the form factors can be written as

$$F_V(E_\gamma) = \frac{e_u M_B f_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma, \mu) + \xi(E_\gamma) + \Delta\xi(E_\gamma)$$

$$F_A(E_\gamma) = \frac{e_u M_B f_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma, \mu) + \xi(E_\gamma) - \Delta\xi(E_\gamma)$$

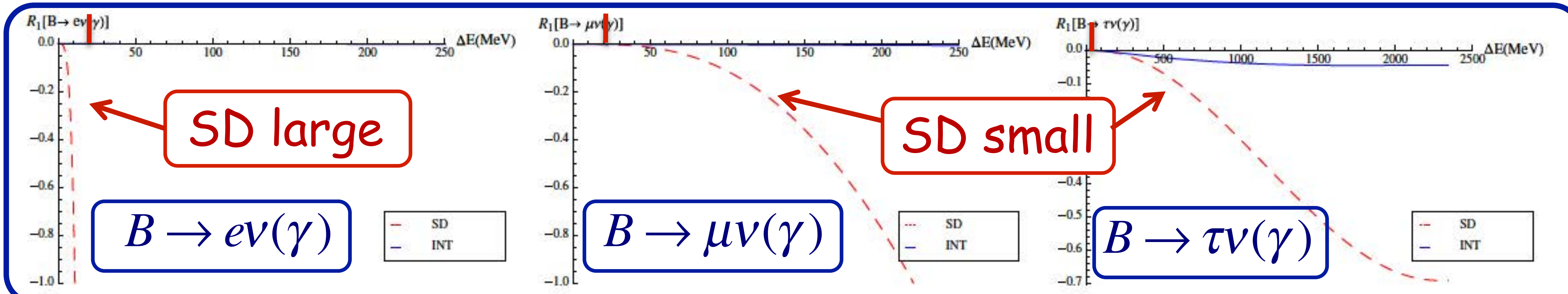


M. Beneke and J. Rohrwild, 2011

Structure dependent contributions to decays of D and B mesons

- For the studies of D and B mesons decays we cannot apply ChPT
- For B mesons in particular we have another small scale, $m_{B^*} - m_B \simeq 45$ MeV
  ➡ the radiation of a soft photon may still induce sizeable SD effects
- A phenomenological analysis based on a simple pole model for F_V and F_A confirms this picture

D. Becirevic *et al.*, PLB 681 (2009) 257



$$F_V \simeq \frac{\tilde{C}_V}{1-(p_B-k)^2/m_{B^*}^2}$$

$$F_A \simeq \frac{\tilde{C}_A}{1-(p_B-k)^2/m_{B_1}^2}$$

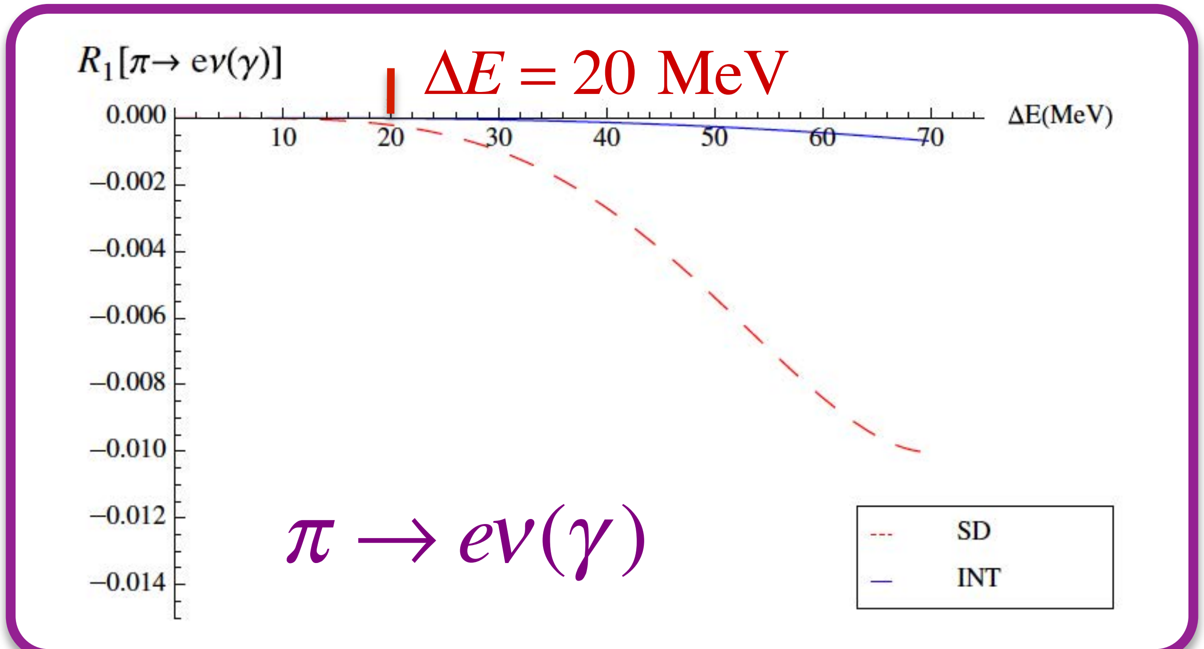
Under this assumption the SD contributions to $B \to e\nu(\gamma)$ for $E_\gamma \simeq 20$ MeV can be very large, but are small for $B \to \mu\nu(\gamma)$ and $B \to \tau\nu(\gamma)$

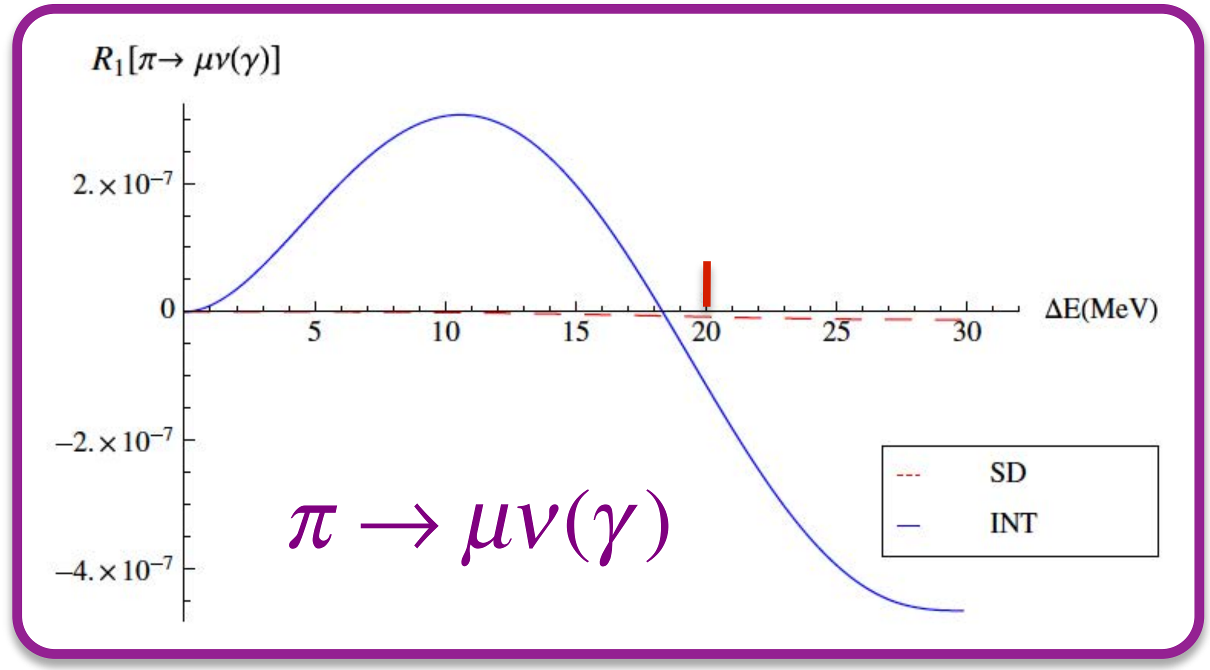
A lattice calculation of F_V and F_A would be very useful

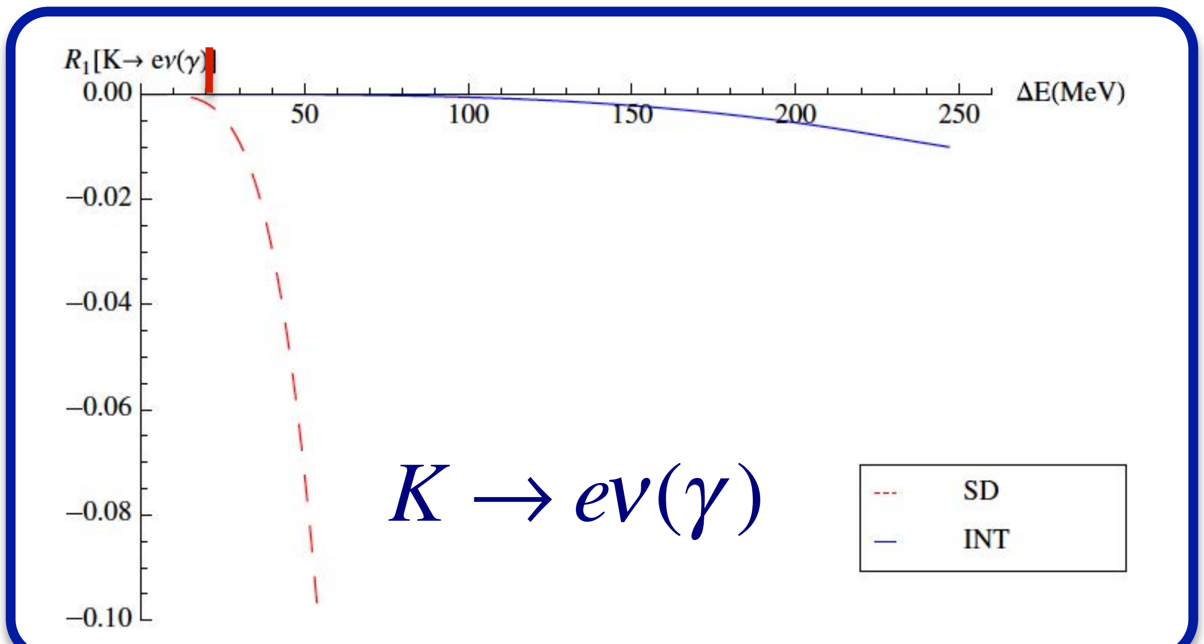
$$R_1^A(\Delta E) = \frac{\Gamma_1^A(\Delta E)}{\Gamma_0^{\alpha,pt} + \Gamma_1^{pt}(\Delta E)} \quad , \quad A = \{SD, INT\}$$

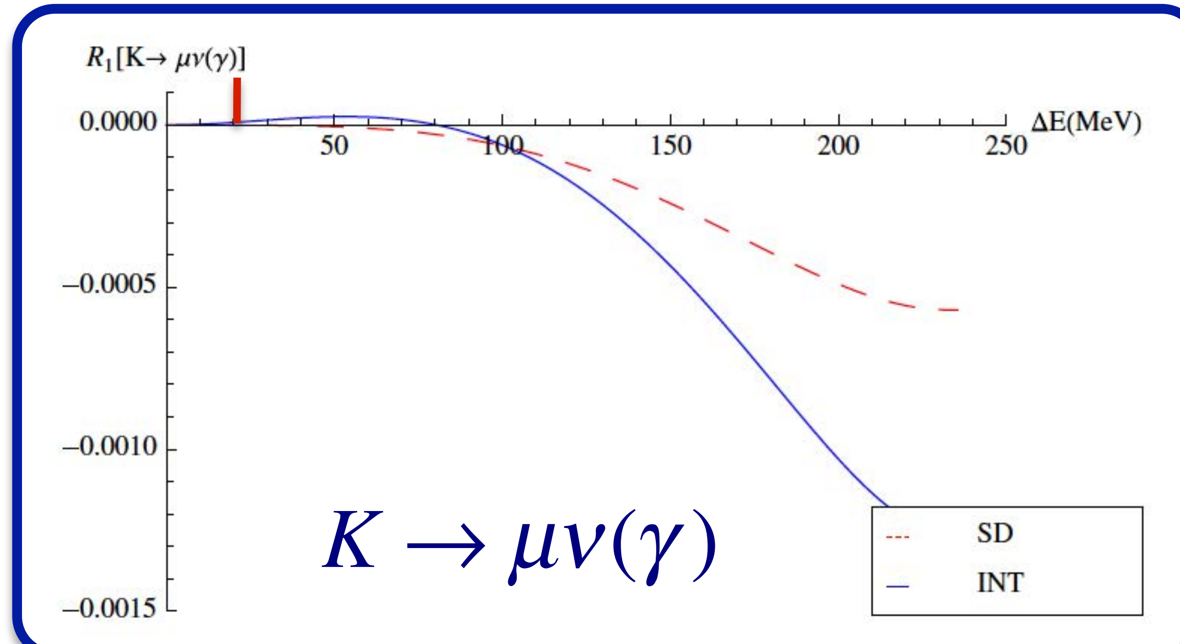
SD = structure dependent

INT = interference





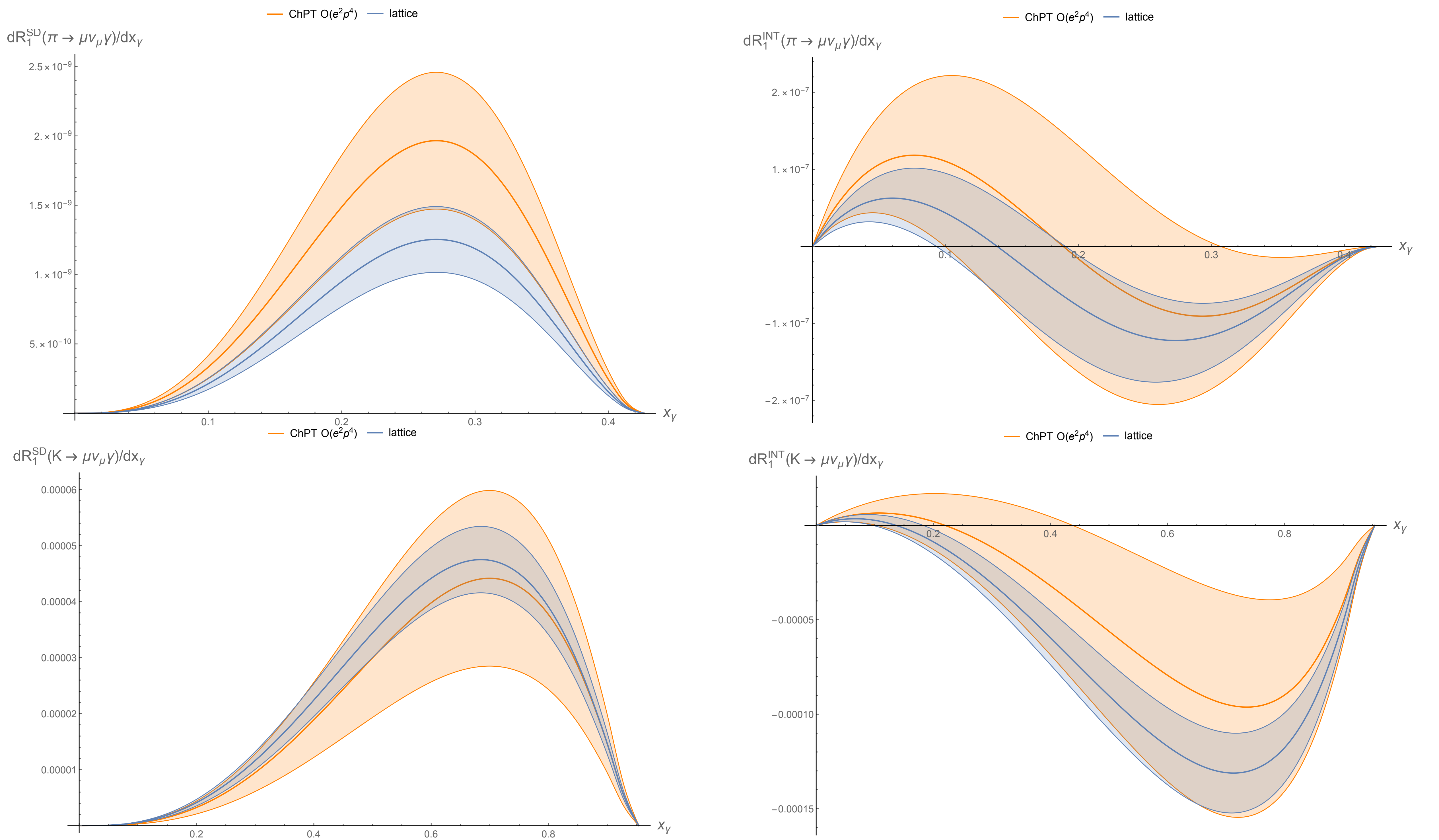




- Interference contributions are negligible in all the decays
- Structure-dependent contributions can be sizable for $K \to e\nu(\gamma)$ but they are negligible for $\Delta E < 20$ MeV (which is experimentally accessible)

$$\frac{4\pi}{\alpha\,\Gamma_0^{\rm tree}}\frac{d\Gamma_1^{\rm SD}}{dx_\gamma} = \frac{m_P^2}{6 f_P^2\, r_\ell^2\,(1-r_\ell^2)^2}\left[F_V(x_\gamma)^2 + F_A(x_\gamma)^2\right] f^{\rm SD}(x_\gamma)$$

$$\frac{4\pi}{\alpha\,\Gamma_0^{\rm tree}}\frac{d\Gamma_1^{\rm INT}}{dx_\gamma} = -\frac{2\,m_P}{f_P\,(1-r_\ell^2)^2}\left[F_V(x_\gamma)\, f_V^{\rm INT}(x_\gamma) + F_A(x_\gamma)\, f_A^{\rm INT}(x_\gamma)\right]$$



3pt function in Euclidean space: time integrals

For large negative t_B,

$$
\begin{aligned}
I^{<}_{\mu\nu}(t_B, T) &= \int_{-T}^{0} dt \;\; e^{E_\gamma t} \; C_{\mu\nu}(t, t_B) \\
&= \langle B(\mathbf{p}_B)|\phi_B^\dagger(0)|0\rangle \frac{1}{2E_B} e^{E_B t_B} \\
&\quad \times \sum_n \frac{1}{2E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)}} \frac{1}{E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} - E_B} \\
&\quad \times \langle 0|J_\nu^{\text{weak}}(0)|n(\mathbf{p}_B-\mathbf{p}_\gamma)\rangle\langle n(\mathbf{p}_B-\mathbf{p}_\gamma)|J_\mu(0)|B(\mathbf{p}_B)\rangle \\
&\quad \times \left(1 - e^{-(E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} - E_B)T}\right)
\end{aligned}
$$

The unwanted exponential $e^{-(E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} - E_B)T}$ goes to zero for large T if $E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} > E_B$.

Because the states $|n(\mathbf{p}_B - \mathbf{p}_\gamma)\rangle$ have the same quark-flavor quantum numbers as the B meson, we have $E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} \geq E_{B,(\mathbf{p}_B-\mathbf{p}_\gamma)} = \sqrt{m_B^2 + (\mathbf{p}_B - \mathbf{p}_\gamma)^2}$.

The inequality becomes $\sqrt{\mathbf{p}_\gamma^2} + \sqrt{m_B^2 + (\mathbf{p}_B - \mathbf{p}_\gamma)^2} > \sqrt{m_B^2 + \mathbf{p}_B^2}$.

This is in fact always satisfied (as long as $\mathbf{p}_\gamma \neq 0$).

3pt function in Euclidean space: time integrals [2]

For large negative t_B,

$$\begin{aligned} I^{>}_{\mu\nu}(t_B, T) &= \int_0^T dt \;\; e^{E_\gamma t}\, C_{\mu\nu}(t, t_B) \\ &= -\langle B(\mathbf{p}_B)|\phi_B^\dagger(0)|0\rangle \frac{1}{2E_B}\, e^{E_B t_B} \\ &\quad \times \sum_n \frac{1}{2E_{m,\mathbf{p}_\gamma}} \frac{1}{E_\gamma - E_{m,\mathbf{p}_\gamma}} \\ &\quad \times \langle 0|J_\mu(0)|m(\mathbf{p}_\gamma)\rangle \langle m(\mathbf{p}_\gamma)|J_\nu^{\text{weak}}(0)|B(\mathbf{p}_B)\rangle \\ &\quad \times \left(1 - e^{(E_\gamma - E_{m,\mathbf{p}_\gamma})T}\right) \end{aligned}$$

The unwanted exponential $e^{(E_\gamma - E_{m,\mathbf{p}_\gamma})T}$ goes to zero for large T if $E_{m,\mathbf{p}_\gamma} > E_\gamma$.

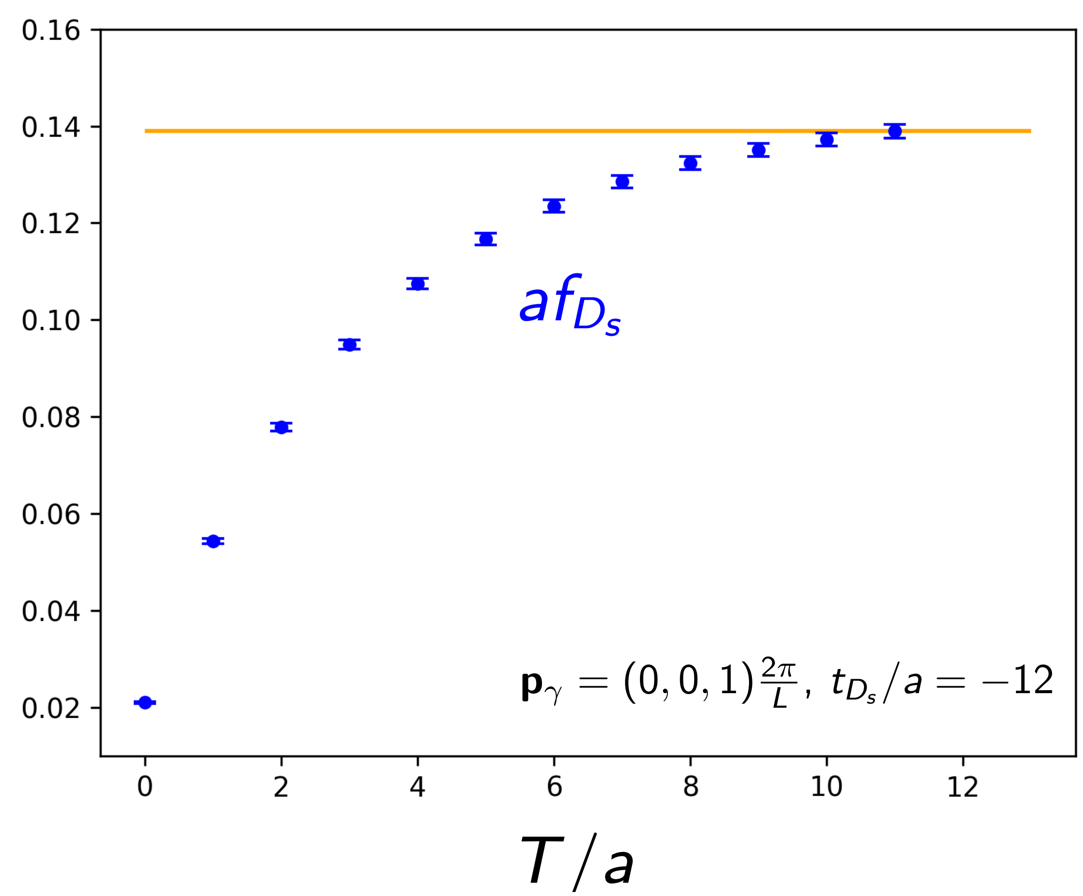
Because the states $|m(\mathbf{p}_\gamma)\rangle$ have a nonzero mass, this is always satisfied.

Cross-checks

Recall

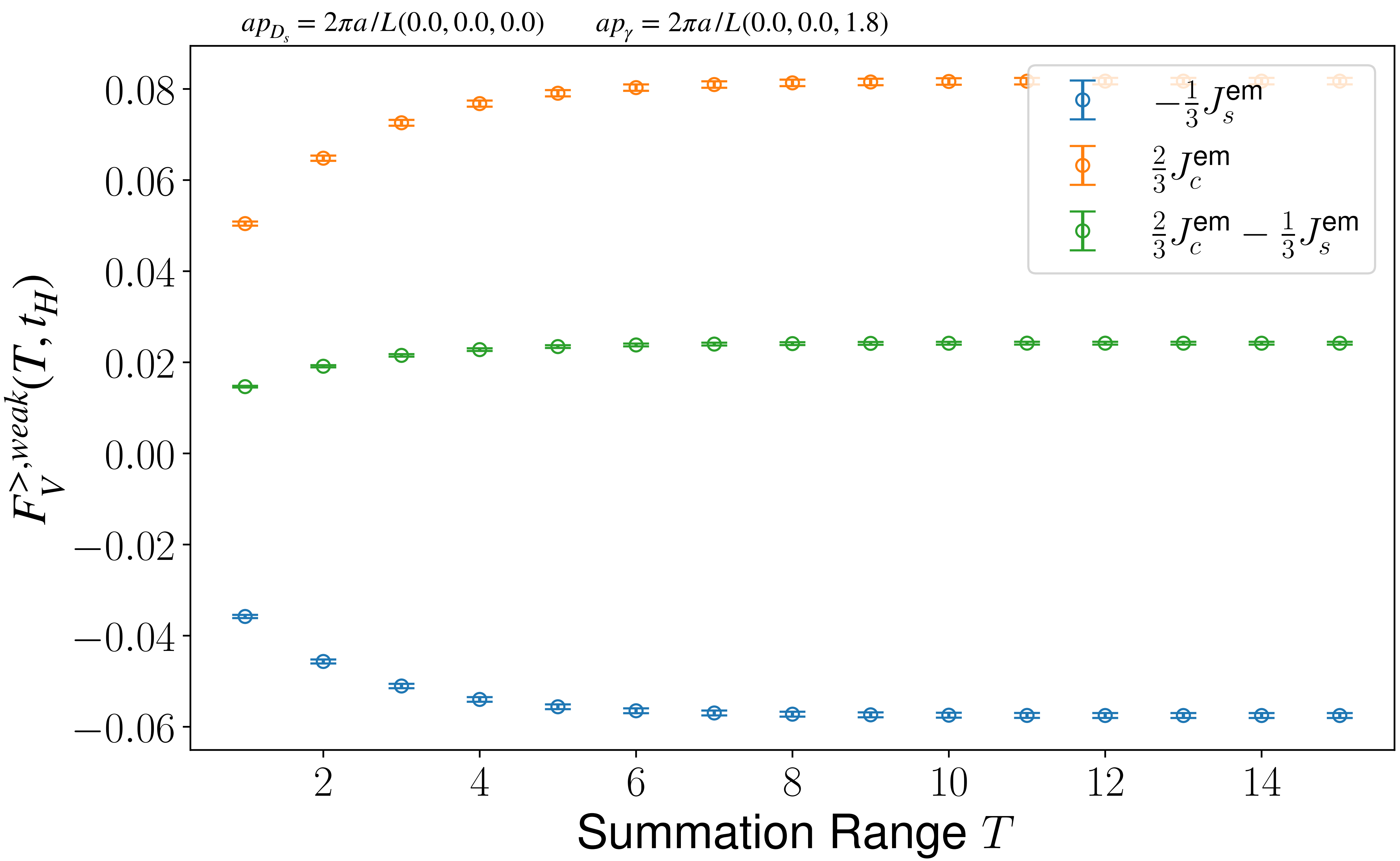
$$T_{\mu\nu} = \epsilon_{\mu\nu\tau\rho} p_\gamma^\tau v^\rho F_V + i[-g_{\mu\nu}(p_\gamma \cdot v) + v_\mu (p_\gamma)_\nu] F_A - i \frac{v_\mu v_\nu}{p_\gamma \cdot v} m_{D_s} f_{D_s}$$
$$+ (p_\gamma)_\mu\text{-terms}$$

$\longrightarrow$ also extract f_{D_s} as a cross-check



Yellow line = FLAG 2021 average

Cancellation between quark components



Fit form: 4d method

Use fit ranges where data has plateaued in t_H, i.e. $t_H \to -\infty$

Include terms to fit
(1) unwanted exponential from first intermediate state

Sum of both time orderings $I_{\mu\nu}(T, t_H) = I^{<}_{\mu\nu}(T, t_H) + I^{>}_{\mu\nu}(T, t_H)$

$$F(t_H, T) = F + B_F^{<} \underbrace{e^{-(E_\gamma - E_H + E^{<})T}}_{t_{em}<0} + B_F^{>} \underbrace{e^{(E_\gamma - E^{>})T}}_{t_{em}>0}$$

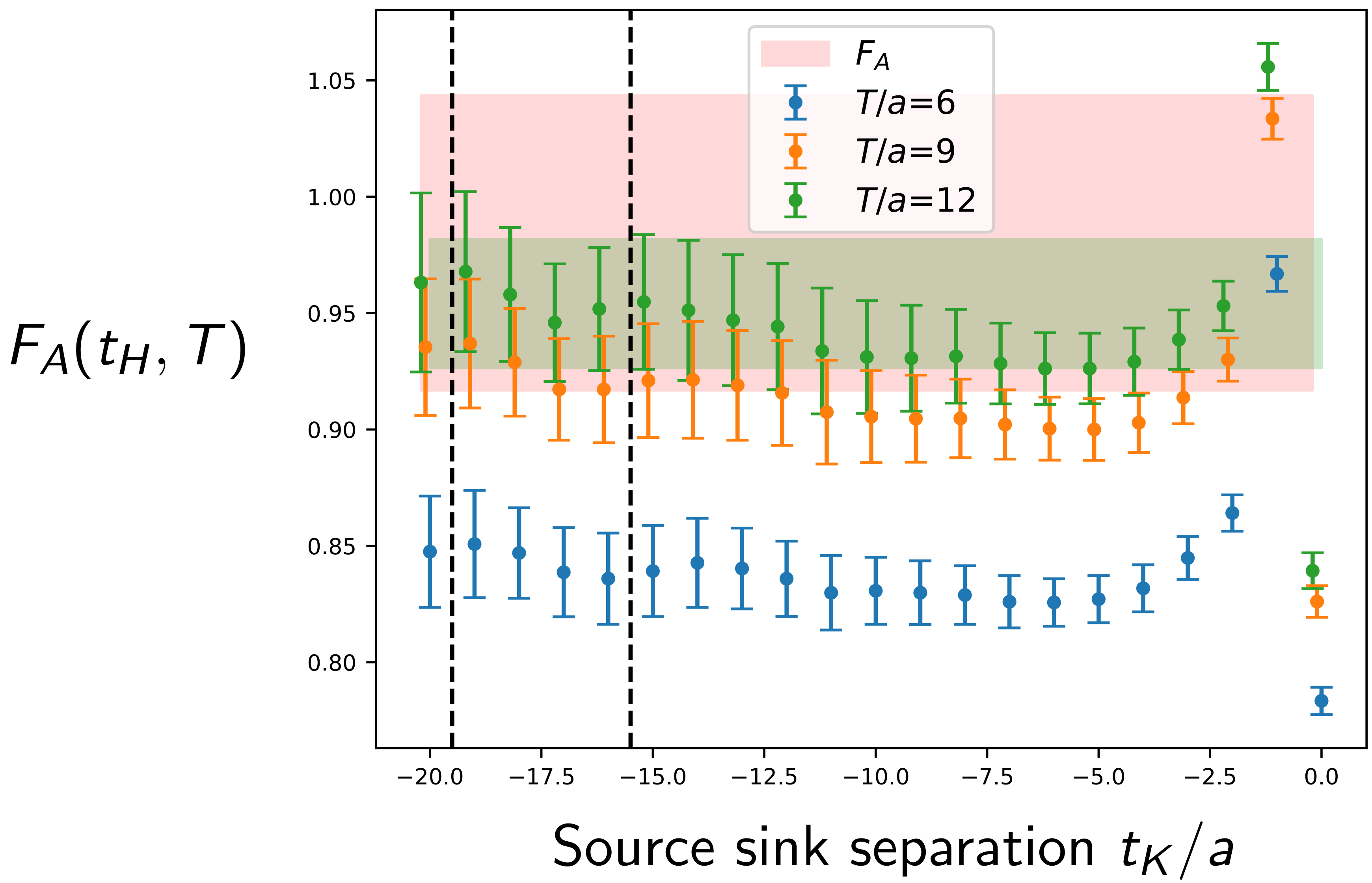
■ fit parameters

Only have three values of T, fitting multiple exponentials not possible
$\to$ Use broad Gaussian prior on $E^{>}$

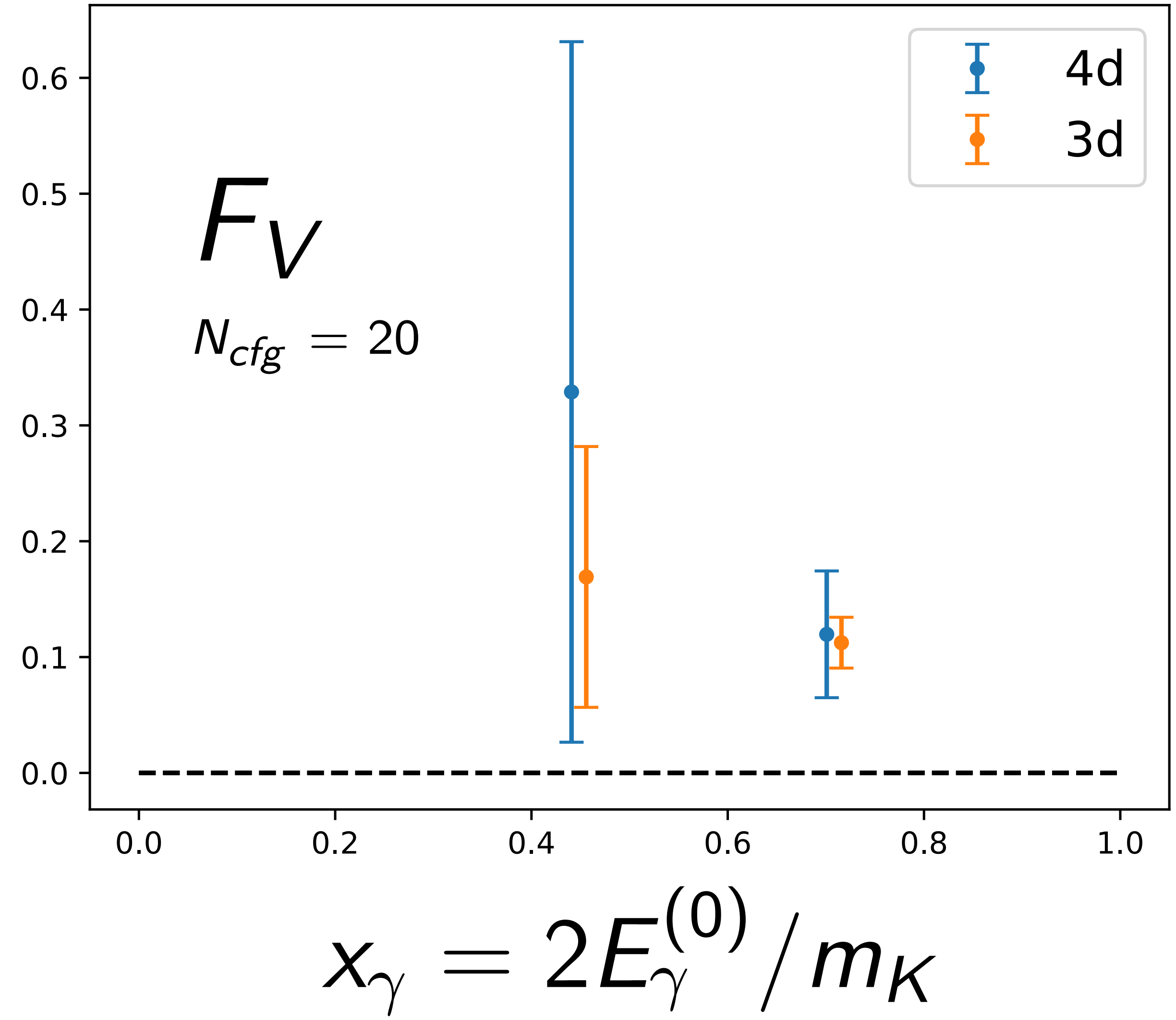
$K \to \ell\nu_\ell\gamma$: 4d method

Sum of both time orderings $t_{em} < 0 + t_{em} > 0$:

$$F_A(t_H, T) = F_A + B^{<}_{F_A} e^{-(E_\gamma - E_K + E^{<}_A)T} + B^{>}_{F_A} e^{(E_\gamma - E^{>}_A)T}$$



$K \to \ell \nu_\ell \gamma$: 3d vs 4d analysis results



4d method cannot resolve the sum of the unwanted exponentials of the separate time orderings