

# A lattice QCD study of the $B \rightarrow \pi\pi \ell\nu$ transition

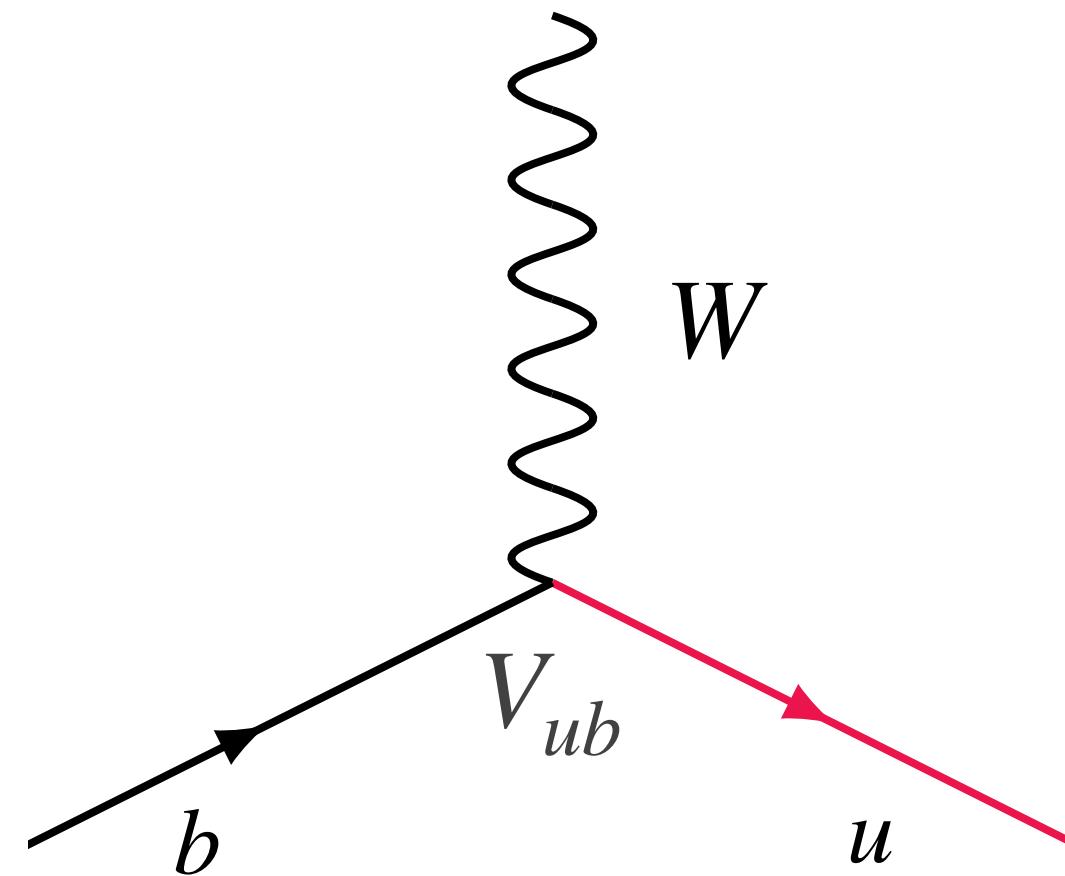
The 39th International Symposium on  
Lattice Field Theory  
Bonn, Germany, August 2022

in collaboration with:

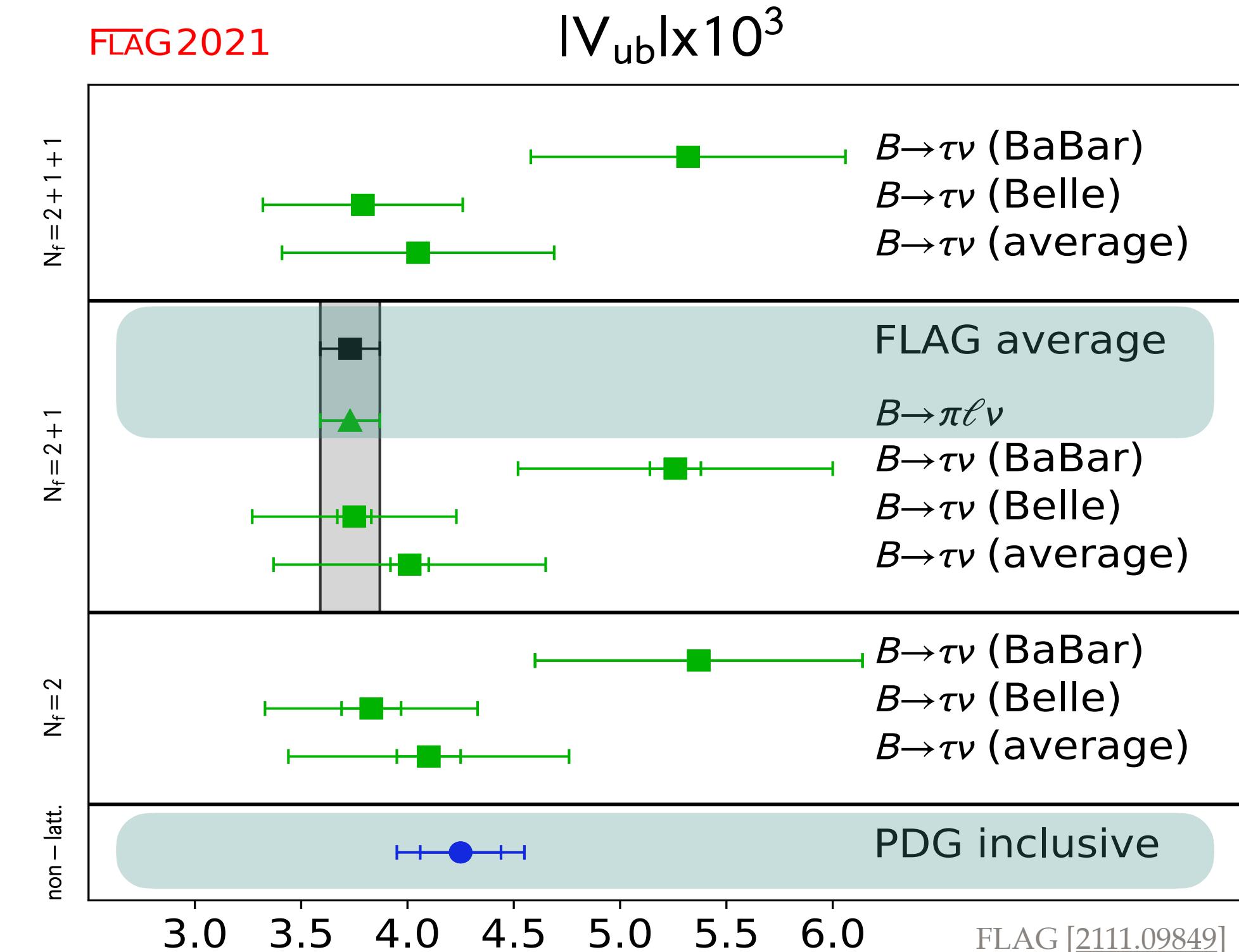
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J.W. Negele, S. Paul, A. Pochinsky, G. Rendon

Luka Leskovec

# why $V_{ub}$ ?

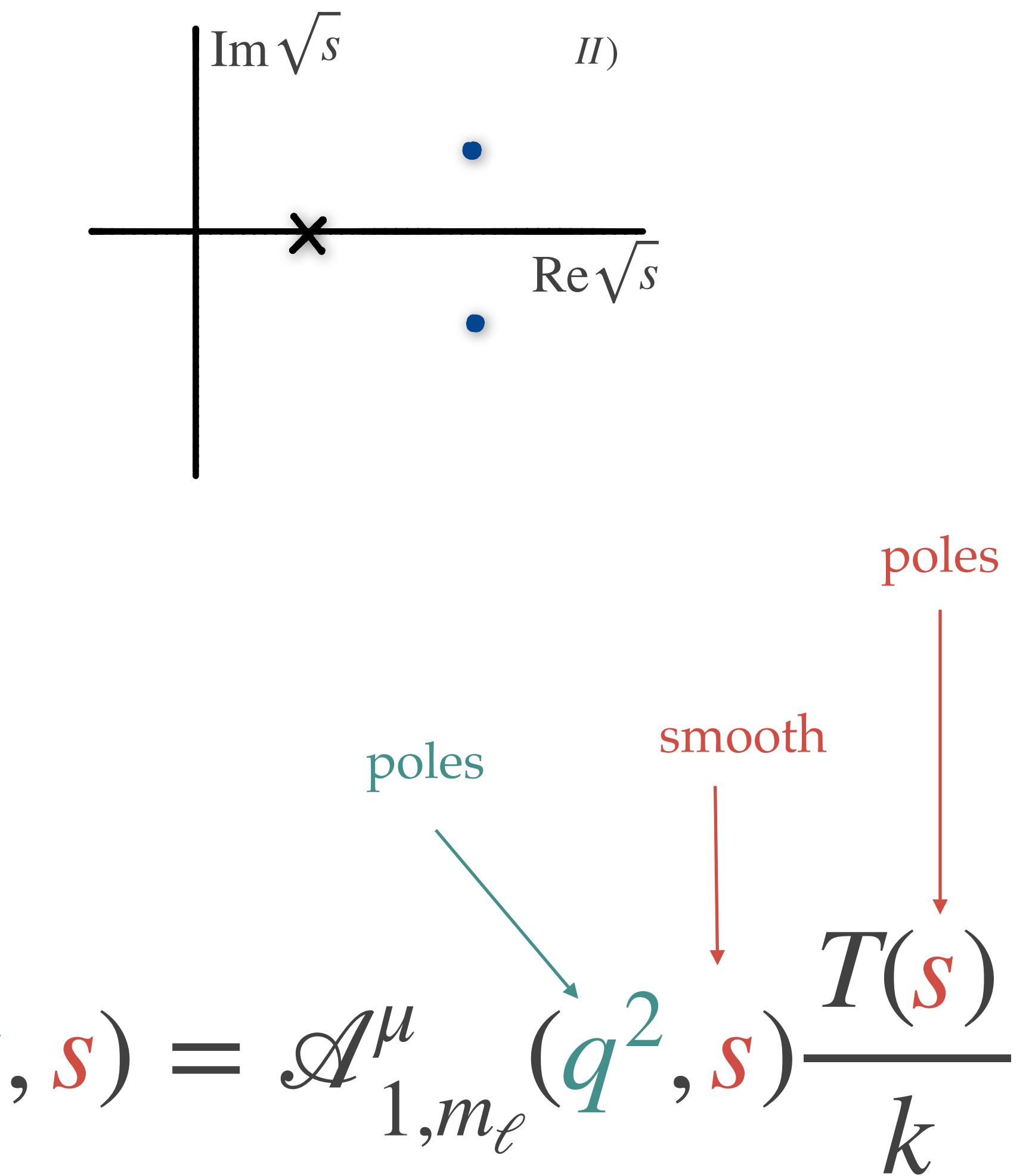
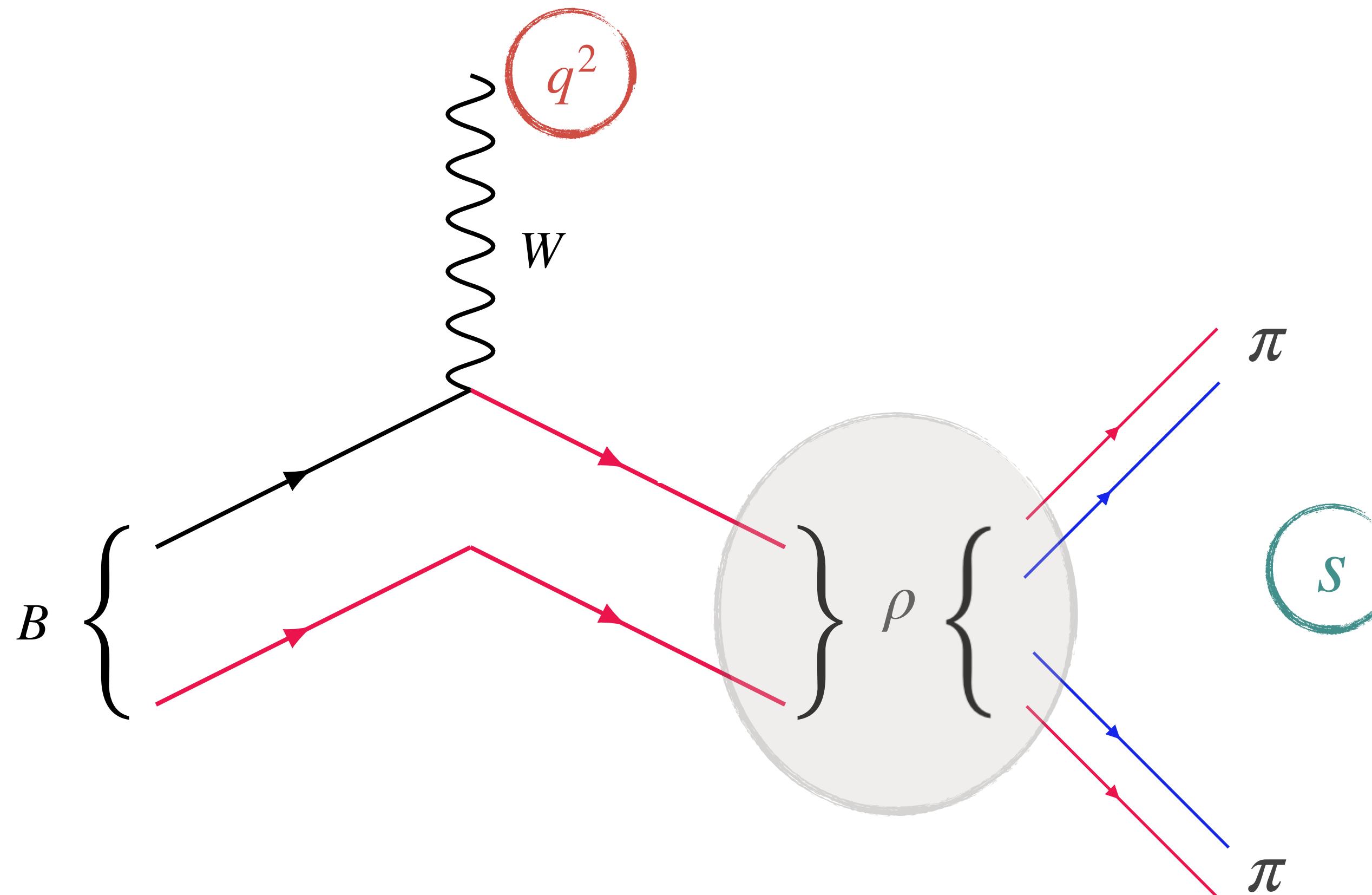


$$\begin{bmatrix} d^W \\ s^W \\ b^W \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$



❖ focus: semileptonic transitions

# $V_{ub}$ from $B \rightarrow \pi\pi\ell\nu$



$$\mathcal{H}_{1,m_\ell}^\mu(q^2, s) = \mathcal{A}_{1,m_\ell}^\mu(q^2, s) \frac{T(s)}{k}$$

# goal?

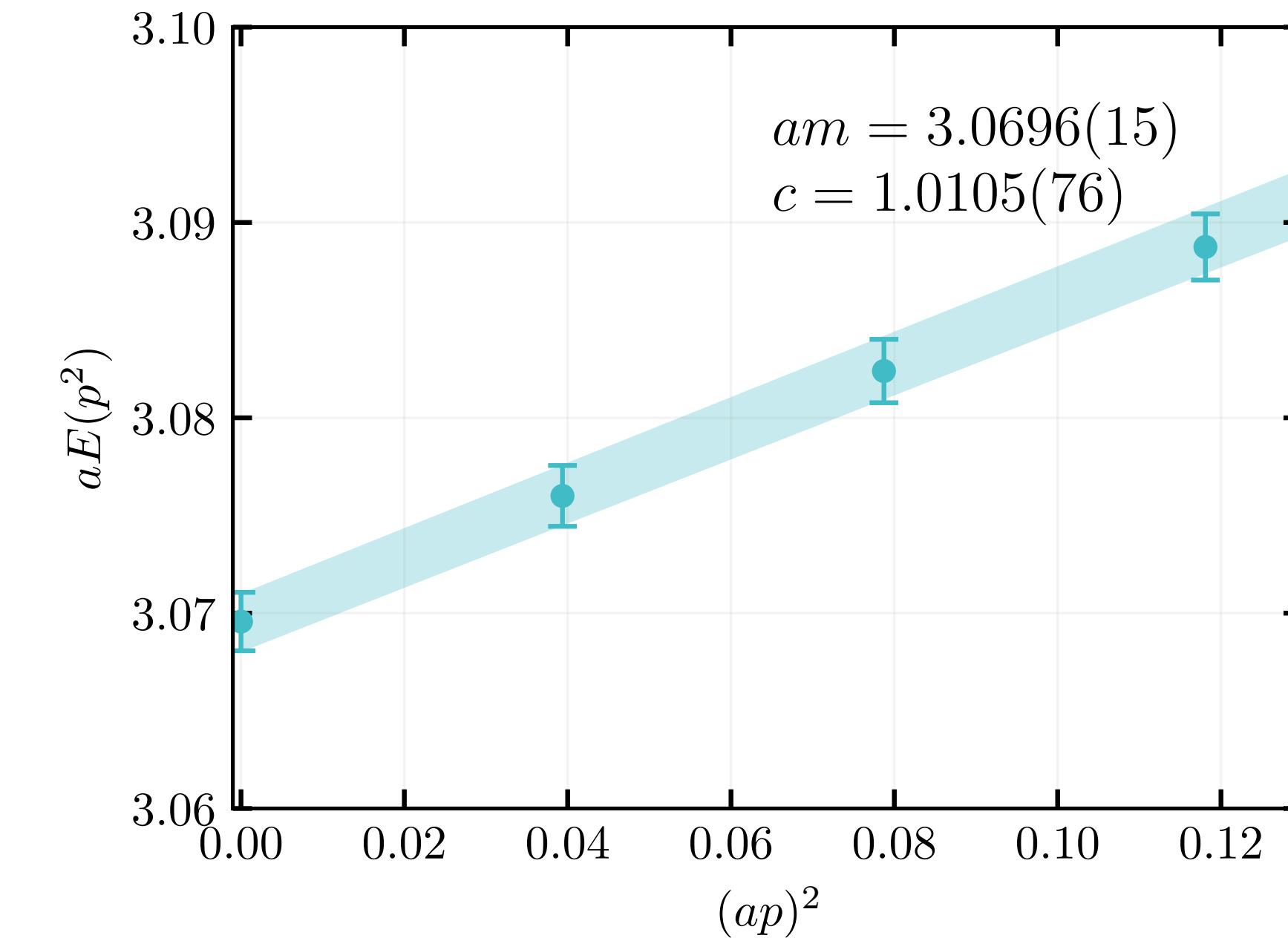
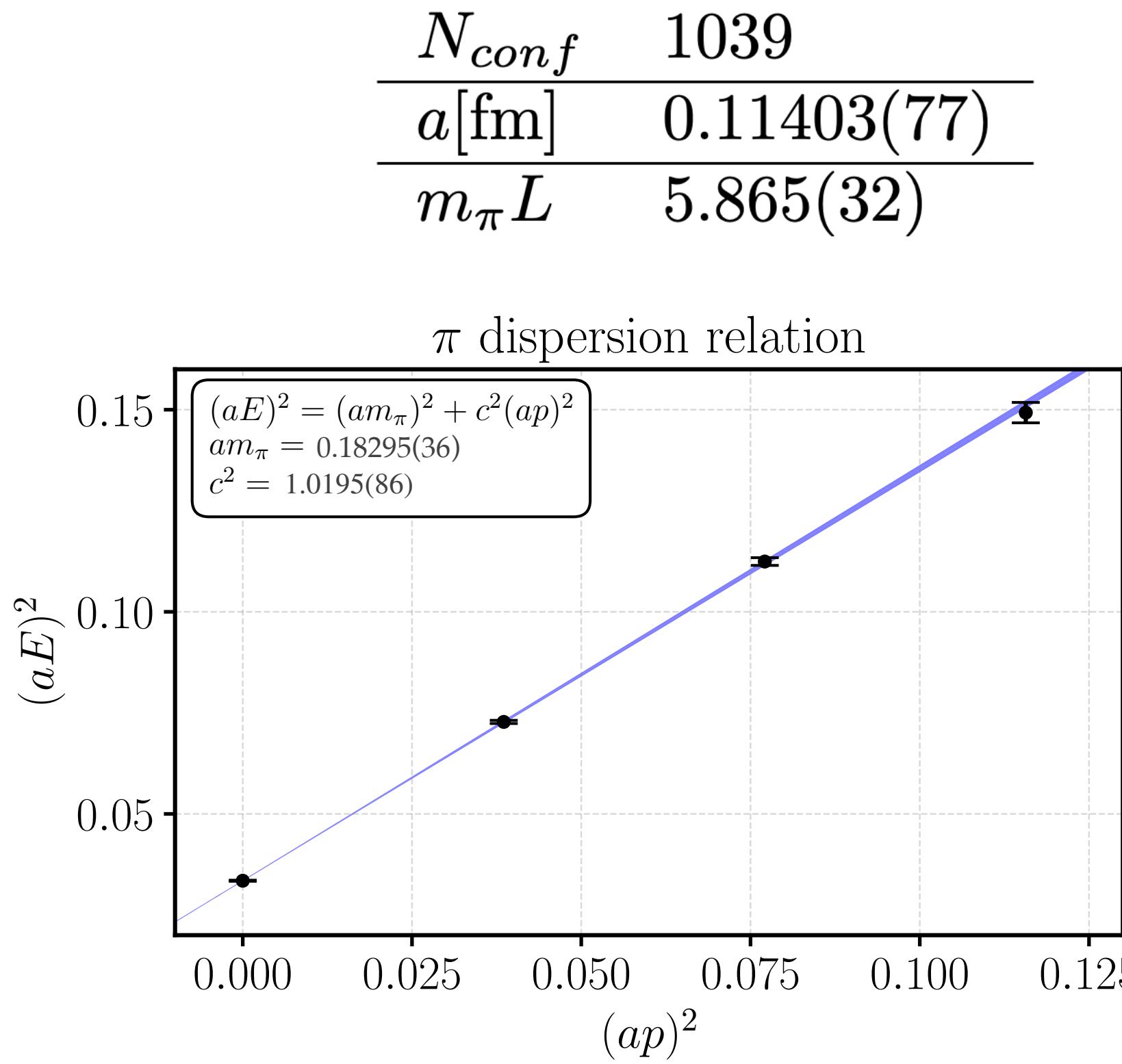
$$\begin{aligned} \mathcal{A}_{1,m_\ell}^\mu &= \frac{2i\textcolor{red}{V}}{m_B} \epsilon^{\mu\nu\alpha\beta} \varepsilon_\nu^*(p_f, m_\ell) (p_f)_\alpha (p_i)_\beta \\ &\quad - 2m_{th} \textcolor{blue}{A}_0 \frac{\varepsilon^*(p_f, m_\ell) \cdot q}{q^2} q^\mu \\ &\quad - (m_B + m_{th}) \textcolor{blue}{A}_1 \left( \varepsilon^{*\mu}(p_f, m_\ell) - \frac{\varepsilon^*(p_f, m_\ell) \cdot q}{q^2} q^\mu \right) \\ &\quad + \textcolor{blue}{A}_2 \frac{\varepsilon(p_f, m_\ell) \cdot q}{m_B + m_{th}} \left( p_i^\mu + p_f^\mu - \frac{m_B^2 - (m_{th})^2}{q^2} q^\mu \right) \end{aligned}$$

$\textcolor{red}{V}, \textcolor{blue}{A}_0, \textcolor{blue}{A}_1, \textcolor{blue}{A}_2$

focus on  $\textcolor{red}{V}$  today

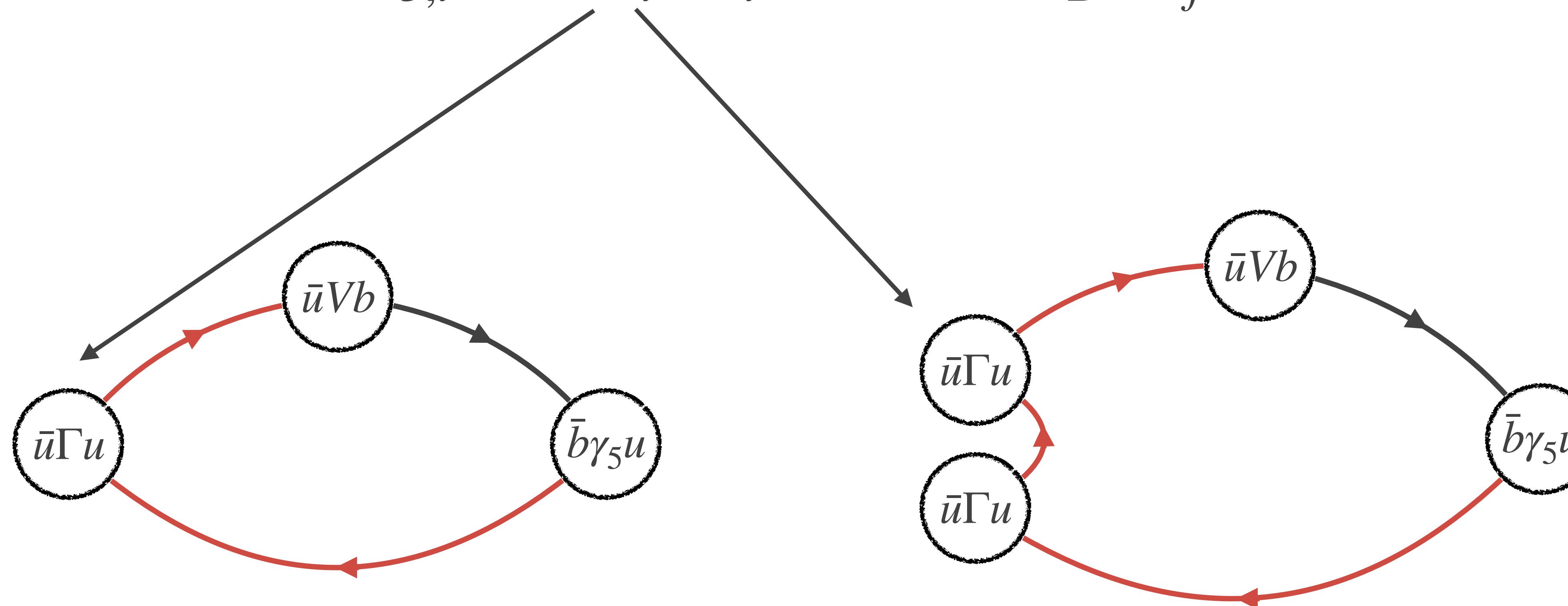
# ensemble

- ❖  $N_f = 2 + 1$  clover-Wilson fermions
- ❖ isotropic
- ❖  $m_\pi = 316.6(6)$  MeV
- ❖ RHQ heavy quarks
- ❖  $m_B = 5319.8(2.6)$  MeV



# the calculation

$$C_{3,i} = \langle O_i(\vec{p}_i, \Lambda) \ V^\mu \ O_B(\vec{p}_f) \rangle$$



... and a lot of technical details I am skipping

# the 3-point functions

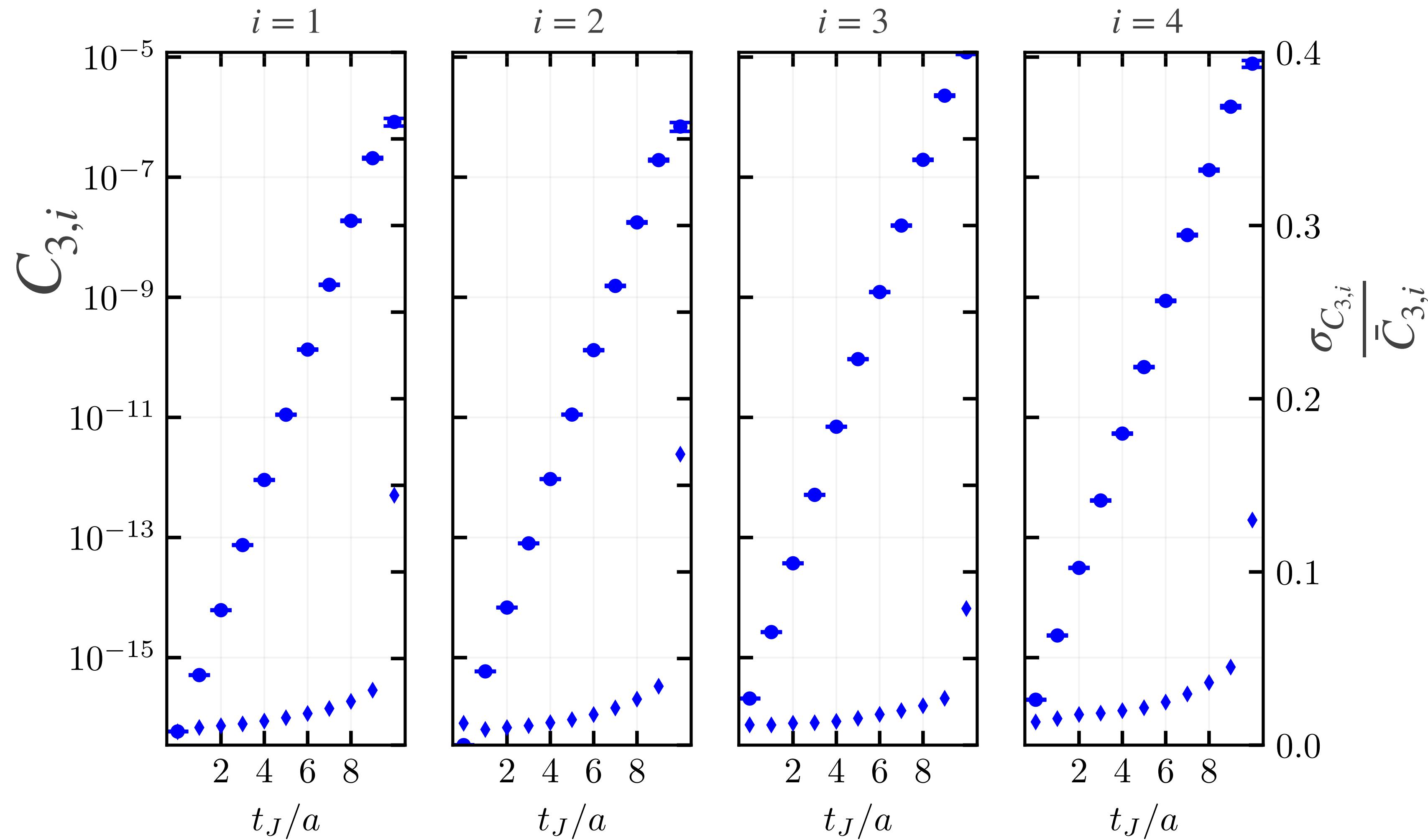
$$\vec{p}_i = \frac{2\pi}{L}[1,0,1]$$

$$\Lambda = B_2$$

$$\mu = z$$

$$\vec{p}_f = \frac{2\pi}{L}[0,0,1]$$

$$C_{3,i} = \langle O_i(\vec{p}_i, \Lambda) V^\mu O_B(\vec{p}_f) \rangle$$



$$O_1 = \bar{u} \Gamma_{B_1} u [\vec{p}_i]$$

$$O_2 = \bar{u} \gamma_t \Gamma_{B_1} u [\vec{p}_i]$$

$$O_3 = \pi^+(\vec{p}_{i,1}) \pi^-(\vec{p}_{i,2})|_{\vec{p}_i}$$

$$O_4 = \pi^+(\vec{p}_{i,1}) \pi^-(\vec{p}_{i,2})|_{\vec{p}_i}$$

# state projection

$$C_{3,i} =$$

$$\sum_{m \in B} \sum_{n \in [\pi\pi]} \langle 0 | O_i | n \rangle \langle n | V | m \rangle \langle m | O_B | 0 \rangle \frac{e^{-E_n(t_f-t)} e^{-E_m^B(t-t_i)}}{2E_n 2E_m^B},$$

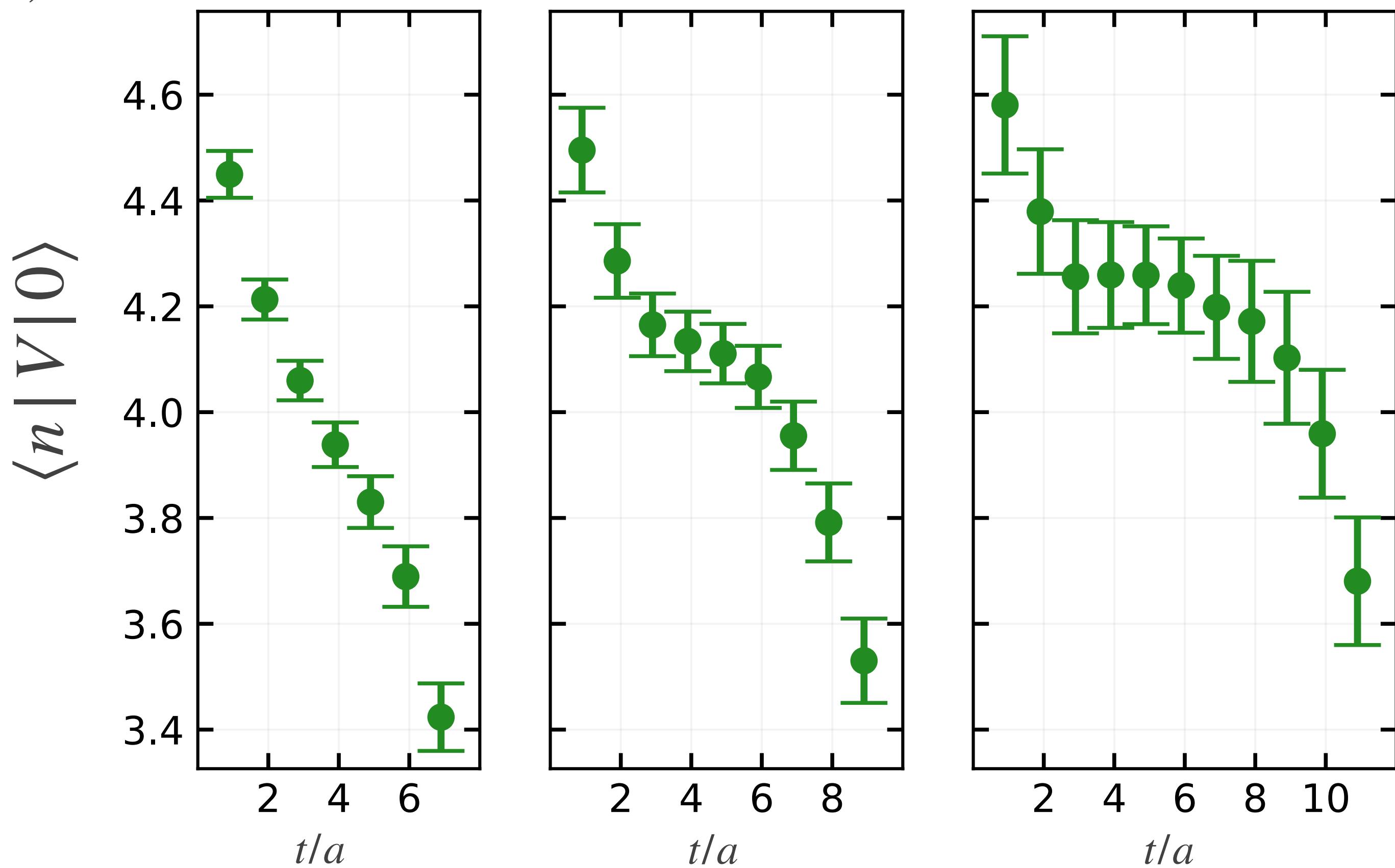
$$C_3^n = u_i^n C_{3,i}$$

weights from  $\pi\pi$  GEVP

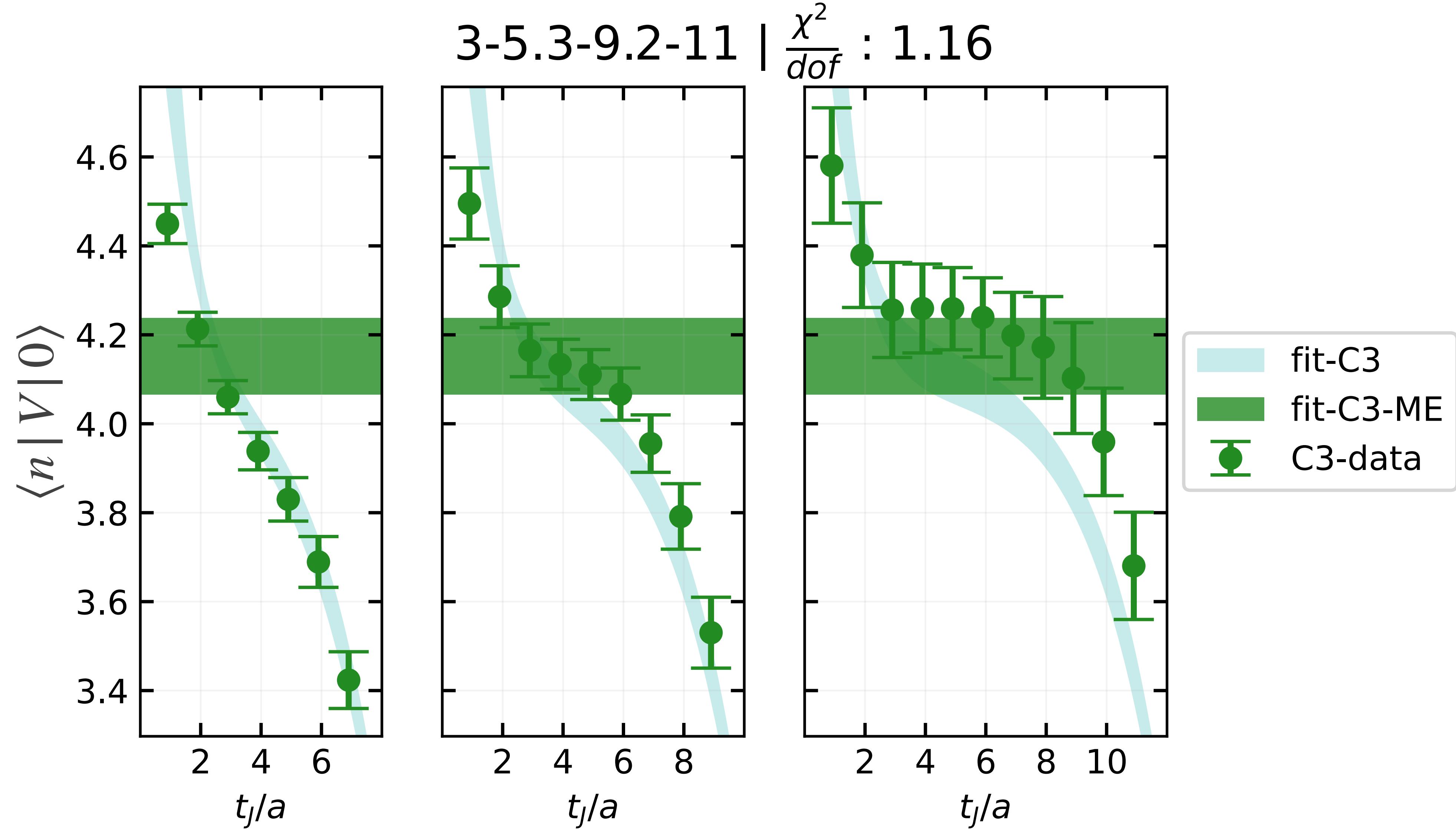
$$C_3^n = \langle n | V | 0 \rangle \langle 0 | O_B | 0 \rangle \frac{e^{-E_n(t_f-t)} e^{-E_0^B(t-t_i)}}{2E_n 2E_0^B}$$

+ excited state cont.

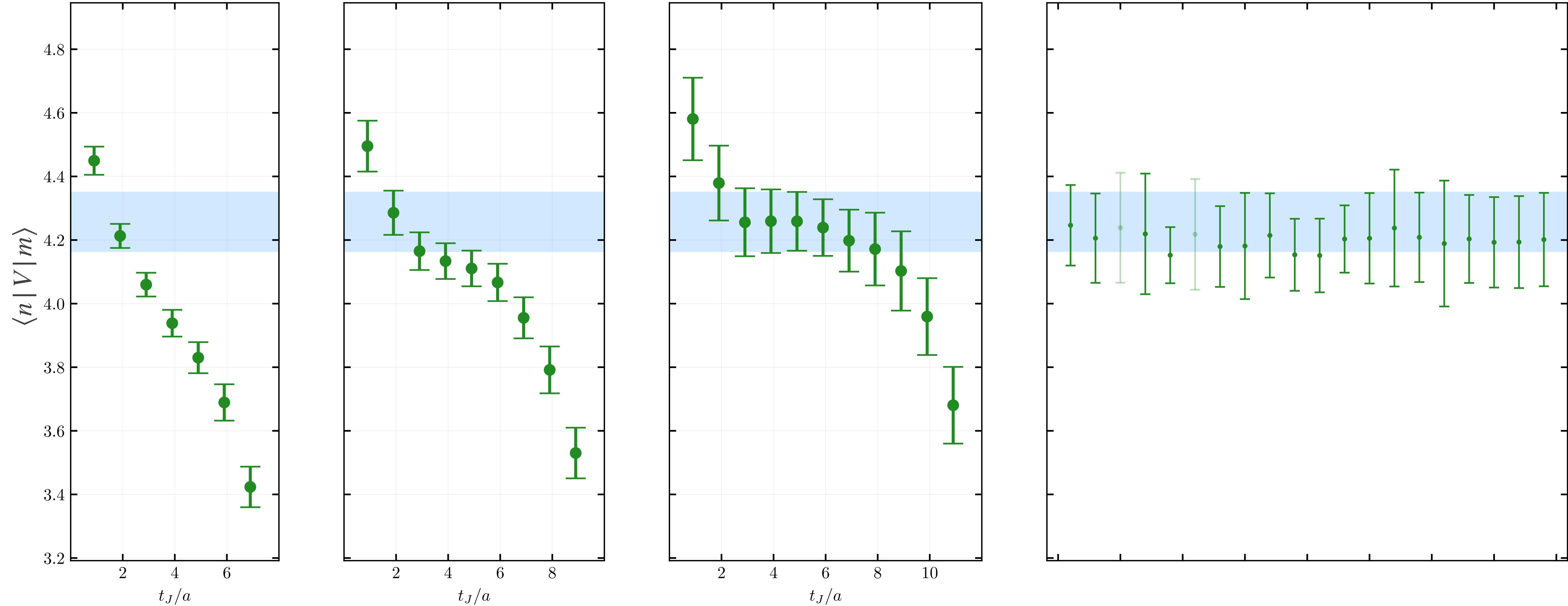
$$\vec{p}_i = \frac{2\pi}{L}[1,0,1] \quad \Lambda = B_1 \quad n = 1 \quad \vec{p}_f = \frac{2\pi}{L}[0,0,1]$$



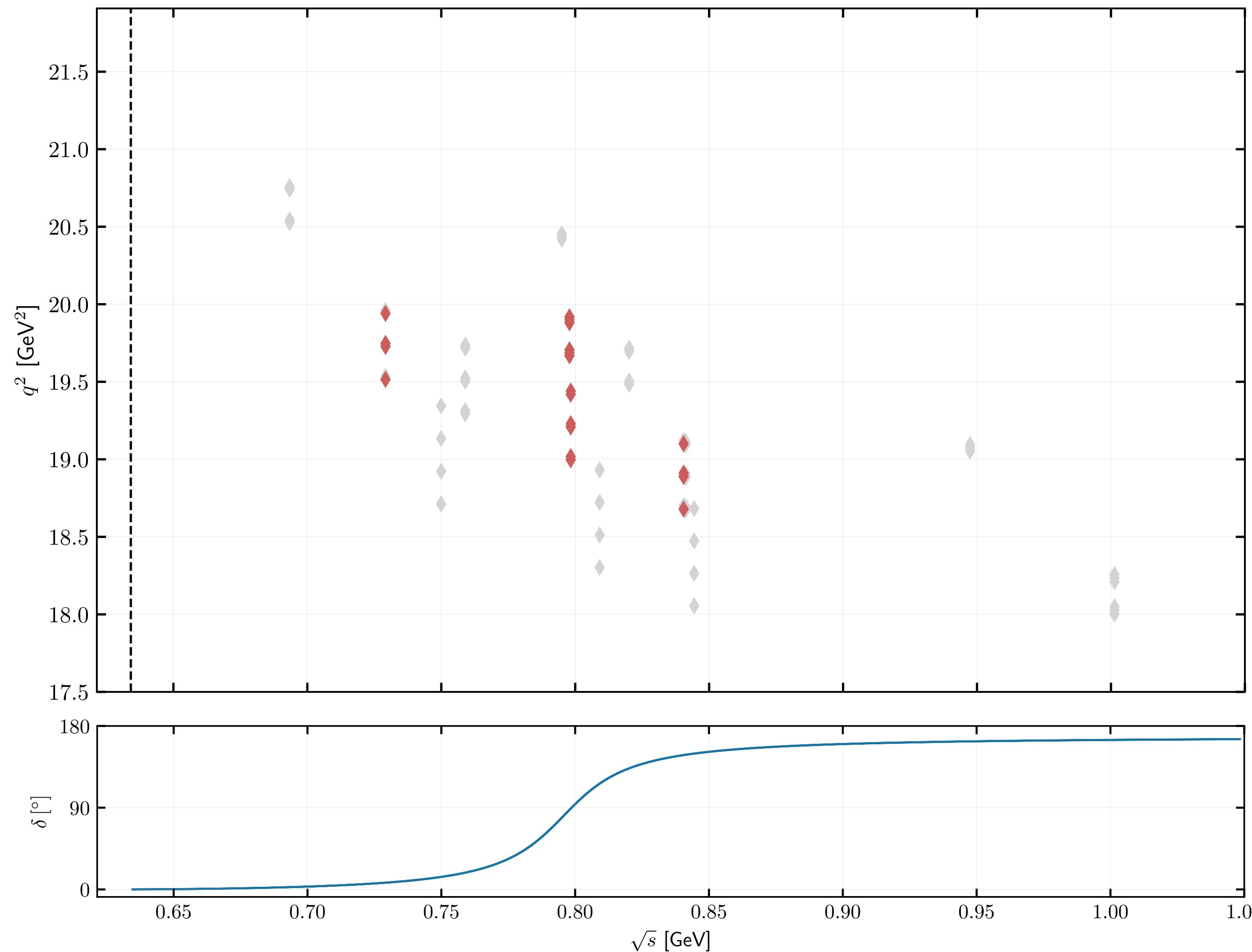
# example fit to data



# a series of fits to data



# what do we have access to



# normalization of the matrix elements

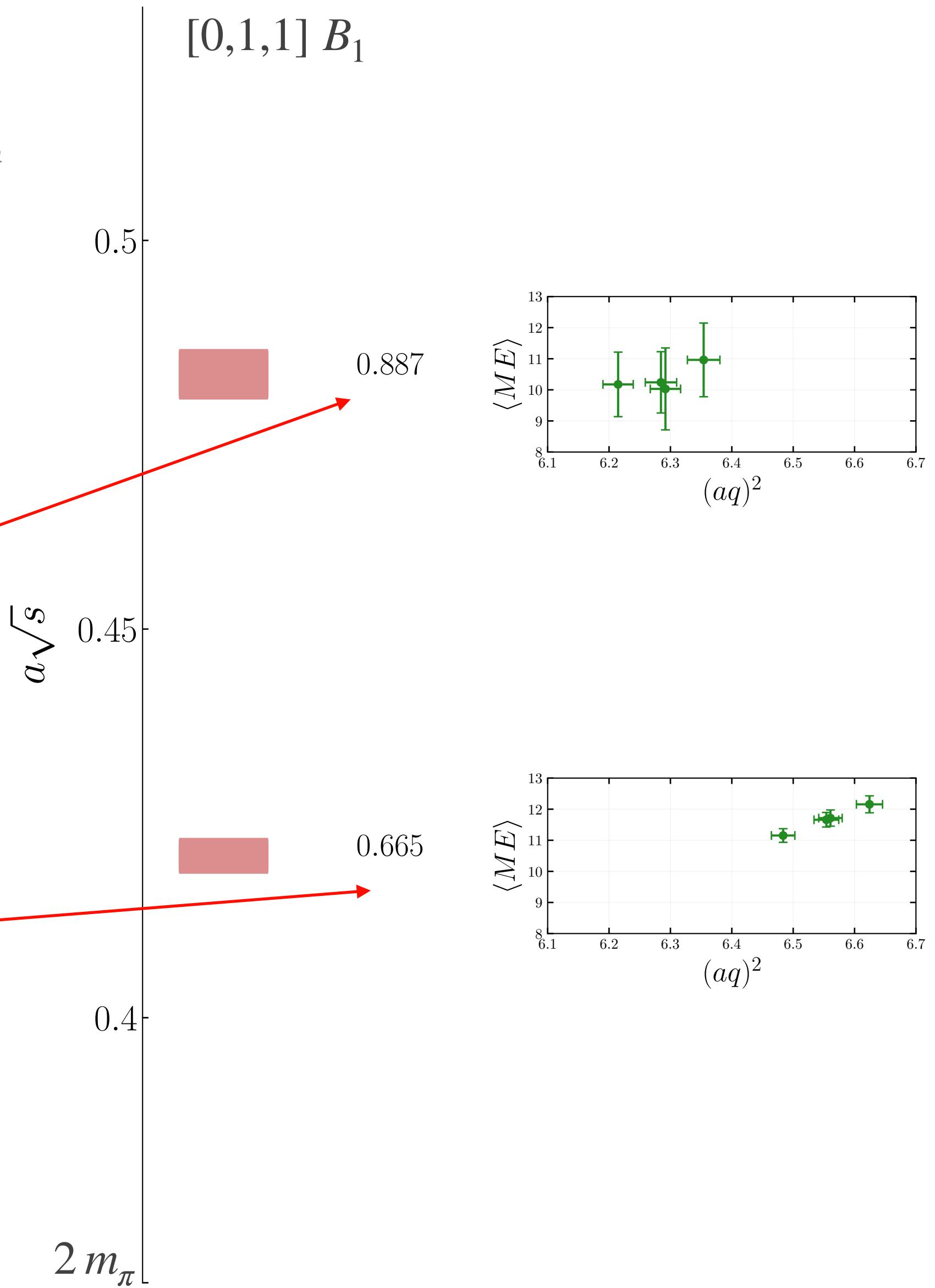
normalization of finite-volume states

$$|\sqrt{s}_n^\Lambda\rangle_L \sim \sqrt{R_n} |\pi\pi(\sqrt{s} = \sqrt{s}_n^\Lambda)\rangle_\infty$$

$$R_n = 2E_n \lim_{E \rightarrow E_n^\Lambda} \frac{E - E_n^\Lambda}{F^{-1} + \mathcal{M}} = \frac{2\sqrt{s}_n^\Lambda}{\mu_0^*} \mathbf{w}_0 \mathbf{w}_0^T \Big|_{\sqrt{s}_n^\Lambda}$$

$$|\langle ME \rangle_L| = \frac{1}{\sqrt{2E_B} \sqrt{2E_n^\Lambda}} \boxed{\sqrt{\frac{2E_n^*}{-\mu_0^*}}} w_0^T \cdot V$$

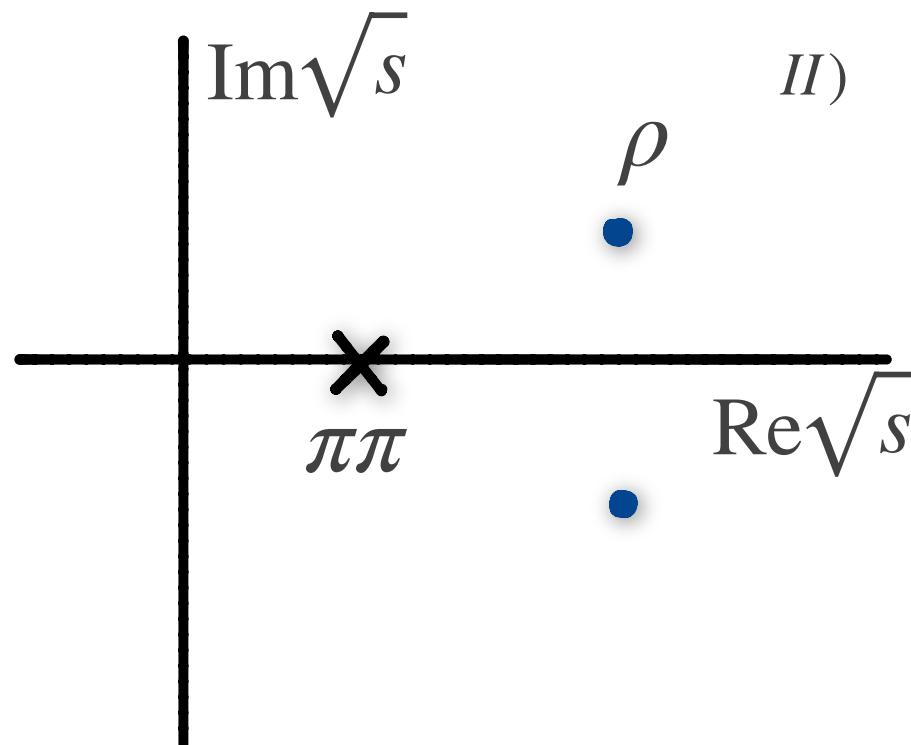
Lellouch, Luscher [hep-lat/0003023](#)  
 Lin, Sachrajda, Testa [hep-lat/0104006](#)  
 ...  
 Briceno, Hansen, Walker-Loud [1406.5965](#)  
 Briceno, Hansen [1502.04314](#)  
 Briceno, Dudek, LL [2105.02017](#)



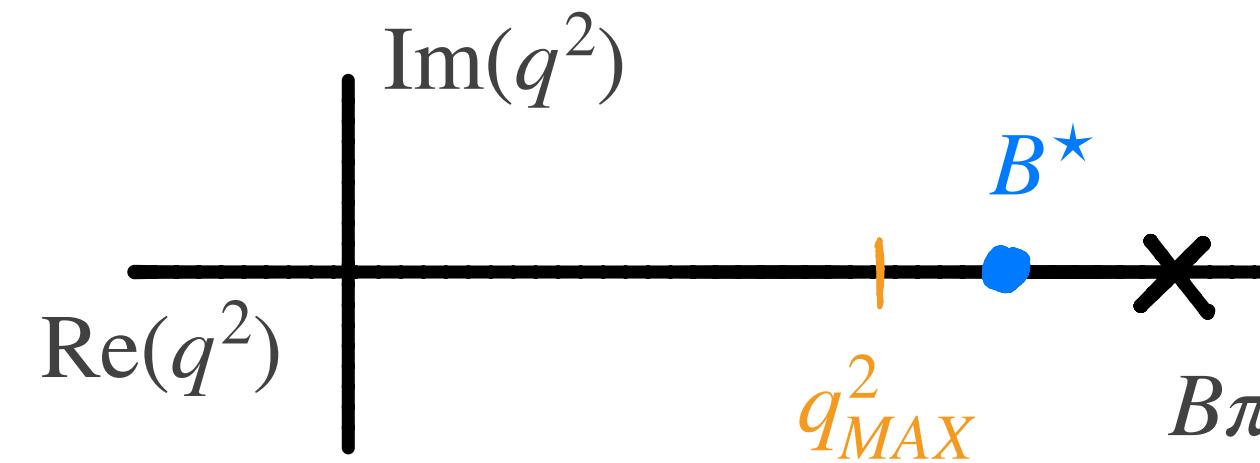
# fitting the matrix elements

$$\mathcal{A} = V(q^2, s) \frac{T(E^\star)}{k}$$

$$V(q^2, s) = \frac{1}{1 - \frac{q^2}{m_{B^\star}^2}} \sum_n \sum_m a_{nm} z^n(q^2) s^m$$



- $T(E^\star)$ :
  - $\pi\pi$  threshold
  - $\rho$  pole



- $V(q^2, E^\star)$ :
  - smooth in  $E$
  - $q^2$  has poles and thresholds
  - use  $z$ -expansion

Boyd, Grinstein, Lebed [hep-ph/9412324](#)  
 Bourrely, Caprini, Lellouch [0807.2722](#)  
 Alexandrou, LL, Meinel et al. [1807.08357](#)

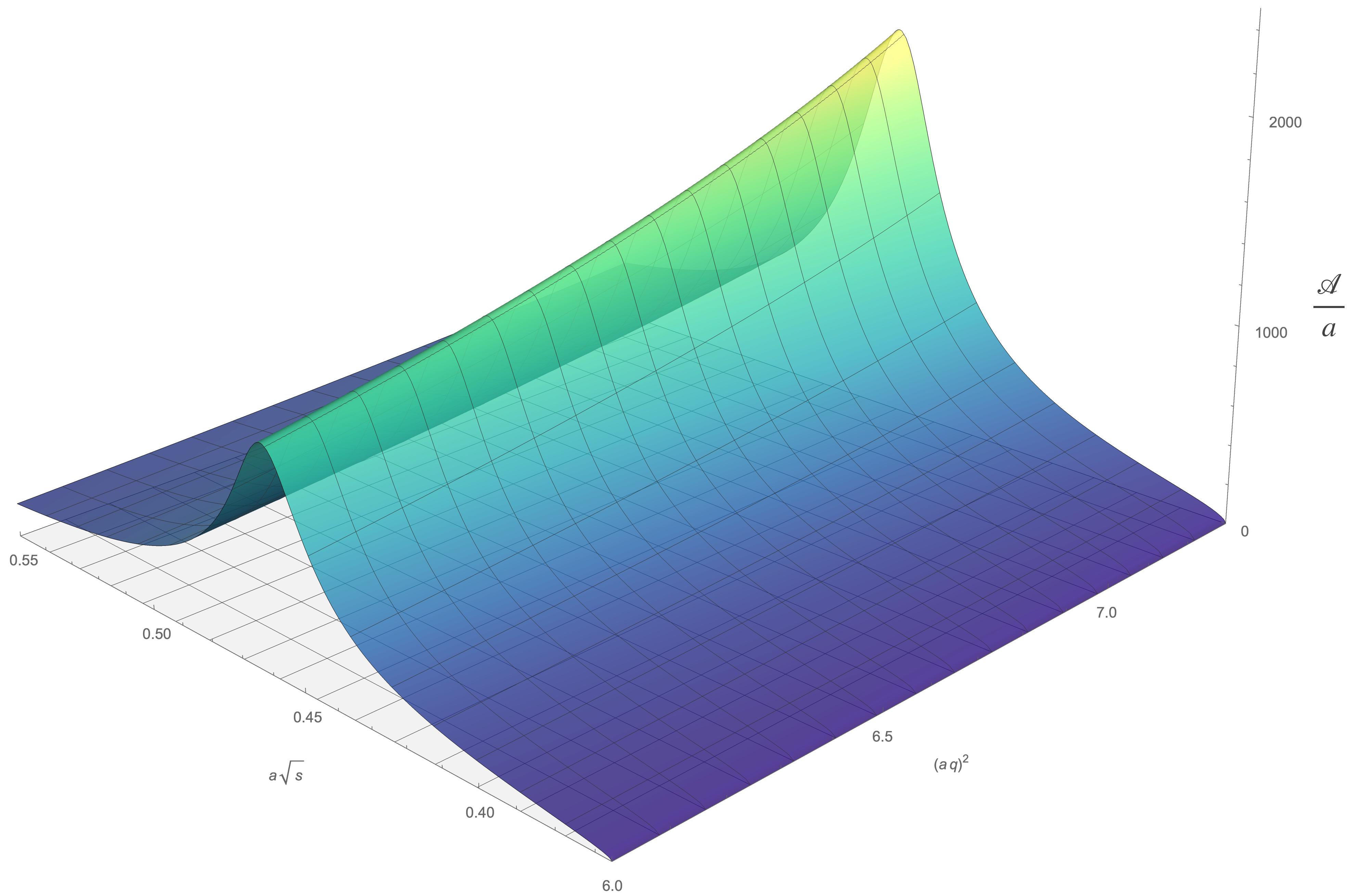
$N_{data} = 20$

$N_{par} = 2$

$\frac{\chi^2}{\text{dof}} = 1.1$

$a_{0,0} = 1.36(20)$

$a_{1,0} = -3.2(1.4)$



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Thank you

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