Neutrino Oscillation and Lattice QCD

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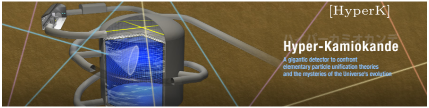
Outline

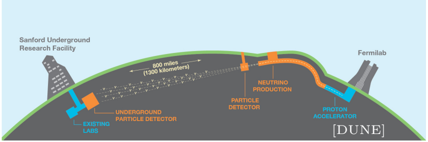
- Introduction
- Quasielastic Scattering from Experiment
  - Deuterium Constraints
- $F_A(Q^2)$ from LQCD
  - Excited States
  - Summary of $F_A(Q^2)$ Calculations
  - T2K/DUNE Implications
- Ongoing Work
  - Axial Form Factor Updates
  - Future Prospects

Note: all references in online slides are hyperlinked

Neutrino Oscillation

Neutrino Physics Goals





Flagship long baseline experiments to measure neutrino oscillation

DUNE: USA, HyperK: Japan

$O(10 - 100 \text{ kton})$ mass

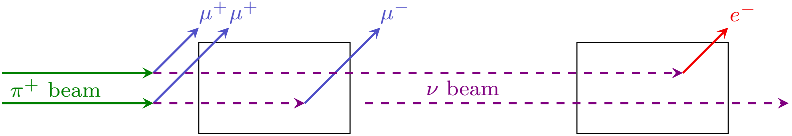
Seek to answer fundamental questions about neutrinos:

- mass ordering ($\Delta m^2_{32} > 0$?)
- octant ($\sin^2\theta_{23} = 0.5$?)
- CP violation ($\delta_{\text{CP}} =$?)
- PMNS unitarity?
- 3 ν flavors?
- precision constraints

Also measure solar and supernova neutrinos

Data collection starts 2028-2029 $\Longrightarrow$ need support from theory!

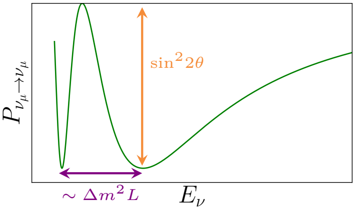
Neutrino Oscillation



$\underbrace{|\nu_\ell\rangle}_{\text{flavor eigenstate}} = \sum_i U^*_{\ell i} \underbrace{|\nu_i\rangle}_{\text{mass eigenstate}}$ $\qquad |\nu_i\rangle \to e^{-iE_i t}|\nu_i\rangle$

2 flavor model:

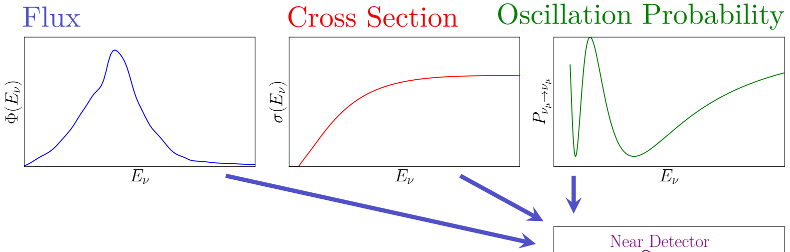
$P_{\nu_\mu \to \nu_e} = \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m^2 [\text{eV}^2] L [\text{km}]}{E_\nu [\text{GeV}]} \right)$

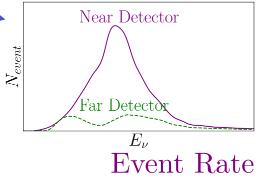


Mass eigenstates – propagation
flavor eigenstates – interaction
} Not the same

Oscillation probability is function of L/E_ν at fixed L

Measuring Oscillations





$$N_{\text{event}} = \int_{\text{bin}} dE_\nu\, \Phi(E_\nu)\sigma(E_\nu)\, \underbrace{\Big[\times P(L/E_\nu) \Big]}_{\text{far detector only}}$$

$$\Big[P(L/E_\nu)\Big]_{\text{bin}} \approx \Big[\frac{N_{\text{event}}[\text{FAR}]}{N_{\text{event}}[\text{NEAR}]} \Big]_{\text{bin}}$$

Neutrinos from tertiary beam ($p \to \pi^+ \to \nu_\mu$): broad flux

Incoming E_ν not known, must determine from measurement

Neutrino Event Topologies

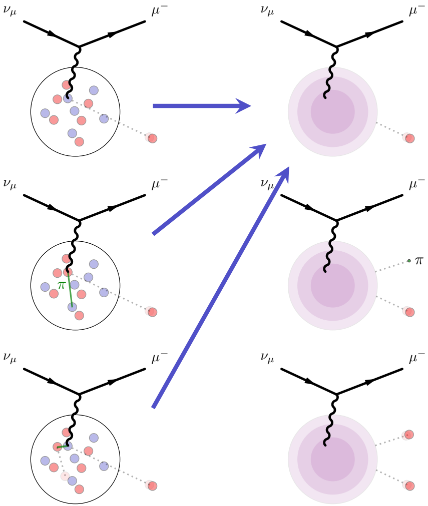
Nuclear environment complicates measurements:

- Many allowed kinematic channels
- Reinteractions within nucleus
- Only final state particles are observable

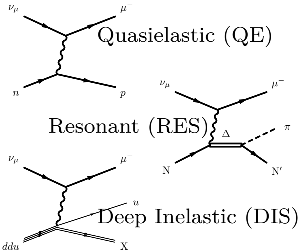
For any event, range of possible E_ν depending upon event topology

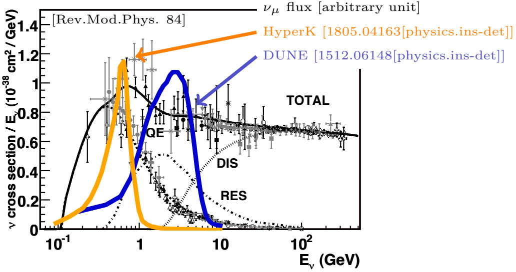
$\Longrightarrow$ Event-by-event E_ν measurements are not possible

"Reconstruct" event distributions using Monte Carlo simulations



Neutrino Cross Sections





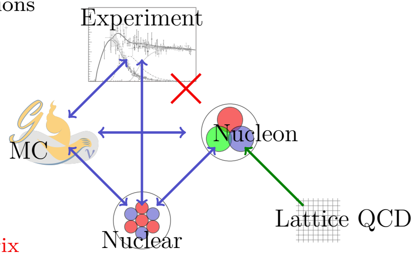
Nucleon amplitudes → *nuclear* cross sections

Energy range spans several *nucleon* interaction topologies

Goal: isolate, quantify, improve *nucleon* amplitudes with LQCD

⟹ inputs to Monte Carlo simulations of *nuclear* systems

⟹ need full form factor parameterizations + covariance matrix

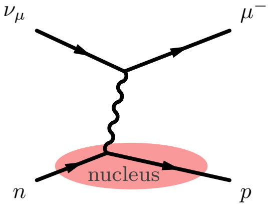


Note: axial radius r_A^2 is useful for comparison, but incomplete picture

QE Experimental Constraints

Quasielastic Form Factors

Quasielastic (QE) scattering assumes quasi-free nucleon inside nucleus



$$\mathcal{M}_{\text{nucleon}} = \langle \ell | \mathcal{J}^\mu | \nu_\ell \rangle \langle N' | \mathcal{J}_\mu | N \rangle$$

$$\langle N'(p') | (V-A)_\mu(q) | N(p) \rangle$$
$$= \bar{u}(p') \Big[\gamma_\mu F_1(q^2) + \frac{i}{2M_N} \sigma_{\mu\nu} q^\nu F_2(q^2) + \gamma_\mu \gamma_5 F_A(q^2) + \frac{1}{2M_N} q_\mu \gamma_5 F_P(q^2) \Big] u(p)$$

- F_1, F_2: constrained by eN scattering
- F_P: subleading in cross section,
  $\propto F_A$ from partially conserved axial current (PCAC) constraint

Axial form factor F_A is leading contribution to nucleon cross section uncertainty

Form Factor Parameterizations

Most common in experimental literature: dipole ansatz —

$$F_A(Q^2) = g_A \left(1 + \frac{Q^2}{m_A^2}\right)^{-2}$$

- Overconstrained by both experimental and LQCD data (revisit later)
- Inconsistent with QCD, requirements from unitarity bounds
- Motivated by $Q^2 \to \infty$ limit, data restricted to low Q^2

Model independent alternative: z expansion [Phys.Rev.D 84 (2011)] —

$$F_A(z) = \sum_{k=0}^{\infty} a_k z^k \qquad z(Q^2; t_0, t_{\rm cut}) = \frac{\sqrt{t_{\rm cut} + Q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} + Q^2} + \sqrt{t_{\rm cut} - t_0}} \qquad t_{\rm cut} \leq (3M_\pi)^2$$

- Rapidly converging expansion
- Controlled procedure for introducing new parameters

Deuterium Constraints on F_A

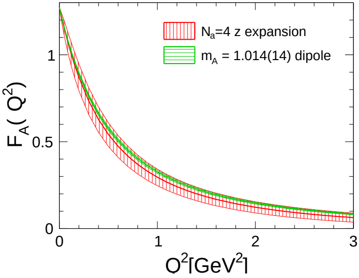
- Outdated bubble chamber experiments:
  - Total $O(10^3)$ ν_μQE events
  - Original data lost
  - Unknown corrections to data
  - Deficient deuterium correction
- Dipole overconstrained by data
  underestimated uncertainty $\times O(10)$
- Prediction discrepancies could be from
  nucleon and/or nuclear origins

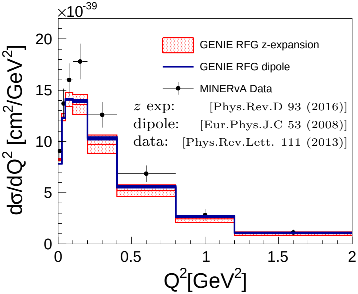
Coming soon:

MINERνA $\bar{\nu}_\mu p \to \mu^+ n$ dataset

Obtained from hydrocarbon target

See [Cai thesis (2021)]





Lattice QCD

Digression - Excited States

Previous concerns about low g_A axial vector coupling

Axial vector coupling/form factor systematics have been extensively studied
Excited states identified as a major cause for concern

Gold standard: $N\pi$ multiparticle interpolating operators
Last major hurdle to solidify confidence in results
$\Longrightarrow$ numerically costly, first studies being done now
$\Longrightarrow$ will demonstrate robustness (or flaws) of previous results

Other studies performed with only 3-quark operators in the meantime — too much to cover in this talk!

Excited States - PCAC Checks

Nontrivial checks from PCAC:

$$\langle N|\mathcal{A}_\mu|N\rangle = \bar{u}\Big[\gamma_\mu\gamma_5 F_A(q^2) + \frac{q_\mu}{2M_N}\gamma_5 F_P(q^2)\Big]u$$

$$\langle N|\mathcal{P}|N\rangle = \bar{u}\Big[\gamma_5 F_5(q^2)\Big]u$$

PCAC operator relation: $q^\mu \mathcal{A}_\mu = 2\hat{m}\mathcal{P}$

$$\implies 2M_N F_A(Q^2) - \frac{Q^2}{2M_N} F_P(Q^2) = 2\hat{m} F_5(Q^2)$$

Previously: Checks succeed for correlators, fail for form factors

Solution: better excited state quantification

$\implies$ extracted something other than $\langle N|\mathcal{A}_\mu|N\rangle$, like $\langle N\pi|\mathcal{A}_\mu|N\rangle$

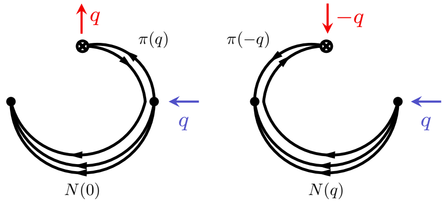
Two general strategies have been used to deal with excited states:

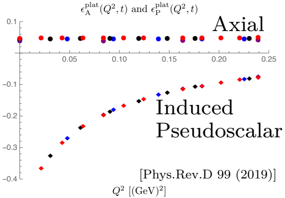
large t_{sep}: ground state saturation; signal-to-noise degradation
e.g. summation method [Phys.Rev.D 86 (2012)]

many t_{sep}: isolate, remove excited states; many fit parameters
[Phys.Rev.C 105 (2022)]

Can we do better?

Excited States - χPT and $N\pi$





Contamination primarily from enhanced $N\pi$, mostly from induced pseudoscalar

Caution: 2-point functions not sensitive to $N\pi$

$\implies$ need simultaneous 2- and 3-point function fits

[Phys.Rev.C 105 (2022)] [Phys.Rev.D 105 (2022)]

Prediction from χPT: [Phys.Rev.D 99 (2019)]

First demonstration by NME: [Phys.Rev.Lett. 124 (2020)]

χPT-inspired fit methods for fitting form factor data
[Phys.Rev.D 105 (2022)] [JHEP 05 (2020) 126]

Alternate fit strategies to remove $N\pi$ (are they comparable?):

- Kinematic constraints ($F_P = 0$)
- include $\mathcal{A}_4$ (strong $N\pi$ coupling)
- explicit $N\pi$ operators

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Status of Lattice QCD Determination of Nucleon Form Factors and Their Relevance for the Few-GeV Neutrino Program

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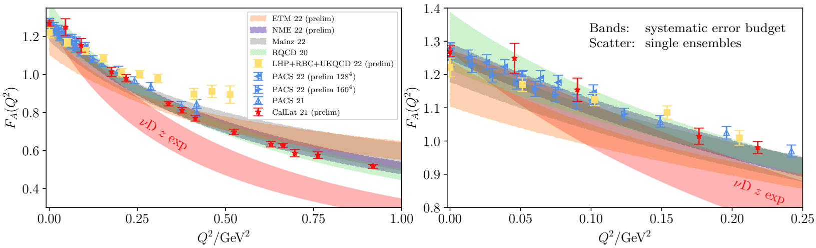
Abstract

Calculations of neutrino–nucleus cross sections begin with the neutrino–nucleon interaction, making the latter critically important to flagship neutrino oscillation experiments despite limited measurements with poor statistics. Alternatively, lattice quantum chromodynamics (LQCD) can be used to determine these interactions from the Standard Model with quantifiable theoretical uncertainties. Recent LQCD results of g_A are in excellent agreement with data, and results for the (quasi-)elastic nucleon form factors with full uncertainty budgets are expected within a few years. We review the status of the field and LQCD results for the nucleon axial form factor, $F_A(Q^2)$, a major source of uncertainty in modeling sub-GeV neutrino–nucleon interactions. Results from different LQCD calculations are consistent but collectively disagree with existing models, with potential implications for current and future neutrino oscillation experiments. We describe a road map to solidify confidence in the LQCD results and discuss future calculations of more complicated processes, which are important to few-GeV neutrino oscillation experiments.

Expected final online publication date for the *Annual Review of Nuclear and Particle Science*, Volume 72 is September 2022. Please see http://www.annualreviews.org/page/journal/pubdates for revised estimates.

Nucleon Axial Form Factor

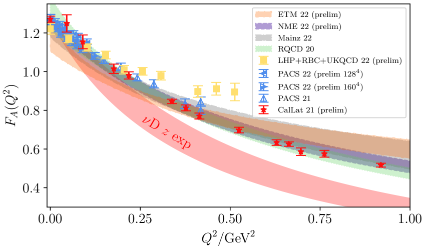


Lots of activity! (Thank you for private communications!)

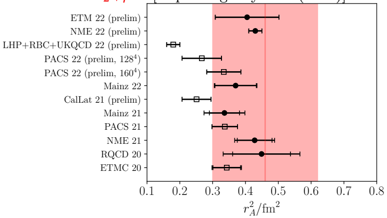
- Excellent agreement among collaborations
- Small systematic effects observed (expectation: largest at $Q^2 \to 0$)
- Collective disagreement with deuterium $\implies$ deuterium corrections?

Remaining: quantify excited state contamination
effects on ground state using $N\pi$ interpolating operators

Axial Radius (r_A^2)



Filled circle: full error budget
Open square: incomplete
$D_2+\mu H$: [Rept.Prog.Phys. 81 (2018)]



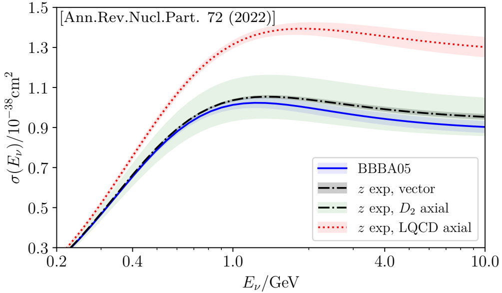
Radius related to slope: $r_A^2 = -\frac{6}{g_A} \frac{dF_A}{dQ^2}\Big|_{Q^2=0}$

Good agreement with radius from experiment, poor agreement with large Q^2

Fixing radius to agree with large Q^2 would bring radius down to $r_A^2 \sim 0.25$ fm^2

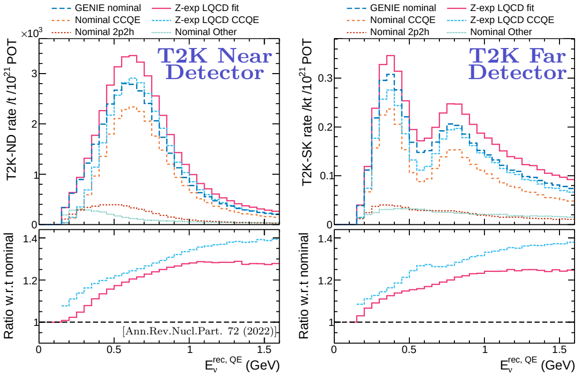
$\implies$ Incompatible with dipole ansatz: would have to be neither or both

Free Nucleon Cross Section



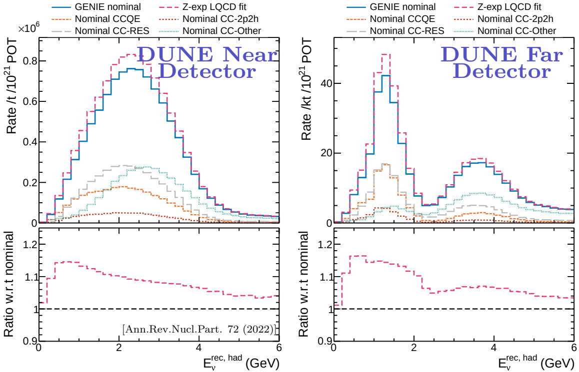
- Representative LQCD dataset (CalLat 21)
- Integral over $Q^2 \implies$ enhancement of discrepancy
- LQCD prefers 30-40% enchancement of ν_μ CCQE cross section
- recent Monte Carlo tunes require 20% enhancement of QE
  [2206.11050 [hep-ph]] [Phys.Rev.D 105 (2022)]

T2K Implications



- Dashed dark blue (GENIE nominal) vs solid magenta (z exp LQCD fit)
- QE enhancements produce 10-20% event rate enhancement, E_ν-dependent
- Monte Carlo tuning makes more detailed comparisons complicated
  $\implies$ All channels are adjusted to compensate for QE changes
- cross section changes at ND $\neq$ *effective* cross section changes at FD:
  insufficient CCQE model freedom $\rightarrow$ bias in FD prediction

DUNE Implications



- Solid dark blue (GENIE nominal) vs dashed magenta (z exp LQCD fit)
- QE enhancements produce 10-20% event rate enhancement, E_ν-dependent
- Monte Carlo tuning makes more detailed comparisons complicated
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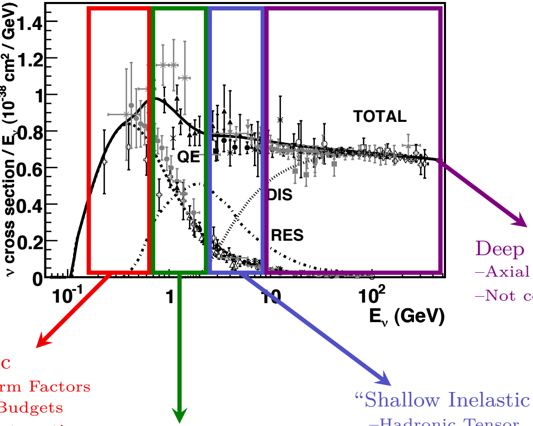
Future Directions

Future Directions



Deep Inelastic Scattering

–Axial quasi/pseudo PDF

–Not covered here

Quasielastic

–Nucleon Form Factors

–Full Error Budgets

–Detailed Systematics

$N \to \Delta, N \to N^*$

–Transition Matrix Elements

–Multiparticle Operators

–Initial Calculations

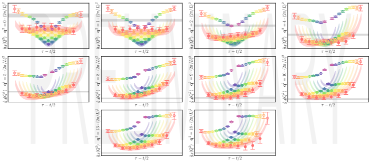
"Shallow Inelastic Scattering" (SIS)

–Hadronic Tensor

–Four Point Functions

–Exploration

Nucleon Form Factor Updates 1/3

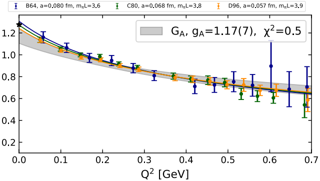


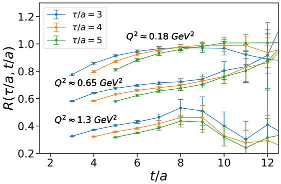
CalLat

- MDWF on HISQ
- Fits to single physical mass ensemble
- Global fit across several Q^2
- Many $t_{\rm sep}$ $\Longrightarrow$ more ability to disentangle excited states
- [PoS LATTICE2021 (2022)]

ETM

- Twisted mass clover
- 3 physical mass ensembles
- Continuum extrapolation + PCAC checks
- [Ferenc Pittler – Fri 2:30pm]





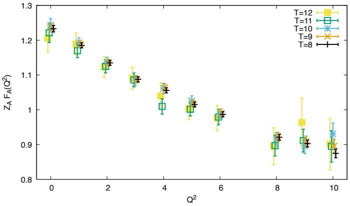
Fermilab Lattice+MILC

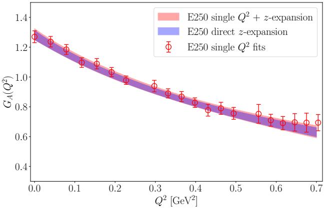
- HISQ on HISQ
- Unphysical pion mass at $Q^2 \neq 0$
- Plateau ratios $\vec{Q} \perp \vec{\mathcal{A}}$
- [Yin Lin – Lattice 2021]

Nucleon Form Factor Updates 2/3

LHP+RBC+UKQCD

- Domain wall fermion
- 1 physical mass ensemble
- axial + electromagnetic form factors
- Tested several form factor parameterizations
- [Shigemi Ohta – Tues 3:00pm]

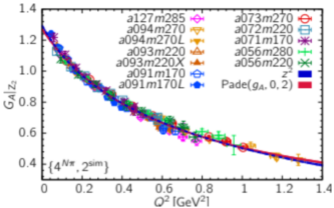




Mainz

- Recent publication [2207.03440 [hep-lat]]
- $O(a)$ improved Wilson
- 14 ensembles
- full error budget
- Fit correlators directly to z exp
- [Jonna Koponen – Tues 3:40pm]

Nucleon Form Factor Updates 3/3

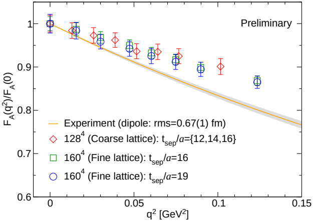


NME

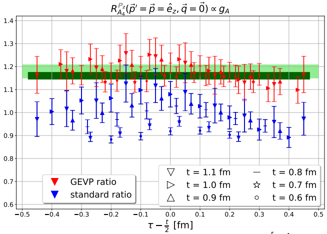
- 7 → 13 ensemble update
- full error budget
- PCAC checks
- extensive excited state studies
- multiple parameterizations tested
- [Rajan Gupta – Fri 3:50pm]

PACS

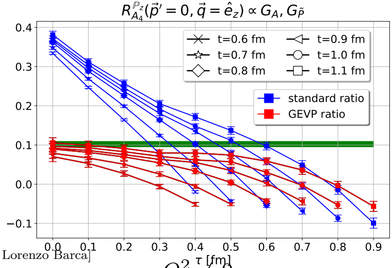
- $O(a)$ improved Wilson
- 2 physical mass ensembles at 10 fm
- axial + electromagnetic form factors
- [Ryutaro Tsuji – Tues 5:10pm]



RQCD - Multiparticle Interpolating Operators



$Q^2 = 0$



[priv.comm. Lorenzo Barca]

$Q^2 \neq 0$

Address primary source of excited state contamination: $N\pi$

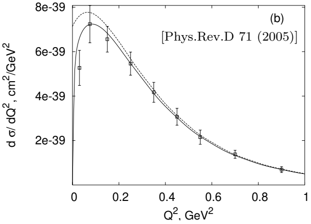
2×2 operator basis, explicit 3- and 5-quark interpolating operators

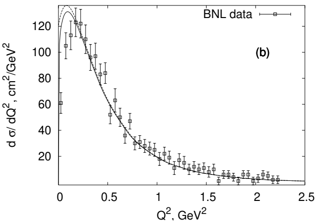
Significantly flatter ratios, simplified analysis

Will analysis with only 3-quark operators be consistent?

[Lorenzo Barca – Fri 4:40pm] RQCD $N\pi$ Operators
[Ferenc Pittler – Fri 2:30pm] ETM $N\pi$ Operators
[André Walker-Loud – Tues 3:20pm] $N\pi$ spectroscopy/scattering

Resonance Production - $N \to \Delta$





$N \to \Delta$ transition form factors are poorly known, but needed

1π production cross section known to 30% [Phys.Rev.C 88 (2013)]

DUNE error budget anticipates $\lesssim$10% precision [2002.03005 [hep-ex]]

Completely unconstrained axial form factors in other $J^P = 3/2^-$ channels
$\implies$ 100% uncertainties from $V - A$, $A - A$ interference terms

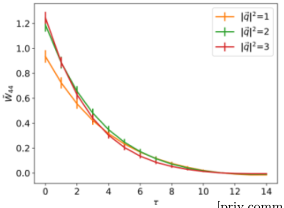
[Phys.Rev.D 74 (2006)]

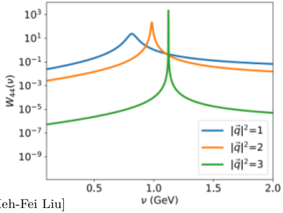
Previous work by ETM: [Phys.Rev.D 83 (2011)] [Phys.Rev.Lett. 98 (2007)]

Formal developments:

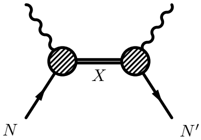
$1 + \mathcal{J} \to 2$	$(N\gamma^* \to N\pi)$	[Phys.Rev.D 103 (2021)]
$1 + \mathcal{J} \to 2 + \mathcal{J}$	$(N\gamma^* \to \Delta \to N\pi\gamma^*)$	[Phys.Rev.D 105 (2022)]

Resonance Production - $N \to N^*$





[priv.comm. Keh-Fei Liu]

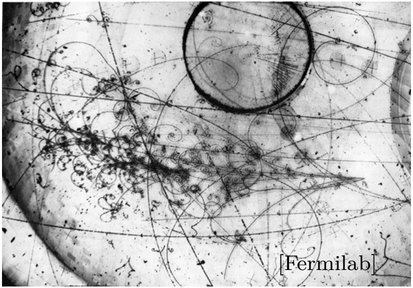


See also: [Phys.Rev.D 101 (2020)]

Four point function with $\langle \mathcal{O}(0)\mathcal{J}_4(-q)\mathcal{J}_4(q)\bar{\mathcal{O}}(0)\rangle$, $M_\pi \sim 370$ MeV
Removed elastic contribution $\implies$ resonant response (strong overlap with Roper)

Hadronic tensor methods for addressing SIS (1.4 GeV $\leq W \leq$ 2.0 GeV)
Large $N\pi$, $N\pi\pi$ contributions; strong interferences between resonant/nonresonant
Currently no practical $Q^2 \neq 0$ data in this region [S.Nakamura - NuSTEC S&DIS]

Outlook



- Nucleon form factor uncertainty significantly underestimated
- Dipole parameterization should be abandoned
- Mounting evidence that enhancement of ν QE cross section needed
- $N\pi$ states are a major player in LQCD calculations
- LQCD is a proxy for missing experimental data
- Unfilled niche: need support for neutrino experimental program
  - resonant transition form factors
  - shallow inelastic scattering

  $\Longrightarrow$ now is the time to start!

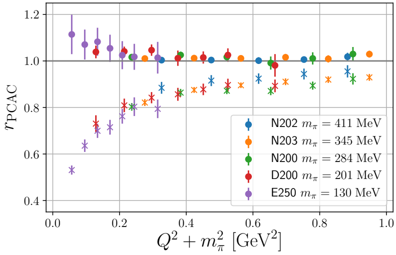
If I have failed to mention your relevant work, please let me know!

Thank you for your attention!

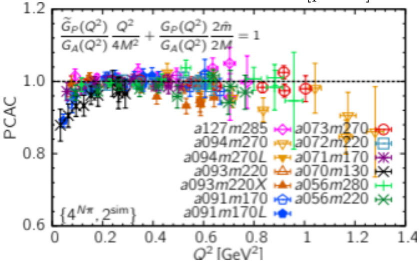
Backup

PCAC Checks

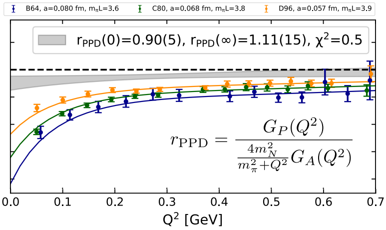
RQCD [JHEP 05 (2020)]



NME [prelim]



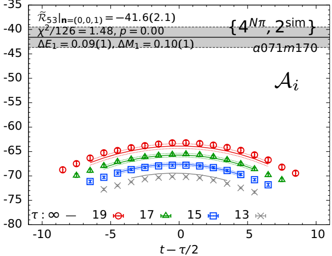
ETMC [prelim]



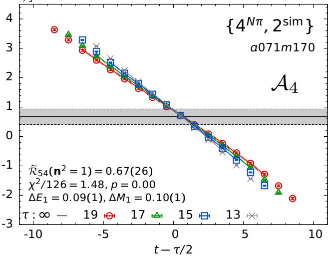
- Relation btw F_A, F_P, $\tilde{F}_P$ via PCAC
- Contamination in F_A and $\tilde{F}_P$, F_P very different [Phys.Rev.D 99 (2019)]
  $\Longrightarrow$ nontrivial consistency check

Excited States in Temporal Axial Current

[Phys.Rev.D 105 (2022)]



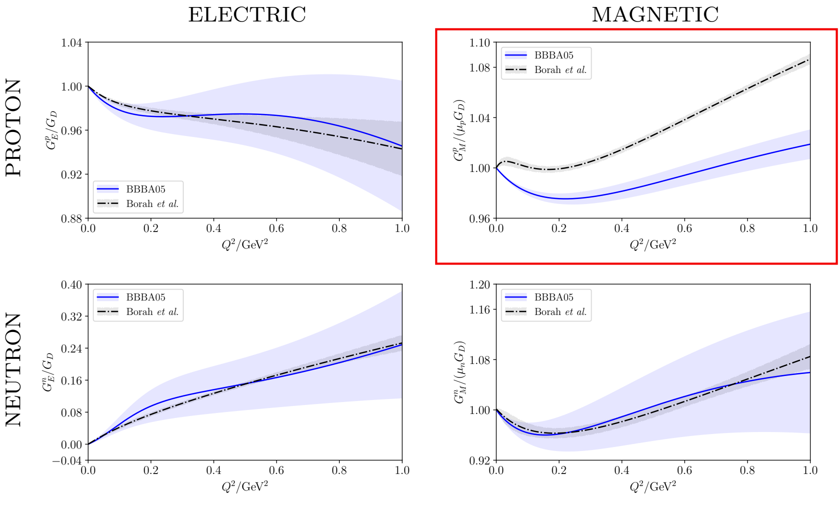
versus



Plotted: ratio combination $\sim \langle N|\mathcal{A}_\mu|N\rangle$

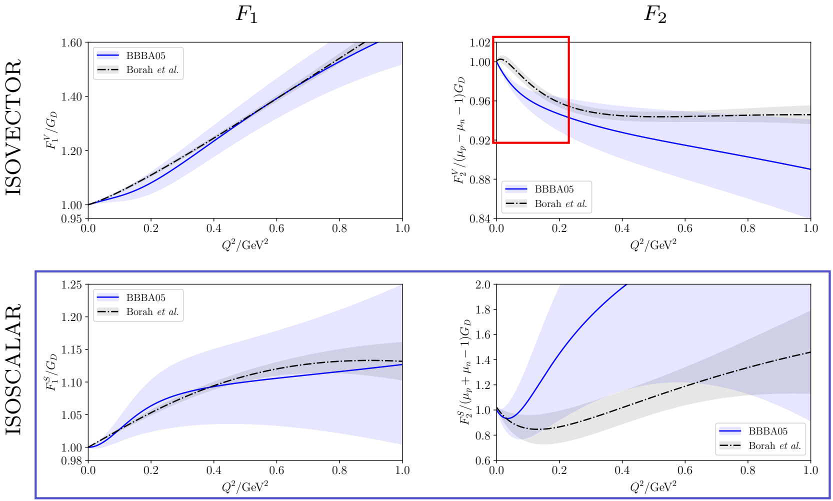
Highly enhanced excited states in $\mathcal{A}_4$, use as handle to quantify $N\pi$

Vector Form Factors - Proton/Neutron



Large tension in proton magnetic form factor

Vector Form Factors - Isospin Symmetric



Uncertain slope of F_2^V

Large uncertainty on isoscalar form factors

z Expansion in χPT

[Ann.Rev.Nucl.Part. 72 (2022)]

$$F_A(z) = \sum_{k=0}^{\infty} a_k z^k \qquad z(Q^2; t_0, t_{\text{cut}}) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}} - t_0}} \qquad t_{\text{cut}} \leq (3M_\pi)^2$$

With nonzero t_0, chiral expansion is about $Q^2 = -t_0$ rather than $Q^2 = 0$:

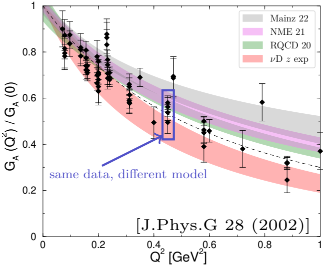
$$x \equiv \frac{Q^2 + t_0}{t_c - t_0} = 4 \sum_{k=1}^{\infty} k z^k \sim O(z)$$

$$(1+x)^{1/2} - 1 = \frac{x}{2} - 4 \sum_{k=2}^{\infty} \frac{(2k-3)!}{k!(k-2)!} \Big(-\frac{x}{4} \Big)^k$$

$$z = \frac{1}{x} \Big((1+x)^{1/2} - 1 \Big)^2 \sim O(x)$$

$$Q^{2m} = \Big((Q^2 + t_0) - t_0 \Big)^m = (t_c - t_0)^m \sum_{n=0}^{m} \binom{m}{n} x^n \Big(\frac{-t_0}{t_c - t_0} \Big)^{m-n}$$

Electro Pion Production



- Large model uncertainty, not included in world averages
- Valid only in $M_\pi \to 0$, $q \to 0$ limits
- Expansion to $O(M_\pi^2, Q^2)$:
  - restricted Q^2 validity
  - lacks shape freedom in Q^2
- Predates Heavy Baryon χPT, no systematic power counting

Modern experiments do not report $F_A(Q^2) \implies$ averages out of date

Possible argument for comparing to r_A^2 from low Q^2; high Q^2 untrustworthy

Effort needed to update prediction from photo/electro pion production