# Nonperturbative Supercurrent Renormalization in $\mathcal{N} = 1$ SYM

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# Motivation: SUSY

Supersymmetric models are theoretically appealing

- Conjectures on confinement and relations to Gauge/Gravity duality

**Main goal:** How much can we extend these conjectures to QCD or Yang-Mills?

- Requires insight on nonperturbative regime $\rightarrow$ Lattice field theory

**Main problem:** How to simulate supersymmetric models on the lattice?

- Can we gain information from perturbation theory?

# $\overline{\text{N}}$=1 SYM

$$\mathcal{L} = tr\left[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + i\bar{\lambda}\gamma^{\mu}D_{\mu}\lambda - m_0\bar{\lambda}\lambda\right]$$

## Field content

- $\lambda$ : One Majorana fermion in the adjoint representation $\rightarrow$ gluino
- $F_{\mu\nu}(A_{\mu})$: Non-Abelian SU($N$) field strength $\rightarrow$ gluons

## Parameters

- $m_0$: Gluino mass $\kappa = \frac{1}{2(m_o+4)}$ $\rightarrow$ breaks supersymmetry softly
- $D = \partial_{\mu} + igA_{\mu}$: Gauge coupling constant $\beta = 2N/g$

# SUSY Ward Identities

## SUSY Ward Identity for two point functions

$$\langle \nabla_\mu S_\mu(x) Q(y) \rangle = 0, \longrightarrow S_\mu \text{ is conserved}$$

$$S_\mu(x) = -\sigma_{\nu\rho}\gamma_\mu F^a_{\mu\nu}(x)\lambda^a(x), \quad \sigma_{\nu\rho} = \frac{1}{2}[\gamma_\nu, \gamma_\rho].$$

# SUSY Ward Identities

## Adding gluino mass term

$$\langle \nabla_\mu S_\mu(x) Q(y) \rangle = m_0 \langle (\chi(x) Q(y) \rangle,$$

$$\chi(x) = \sigma_{\mu\nu} F^a_{\mu\nu}(x) \lambda^a(x)$$

# SUSY Ward Identities

## Discretizing the action

$$\langle \nabla_\mu S_\mu(x) Q(y) \rangle = m_0 \langle (\chi(x) Q(y) \rangle + \langle X_S(x) Q(y) \rangle .$$

# SUSY Ward Identities

After renormalization $X_S(x)$ accounts for a redifinition of renormalization factors

$$Z_S \langle \nabla_\mu S_\mu(x) Q(y) \rangle + Z_T \langle \nabla_\mu T_\mu(x) Q(y) \rangle = m_S \langle (\chi(x)) Q(y) \rangle + \mathcal{O}(a),$$

$$T_\mu = 2\gamma_\nu F^a_{\mu\nu}(x) \lambda^a(x).$$

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SUSY WI recovered along with SUSY at $a \to 0$ by tuning to $m_S = 0$, [1]

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**Theories with more parameters (like SQCD) ?**

**Idea:** Tune extra parameters to match their perturbative result

**Test:** Compare perturbative and non-perturbative value of $Z_T/Z_S$ in $\mathcal{N} = 1$ SYM

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# GIRS: Gauge-Invariant-Renormalization-Scheme[2]

[2] M. Costa et. al [ArXiv:2102.00858]

# GIRS: Gauge-Invariant-Renormalization-Scheme[2]

**Definition:** Renormalization condition for two-point function

$$\langle \mathcal{O}_X^{\mathrm{GIRS}}(x)\mathcal{O}_Y^{\mathrm{GIRS}}(y)\rangle|_{x-y=z} \equiv Z_X^{\mathrm{B,GIRS}} Z_Y^{\mathrm{B,GIRS}} \langle \mathcal{O}_X^{\mathrm{B}}(x)\mathcal{O}_Y^{\mathrm{B}}(y)\rangle|_{x-y=z} = \langle \mathcal{O}_X(x)\mathcal{O}_Y(y)\rangle^{\mathrm{tree}}|_{x-y=z}$$

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**Advantages:**

- Can be used both perturbatively and non-perturbatively
- Only gauge invariant operators involved: $S_\mu$, $T_\mu$, and $\mathcal{O} = -\sigma_{\mu\nu} F^a_{\mu\nu}(x)\lambda^a(x)$

$$S_\mu^{Ren} = Z_{SS} S_\mu + Z_{ST} T_\mu, \qquad \mathcal{O}^{Ren} = Z_{\mathcal{O}}\mathcal{O}$$

$$T_\mu^{Ren} = Z_{TS} S_\mu + Z_{TT} T_\mu,$$

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# GIRS scheme

The GIRS renormalization condition for $G^{ST}_{\mu\nu} = \langle S_\mu(x)\overline{T}_\nu(y)\rangle$

$$\int d^3z\, \mathrm{Tr}\left[G^{ST,\,GIRS}_{\mu\nu}(x,y)\; P_{\nu\mu}\right] = \int d^3z\, \mathrm{Tr}\left[G^{ST,\,\mathrm{tree}}_{\mu\nu}(x,y)\; P_{\nu\mu}\right],$$

$P_{\mu\nu} = \gamma_\mu\gamma_0\gamma_\nu \rightarrow$ Projector correct symmetries

Renormalized two point-functions

$$G^{ST,GIRS}_{\mu\nu} = Z_{SS}Z_{TT}G^{ST,\,\mathrm{B}}_{\mu\nu} + Z_{ST}Z_{TT}G^{TT,\,\mathrm{B}}_{\mu\nu} + Z_{SS}Z_{TS}G^{SS,\,\mathrm{B}}_{\mu\nu} + Z_{ST}Z_{TS}G^{TS,\,\mathrm{B}}_{\mu\nu}$$

# GIRS on the lattice

$$\frac{1}{3L^3}\sum_{\vec{x},\vec{y}}\sum_{i} \mathrm{Tr}\left[G_{ii}^{SS,\,GIRS}((\vec{x},t),(\vec{y},0))\,\gamma_i\gamma_4\gamma_i\right] = \frac{2(N_c^2-1)t}{\pi^2|t|^5},$$

$$\frac{1}{3L^3}\sum_{\vec{x},\vec{y}}\sum_{i} \mathrm{Tr}\left[G_{ii}^{TT,\,GIRS}((\vec{x},t),(\vec{y},0))\,\gamma_i\gamma_4\gamma_i\right] = \frac{5(N_c^2-1)t}{4\pi^2|t|^5},$$

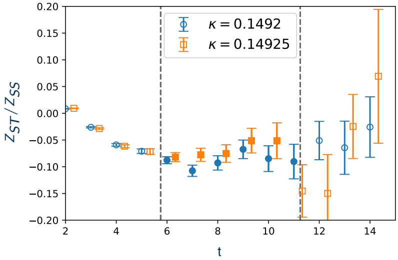
$$\frac{1}{3L^3}\sum_{\vec{x},\vec{y}}\sum_{i} \mathrm{Tr}\left[G_{ii}^{ST,\,GIRS}((\vec{x},t),(\vec{y},0))\,\gamma_i\gamma_4\gamma_i\right] = \frac{(N_c^2-1)t}{\pi^2|t|^5},$$

$$\frac{1}{3L^3}\sum_{\vec{x},\vec{y}}\sum_{i} \mathrm{Tr}\left[G_{i}^{S\mathcal{O},\,GIRS}((\vec{x},t),(\vec{y},0))\,\gamma_4\gamma_i\right] = 0.$$

$t \longrightarrow$ GIRS renormalization scale

# GIRS renormalization factors

Artifacts Signal Noise




## Parameters

Wilson fermions

Tree-level Symanzik improved gauge action

| Group | $\beta$ | $\kappa$ | L | T |
|---|---|---|---|---|
| SU(2) | 1.75 | 0.14920 | 24 | 48 |
| SU(2) | 1.75 | 0.14925 | 24 | 48 |
| SU(2) | 1.75 | 0.14940 | 48 | 64 |
| SU(3) | 5.6 | 0.16550 | 24 | 48 |

- In GIRS the Z factors depend on the GIRS scale $t \rightarrow$ difficult to fit
- More results are known in $\overline{\text{MS}}$ scheme

# Conversion factors

Conversion factors relate different schemes -> Regularization independent

$$\begin{pmatrix} C_{SS}^{\text{GIRS},\,\overline{\text{MS}}} & C_{ST}^{\text{GIRS},\,\overline{\text{MS}}} \\ C_{TS}^{\text{GIRS},\,\overline{\text{MS}}} & C_{TT}^{\text{GIRS},\,\overline{\text{MS}}} \end{pmatrix} \cdot \begin{pmatrix} Z_{SS}^{\text{R},\,\text{GIRS}} & Z_{ST}^{\text{R},\,\text{GIRS}} \\ Z_{TS}^{\text{R},\,\text{GIRS}} & Z_{TT}^{\text{R},\,\text{GIRS}} \end{pmatrix} = \begin{pmatrix} Z_{SS}^{\text{R},\,\overline{\text{MS}}} & Z_{ST}^{\text{R},\,\overline{\text{MS}}} \\ Z_{TS}^{\text{R},\,\overline{\text{MS}}} & Z_{TT}^{\text{R},\,\overline{\text{MS}}} \end{pmatrix}$$

How to use them:

1. Compute the Z factors in perturbation theory in DR for GIRS and $\overline{\text{MS}}$
2. Find the conversion factors between
3. Apply them on the lattice regularization

# Perturbative conversion factors

To one-loop in perturbation theory:

$$C_{SS}^{\mathrm{GIRS},\,\overline{\mathrm{MS}}} = 1 - \frac{g_{\overline{\mathrm{MS}}}^2}{16\pi^2}\frac{17N_c}{6} + \mathcal{O}(g_{\overline{\mathrm{MS}}}^4),$$

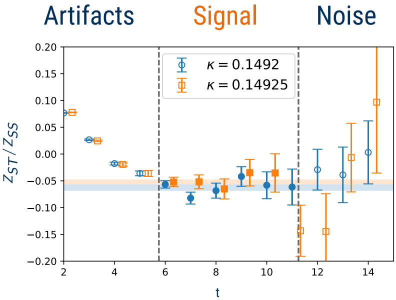
$$C_{ST}^{\mathrm{GIRS},\,\overline{\mathrm{MS}}} = \frac{g_{\overline{\mathrm{MS}}}^2}{16\pi^2}4N_c + \mathcal{O}(g_{\overline{\mathrm{MS}}}^4),$$

$$C_{TS}^{\mathrm{GIRS},\,\overline{\mathrm{MS}}} = -\frac{g_{\overline{\mathrm{MS}}}^2}{16\pi^2}\frac{3N_c}{2}\left(\frac{2}{3} + 2\gamma_E + \ln(\bar{\mu}^2 a\, t^2)\right) + \mathcal{O}(g_{\overline{\mathrm{MS}}}^4),$$

$$C_{TT}^{\mathrm{GIRS},\overline{\mathrm{MS}}} = 1 + \frac{g_{\overline{\mathrm{MS}}}^2}{16\pi^2}N_c\left(\frac{7}{6} + 6\gamma_E + 3\ln(\bar{\mu}^2 a\, t^2)\right) + \mathcal{O}(g_{\overline{\mathrm{MS}}}^4).$$

$$g_{\overline{\mathrm{MS}}}^2 = g^{\mathrm{B}} + \mathcal{O}(g_{\overline{\mathrm{MS}}}^4), \qquad \bar{\mu} = 2\ \mathrm{GeV}, \qquad a = 0.1\ \mathrm{fm}, \qquad N_c = 2$$

# $\overline{\text{MS}}$ renormalization factors




## Non-perturbative

$$\frac{Z_{ST}}{Z_{SS}} = -0.0517(84),\ \kappa = 0.14920$$

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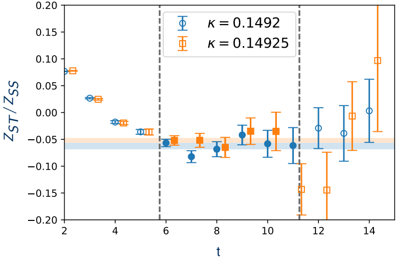
## Perturbative

$$\frac{Z_{ST}}{Z_{SS}} = 0.1008$$

# $\overline{\text{MS}}$ renormalization factors

Artifacts Signal Noise

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## Perturbative

$$\frac{Z_{ST}}{Z_{SS}} = 0.1008$$

**Warning:** Results are in high tension !

# Conclusions

## Summary

- SUSY WI can be recovered by properly tuning the parameters

  **Idea:** Can we use perturbation theory?
- GIRS as a non-perturbative scheme
- Computed the Z factors for $\mathcal{N} = 1$ SYM $\rightarrow$ Far from perturbative value

## Conclusion: Need for Improvement

- Use clover improved fermions $\rightarrow \mathcal{O}(a)$ improved action
- Include smearing?
- Can we simulate closer to a perturbative regime?

# Thanks a lot for your attention!

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# Smearing

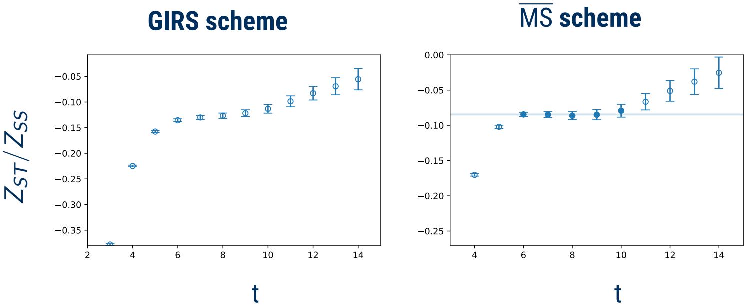


Figure: $V = 24^3 \times 48$ lattice with $\kappa = 0.14920$ with smearing. A more pronounced plateau is observed.

# Higher perturbative effects

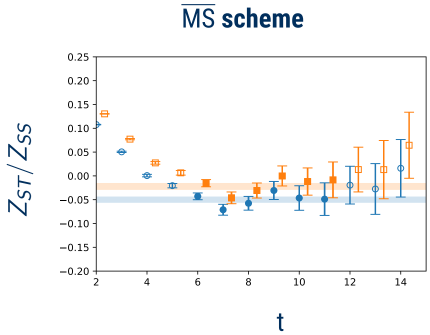


Figure: $Z_{ST}^{\overline{\text{MS}}}/Z_{SS}^{\overline{\text{MS}}}$ computed with $C(g^2)^{\text{GIRS},\overline{\text{MS}}}$ blue dots, and $C^{-1}(-g^2)^{\text{GIRS},\overline{\text{MS}}}$ orange squares. The difference between the two computations is due to higher loop corrections.

# Tree level all $\mathcal{O}(a)$ improvement

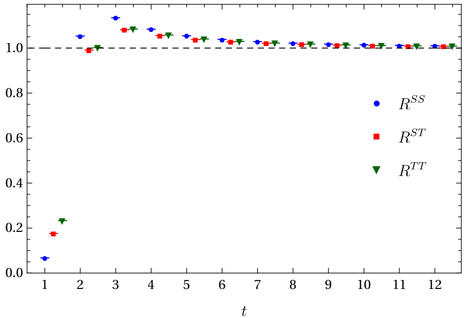


Figure: Ratios between lattice (all orders in $a$) and continuum tree-level values: $R^{AB}(t) \equiv \mathrm{Tr}[(G_{ii}^{AB}(t))^{\text{tree lat.}}\gamma_i\gamma_4\gamma_i]/\mathrm{Tr}[(G_{ii}^{AB}(t))^{\text{tree cont.}}\gamma_i\gamma_4\gamma_i]$, for $AB = SS, ST, TT$. For better visibility, $R^{ST}(t)$ and $R^{TT}(t)$ are shifted in $t$ by +0.25 and +0.50, respectively.