Dark Isosinglet Mesons in $Sp(4)$ Gauge Theory with $N_f = 2$

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$Sp(2N)_c$ Dark Matter models

- $N_f = 2$ provides a minimal SIMP model
  2 massive fundamental fermions in pseudoreal repr.
  $\Rightarrow$ 5 pseudo-Goldstones π
- Dark pions π are DM candidates.

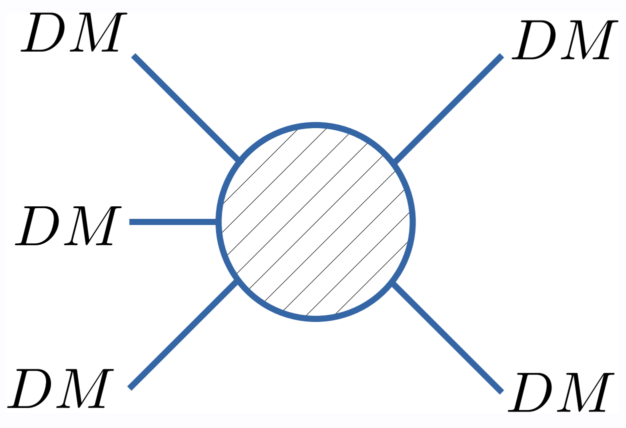
WIMPs: $2\mathrm{DM} \to 2\mathrm{SM} \ \Rightarrow m_D \approx \mathrm{TeV}$

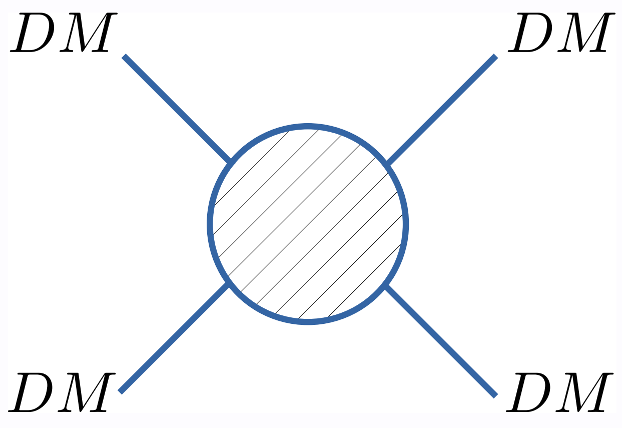
SIMPs: $3\mathrm{DM} \to 2\mathrm{DM} \Rightarrow m_D \approx \mathcal{O}(100)\mathrm{MeV}$ **[1]**

[1] Hochberg et. al. [1402.5143]

Dark Matter Self-Scattering

- $3\pi \rightarrow 2\pi$ scattering sets the DM relic density
- DM self-interactions can address structural issues
  $2\pi \rightarrow 2\pi$ could address "cusp vs. core" problem





Hochberg et. al. [1411.3727] [1512.07917]

Dark sector Lagrangian

- pseudoreal fermion repr. of the "dark quarks"
- Both dark quarks are **massive**
- Currently phenomenologically preferred regime

$\Rightarrow m_\pi \approx \mathcal{O}(100\ \mathrm{MeV})$ (relic density: $3\pi \rightarrow 2\pi$)

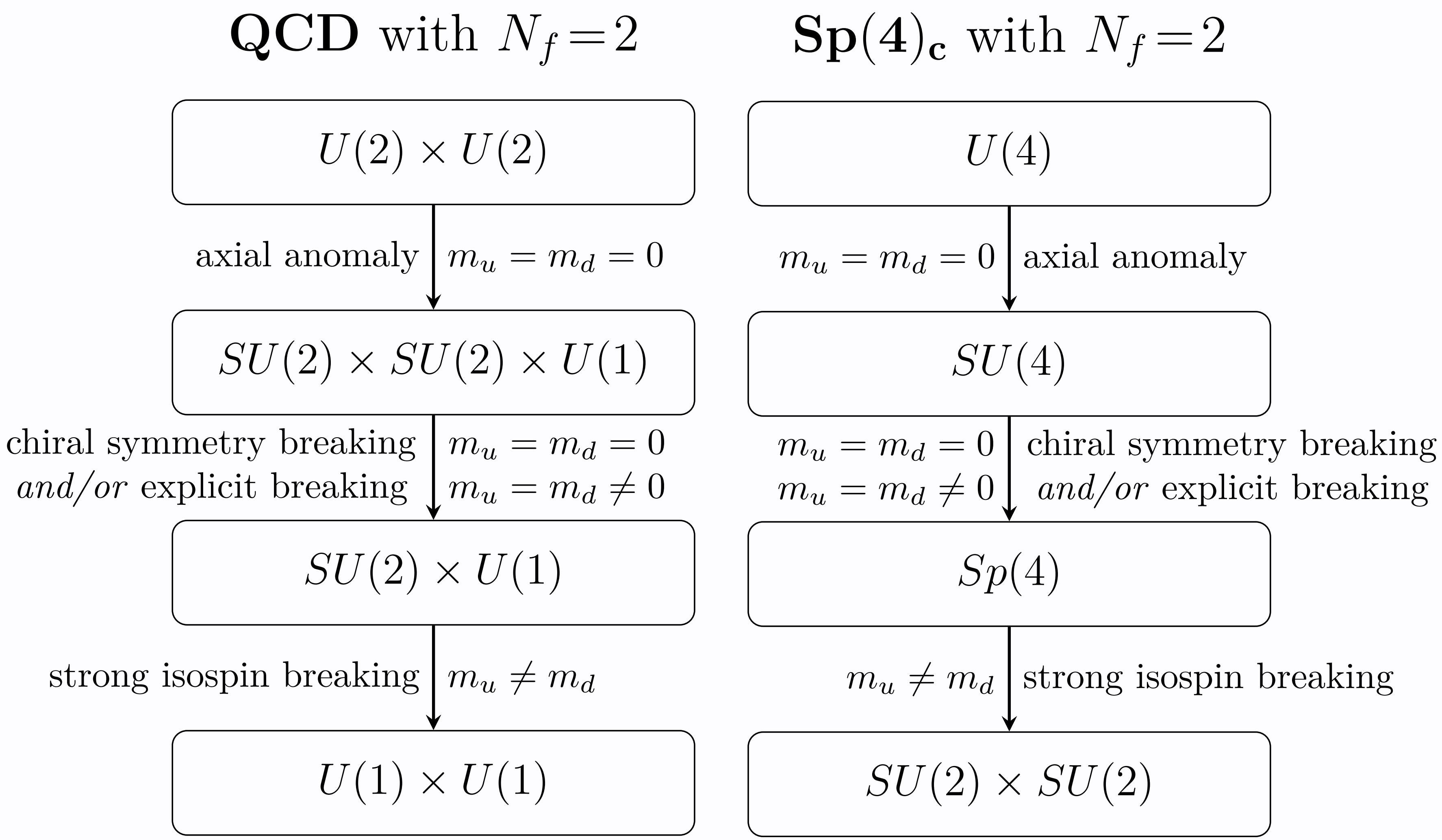
$\Rightarrow 2\pi \rightarrow 2\pi$ cross-section constrained **[1]**

velocity dependence for DM structure problems?

$$\mathcal{L}_{\mathrm{dark}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{f=u,d} \bar{\psi}_f (i \not{D} + m_f) \psi_f$$

[1] Andrade et. al. 2012.06611 Eckert et. al. [2205.01123]

Global symmetries



see e.g. Kogut et. al. [hep-ph/0001171], von Smekal [1205.4205]

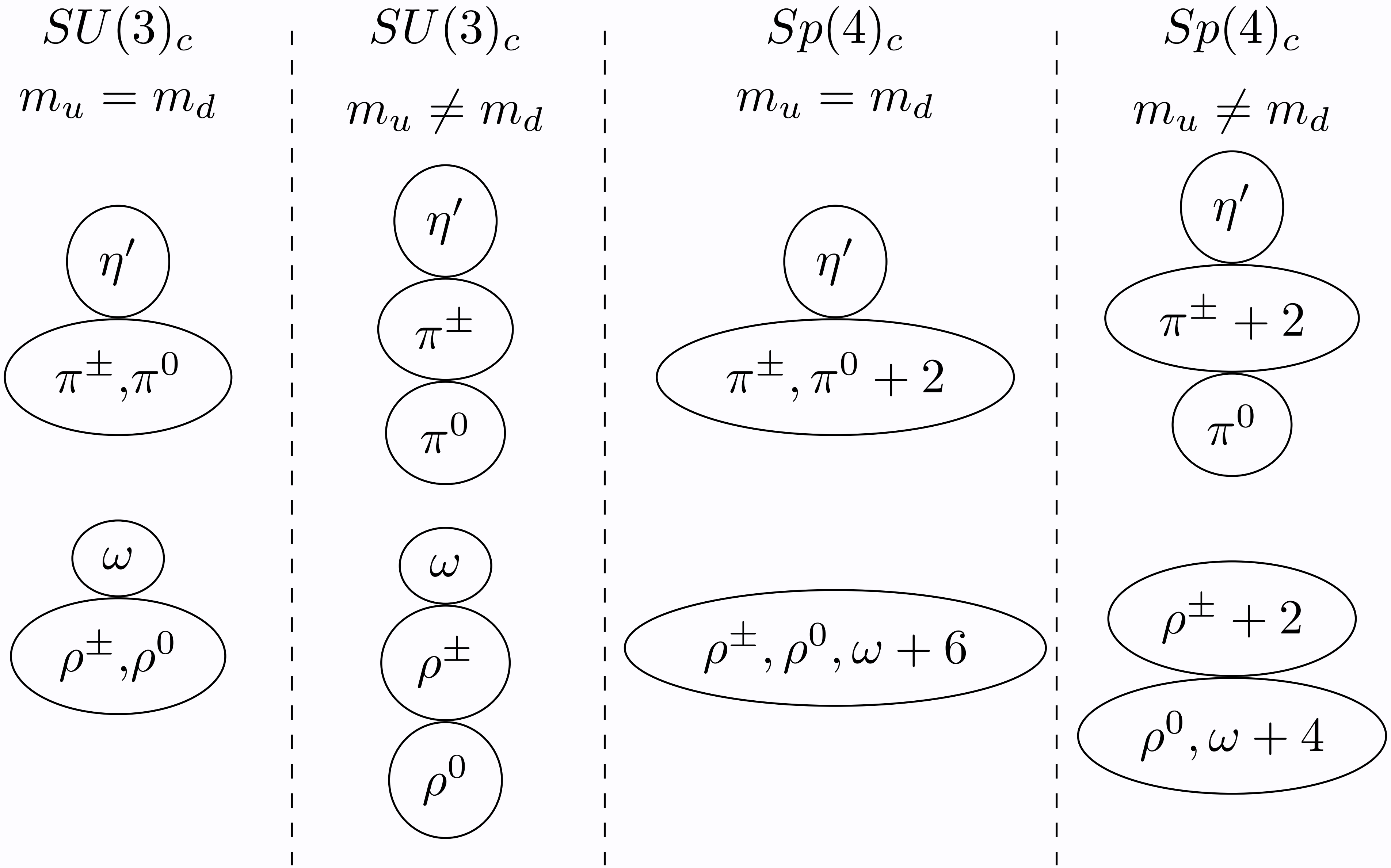
Singlet mesons in $Sp(4)$ with $N_f = 2$

- $m_u = m_d$: Every 5-plet has a singlet equvivalent
  e.g. pseudoscalar η', scalar σ / f_0
- No mesonic singlets for J^P associated with 10-plet
  e.g. the vector meson ω / ϕ is part of multiplet **[1]**

- $m_u \neq m_d$: 5-plet $\to 4 + 1$, 10-plet $\to 4 + 6$
  e.g. π^0 becomes a singlet **[2]**

[1] Bennett et. al. [1912.06505] [2] Kulkarni et. al. [2202.05191]

Pseudoscalar (PS) and vector (V) multiplets

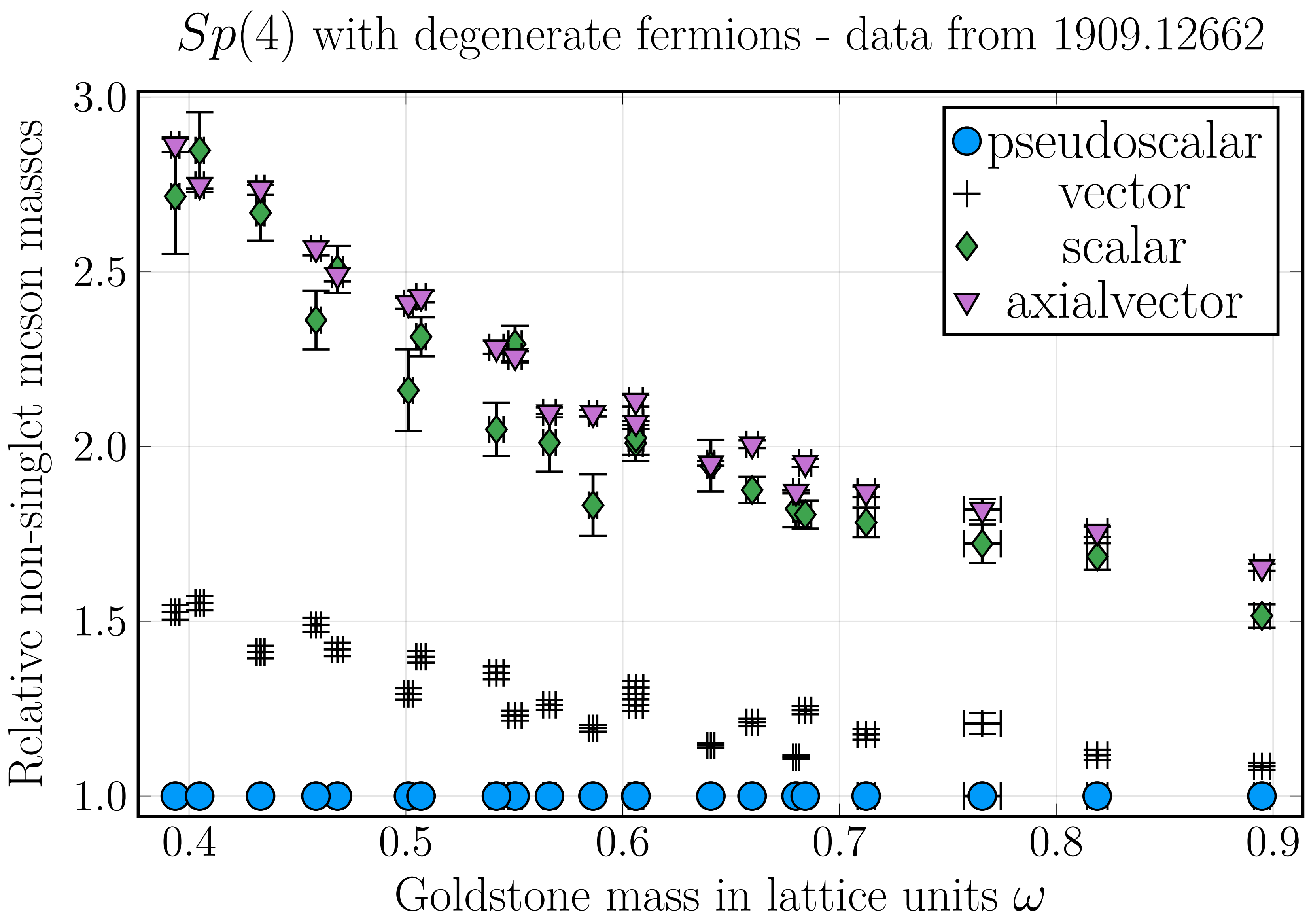


Singlets are relevant for DM!

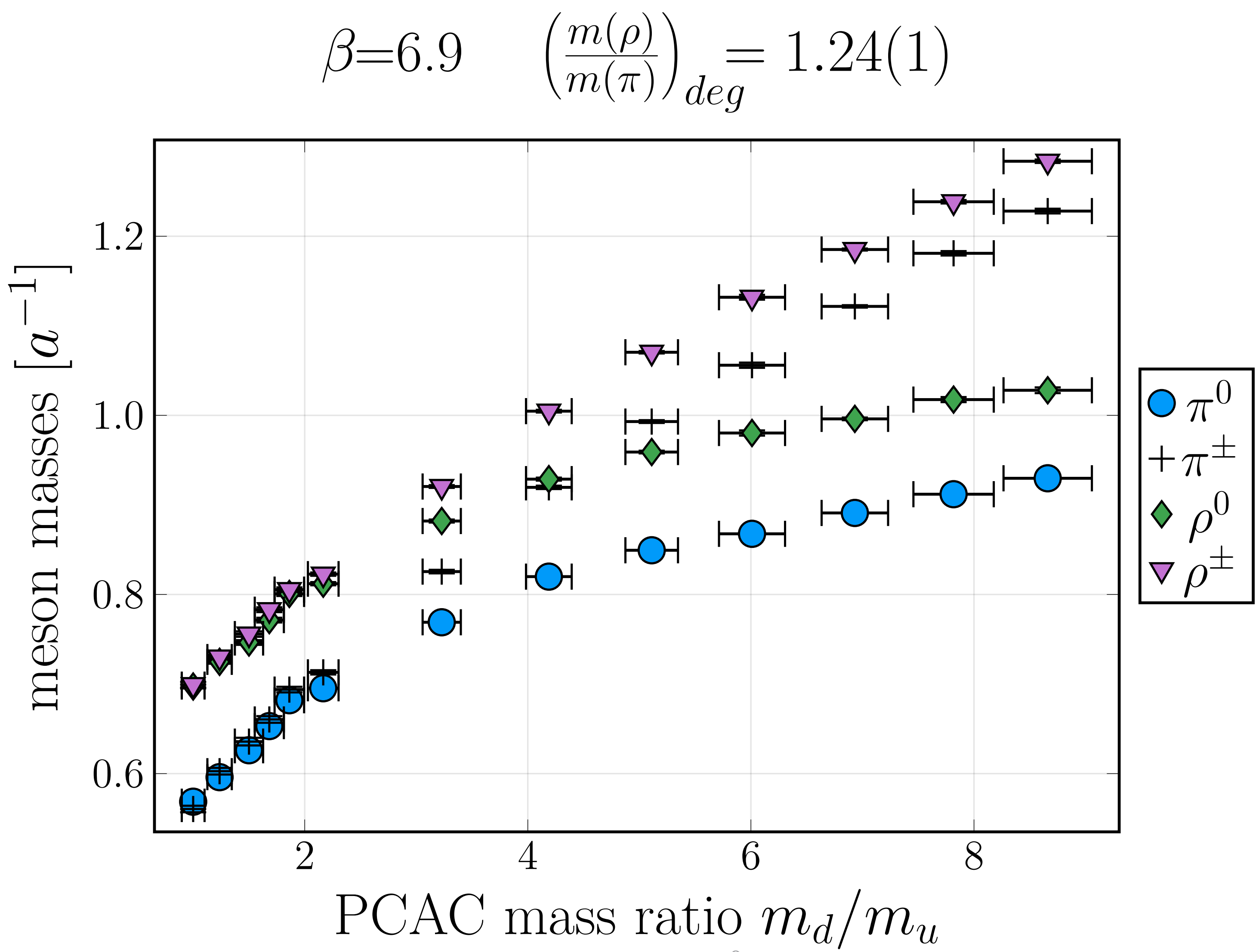
- Not protected by symmetries $\Rightarrow$ decay into SM
- Useful in construction of EFTs?
- Can be involved in scattering processes

- So far never studied in $Sp(2N)$ gauge theory on the lattice

- Non-singlets: $m_u = m_d$ [1909.12662]



- Non-singlets: $m_u \neq m_d$ [2202.05191]



(singlet contribution to π^0 neglected)

Pseudoscalar singlet η' in other theories

- What about singlet mesons in other theories?
- Two-flavour QCD ($SU(3)_c$ with $N_f = 2$) **[1]**
  - Different regimes with $m_{\eta'} \approx m_\rho$ and $m_{\eta'} < m_\rho$
  - Absence of strange: $m_{\eta'}$ decreases by $\approx 200\mathrm{MeV}$
- $SU(2)_c$ results are available (light quarks) **[2]**
  - chiral limit: $m_\pi < m_\rho \approx m_{\eta'}$

[1] e.g. ETMC [0710.1517] [0804.3871] [2] Arthur et. al. [1607.06654]

Strategy: Start with η' (and π^0 for $m_u \neq m_d$)

- DM candidates are pseudoscalar
  $\Rightarrow \eta'$ possibly relevant for EFT
- Scalar singlet σ / f_0 are technically involved
  $\Rightarrow$ identify useful variance reduction techniques
- Configurations and measurements using `HiRep` **[1]**

[1] Del Debbio et. al. [0805.2058] extended for $Sp(4)$ (github.com/sa2c/HiRep)

Obtaining a signal

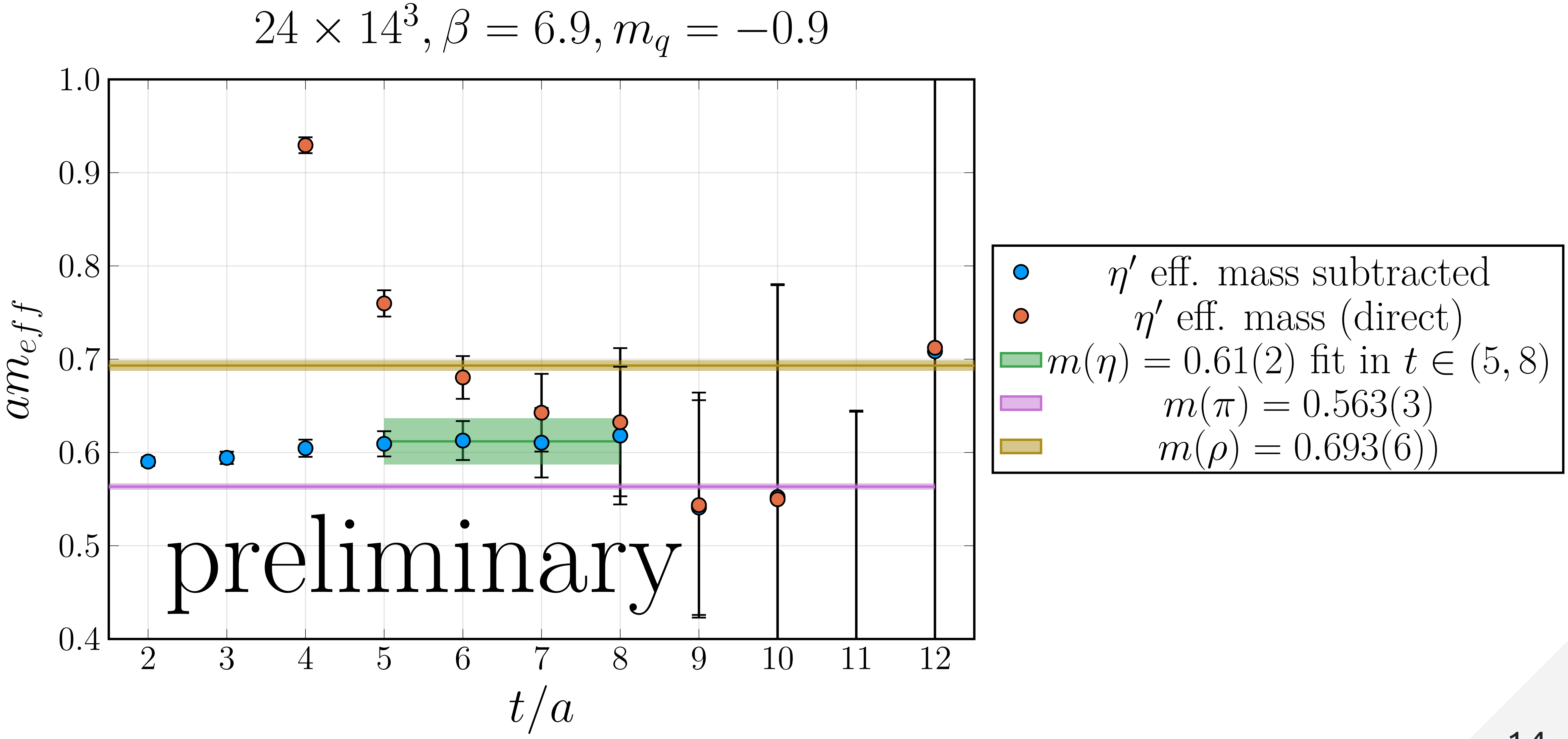
- Diluted noisy sources (Z_2 noise, spin dilution) **[1]**
- Excited state subtraction in connected pieces **[2]**
- Unbiased disconnected correlator as in **[3]**
- Smaller and coarser lattices, heavier fermions

Downsides and current limitations:

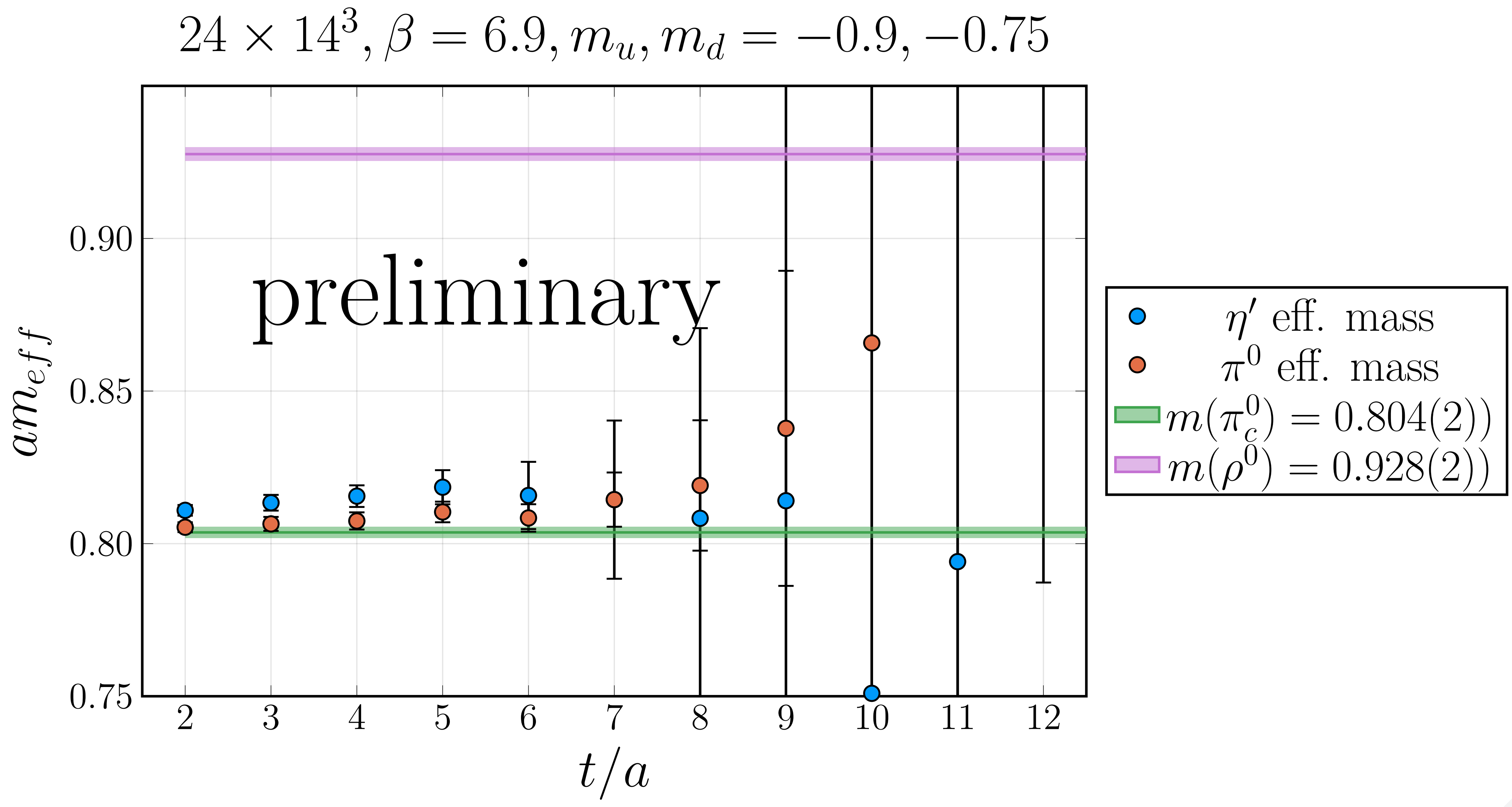
- No analysis of systematics
  $\Rightarrow$ finite volume and spacing effects expected
- Only a few timeslices of signal

[1] Foley et. al. [hep-lat/0505023] [2] Neff et. al. [hep-lat/0106016] [3] Arthur et. al. [1607.06654]

- η' : Degenerate fermions with $m_\rho / m_\pi = 1.24(1)$



- Non-degenerate: π^0 and η'



Self-interactions: Pion scattering length a_0

- First estimate on the ensemble $m_\rho / m_\pi = 1.24(1)$ $\beta = 6.9, 24 \times 14^3, m_u = m_d$:
- Energy shift extracted as in **[1]** from

$$R(t) = \frac{C_{\pi\pi}(t) - C_{\pi\pi}(t+1)}{C_\pi^2(t) - C_\pi^2(t+1)}$$

- **no systematics**, small lattice, not a full analysis!

$$a_0 = 1.0(5)$$

[1] Arthur et. al. [1412.4771]

Experimental constraints: DM self-interaction

- current upper limits at $\sigma/m_D < 0.19\mathrm{cm}^2\mathrm{g}^{-1}$ **[1]** and $\sigma/m_D < 0.13\mathrm{cm}^2\mathrm{g}^{-1}$ **[2]**
- our rough analysis suggests $m_D \geq 100\mathrm{MeV}$
  - compatible with relic density constraints
  - $\sigma(v)$ for core-vs-cusp problem might be needed

[1] Andrade et. al. 2012.06611 [2] Eckert et. al. [2205.01123]

Summary/Conclusion

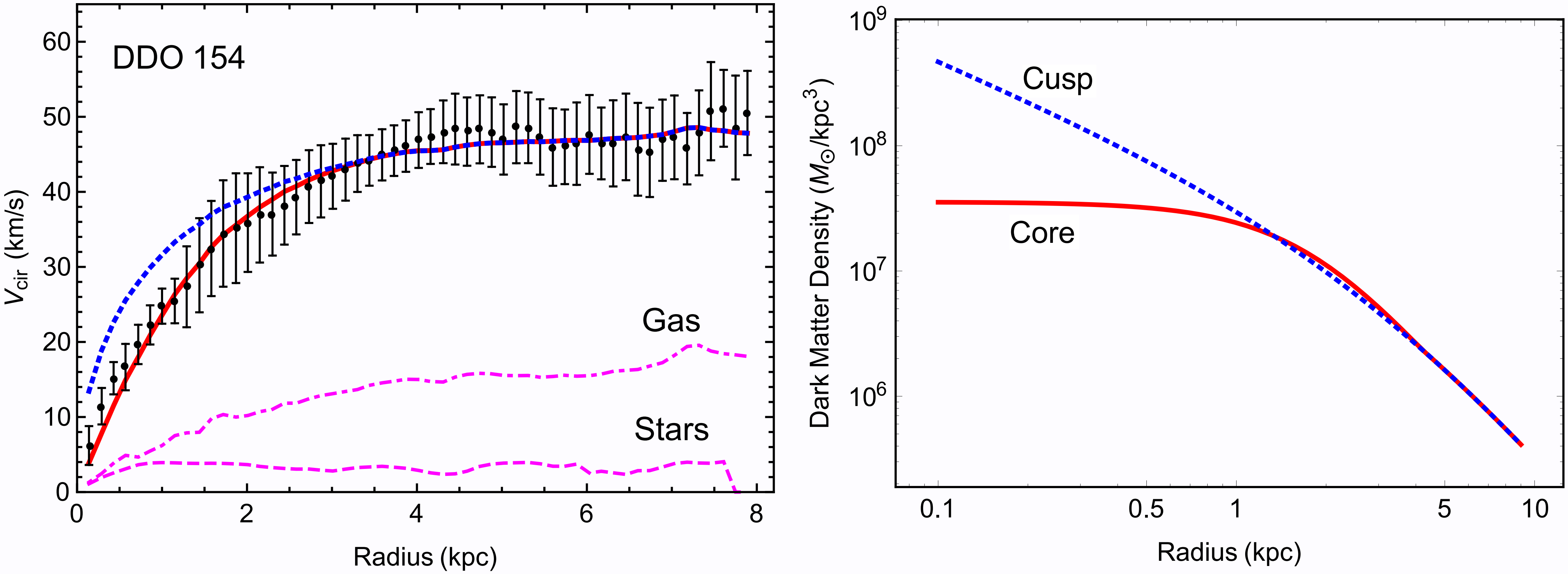
- First look at isosinglet mesons in $Sp(4)$
  - important in composite DM models
  - not always present - e.g. no vector isosinglet
  - first lattice results for η' and π^0
- Estimate on the π scattering length
  - our ensembles are of phenomenological interest

Early, exploratory study. A lot more to do.

Thank you!

Backup slides

Core-vs-cusp problem



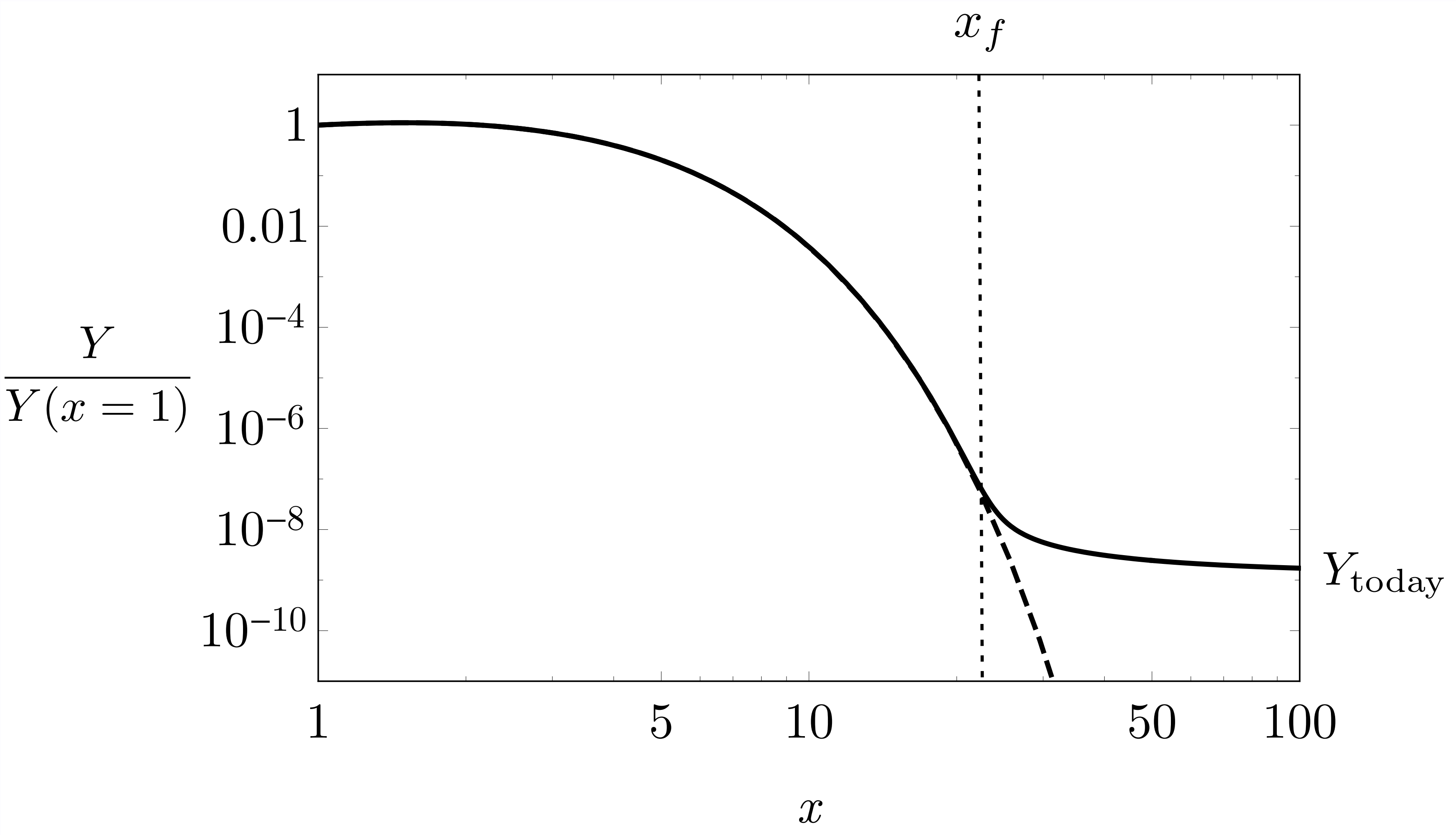
taken from: Tulin and Yu 1705.02358v2

Thermal Dark Matter - Freeze out

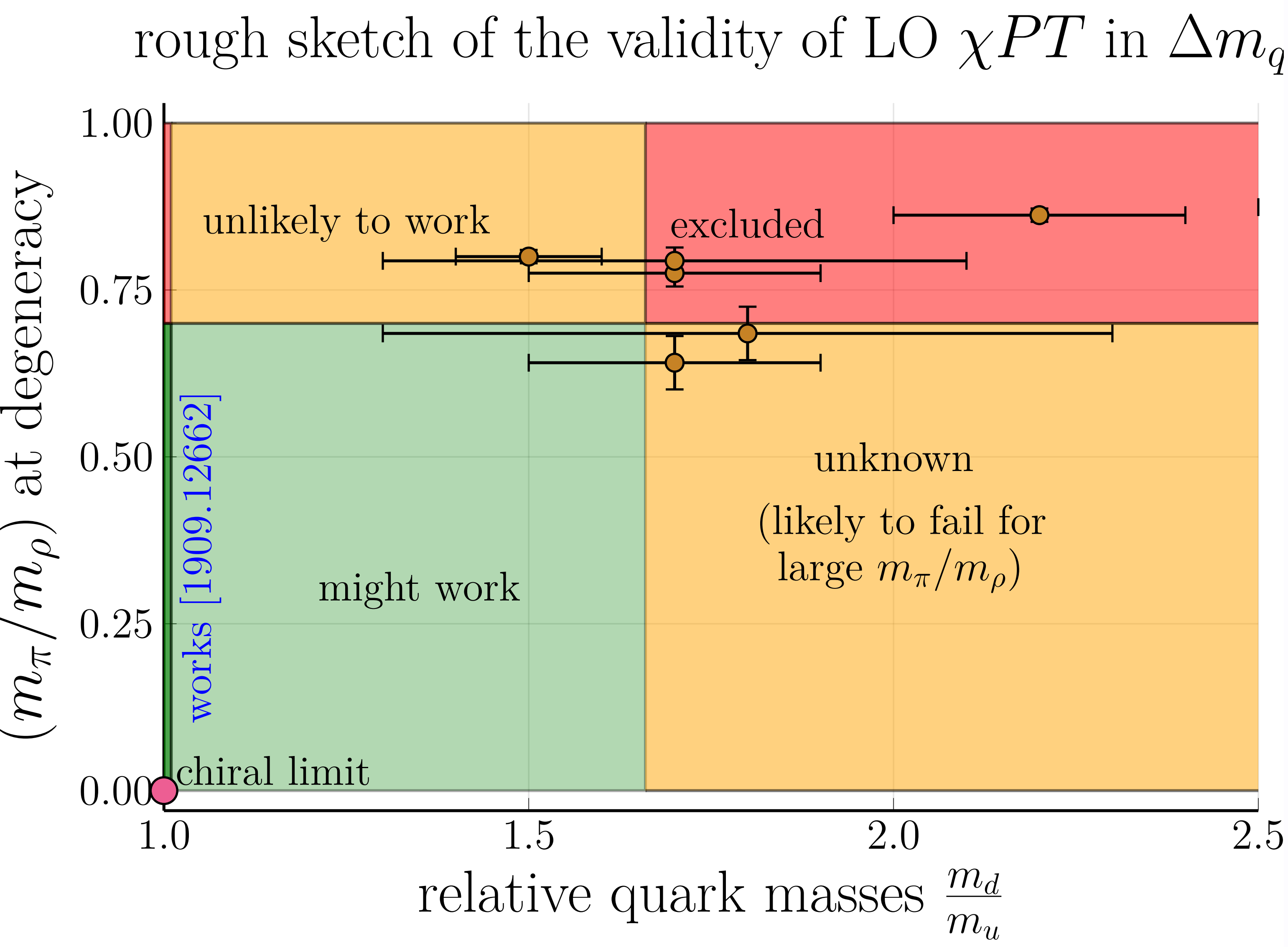
- Early universe: Thermal and chemical equilibrium
- DM depletion process, e.g. $\chi\chi_{DM} \leftrightarrow \psi\psi_{SM}$
  - Universe cools: $\psi\psi \rightarrow \chi\chi$ kinematically forbidden
  - *Dark Matter number decreases* through $\chi\chi \rightarrow \psi\psi$
  - Universe expands: χ annihilation stops

- DM eventually thermally decoupled: **freeze out**

Freeze-out process



- Y ... DM number density, $x = m_{DM}/T$



- data points: breakdown of LO χPT in Δm on the lattice
- $\frac{m_d}{m_u} = 1 + \Delta m$
- $\frac{m_\pi}{m_\rho}$ fixed at degeneracy

Meson Spectroscopy

- Obtain 2-point function of Operator $\mathcal{O}(x)$

$$\mathcal{O}_{\pi^+} = \bar{u}\gamma_5 d$$

$$\mathcal{O}_{\pi^-} = \bar{d}\gamma_5 u$$

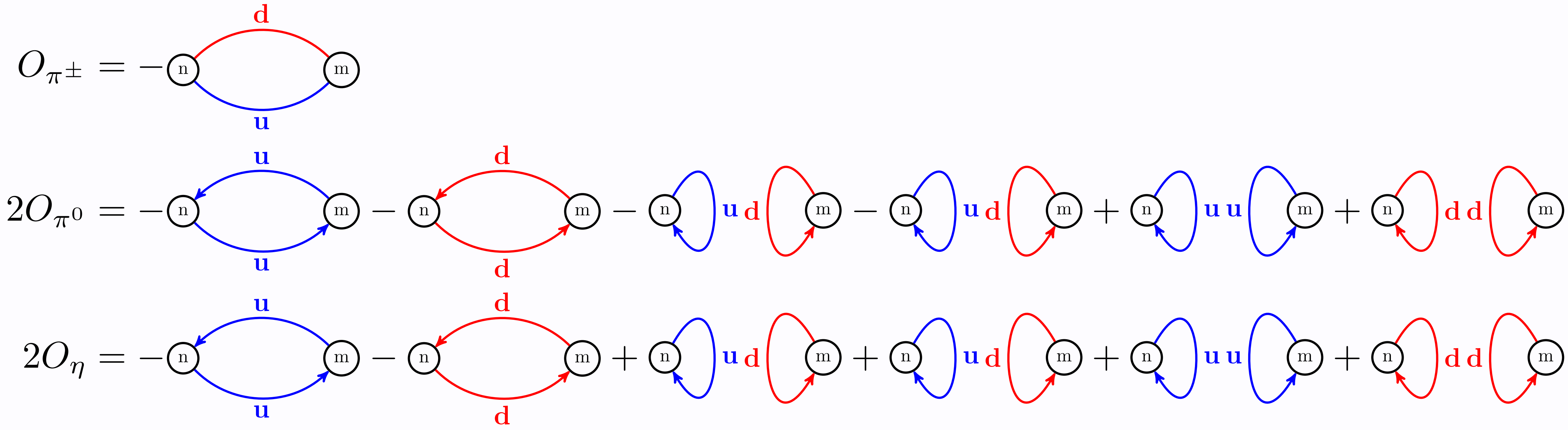
$$\mathcal{O}_{\pi^0} = \left(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d\right) / \sqrt{2}$$

$$\mathcal{O}_{\eta} = \left(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d\right) / \sqrt{2}$$

- For scalars replace $\gamma_5 \to \mathbb{I}$, for vectors $\gamma_5 \to \gamma_i$

$$\langle O(\vec{x}, t)\bar{O}(0)\rangle = \sum_n \langle 0|\hat{O}|n\rangle\langle n|\hat{O}^\dagger|0\rangle e^{-tE_n}$$

Diagrammatical representation



- Quark lines "come back" $\Rightarrow$ **disconnected** diagrams
- Cancellation for π^0 in symmetric limit

Always present for singlet mesons!

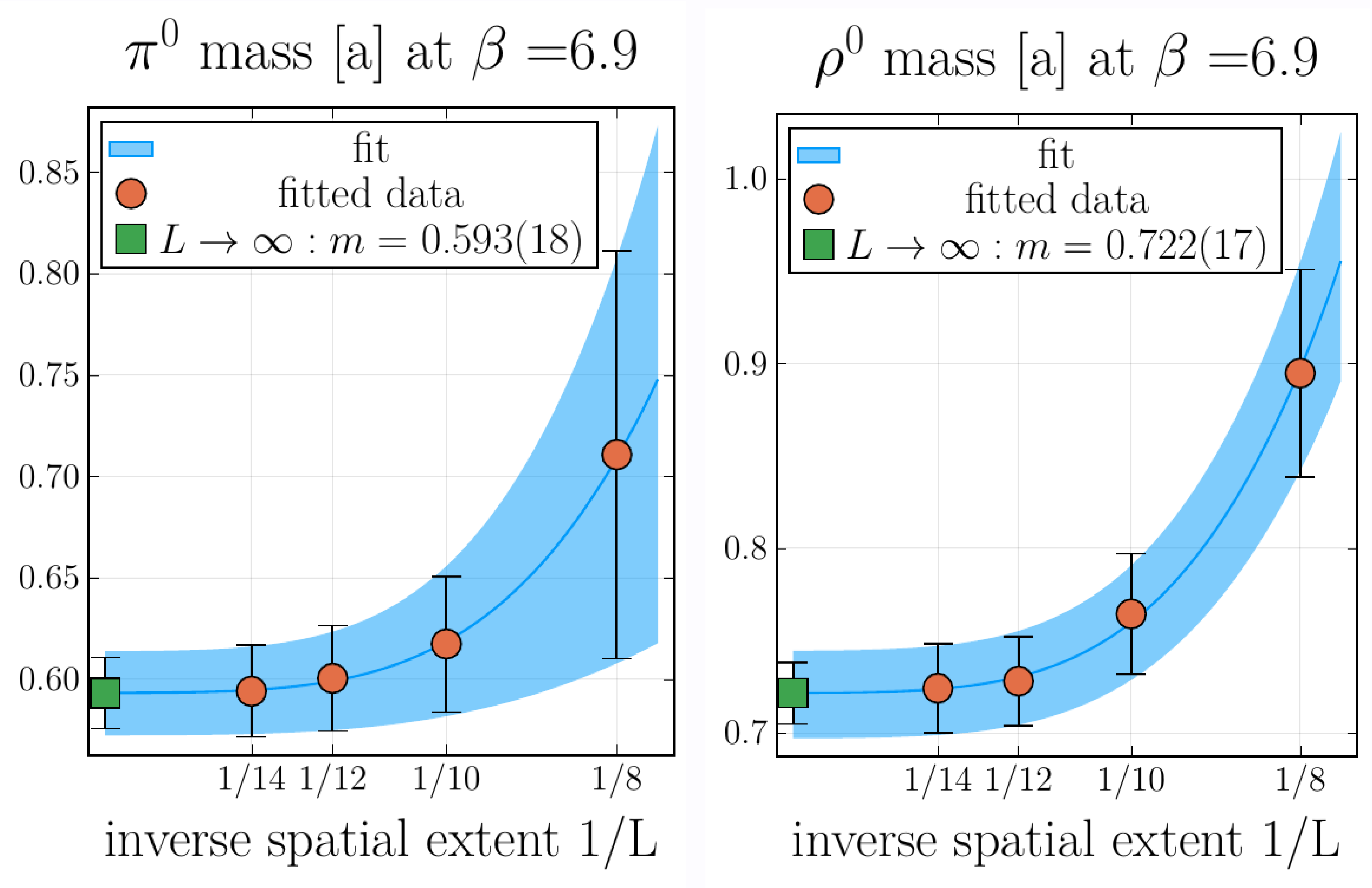
Goldstone bosons and parity

$$P : \psi(x, t) \to \gamma_0 \psi(-x, t)$$

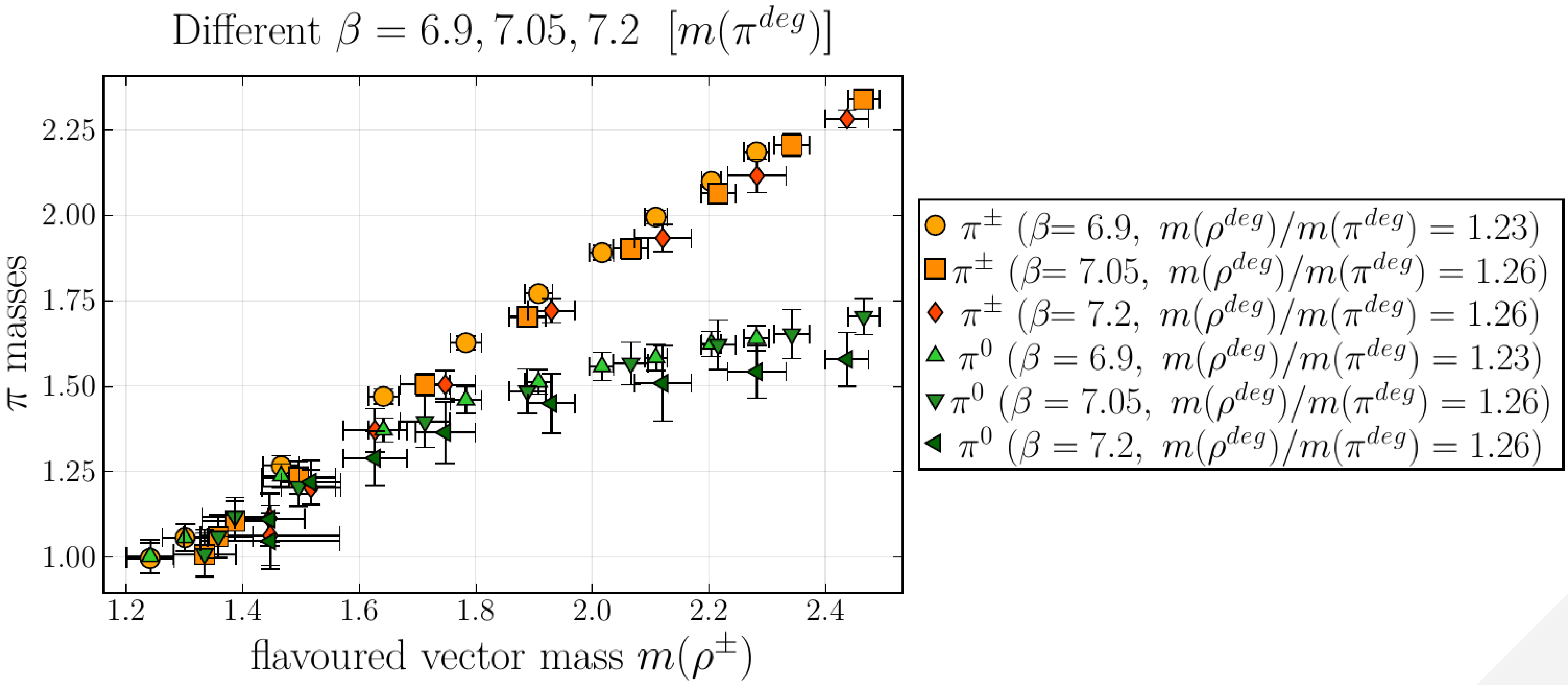
$$D : \psi(x, t) \to \pm i\gamma_0 \psi(-x, t)$$

name	operator	J^P	J^D
π^-	$\bar{u}\gamma_5 d$	0^-	0^-
π^+	$\bar{d}\gamma_5 u$	0^-	0^-
π^0	$\bar{u}\gamma_5 u - \bar{d}\gamma_5 d$	0^-	0^-
π_{qq}	$u^T SC\gamma_5 d$	0^+	0^-
$\pi_{\bar{q}\bar{q}}$	$\bar{u}SC\gamma_5 \bar{d}^T$	0^+	0^-

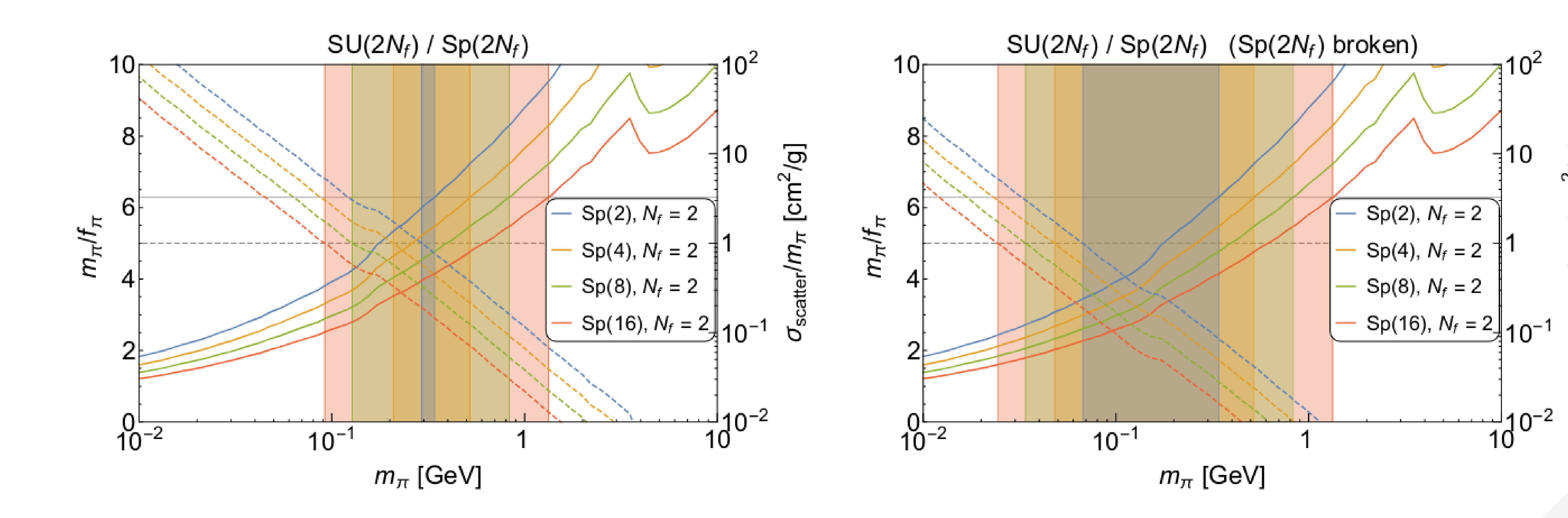
Finite Volume: Meson masses



Spacing effects: Goldstone masses



Effects of varying N_c and $Sp(2N_f)$ breaking



taken from [1411.3727]