Noether supercurrent operator mixing from lattice perturbation theory

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General Outline

1. Introduction to SYM and definitions of operators
2. Computational setup for the renormalization of the supercurrent operator

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For a more detailed description, see: Phys. Rev. D 106, 034502 [arXiv:2205.02012 [hep-lat]].

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Introduction to the Supersymmetric $\mathcal{N}=1$ Yang-Mills (SYM) theory with gauge group $SU(N_c)$

- It describes the strong interactions between gluons and gluinos.
- SYM shares some of the fundamental properties of supersymmetric theories.
- It is amenable to high-accuracy nonperturbative investigations.

Lagrangian of SYM

The Lagrangian of SYM in the continuum is:

$$\mathcal{L}_{\rm SYM} = -\frac{1}{4} u^\alpha_{\mu\nu} u^\alpha_{\mu\nu} + \frac{i}{2} \bar{\lambda}^\alpha \gamma^\mu \mathcal{D}_\mu \lambda^\alpha \quad ,$$

$$u_{\mu\nu} = \partial_\mu u_\nu - \partial_\nu u_\mu + ig[u_\mu, u_\nu] \quad , \quad \mathcal{D}_\mu \lambda = \partial_\mu \lambda + ig[u_\mu, \lambda].$$

This definition includes the coupling constant g, the field strength tensor $u_{\mu\nu} = u^\alpha_{\mu\nu} T^\alpha$ and the gluino field $\lambda = \lambda^\alpha T^\alpha$.
T^α are generators of the $SU(N_c)$ algebra.

• $\mathcal{L}_{\rm SYM}$ is invariant, up to a total derivative, under the following supersymmetric transformation:

$$\begin{aligned} \delta_\xi u^\alpha_\mu &= -i\bar{\xi}\gamma^\mu \lambda^\alpha, \\ \delta_\xi \lambda^\alpha &= \frac{1}{4} u^\alpha_{\mu\nu} [\gamma^\mu, \gamma^\nu] \xi. \end{aligned}$$

In order to quantize the theory, one should also include the gauge-fixing term and the corresponding term involving the ghost field c^α which arises from the Faddeev-Popov procedure.

Supercurrent Definition

The supercurrent S_μ stems from the application of Noether's theorem to supersymmetric transformations of $\mathcal{L}_{\rm SYM}$. In Euclidean space S_μ takes the form:

$$S_\mu = -\frac{1}{2}\,\mathrm{tr}_c(u_{\rho\,\sigma}[\gamma_\rho,\gamma_\sigma]\gamma_\mu\lambda)$$

Its lattice discretization is not unique. A standard definition which we adopt in this work is:

$$S_\mu = -\frac{1}{2}\mathrm{tr}_c(\hat{F}_{\rho\sigma}[\gamma_\rho,\gamma_\sigma]\gamma_\mu\lambda),$$

where

$$\hat{F}_{\mu\nu} = \frac{1}{8ig}(Q_{\mu\nu} - Q_{\nu\mu})$$

$$\begin{aligned} Q_{\mu\nu} = {} & U_{x,x+\mu}U_{x+\mu,x+\mu+\nu}U_{x+\mu+\nu,x+\nu}U_{x+\nu,x} \\ & + U_{x,x+\nu}U_{x+\nu,x+\nu-\mu}U_{x+\nu-\mu,x-\mu}U_{x-\mu,x} \\ & + U_{x,x-\mu}U_{x-\mu,x-\mu-\nu}U_{x-\mu-\nu,x-\nu}U_{x-\nu,x} \\ & + U_{x,x-\nu}U_{x-\nu,x-\nu+\mu}U_{x-\nu+\mu,x+\mu}U_{x+\mu,x} \end{aligned}$$

Operator mixing upon renormalization

At the operator level, the supercurrent suffers from mixing with both gauge invariant (class G) and noninvariant operators (classes A, B, C), which respect the same global symmetries.

Class G: Gauge-invariant operators.

Class A: BRST variation of operators.

Class B: Operators which vanish by the equations of motion.

Class C: Other operators which do not belong to the above classes.

The mixing operators:

- Must have the same index structure as S_μ, i.e., one free spinor index, one Lorentz index, no free color and zero ghost number.
- Their dimensionality must not exceed $7/2$.
- Must respect symmetries of the employed action.

Computational setup for the renormalization of the supercurrent operator – Methodology

1. Produce a list of candidate operators which mix with S_μ.
2. Compute (careful selection of) appropriate Green's functions containing S_μ. [Specific choices of the external momenta for the Green's functions.]
3. Apply renormalization conditions in the $\overline{\mathrm{MS}}$ scheme.
4. Extract of all mixing coefficients and renormalization constants of the operator S_μ unambiguously.

Calculation setup - List of candidate operators which mix with S_μ

In particular, class G contains another dimension 7/2 gauge invariant operator. In the literature, it is denoted as:

$$T_\mu = 2\,\mathrm{tr}_c(u_{\mu\,\nu}\gamma_\nu\lambda)$$

Calculation setup - List of candidate operators which mix with S_μ

We present all candidate gauge noninvariant operators which can mix with S_μ and belong to classes A, B, C[1]:

$$\mathcal{O}_{A1} = \frac{1}{\alpha}\mathrm{tr}_c((\partial_\nu u_\nu)\gamma_\mu\lambda) - ig\,\mathrm{tr}_c([c,\bar{c}]\gamma_\mu\lambda)$$

$$\mathcal{O}_{B1} = \mathrm{tr}_c(u_\mu \not{D}\lambda),\ \mathcal{O}_{B2} = \mathrm{tr}_c(\not{u}\gamma_\mu\not{D}\lambda)$$

$$\mathcal{O}_{C1} = \mathrm{tr}_c(u_\mu\lambda),\ \mathcal{O}_{C2} = \mathrm{tr}_c(\not{u}\gamma_\mu\lambda),\ \mathcal{O}_{C3} = \mathrm{tr}_c(\not{u}\partial_\mu\lambda),\ \mathcal{O}_{C4} = \mathrm{tr}_c((\partial_\mu\not{u})\,\lambda)$$

$$\mathcal{O}_{C5} = \mathrm{tr}_c((\partial_\nu u_\nu)\gamma_\mu\lambda),\ \mathcal{O}_{C6} = \mathrm{tr}_c(u_\nu\gamma_\mu\partial_\nu\lambda),\ \mathcal{O}_{C7} = i\,g\ \mathrm{tr}_c([u_\rho\,,u_\sigma][\gamma_\rho,\gamma_\sigma]\gamma_\mu\lambda)$$

$$\mathcal{O}_{C8} = i\,g\ \mathrm{tr}_c([u_\mu,u_\nu]\gamma_\nu\lambda),\ \mathcal{O}_{C9} = i\,g\ \mathrm{tr}_c([c,\bar{c}]\gamma_\mu\lambda)$$

[1] Operators $\mathcal{O}_{C5}$ and $\mathcal{O}_{C9}$, taken together with $\mathcal{O}_{A1}$, are linearly dependent; however, keeping both of them in the list affords us with additional consistency checks.

Renormalization of S_μ

- The renormalized supercurrent can be written as a linear combination of these operators:

$$S_\mu^R = Z_{SS} S_\mu^B + Z_{ST}\, {T_\mu}^B + Z_{SA1}\, {\mathcal{O}_{A1}}^B + \sum_{i=1}^{2} Z_{SBi}\, {\mathcal{O}_{Bi}}^B + \sum_{i=1}^{9} Z_{SCi}\, {\mathcal{O}_{Ci}}^B$$

- We calculate bare Green's functions of S_μ with external elementary quantum (and ghost) fields in momentum space both in lattice and dimensional regularizations at one-loop order. The Green's functions which we calculate have external gluinos, gluons and ghosts:

$$\langle u_\nu^{\alpha_1}(-q_1)\, S_\mu(x)\, \bar\lambda^{\alpha_2}(q_2)\rangle_{amp},$$

$$\langle u_\nu^{\alpha_1}(-q_1)\, u_\rho^{\alpha_2}(-q_2)\, S_\mu(x)\, \bar\lambda^{\alpha_3}(q_3)\rangle_{amp},$$

$$\langle c^{\alpha_3}(q_3)\, S_\mu(x)\, \bar c^{\alpha_2}(q_2)\bar\lambda^{\alpha_1}(-q_1)\rangle_{amp}.$$

Operators	Tree Level two-point Green's function (external legs: $u_\nu(-q_1)\,\bar\lambda(q_2)$)	Tree Level three-point Green's function (external legs: $u_\nu\, u_\rho\, \bar\lambda$)	Tree Level three-point Green's function (external legs: $c\,\bar c\,\bar\lambda$)
S_μ	$-i(\not{q}_1\gamma_\nu - q_{1\nu})\gamma_\mu$	$g\,[\gamma_\nu\,,\gamma_\rho]\gamma_\mu/2$	0
T_μ	$i(q_{1\mu}\gamma_\nu - \not{q}_1\delta_{\mu\nu})$	$-\,g\,(\delta_{\mu\nu}\gamma_\rho + \delta_{\mu\rho}\gamma_\nu)$	0
$\mathcal{O}_{A1}$	$i\,q_{1\nu}\gamma_\mu/(2\alpha)$	0	$(g/2)\gamma_\mu$
$\mathcal{O}_{B1}$	$i\,\delta_{\mu\nu}\not{q}_2/2$	$-\,g\,(\delta_{\nu\mu}\gamma_\rho + \delta_{\rho\mu}\gamma_\nu)/2$	0
$\mathcal{O}_{B2}$	$i\,\gamma_\nu\gamma_\mu\not{q}_2/2$	$-2\,g\,\gamma_\nu\gamma_\mu\gamma_\rho$	0
$\mathcal{O}_{C1}$	$\delta_{\mu\nu}/2$	0	0
$\mathcal{O}_{C2}$	$\gamma_\nu\gamma_\mu/2$	0	0
$\mathcal{O}_{C3}$	$i\,\gamma_\nu q_{2\mu}/2$	0	0
$\mathcal{O}_{C4}$	$i\,\gamma_\nu q_{1\mu}/2$	0	0
$\mathcal{O}_{C5}$	$i\,\gamma_\mu q_{1\nu}/2$	0	0
$\mathcal{O}_{C6}$	$i\,\gamma_\mu q_{2\nu}/2$	0	0
$\mathcal{O}_{C7}$	0	$-g\,[\gamma_\nu\,,\gamma_\rho]\gamma_\mu$	0
$\mathcal{O}_{C8}$	0	$-\,g\,(\delta_{\nu\mu}\gamma_\rho + \delta_{\rho\mu}\gamma_\nu)/2$	0
$\mathcal{O}_{C9}$	0	0	$-(g/2)\gamma_\mu$

Table 1: The two-point and three-point tree-level Green's functions of S_μ and T_μ as well as of gauge noninvariant operators which may mix with S_μ. The Green's functions are shown apart from overall exponential and color factors.

Lattice Action and choices of the external momenta

- We make use of the Wilson formulation on the lattice, with the addition of the clover (SW) term for gluino fields.
- The same operators may mix with T_μ given that they share the same quantum numbers; we compute the renormalization factor and the mixing coefficients for T_μ as well.
- Sufficient set of choices for external momenta are:
  - Three choices for 2-pt (external $u(q_1)\lambda(q_2)$): $q_2 = 0$, $q_1 = 0$, $q_2 = -q_1$.
  - One choice for 3-pt (external $u(q_1)u(q_2)\lambda(q_3)$): ($q_2 = 0$, $q_3 = -q_1$.)
  - One choice for 3-pt (external $\lambda(q_1)\bar{c}(q_2)c(q_3)$): ($q_2 = q_1$, $q_3 = 0$).

Feynman diagrams for $\langle u_\nu S_\mu \bar{\lambda}\rangle$ and $\langle u_\nu T_\mu \bar{\lambda}\rangle$

The one-loop Feynman diagrams (one-particle irreducible (1PI)) contributing to corresponding Green's functions are shown below.

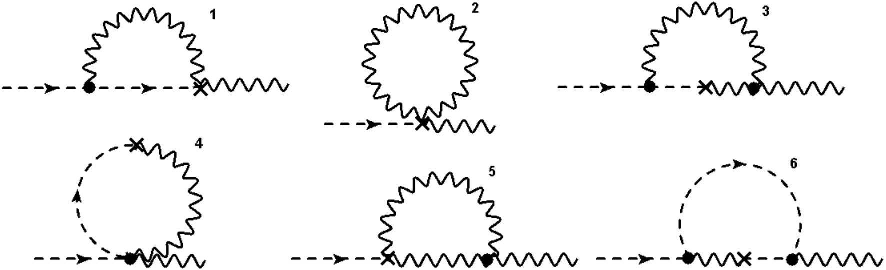

Figure 1: One-loop Feynman diagrams contributing to the two-point Green's functions $\langle u_\nu S_\mu \bar{\lambda}\rangle$ and $\langle u_\nu T_\mu \bar{\lambda}\rangle$. A wavy (dashed) line represents gluons (gluinos). A cross denotes the insertion of $S_\mu(T_\mu)$. Diagrams 2, 4 do not appear in dimensional regularization; they do however show up in the lattice formulation.

Feynman diagrams for $\langle u_\nu u_\rho S_\mu \bar{\lambda} \rangle$ and $\langle u_\nu u_\rho T_\mu \bar{\lambda} \rangle$

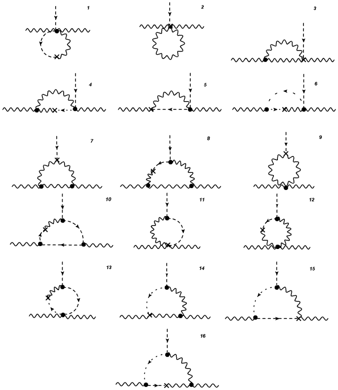

Figure 2: One-loop Feynman diagrams contributing to the three-point Green's functions $\langle u_\nu u_\rho S_\mu \bar{\lambda} \rangle$ and $\langle u_\nu u_\rho T_\mu \bar{\lambda} \rangle$. Diagrams 1, 2, 3, 5, 6, 11, and 13 do not appear in dimensional regularization but they contribute in the lattice regularization. A mirror version of diagrams 3, 4, 5, 6, 8, 10, 14, 15 and 16 must also be included.

Feynman diagrams for $\langle c\, S_\mu\, \bar{c}\, \bar{\lambda} \rangle$ and $\langle c\, T_\mu\, \bar{c}\, \bar{\lambda} \rangle$

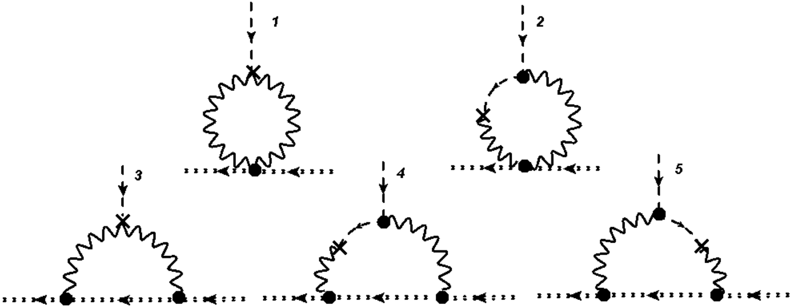

Figure 3: One-loop Feynman diagrams contributing to the three-point Green's functions $\langle c\, S_\mu\, \bar{c}\, \bar{\lambda} \rangle$ and $\langle c\, T_\mu\, \bar{c}\, \bar{\lambda} \rangle$. The "double dashed" line is the ghost field. Diagrams 1 and 2 do not appear in dimensional regularization; they do however show up in the lattice formulation.

Results for Green's functions and for the mixing matrix on the lattice

- Both $\overline{\mathrm{MS}}$-renormalized and bare Green's functions have the same tensorial structures.
- The difference between the one-loop $\overline{\mathrm{MS}}$- renormalized Green's functions (Green's functions in dimensional regularization, $D = 4 - 2\epsilon$, with $1/\epsilon \to 0$) and the corresponding bare lattice Green's functions is polynomial in the external momenta.
- The difference is proportional to the tree level value of the Green's functions of the operators.

Results for 2pt Green's function of S_μ for the choice $q_2 = 0$

The resulting expression for the difference between the two-point $\overline{\mathrm{MS}}$-renormalized and lattice bare Green's functions of S_μ is (for $q_2 = 0$):

$$\langle u_\nu^{\alpha_1}(-q_1)\, S_\mu\, \bar\lambda^{\alpha_2}(q_2)\rangle_{amp}\Big|^{\overline{\mathrm{MS}}}_{q_2=0} - \langle u_\nu^{\alpha_1}(-q_1)\, S_\mu\, \bar\lambda^{\alpha_2}(q_2)\rangle_{amp}\Big|^{LR}_{q_2=0} =$$

$$i\frac{g^2}{16\pi^2}\frac{1}{2}\delta^{\alpha_1\,\alpha_2} e^{iq_1 x} \times \Bigg[\frac{39.47842}{N_c}(\gamma_\nu\gamma_\mu \not{q}_1 + \gamma_\mu q_{1\nu}) - \frac{78.95683}{N_c}\gamma_\nu q_{1\mu}$$

$$+N_c\Big(- 5.99999\,\not{q}_1\delta_{\mu\,\nu} + \gamma_\nu q_{1\mu} 5.99722$$

$$+(\gamma_\nu\gamma_\mu\not{q}_1 + \gamma_\mu q_{1\nu} - 2\gamma_\nu q_{1\mu})\big(- 30.57429 + 5.17830\alpha$$

$$-4.55519 c_{\mathrm{SW}}^2 + 5.3771 c_{\mathrm{SW}} r + \frac{3}{2}(1-\alpha)\log\left(a^2\bar\mu^2\right)\big)\Big)\Bigg]$$

- Absence of q-independent terms means that the lower-dimensional operators do not mix with S_μ.

Renormalization condition for the 2-pt Green's functions of S_μ

The condition applied to the gluino-gluon Green's function of the operator S_μ reads to one loop:

$$\begin{aligned}
\langle u_\nu^R\, S_\mu^R\, \bar\lambda^R\rangle_{amp} &= Z_\lambda^{-1/2}\, Z_u^{-1/2} \langle u_\nu^B\, S_\mu^R\, \bar\lambda^B\rangle_{amp} \\
&= Z_\lambda^{-1/2}\, Z_u^{-1/2} Z_{SS} \langle u_\nu^B\, S_\mu^B\, \bar\lambda^B\rangle_{amp} \\
&+ Z_{ST}\langle u_\nu^B\, {T_\mu}^B\, \bar\lambda^B\rangle_{amp}^{tree} + Z_{SA1}\langle u_\nu^B\, {\mathcal{O}_{A1}}^B\, \bar\lambda^B\rangle_{amp}^{tree} \\
&+ \sum_{i=1}^{2} Z_{SBi}\langle u_\nu^B\, {\mathcal{O}_{Bi}}^B\, \bar\lambda^B\rangle_{amp}^{tree} \\
&+ \sum_{i=1}^{6} Z_{SCi}\langle u_\nu^B\, {\mathcal{O}_{Ci}}^B\, \bar\lambda^B\rangle_{amp}^{tree} + \mathcal{O}(g^4)
\end{aligned}$$

Results for 2pt Green's function of S_μ for the choice $q_2 = 0$

From the choice $q_2 = 0$ we extract these:

$$Z_{SS}^{LR,\overline{\rm MS}} = 1 + \frac{g^2}{16\pi^2}\Big(\frac{-9.86960}{N_c} + N_c(-2.3170 + 14.49751 c_{\rm SW}^2 - 1.23662 c_{\rm SW}\, r)\Big)$$

$$Z_{ST}^{LR,\overline{\rm MS}} = \frac{g^2}{16\pi^2} 3N_c$$

$$Z_{SA1}^{LR,\overline{\rm MS}} = Z_{SC1}^{LR,\overline{\rm MS}} = Z_{SC2}^{LR,\overline{\rm MS}} = Z_{SC4}^{LR,\overline{\rm MS}} = Z_{SC5}^{LR,\overline{\rm MS}} = 0$$

- $Z_{SS}^{LR,\overline{\rm MS}}$ is finite: this is in line with its classical conservation.
- $Z_{ST}^{LR,\overline{\rm MS}}$ is related to the γ-trace anomaly corresponding to super-conformal symmetry breaking and is identical to the one loop level β-function.
- No mixing with $\mathcal{O}_{A1}$, $\mathcal{O}_{C1}$, $\mathcal{O}_{C2}$, $\mathcal{O}_{C4}$, $\mathcal{O}_{C5}$.

2-pt Green's function of S_μ for the choice $q_1 = 0$

$$\langle u_\nu^{\alpha_1}(-q_1)\, S_\mu\, \bar\lambda^{\alpha_2}(q_2)\rangle_{amp}\Big|_{q_1=0}^{\overline{\rm MS}} - \langle u_\nu^{\alpha_1}(-q_1)\, S_\mu\, \bar\lambda^{\alpha_2}(q_2)\rangle_{amp}\Big|_{q_1=0}^{LR} = i\,\frac{g^2 N_c}{16\pi^2} \times$$

$$\frac{1}{2}\delta^{\alpha_1\,\alpha_2} e^{iq_2 x} \times \left[\gamma_\nu\gamma_\mu \not{q}_2 \left(0.80802 - \frac{1}{2}\log\left(a^2\bar\mu^2\right)\right) - \not{q}_2 \delta_{\mu\,\nu}\left(0.38395 + \log\left(a^2\bar\mu^2\right)\right)\right]$$

From the choice $q_1 = 0$ we determine the following:

$$Z_{SB1}^{LR,\overline{\rm MS}} = \frac{g^2}{16\pi^2} N_c \left(-0.38395 - \log\left(a^2\bar\mu^2\right)\right)$$

$$Z_{SB2}^{LR,\overline{\rm MS}} = \frac{g^2}{16\pi^2} N_c \left(0.80802 - \frac{1}{2}\log\left(a^2\bar\mu^2\right)\right)$$

$$Z_{SC3}^{LR,\overline{\rm MS}} = Z_{SC6}^{LR,\overline{\rm MS}} = 0$$

- $Z_{SB1}^{LR,\overline{\rm MS}}$ and $Z_{SB2}^{LR,\overline{\rm MS}}$ are logarithmically divergent mixing coefficients.
- No mixing with $\mathcal{O}_{C3}$, $\mathcal{O}_{C6}$.

2-pt Green's function of S_μ for the choice $q_2 = -q_1$

The choice $q_2 = -q_1$ serves as a consistency check for the above results.

$$\langle u_\nu^{\alpha_1}(-q_1)\, S_\mu\, \bar\lambda^{\alpha_2}(q_2)\rangle_{amp}\Big|^{\overline{\rm MS}}_{q_2=-q_1} - \langle u_\nu^{\alpha_1}(-q_1)\, S_\mu\, \bar\lambda^{\alpha_2}(q_2)\rangle_{amp}\Big|^{LR}_{q_2=-q_1} =$$

$$i\,\frac{g^2}{16\pi^2}\,\frac{1}{2}\delta^{\alpha_1\,\alpha_2} \times \Bigg[\frac{39.47842}{N_c}(\gamma_\nu\gamma_\mu\not{q}_1 + \gamma_\mu q_{1\nu}) - \frac{78.95683}{N_c}\gamma_\nu q_{1\mu}$$

$$+N_c\Bigg(0.80802\gamma_\mu q_{1\nu} + 4.38396\gamma_\nu q_{1\mu} + \frac{1}{2}\gamma_\nu\gamma_\mu\not{q}_1 \log\left(a^2\bar\mu^2\right)$$

$$+(\gamma_\nu\gamma_\mu\not{q}_1 + \gamma_\mu q_{1\nu} - 2\gamma_\nu q_{1\mu})\Big(-31.38231 + 5.17830\alpha$$

$$-4.55519c_{\rm SW}^2 + 5.37708c_{\rm SW}\,r + 2\log\left(a^2\bar\mu^2\right) - \frac{3}{2}\alpha\log\left(a^2\bar\mu^2\right)\Big)$$

$$+\not{q}_1\delta_{\mu\,\nu}\left(-5.61605 + \log\left(a^2\bar\mu^2\right)\right)\Bigg)\Bigg]$$

Renormalization conditions for the 3-pt Green's functions of S_μ

$$\begin{aligned}\langle u_\nu^R u_\rho^R S_\mu^R \bar\lambda^R\rangle_{amp} &= Z_\lambda^{-1/2} Z_u Z_{SS} \langle u_\nu^B u_\rho^B S_\mu^B \bar\lambda^B\rangle_{amp} + Z_{ST}\langle u_\nu^B u_\rho^B {T_\mu}^B \bar\lambda^B\rangle_{amp}^{tree} \\ &+ \sum_{i=1}^{2} Z_{SBi}\langle u_\nu^B u_\rho^B {\mathcal{O}_{Bi}}^B \bar\lambda^B\rangle_{amp}^{tree} \\ &+ \sum_{i=7}^{8} Z_{SCi}\langle u_\nu^B u_\rho^B {\mathcal{O}_{Ci}}^B \bar\lambda^B\rangle_{amp}^{tree} + \mathcal{O}(g^4)\end{aligned}$$

$$\begin{aligned}\langle c^R S_\mu^R \bar c^R \bar\lambda^R\rangle_{amp} &= Z_c^{-1} Z_\lambda^{-1/2} Z_{SS} \langle c^B S_\mu^R \bar c^b \bar\lambda^B\rangle_{amp} + Z_{ST}\langle c^B {T_\mu}^B \bar c^B \bar\lambda^B\rangle_{amp}^{tree} \\ &+ Z_{SA1}\langle c^B {\mathcal{O}_{A1}}^B \bar c^B \bar\lambda^B\rangle_{amp}^{tree} \\ &+ Z_{SC9}\langle c^B {\mathcal{O}_{C9}}^B \bar c^B \bar\lambda^B\rangle_{amp}^{tree} + \mathcal{O}(g^4)\end{aligned}$$

3-pt Green's functions of S_μ

The three-point Green's functions determine the mixing with $\mathcal{O}_{C7}$, $\mathcal{O}_{C8}$ and $\mathcal{O}_{C9}$.
The results for the first one is shown below:

$$\langle u_\nu^{\alpha_1}(-q_1) u_\rho^{\alpha_2}(-q_2)\, S_\mu\, \bar\lambda^{\alpha_3}(q_3)\rangle\Big|^{\overline{\rm MS}}_{q_2=0,q_3=-q_1} - \langle u_\nu^{\alpha_1}(-q_1) u_\rho^{\alpha_2}(-q_2)\, S_\mu\, \bar\lambda^{\alpha_3}(q_3)\rangle\Big|^{LR}_{q_2=0,q_3=-q_1} =$$

$$\frac{g^3 N_c}{16\pi^2}\, f^{\alpha_1\alpha_2\alpha_3}\Bigg[(\delta_{\nu\rho}\gamma_\mu - \gamma_\nu\gamma_\rho\gamma_\mu)\Big(\frac{19.73920}{N_c^2} - 12.48660 + 3.28231\alpha - 2.27761 c_{\rm SW}^2 + 2.68854 c_{\rm SW}\, r + \frac{1-2\alpha}{2}\log\left(a^2\bar\mu^2\right)\Big)\Bigg]$$

- No mixing with $\mathcal{O}_{C7}$ and $\mathcal{O}_{C8}$.

The lattice Green's function containing gluino-ghost-antighost external fields is identical to the continuum one.

- No mixing with $\mathcal{O}_{C9}$.

Summary – Conclusion

The first two rows of the mixing matrix are derived from Lattice Perturbation theory in the $\overline{\mathrm{MS}}$ scheme.

$$\begin{bmatrix} S_\mu^R \\ T_\mu^R \\ \mathcal{O}_{A1}^R \\ \mathcal{O}_{B1}^R \\ \mathcal{O}_{B2}^R \\ \mathcal{O}_{C1}^R \\ \vdots \\ \mathcal{O}_{C9}^R \end{bmatrix} = \begin{bmatrix} Z_{SS} & Z_{ST} & 0 & Z_{SB1} & Z_{SB2} & 0 \cdots & 0 \\ Z_{TS} & Z_{TT} & 0 & Z_{TB1} & Z_{TB2} & 0 \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \cdots & \vdots \end{bmatrix} \times \begin{bmatrix} S_\mu \\ T_\mu \\ \mathcal{O}_{A1} \\ \mathcal{O}_{B1} \\ \mathcal{O}_{B2} \\ \mathcal{O}_{C1} \\ \vdots \\ \mathcal{O}_{C9} \end{bmatrix}$$

In our ongoing work, we utilize the GIRS scheme; this scheme is more accesible via non-perturbative calculations.

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Thank you for your attention!

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FOUNDATION

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