Supersymmetric Yang-Mills on the Lattice

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Overview

3d Maximal SYM

Extension of previous work[1]

- Lattice Supersymmetry Motivation
- Formulation background
- Results for 3d Maximal SYM

[1] arXiv:2010.00026 by S. Catterall, J. Giedt, R. G. Jha, D. Schaich and T. Wiseman

Motivation

Lattice field theory promises first-principles predictions for strongly coupled supersymmetric QFTs

- Better understanding of quantum mechanics
- Holography
- BSM physics

Supersymmetry on the lattice

Formally $\langle \mathcal{O} \rangle = \mathcal{Z}^{-1} \int \mathcal{D}\phi \ \mathcal{O}(\phi) \ e^{-S[\phi]}$
Regularize by formulating theory in finite, discrete, euclidean space-time with spacing a
At the end take continuum limit ($a \to 0$)

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Supersymmetry is a space-time symmetry – spinor generators Q^I_α and $\overline{Q}^I_{\dot\alpha}$
$\left\{ Q^I_\alpha, \overline{Q}^J_{\dot\beta} \right\} = 2\delta^{IJ} \sigma^\mu_{\alpha\dot\beta} P_\mu$ broken in discrete space-time

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Idea

Preserve susy sub-algebra in discrete lattice space-time
$\longrightarrow$ correct continuum limit with substantially less fine tuning

3d Maximal SYM

3d maximal SYM can be treated as dimensional reduction of 4d maximal SYM.

4d

Gauge field A_μ plus 6 scalars Φ^{IJ}

$\mathcal{N} = 4$ four-component fermions $\Psi^{\mathrm{I}} \longleftrightarrow$ 16 supersymmetries Q^{I}_{α} and $\overline{Q}^{\mathrm{I}}_{\dot{\alpha}}$

Global $\mathrm{SU}(4) \ \sim \ \mathrm{SO}(6)$ R symmetry

All fields massless and in adjoint rep. of SU(N) gauge group

Symmetries relate kinetic, Yukawa and Φ^4 terms $\longrightarrow$ single coupling $\lambda = g^2 N$.

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3d

Gauge field A_μ plus 7 scalars Φ

$\mathcal{N} = 8$ two-component fermions $\Psi^{\mathrm{I}} \longleftrightarrow$ 16 supersymmetries Q^{I}_α and $\overline{Q}^{\mathrm{I}}_{\dot\alpha}$

Global $\mathrm{SO}(8)$ R symmetry

All fields massless and in adjoint rep. of $\mathrm{SU}(N)$ gauge group

Symmetries relate kinetic, Yukawa and Φ^4 terms $\longrightarrow$ single coupling $\lambda = g^2 N$.

Twisting SYM

Construction from 'topological' twisting

Decompose fields in terms of reps of $\mathrm{SO(d)}_{\mathrm{tw}} \equiv \mathrm{diag}\left[\mathrm{SO(d)}_{\mathrm{rot}} \otimes \mathrm{SO(d)}_{R}\right]$

Change of variables $\longrightarrow$ $\mathcal{Q}$s transform with integer "spin" under $\mathrm{SO(d)}_{\mathrm{tw}}$

Need $Q = 2^d$ supersymmetries in d dimensions

Twisting SYM

4d maximal SYM

$$\begin{pmatrix} Q^1_\alpha & Q^2_\alpha & Q^3_\alpha & Q^4_\alpha \\ \overline{Q}^1_{\dot\alpha} & \overline{Q}^2_{\dot\alpha} & \overline{Q}^3_{\dot\alpha} & \overline{Q}^4_{\dot\alpha} \end{pmatrix} = \mathcal{Q} + \mathcal{Q}_\mu\gamma_\mu + \mathcal{Q}_{\mu\nu}\gamma_\mu\gamma_\nu + \overline{\mathcal{Q}}_\mu\gamma_\mu\gamma_5 + \overline{\mathcal{Q}}\gamma_5$$

$$\longrightarrow \mathcal{Q} + \mathcal{Q}_a\gamma_a + \mathcal{Q}_{ab}\gamma_a\gamma_b$$

with $a, b = 1, \cdots, 5$

$\{\mathcal{Q}, \mathcal{Q}\} = 2\mathcal{Q}^2 = 0 \quad \longrightarrow \quad \mathcal{Q}$ subalgebra can be preserved in discrete space-time

Twisting SYM

3d maximal SYM

$\{\mathcal{Q}, \mathcal{Q}_a, \mathcal{Q}_{ab}\} \longrightarrow \{\mathcal{Q}, \mathcal{Q}_5, \mathcal{Q}_a, \mathcal{Q}_{a5}, \mathcal{Q}_{ab}\}$ with $a, b = 1, \cdots, 4$

Same decomposition but now two closed subalgebras
$\{\mathcal{Q}, \mathcal{Q}\} = 2\mathcal{Q}^2 = 0$ and $\{\mathcal{Q}_5, \mathcal{Q}_5\} = 2\mathcal{Q}_5^2 = 0$

Twisting SYM

Fields decomposed in similar way

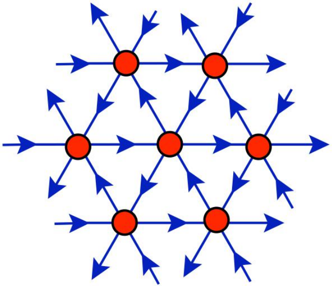
Fields also transform with integer spin under $\mathrm{SO}(3)_{\mathrm{tw}}$ — no spinors

$$\Psi \quad \text{and} \quad \overline{\Psi} \longrightarrow \eta, \psi_5, \psi_a, \chi_{a5} \text{ and } \chi_{ab}$$

$$A_\mu \quad \text{and} \quad \Phi \longrightarrow \text{complexified gauge field } \mathcal{A}_a \text{ and } \overline{\mathcal{A}}_a \longrightarrow \mathrm{U}(N) \text{ gauge theory}$$

$d+1$ links in d dimensions

Use A_d^* lattice - d-dimensional analog of 2d triangular lattice



Basis vectors linearly dependent and not orthogonal
Large S_{d+1} point group symmetry

S_{d+1} ireps match onto irreps of $\mathrm{SO}(d)_{\mathrm{tw}}$

Deformations to stabilize lattice calculations

Dimensional reduction stabilization (4d code with $N_x = 1$)

1) Potential $\propto \text{Tr}\left[(\varphi - \mathbb{I}_N)^\dagger (\varphi - \mathbb{I}_N)\right]$ to break center symmetry in reduced dir(s)

φ - link in reduced dir

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2) Add SU(N) scalar potential $\propto \mu^2 \sum_a \text{Tr}\left[\left(\mathcal{U}_a\overline{\mathcal{U}}_a - \mathbb{I}_N\right)^2\right]$

Softly breaks susy $\longrightarrow$ $\mathcal{Q}$-violating operators vanish $\propto \mu^2 \to 0$

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3) Constrain U(1) plaquette determinant $\sim \zeta \sum_{a<b} (\det \mathcal{P}_{ab} - 1)$

Implemented supersymmetrically by modifying auxiliary field equations of motion

3d maximal SYM thermodynamics

arXiv:2010.00026

Formulate on $r_1 \times r_2 \times r_\beta$ (skewed) 3-torus

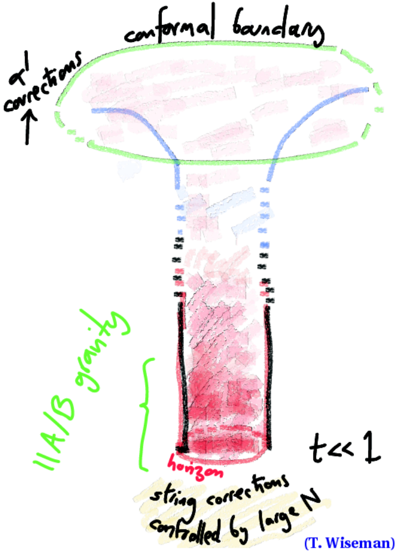
Thermal boundary conditions

$\longrightarrow$ dimensionless temperature $t = \frac{T}{\lambda} = \frac{1}{r_\beta}$

Low temperatures t at large N

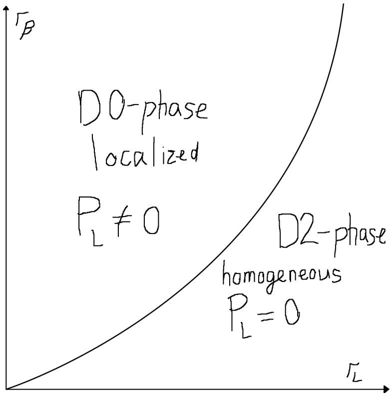
$\updownarrow$

Black branes in dual supergravity



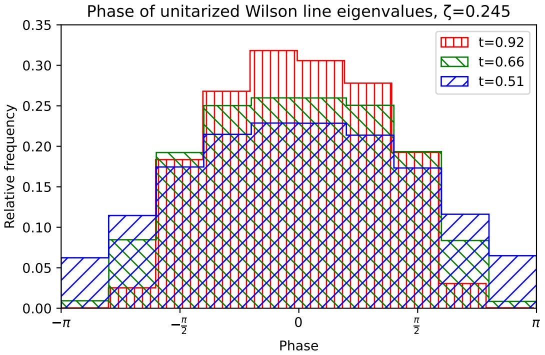
(T. Wiseman)

Phase transition: expectations



Wilson line P_L signals the transition

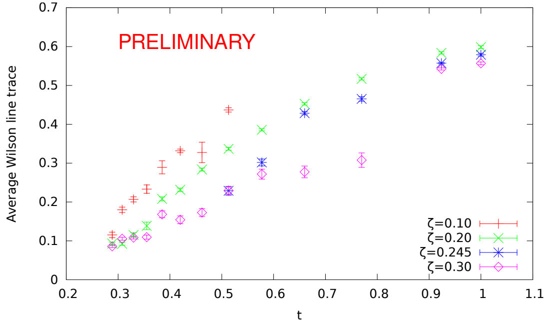
Phase transition

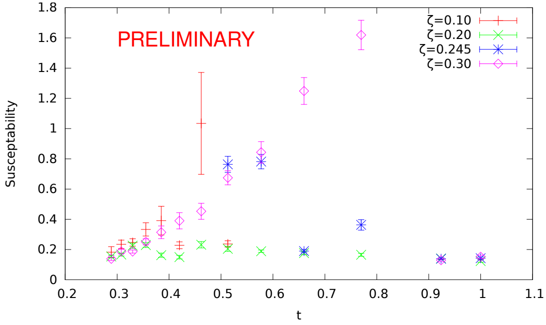


Wilson line eigenvalue phases sensitive to transition

Temperature decreases $\longrightarrow$
Wilson line eigenvalues phases more uniform

Phase transition



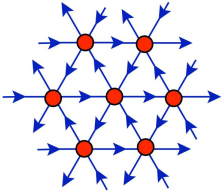


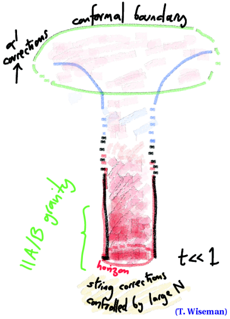
Recap

Preserving susy sub-algebra enables lattice calculations

Localized-homogeneous phase transition

Larger volumes and lattices with $N_L \neq N_t$ in the future





Thank you for your attention!

Collaborators

Simon Catterall, Joel Giedt, Raghav Jha, David Schaich, Toby Wiseman