Reconstruction of spectral densities in a composite-Higgs model

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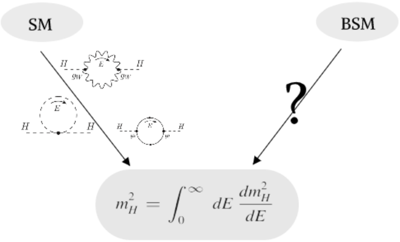
Composite Higgs models

- The identification of the Higgs as a pseudo–NG boson emerging from the breaking of a global symmetry offers a possible solution to the Naturalness problem.
- Such symmetry describes the flavor structure of a new strongly–interacting sector, whose fermions eventually confine into light bound states, including the Higgs boson.
- Higgs boson's mass is understood in terms of Goldstone dynamics
- Strongly–interacting dynamics requires a lattice study.



Lattice study

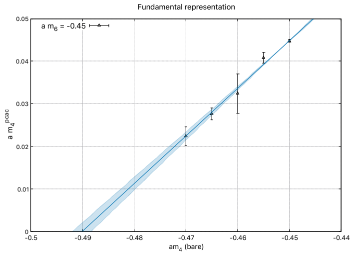
- We consider the Ferretti model, an $SU(4)$ gauge theory. We simplify the matter content to ease the lattice study.
- Two Dirac fermions in the fundamental representation instead of three

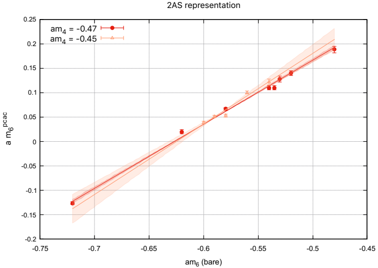
$$\frac{SU(3) \times SU(3)}{SU(3)} \to \frac{SU(2)_L \times SU(2)_R}{SU(2)_V}$$

- Two Dirac fermions instead of five Majorana in the two index anti-symmetric (2AS) representation.

$$\frac{SU(5)}{SO(5)} \to \frac{SU(4)}{SO(4)}$$

- We adopt the Wilson discretisation of the action with $O(a)$ clover improvement term





Spectral densities

Spectral densities encode informations about the spectrum. They are related to lattice correlators

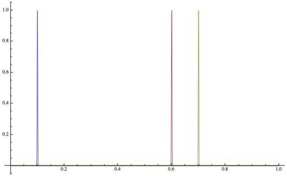
$$c(t) = \int dE\, e^{-tE}\, \rho(E)$$

We explore the possibility of fitting $\rho(E)$ instead of $c(t)$ to get E_i and w_i.

$$c(t) = w_0 e^{-tE_0} + w_1 e^{-tE_1} + \dots$$

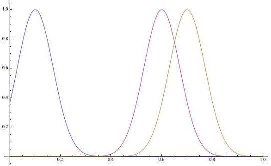
$$\rho(E) = w_0 \delta(E - E_0) + w_1 \delta(E - E_1) + \dots$$

We cannot deal with distributions: we directly extract smeared spectral densities



$$\rho(E) \to \rho_\sigma(E) = \int dE'\, \Delta_\sigma(E - E')\rho(E')$$

σ = *smearing width*



Calculation of smeared spectral densities from lattice data

To solve the inverse problem we use a Backus Gilbert type regularisation (Hansen, Lupo, Tantalo 2019)

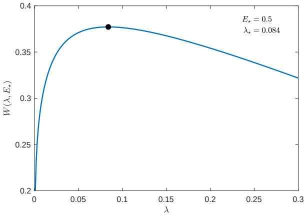
$$c(t) = \int dE\ e^{-tE}\ \rho(E)$$

1 Choose a smearing kernel $\Delta_\sigma(E - E')$

2 Span the smearing kernel with the function in the spectral representation $\bar{\Delta}_\sigma(E - E') = \sum_t g_t(E')\ e^{-tE}$

3 The smeared spectral density is $\rho_\sigma(E) = \sum_t g_t(E)\ c(t)$

★ We find g_t by minimising the functional $\mathcal{W}[g] = (1 - \lambda)\ A[g] + \lambda\ B[g]$



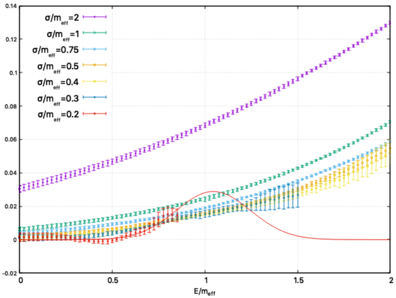
$$\underbrace{A[g] = \int dE\ e^{\alpha E}\ \left|\bar{\Delta}_\sigma(E - E') - \Delta_\sigma(E - E')\right|^2}_{\textit{Controls systematic error}}, \qquad \underbrace{B[g] = g^T \cdot Cov \cdot g}_{\textit{Controls statistical error}}$$

Spectral densities in composite Higgs models

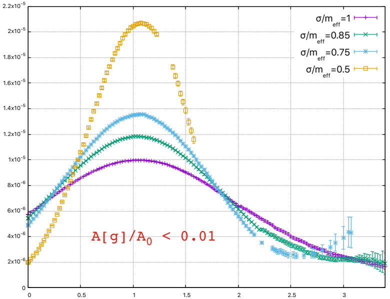
Studying lattice data in "energy-space" can be useful in BSM physics where we lack hints from phenomenology

$$c(t) = \sum_n \frac{e^{-E_n t}}{2E_n} \langle 0|\, \mathcal{O}(t) |n\rangle \langle n| \bar{\mathcal{O}}(0)\, |0\rangle \quad \longrightarrow \quad \rho_\sigma(E) = \sum_n \frac{\langle 0|\, \mathcal{O}(0) |n\rangle \langle n| \bar{\mathcal{O}}(0)\, |0\rangle}{2E_n} \Delta_\sigma(E - E_n)$$

✘ We find excited states to be too dominant to resolve single energy contributions



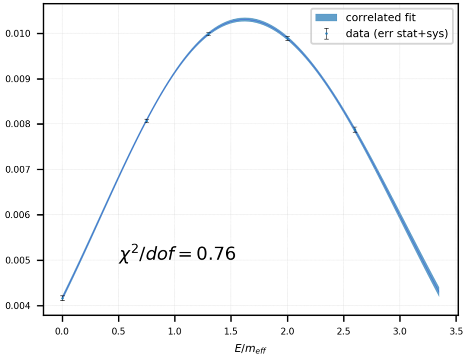
✔ We modify the operator to better overlap with the ground state. We explicitly see excited states suppression



Fit strategy

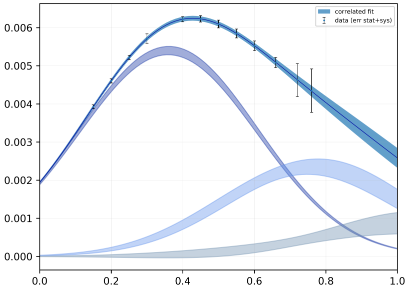
- Instead of fitting the correlator to a sum of exponentials, we fit the spectral density to a sum of Gaussians.
- Energy levels and matrix elements are the parameters to be determined by the fits.

Two Gaussian fit (4 parameters)



We can explicitly see excited states suppression

Three Gaussian fit (6 parameters)

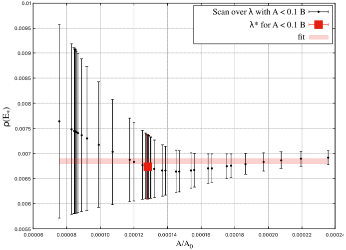


Controlling the systematic error

$$\mathcal{W} = \underbrace{(1-\lambda)\ A[g]}_{\textit{systematic}} + \underbrace{\lambda\ B[g]}_{\textit{statistical}}$$

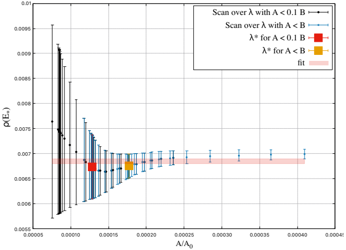
At a given energy, start with the most conservative estimate of the error and fit

small A, large B



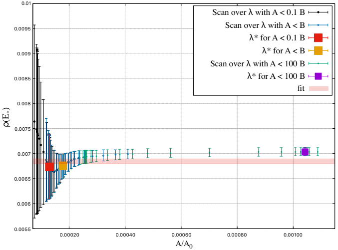
Increase A in favor of a smaller B. If the old fit still lies within the new error, continue

$A \simeq B$



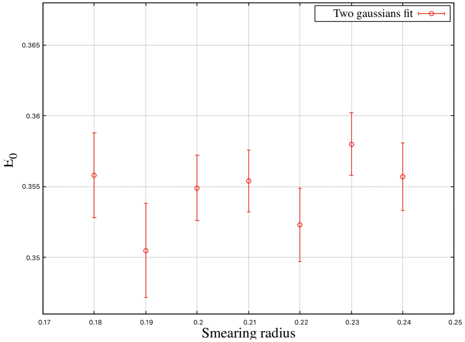
A too large, error underestimated conservative fit no longer compatible with the data

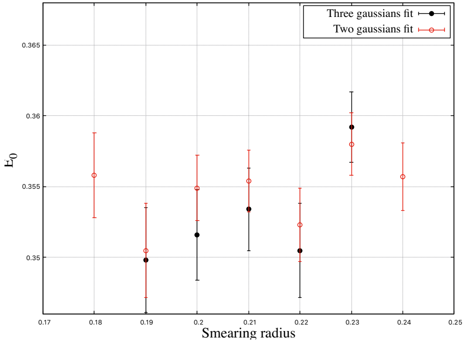
$A < 100B$



Excited state contamination

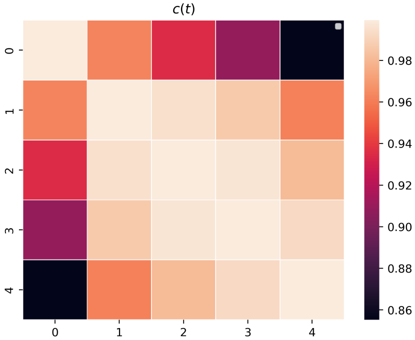
- We check there is no dependence on the smearing radius of the Gaussians.
- For larger radii there can be contamination from excited state: we check that the energy levels from the fits remain consistent by adding an extra Gaussian to the model function.
- Results for the ground state:

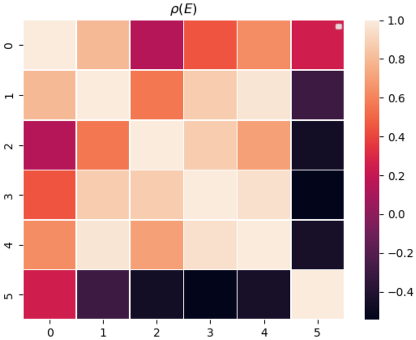




Comparison with exponential fitting

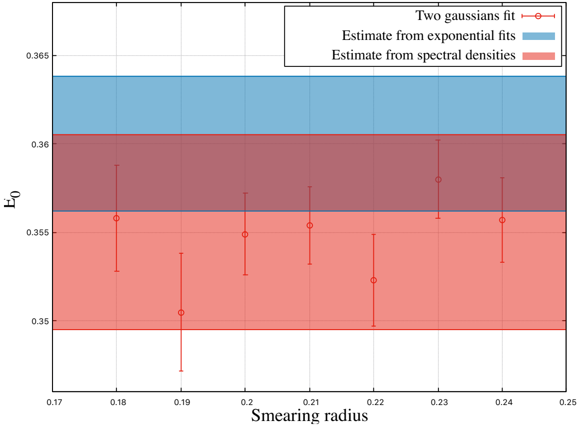
- We perform correrelated fit both of lattice correlators and spectral densities
- Interestingly, highly correlated data appears less correlated in energy space





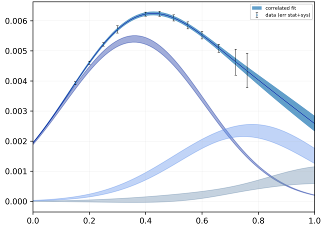
Results for the ground state

- ✔ We find agreement between fits of exponentials and fits of spectral densities
- ✔ Errors of the same order of magnitude ($\simeq$ 1%)



Conclusions

- ✔ We have been exploring the possibility to extract energy levels and matrix elements by fitting spectral densities
- ✔ We understood the strategies involved in the spectral reconstructions and their fits
- ✔ Agreement with exponential fits, systematics well under control.
- See also talk A. De Santis & poster by A. Evangelista



This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 813942.