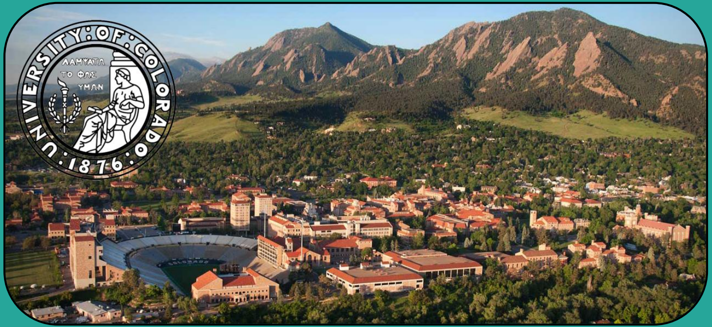

Gradient flow anomalous dimensions for ten-flavor SU(3) gauge theory

Curtis Taylor Peterson, Anna Hasenfratz and Oliver Witzel

On behalf of the Lattice Strong Dynamics (LSD) collaboration

Ten-flavor SU(3) gauge theory and the renormalization group (RG)

[Hasenfratz, Rebbi, Witzel *PRD* **101**, 114508 (2020)]
[Kuti, Fodor, Holland, Wong *PoS* LATTICE2021 (2021) 396]
[LSD *PRD* **103**, 014504 (2020)]
[Cacciapaglia, Pica, Sannino *Phys. Rep.* **877** (2020)]

- Ten-flavor SU(3) gauge theory
  - Interesting for composite Higgs modelling (e.g., LSD 4+6 system)
  - There is ongoing debate over the existence of an infrared fixed point (IRFP)
- Of particular interest are mesonic and baryonic running operator anomalous dimensions

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Energy scale

RG beta-function

$$\beta(\mu; g^2) \equiv \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} g^2(\mu)$$

Operator anomalous dimension

$$\gamma_{\mathcal{O}}(\mu; g^2) \equiv \mu \frac{\mathrm{d}}{\mathrm{d}\mu} \log Z_{\mathcal{O}}(\mu)$$

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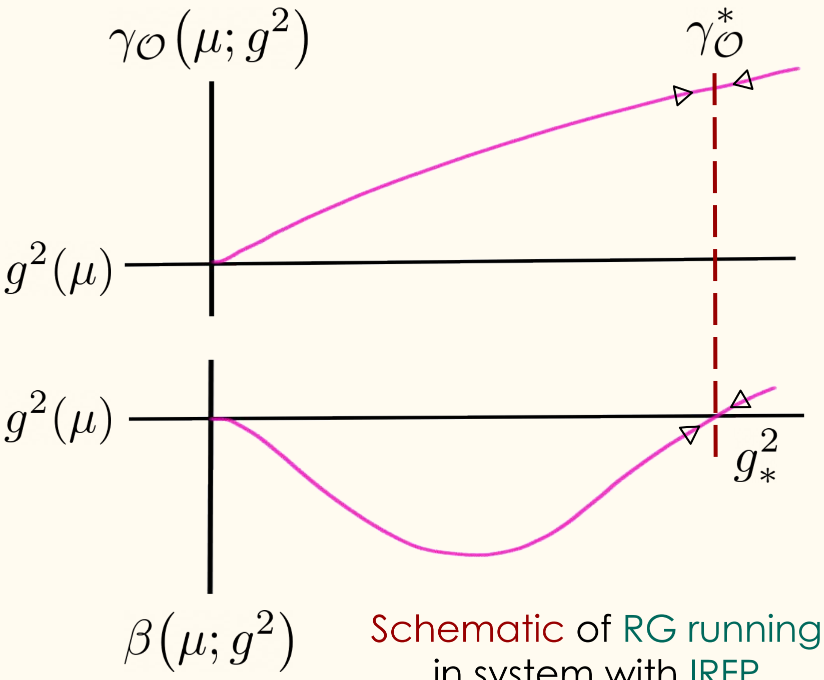

Schematic of RG running in system with IRFP

Gradient flow (GF) and operator anomalous dimensions

[Carosso, Hasenfratz, Neil *PRL* **121**, 201601 (2018)]

- Gradient flow describes an RG transformation when combined with a rescaling step in the calculation of expectation values
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$$\mu^2 \sim 1/8t$$

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$$G_{\mathcal{O}}(t, \hat{x}_4; g_0^2) \equiv \int_{\hat{\mathbf{x}}} \left\langle \mathcal{O}_t(\hat{\mathbf{x}}, \hat{x}_4) \mathcal{O}(\hat{\mathbf{x}}, 0) \right\rangle_{g_0^2}$$

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$$\frac{G_{\mathcal{O}}(t, \hat{x}_4; g_0^2)}{G_{\mathcal{O}}(t', \hat{x}_4; g_0^2)} \sim (t/t')^{-\gamma_{\mathcal{O}}/2 + n_{\mathcal{O}} \tilde{\eta}}$$

$8t/a^2, 8t'/a^2 \gg 1$

and

$\hat{x}_4 \gg \sqrt{8t/a^2}, \sqrt{8t'/a^2}$

Comes from Z_χ

Gradient flow (GF) and operator anomalous dimensions

[Carosso, Hasenfratz., Neil *PRL* **121**, 201601 (2018)]
[Hasenfratz, Monahan, Rizik, Schindler, Witzel *PoS* LATTICE2021 (2021) 155]

GF can be used to define a scheme for the renormalization of local operators

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Matches onto tree-level $\overline{\mathrm{MS}}$ coupling

$$g^2_{\mathrm{GF}}(t; g_0^2) \equiv \mathcal{N} \langle t^2 E(t) \rangle_{g_0^2}$$

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Independent of $\hat{x}_4$ for $\hat{x}_4 \gg \sqrt{8t}/a$

Cancels off $n_{\mathcal{O}}\tilde{\eta}$

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We can trade t dependence for $g^2_{\mathrm{GF}}(t;g_0^2)$ dependence

See also:
[O. Witzel: *Wed. 2:00 PM*]
[A. Shindler: *Wed. 4:30 PM*]
[C. Monahan: *Wed. 4:50*]
[C.H. Wong: *Thu. 12:10 PM*]
[R. Harlander: *Thu. 12:30 PM*]
[A. Hasenfratz: *Fri. 8:50 AM*]

Simulation details

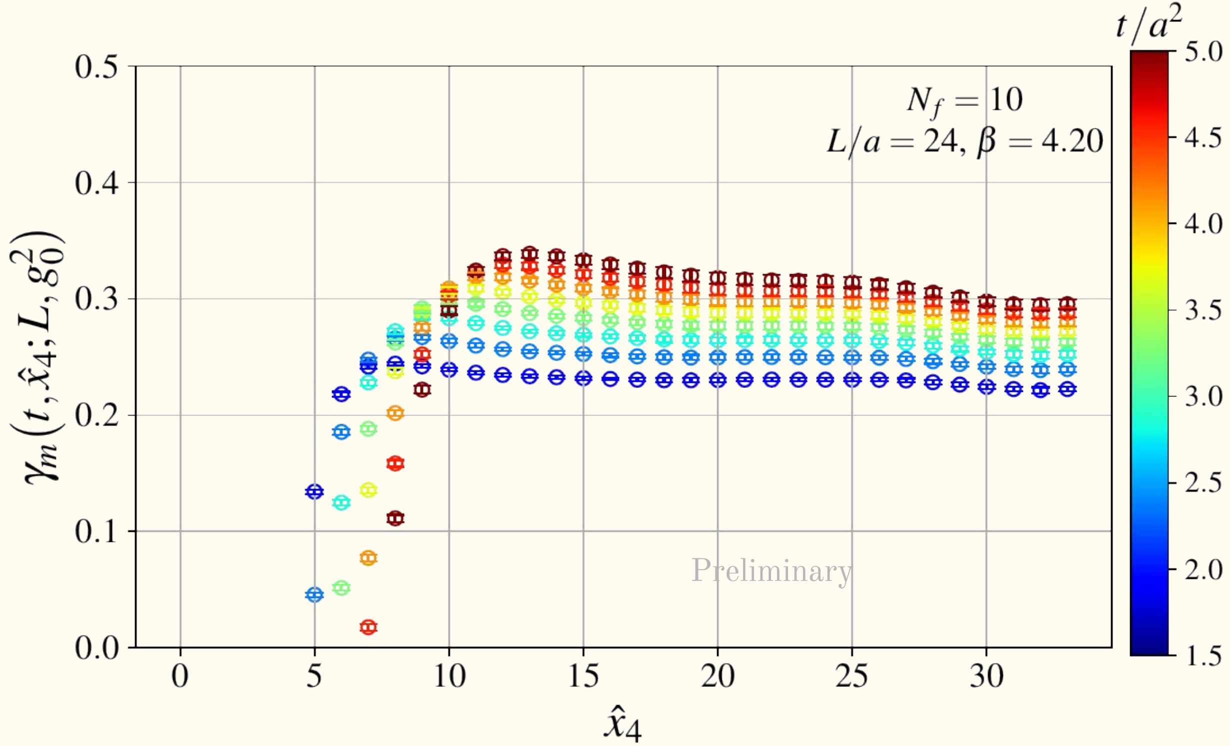
[Boyle, Yamaguchi, Cossu, Portelli LATTICE2015 (2015) 023]
[Pochinsky *PoS* LATTICE2008 (2008) 040]
[Fodor, Holland, Kuti, Mondal, Nogradi, *JHEP* **09** (2014) 018]

- Ten-flavor simulations are performed using Symanzik gauge action with stout smeared Möbius domain wall fermions (DWF) using GRID
  - Bare gauge couplings $\beta \equiv 6/g_0^2 = 5.00, 6.00, 4.20, 4.60$ $4.10, 4.05$
  - Volumes $24^3 \times 64$ and $32^3 \times 64$
- Gradient flow performed with Wilson flow using QLUA
  - We define the gradient flow coupling in finite volume using tree-level normalization
    - Corrects for gauge zero modes and tree-level cutoff effects

$$g^2_{\rm GF}\left(t; L, g_0^2\right) \equiv \underbrace{\frac{\mathcal{N}}{C(t,L)}}\left\langle t^2 E(t)\right\rangle_{L,g_0^2}$$

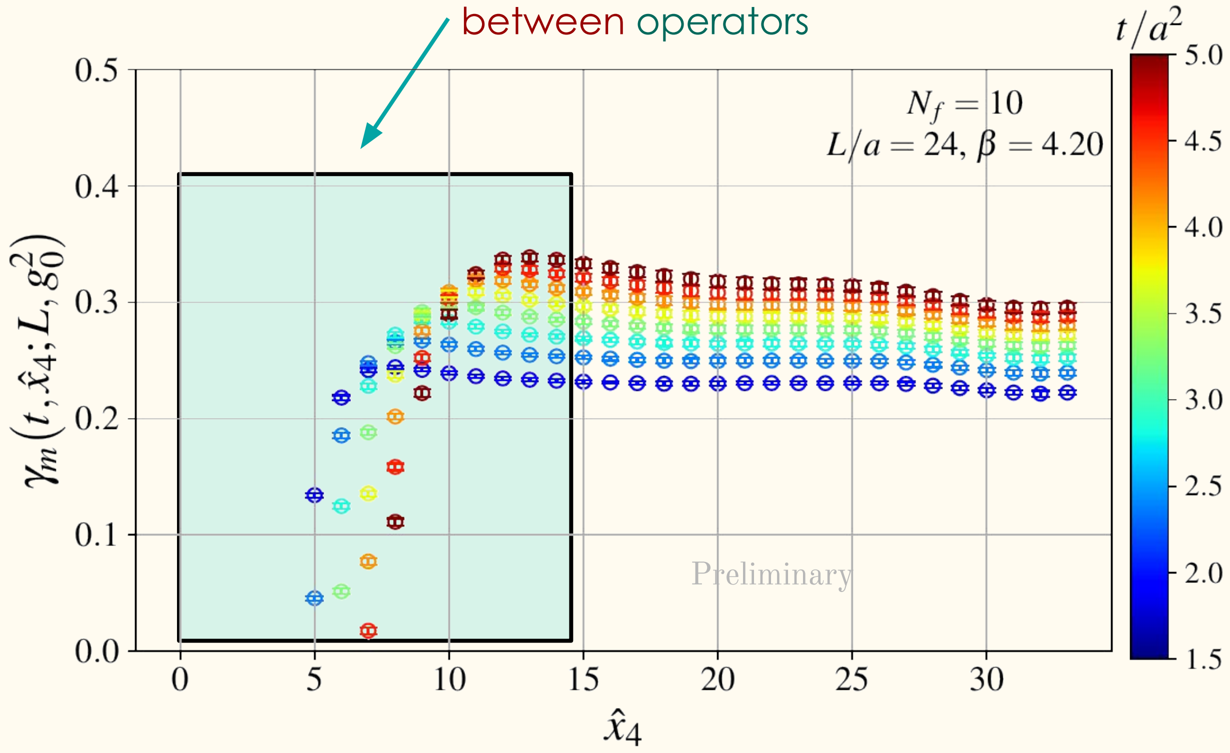
Tree-level normalization factor

Anomalous dimension $\hat{x}_4$-independence



Anomalous dimension $\hat{x}_4$-independence

Insufficient separation between operators



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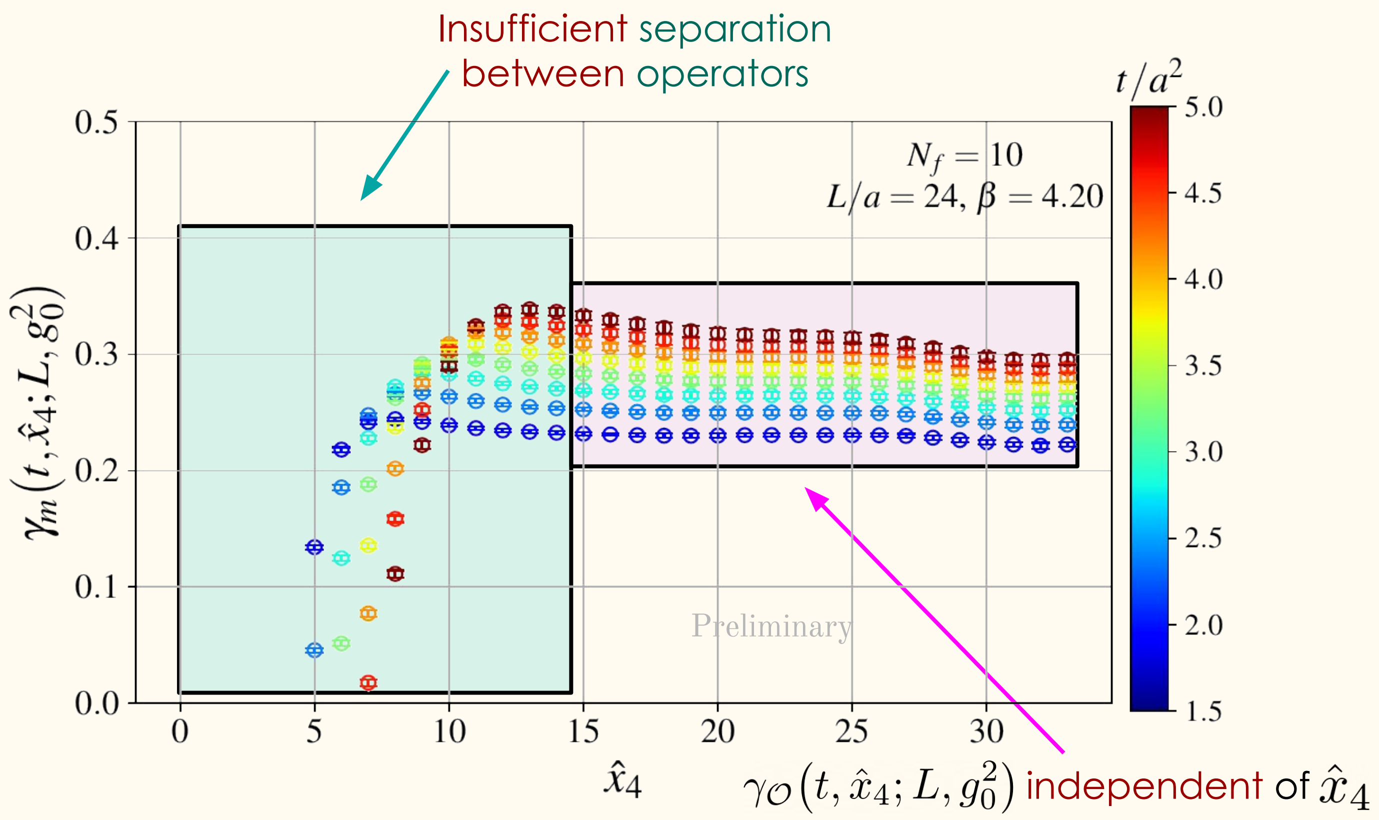
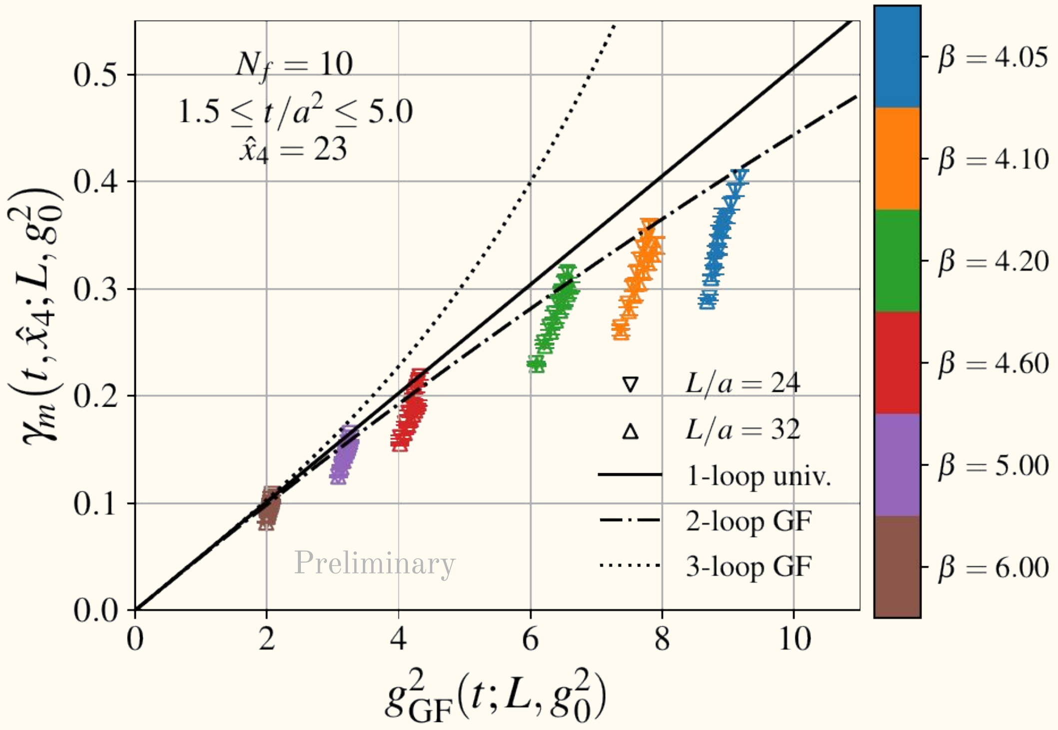


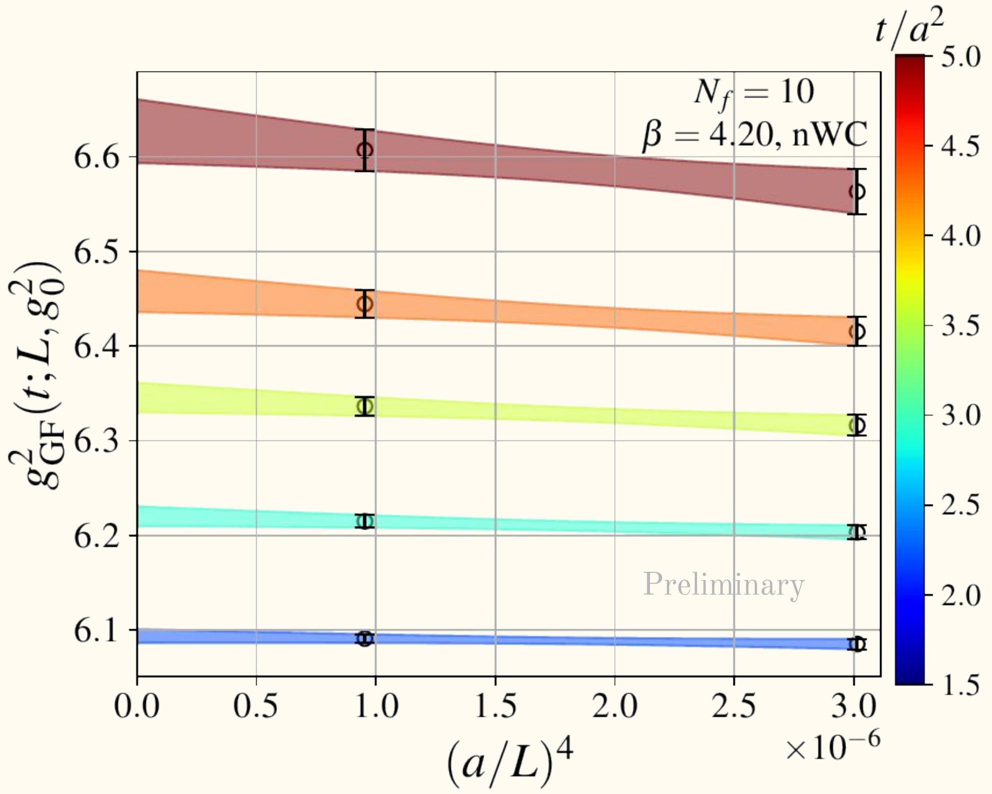
Anomalous dimension (no extrapolations)

[Artz, Harlander, Lange, Neumann, Prausa *JHEP* **06** (2019) 121]

- Finite volume effects appear to be small
- Each bare gauge coupling appears to approach the 1- and 2-loop GF curves from perturbation theory



Infinite volume limit (GF coupling)



Leading-order finite volume effects predicted from scaling of Yang-Mills energy density

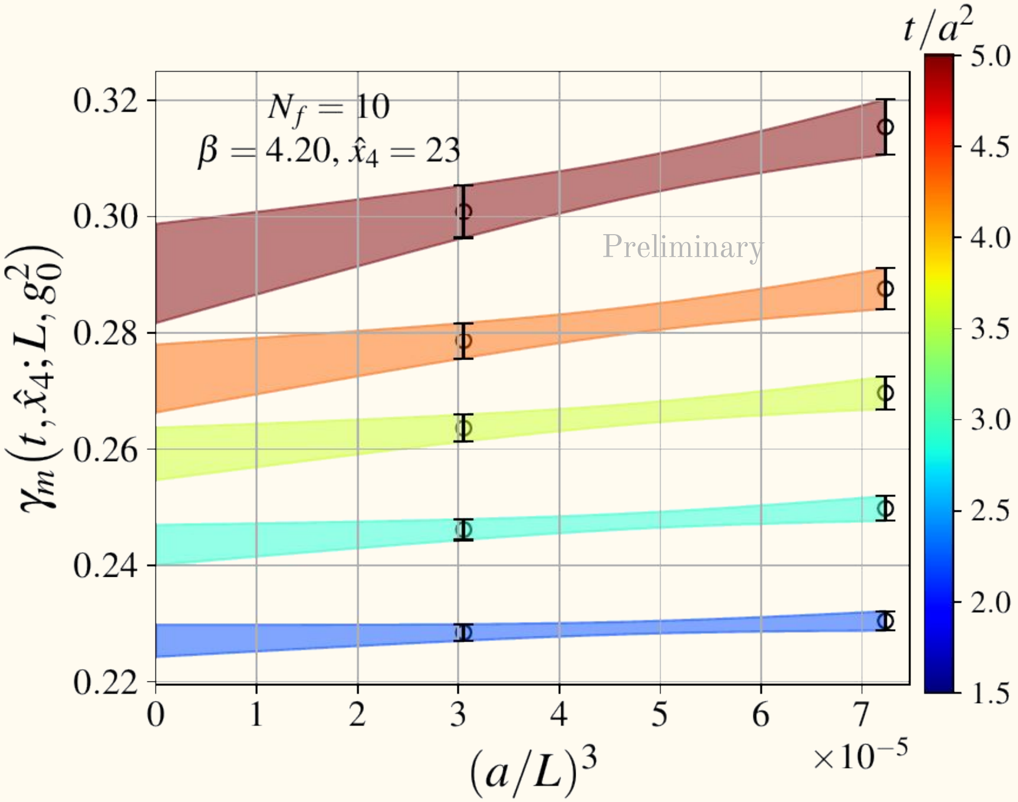
$$g^2_{\mathrm{GF}}(t; L, g_0^2) \approx g^2(t; g_0^2) + k\left(t^2/L^4\right)$$

Infinite volume limit (anomalous dimension)

Leading finite volume effects predicted by scaling of two-point function

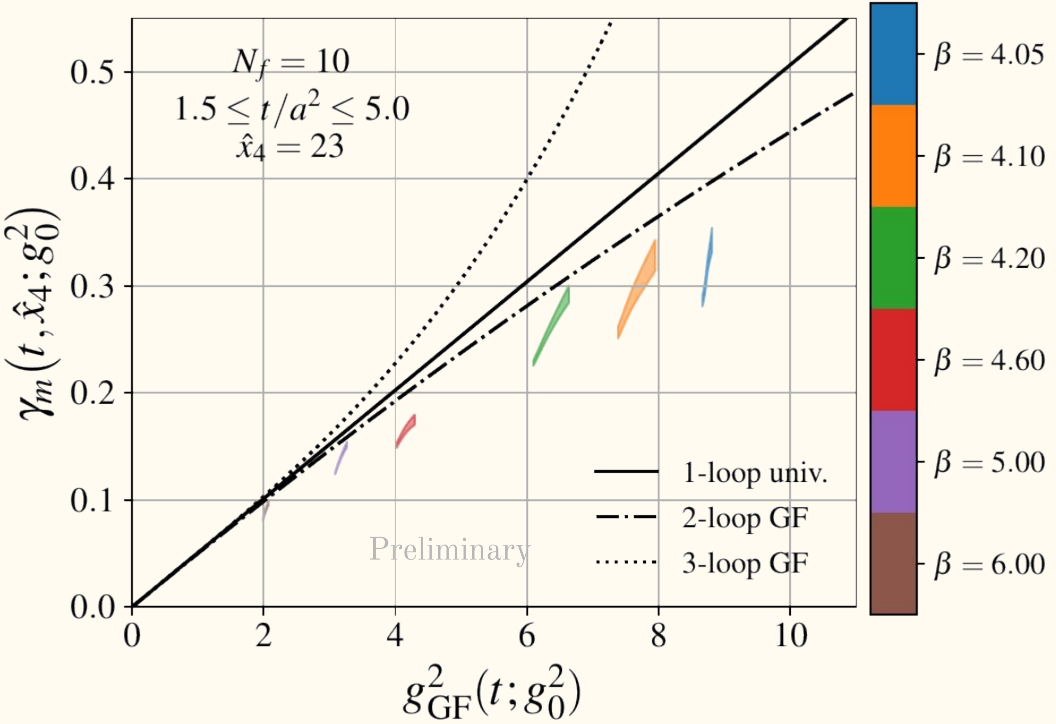
$$\gamma_{\mathcal{O}}(t, \hat{x}_4; L, g_0^2)$$
$$\approx \gamma_{\mathcal{O}}(t, \hat{x}_4; L, g_0^2) + \kappa (t/L^2)^{\delta_{\mathcal{O}}/2}$$

$$\delta_{\mathcal{O}} \approx 3 \quad \text{(meson)}$$
$$\approx 9/2 \quad \text{(baryon)}$$



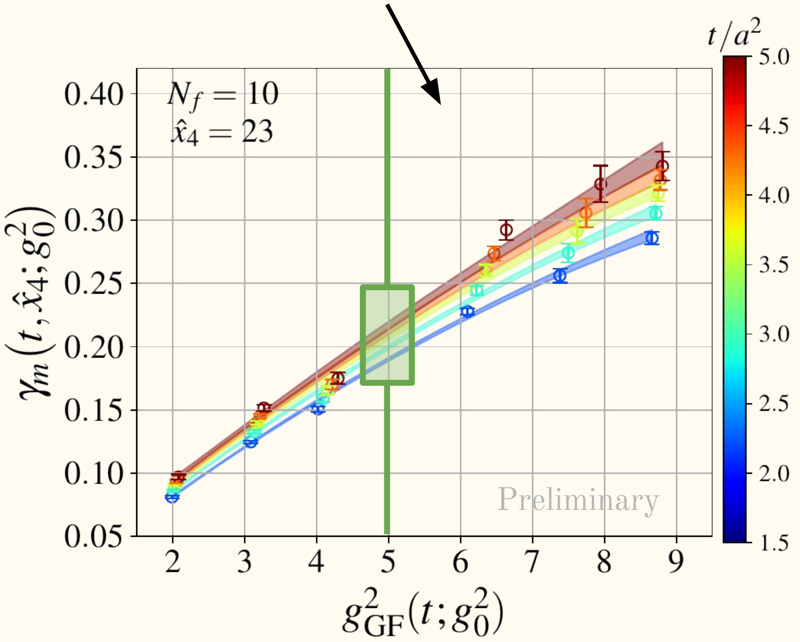
Infinite volume limit (final result)

- Bare gauge couplings do not overlap
  - Must interpolate between bare gauge coupling



Continuum limit (extrapolation)

Interpolation between bare gauge couplings at fixed gradient flow time



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$$\gamma_{\mathcal{O}}(t, \hat{x}_4; g_0^2) \approx \gamma_{\mathcal{O}}(t, \hat{x}_4) + c(a^2/t)$$

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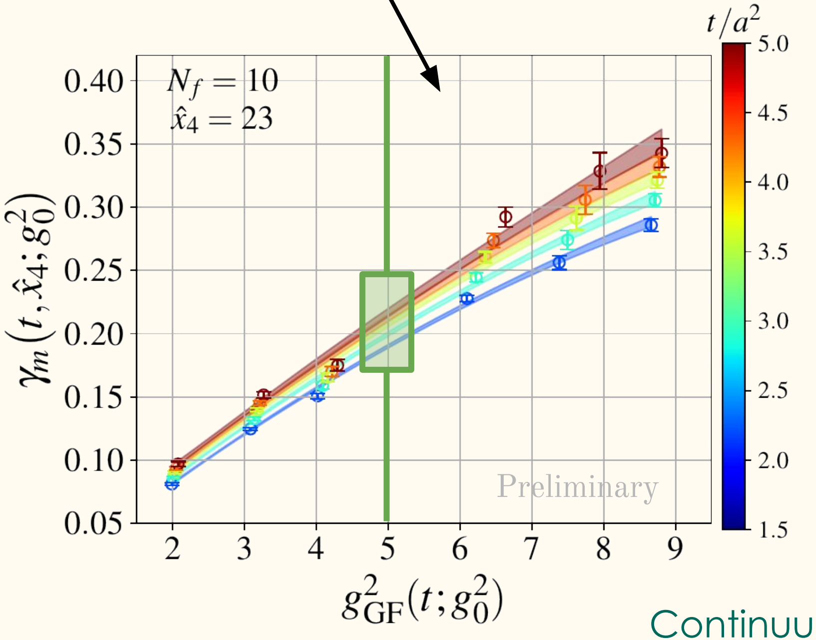
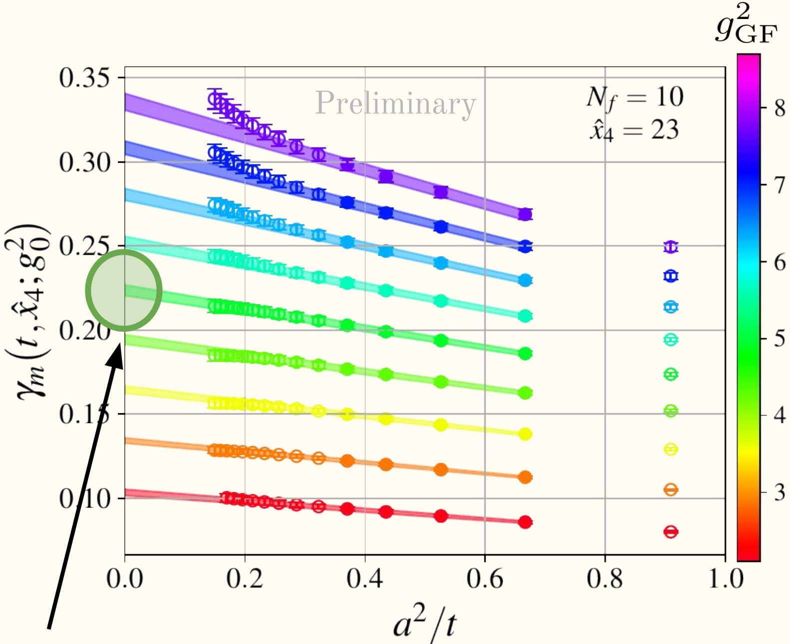
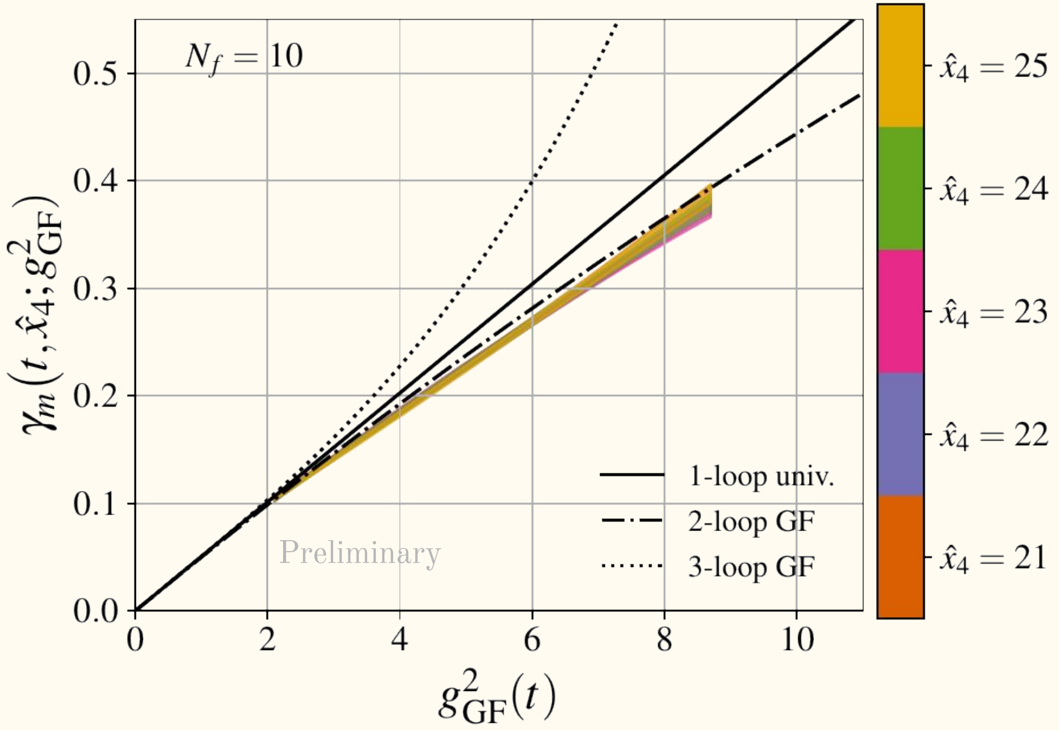




Continuum extrapolation at fixed gradient flow coupling

Continuum limit (final result)

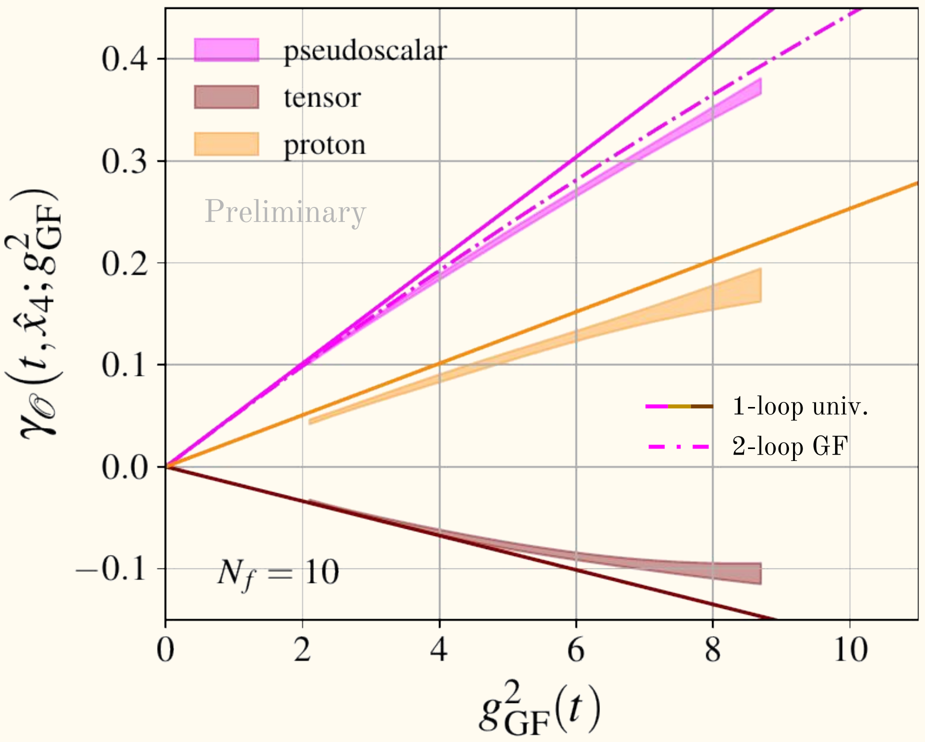
[Artz, Harlander, Lange, Neumann, Prausa *JHEP* **06** (2019) 121]



Continuum mass anomalous dimension appears to closely follow 2-loop GF

Conclusions and future work

- We have also calculated proton and tensor anomalous dimensions



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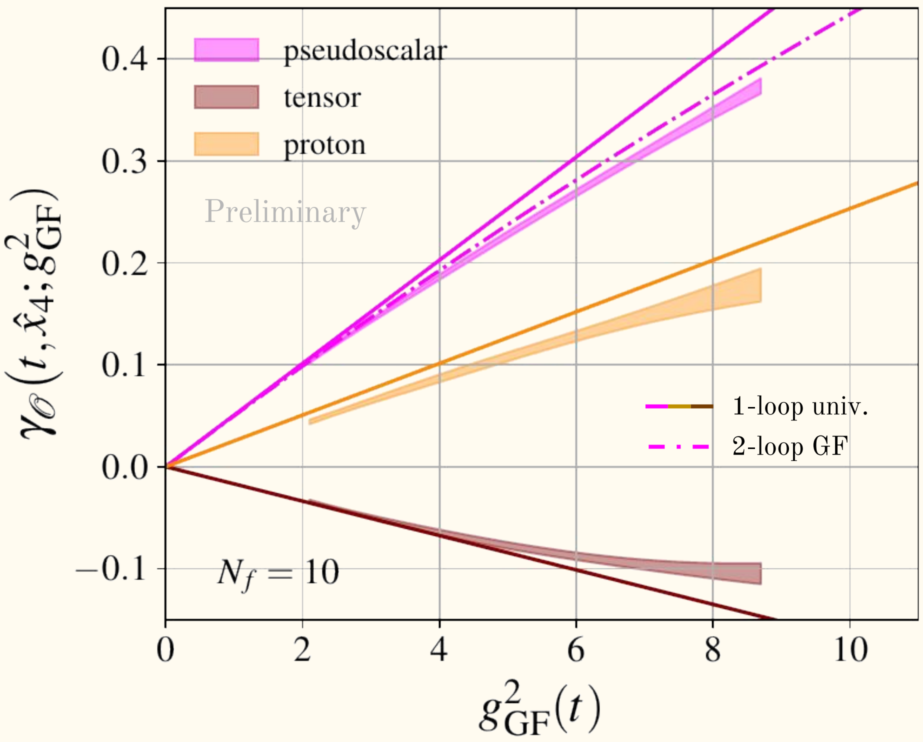
- Gradient flow anomalous dimension closely follow perturbative curves over our current range of couplings
  - No evidence for non-perturbative enhancement over our current range of couplings



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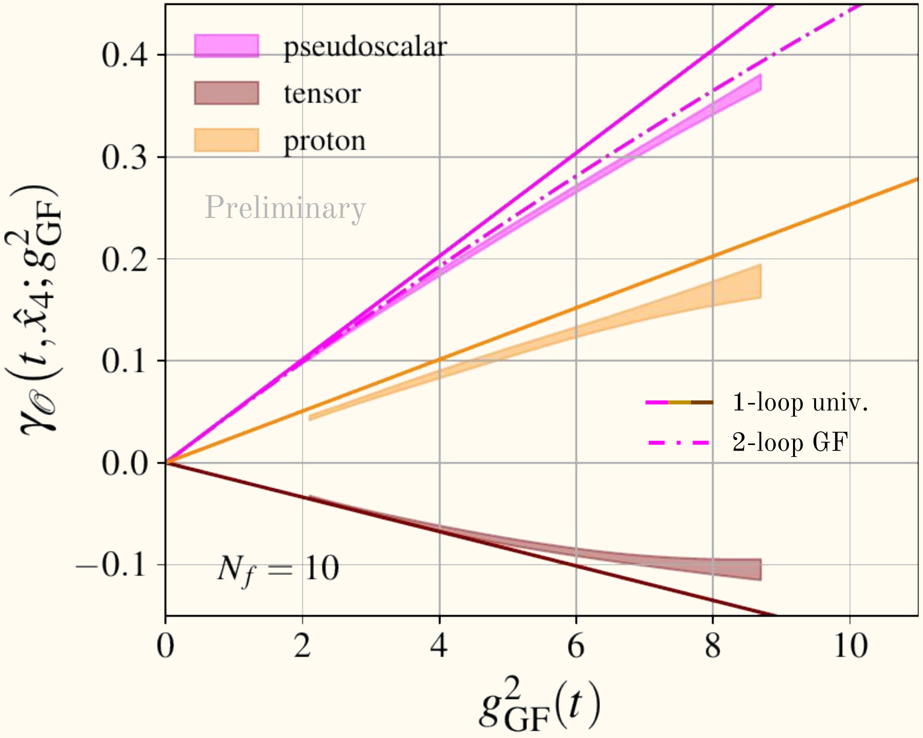

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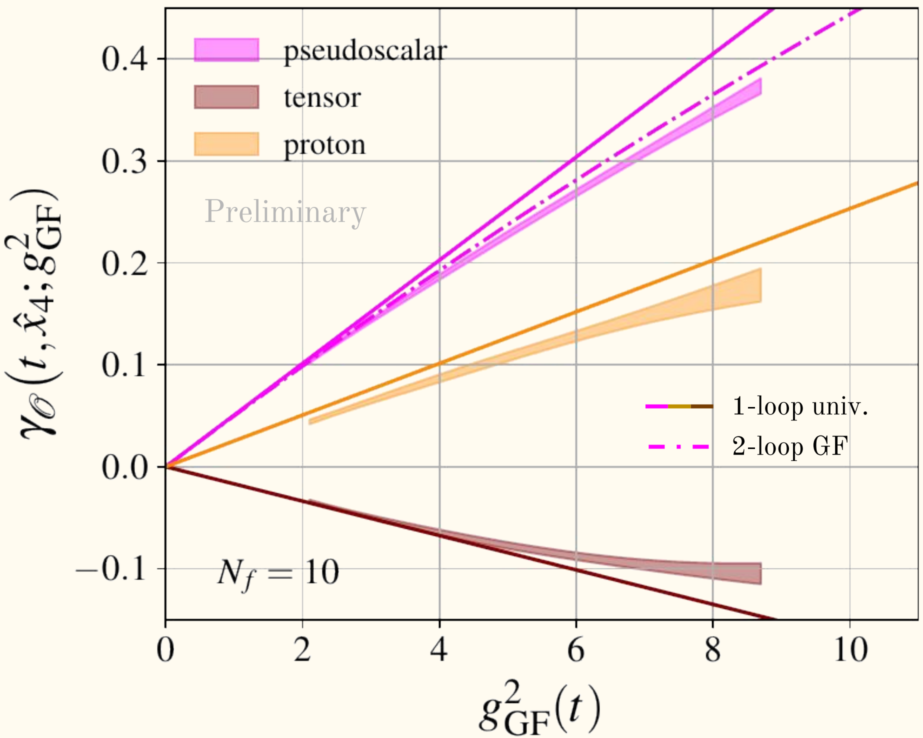
- We are currently running one more bare gauge coupling at $\beta = 4.03$ for L/a = 32



Acknowledgements

U. Colorado: RMACC Summit

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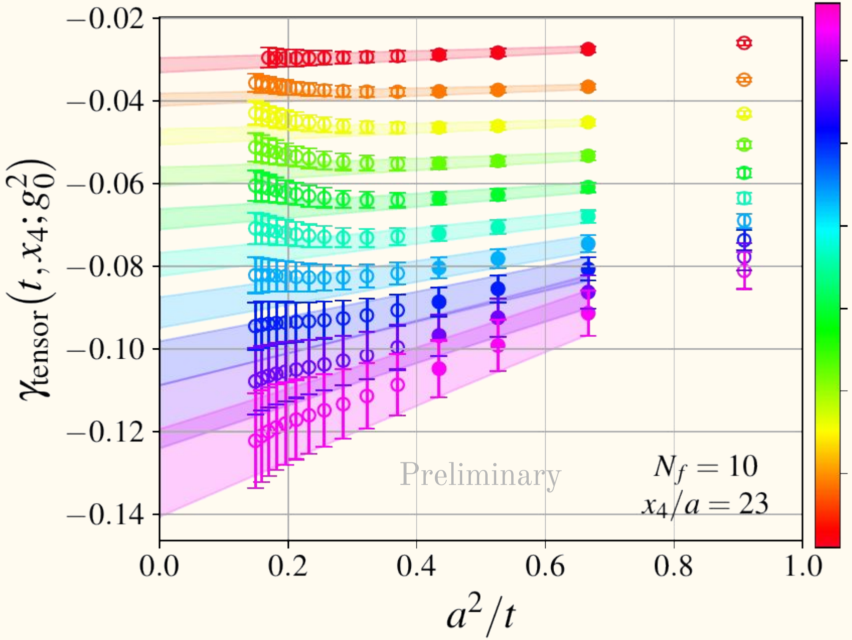
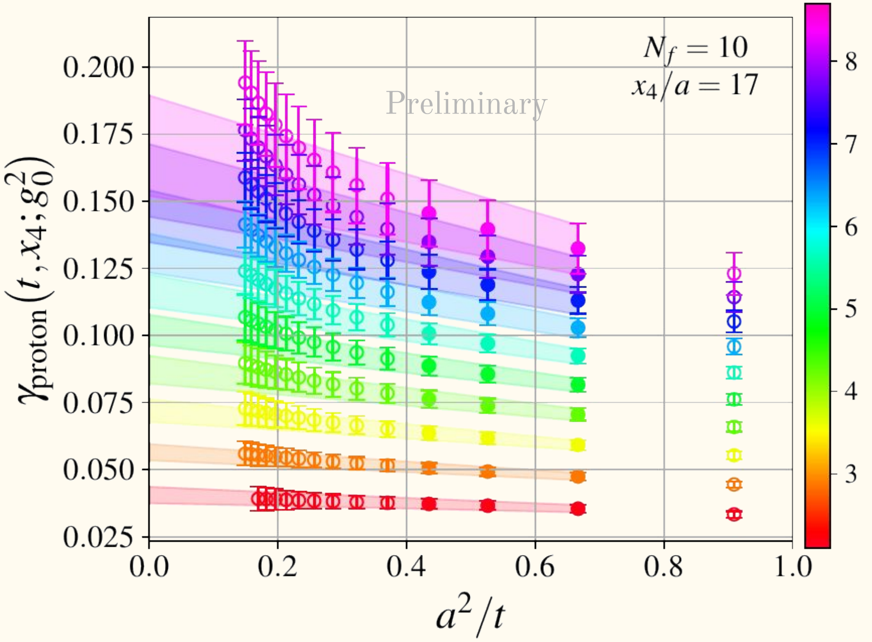
U. Siegen: OMNI

NSF GRFP

LSD Collaboration

Continuum extrapolation of tensor and proton





Tensor and proton anomalous dimension

Starting to converge on
1-loop