Spectroscopy of *Sp*(4) gauge theory with n_f=3 antisymmetric fermions

Jong-Wan Lee
(Pusan National University)

In collaboration with E. Bennett, D. K. Hong, H. Hsiao, C.-J. D. Lin, B. Lucini, M. Piai and D. Vadacchino

Lattice 2022 @ Bonn, Germany
August 10, 2022

Global symmetry and pNGBs

- Consider $Sp(4)$ gauge group + 3 two-index antisymmetric Dirac flavors

- Assumed that the global symmetry is broken explicitly by fermion mass and/or spontaneously by the fermions condensate

$$SU(6) \longrightarrow SO(6)$$

- A large coset: 20 pseudo Nambu Goldstone Bosons (pNGBs)

- A natural subgroup of $SU(6)$ is $SU(3)_L \times SU(3)_R$, where the diagonal component can be embedded in the unbroken subgroup, $SU(3)_D \subset SO(6)$

Motivation

- Relevant to pheno. model buildings for BSM based on $SU(6)/SO(6)$ coset

  – Composite Higgs (CH) & top partial compositeness

G. Ferretti & T. Karataev, arXiv:1312:5330;
J. Bernard, T. Gherghetta & T. S. Ray, arXiv:1311.6562

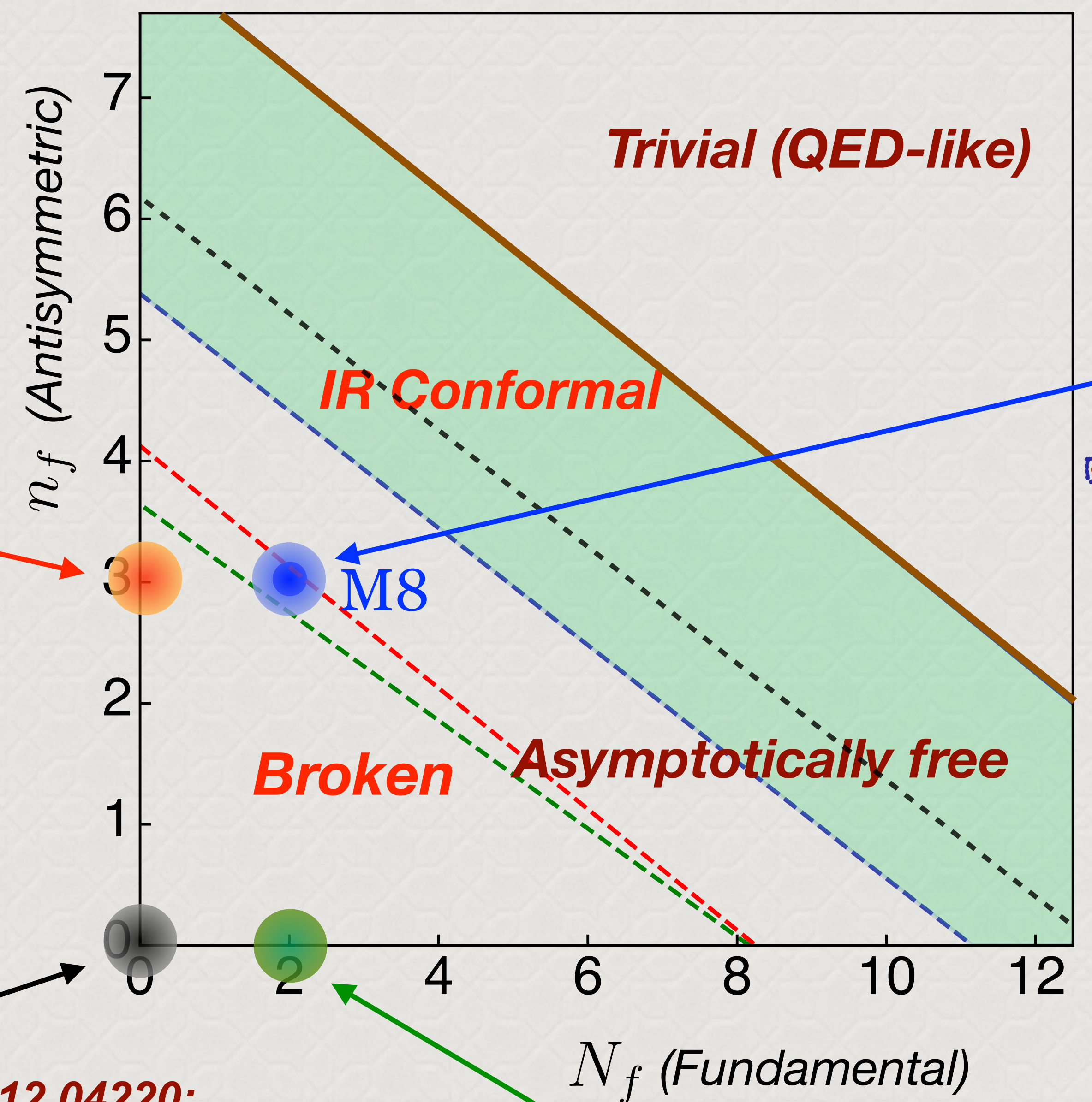
Coset	HC	ψ	χ	$-q_\chi/q_\psi$	Baryon	Name	Lattice
$\frac{\mathrm{SU}(4)}{\mathrm{Sp}(4)} \times \frac{\mathrm{SU}(6)}{\mathrm{SO}(6)}$	Sp(4)	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	1/3	$\psi\psi\chi$	M8	$\surd$
	SO(11)	$4 \times \mathbf{Sp}$	$6 \times \mathbf{F}$	8/3		M9	

  – CH & Dark matter - an extension of the minimal $SU(5)/SO(5)$ CH

	$SU(2)_L \times U(1)_Y$	$SU(2)_L \times SU(2)_R$	$\mathbb{Z}_2$
H_1	$(2, \pm 1/2)$	$(2, 2)$	$+$
H_2	$(2, \pm 1/2)$	$(2, 2)$	$-$
Λ	$(3, \pm 1)$	$(3, 3)$	$+$
φ	$(3, 0)$		
η_1	$(1, 0)$	$(1, 1)$	$+$
η_2	$(1, 0)$	$(1, 1)$	$-$
η_3	$(1, 0)$	$(1, 1)$	$+$

G. Cacciapaglia, H. Cai, A. Deandrea, A. Kushwaha, arXiv:1904:09301;
H. Cai, G. Cacciapaglia, arXiv:2007.04338

Theory space of $Sp(4)$



J.-W. Lee et al, LATTICE 2018

E. Bennett et al, LATTICE 2021

- ☑ Preliminary studies on the bulk phase structure and FV effects

E. Bennett et al (2022), arXiv:2202.05516

- ☑ Exploratory studies of model M8: mesons & chimera baryons

B. Kim, D. Hong & JWL (2020), arXiv:2001.02690

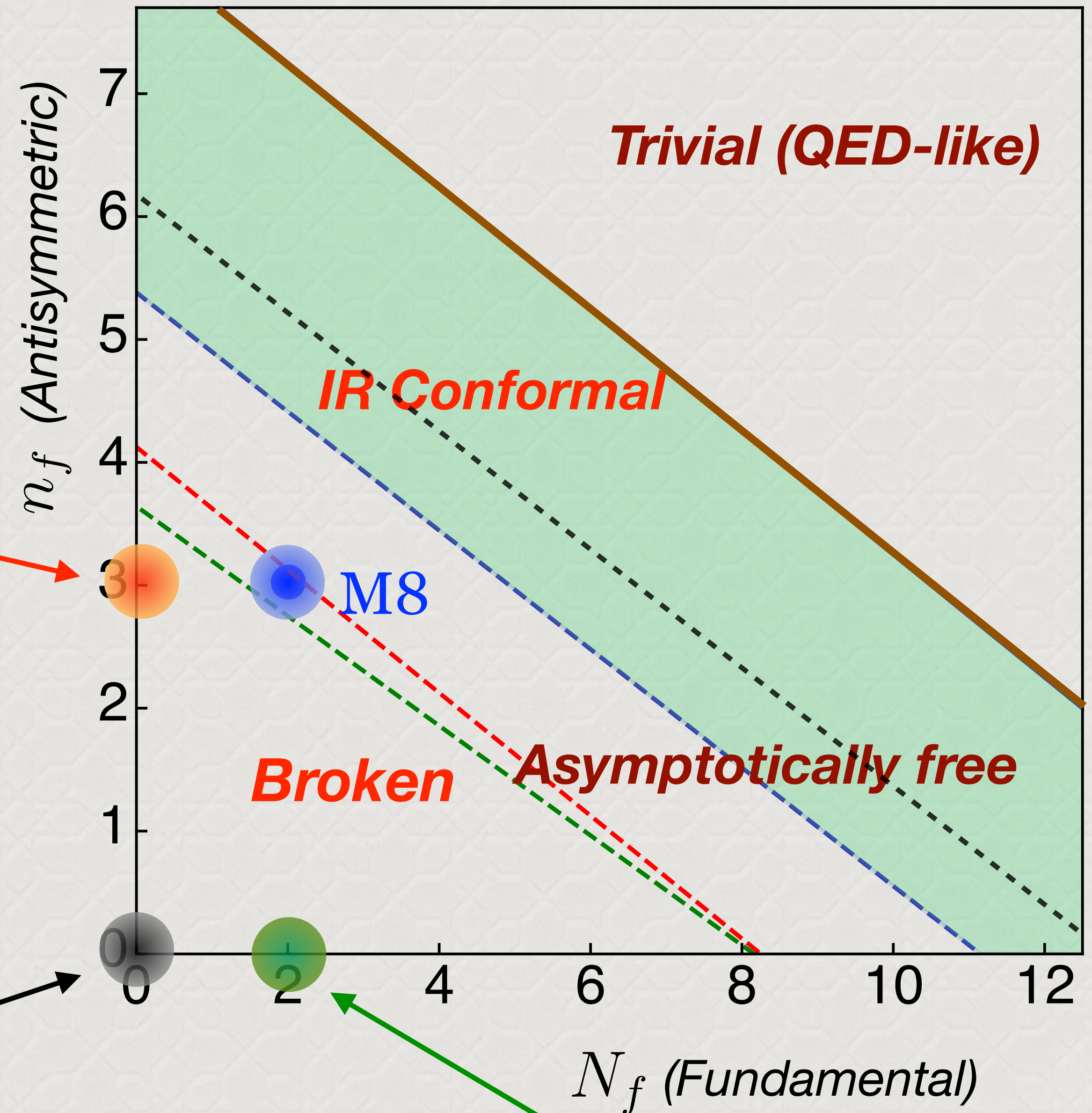
E. Bennett et al, arXiv:1712.04220; (2017, 2019) arXiv:1912.06505.

- ☑ Glueballs & quenched meson spectrum

E. Bennett et al (2019), arXiv:1909.12662

- ☑ Meson spectrum in N_f=2 dynamical simulations

Theory space of $Sp(4)$



This talk

- ☑ Meson spectrum in n_f=3 dynamical simulations

B. Kim, D. Hong & JWL (2020), arXiv:2001.02690

D. Vadacchino, Tues. @ 17:10

H. Hsiao, Wed. @ 14:40

F. Zierler, Mon. @ 18:10

Lattice action and simulation details

- Lattice formulation with the standard Wilson gauge & fermion actions

$$S \equiv \beta \sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{4} \mathrm{Re}\,\mathrm{Tr}\, U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)\right) + a^4 \sum_x \overline{\Psi}_k(x) D^{AS} \Psi_k(x)$$

with $\beta = 4N/g^2$

- The Wilson-Dirac operator is given by

$$D^{AS}\Psi_k(x) \equiv (4/a + m_0^{as})\Psi_k(x) - \frac{1}{2a}\sum_\mu \left\{(1-\gamma_\mu)U_\mu^{AS}(x)\Psi_k(x+\hat{\mu}) + (1+\gamma_\mu)U_\mu^{AS}(x-\hat{\mu})\Psi_k(x-\hat{\mu})\right\}$$

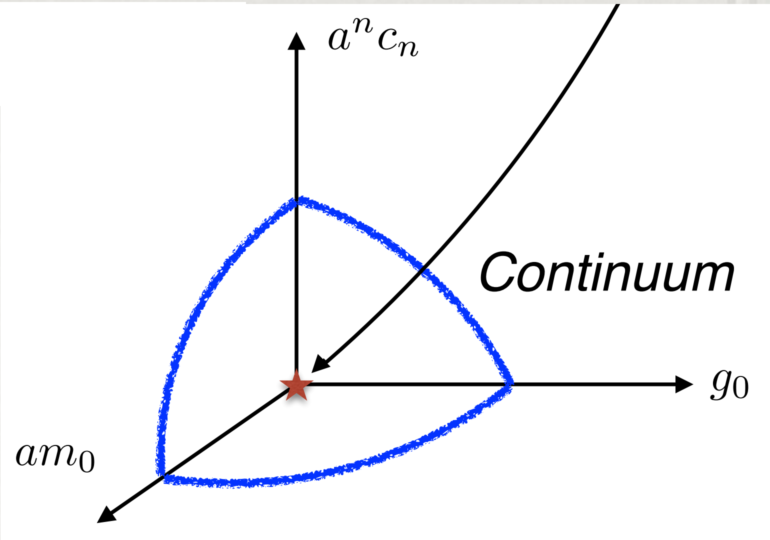
where $\left(U_\mu^{AS}\right)_{(ab)(cd)}(x) \equiv \mathrm{Tr}\left[(e_{AS}^{(ab)})^\dagger U_\mu(x) e_{AS}^{(cd)} U_\mu^{\mathrm{T}}(x)\right], \quad \text{with } a<b,\ c<d.$

$$U_\mu(x) = U_\mu^F(x) \in Sp(4)$$

- Simulations using (R)HMC algorithms implemented in the HiRep code

Del Debbio, Patella & Pica (2008)

Lattice setup



- $Sp(4)$ theory with fermions: Weak and strong coupling regimes are separated by 1st order phase transition.

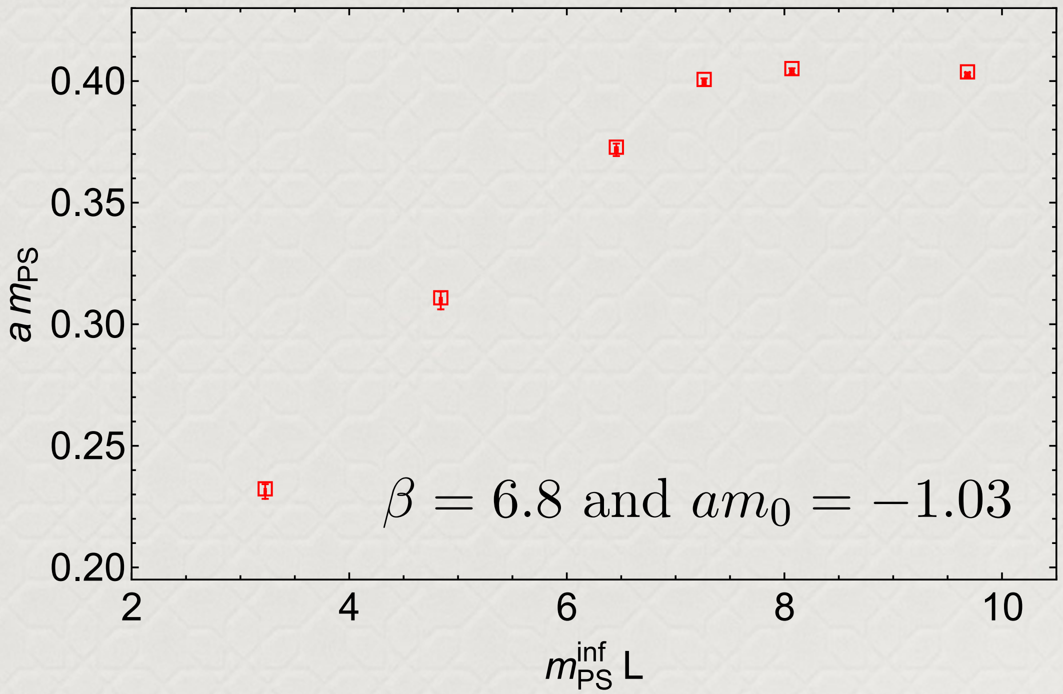
$$n_f\text{=}3 \text{ AS } Sp(4): \quad \beta \gtrsim 6.5$$

J.-W. Lee et al, Lattice (2018) 192

- Finite volume corrections are statistically negligible if $m_{\mathrm{PS}}L \gtrsim 7.5$.

E. Bennett et al, Lattice (2021) 274

  - negative FV contribution can be understood from the NLO corrections in the low-energy chPT. **Bijnens & Lu (2009)**



- Scale setting: Gradient flow method **Luscher (2010)** **Luscher & Wiese (2011)**

$$\hat{a} \equiv a/w_0 \qquad \hat{m} \equiv m^{\mathrm{lat}} w_0^{\mathrm{lat}} = m w_0$$

Borsarnyi et al (2012), arXiv: 1203.4469

Interpolating operators and measurements

- Flavor non-singlet spin 0 and 1 mesons, i.e. $i \neq j$

Label M	Interpolating operator $\mathcal{O}_{\mathrm{M}}$	Mesons in QCD	J^P	$SO(6)$
PS	$\overline{\Psi^i}\gamma_5\Psi^j$	π	0^-	20
S	$\overline{\Psi^i}\Psi^j$	a_0	0^+	20
V	$\overline{\Psi^i}\gamma_\mu\Psi^j$	ρ	1^-	15
T	$\overline{\Psi^i}\gamma_0\gamma_\mu\Psi^j$	ρ	1^-	15
AV	$\overline{\Psi^i}\gamma_5\gamma_\mu\Psi^j$	a_1	1^+	20
AT	$\overline{\Psi^i}\gamma_5\gamma_0\gamma_\mu\Psi^j$	b_1	1^+	15

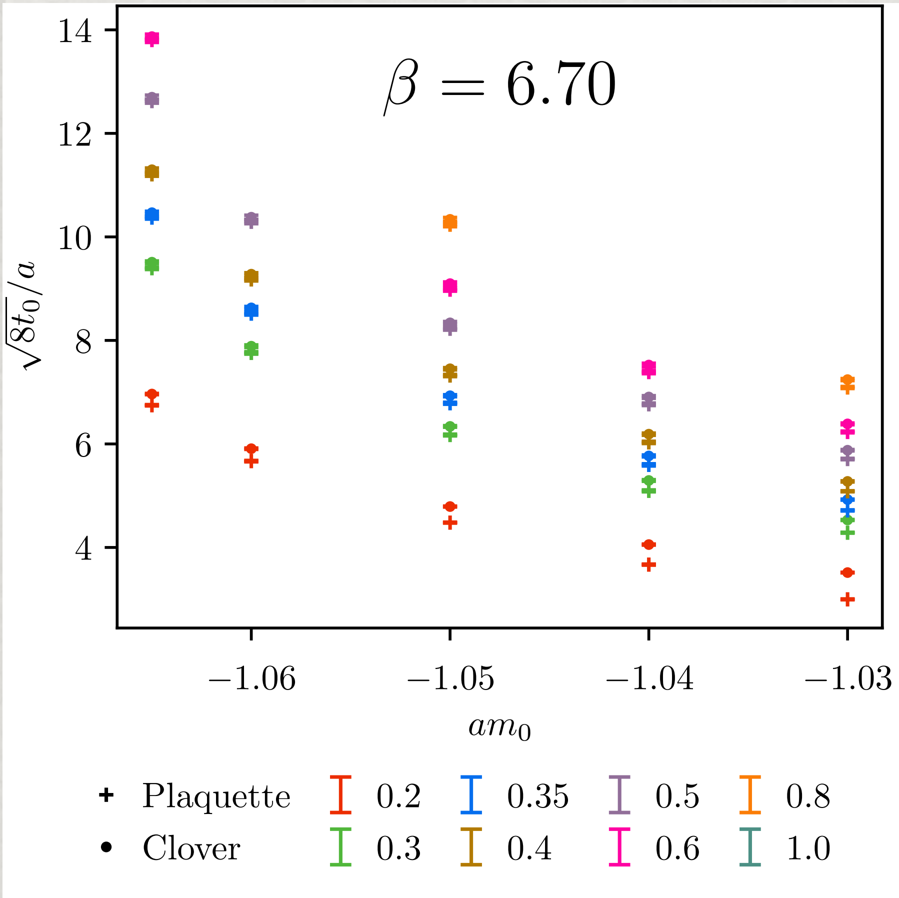
- We use the Z_2xZ_2 single time slice stochastic wall sources with hit number 3

- We extract the mass of the first excited state of vector meson by solving the generalized eigenvalue problem (GEVP) for correlation functions built from two independent interpolating operators.

List of ensembles

Ensemble	Volume	β	am_0	N_{configs}	$\langle P \rangle$	w_0/a
ASB1M1	48×18^3	6.65	-1.05	128	0.579862(30)	1.6268(42)
ASB1M2	48×18^3	6.65	-1.063	135	0.585145(32)	2.142(8)
ASB1M3	48×24^3	6.65	-1.07	137	0.587787(17)	2.603(8)
ASB1M4	48×28^3	6.65	-1.075	170	0.589623(11)	3.074(11)
ASB1M5	48×32^3	6.65	-1.08	120	0.591450(13)	3.636(24)
ASB2M1	54×16^3	6.7	-1.0	90	0.570927(46)	1.1366(17)
ASB2M2	48×16^3	6.7	-1.02	200	0.578740(25)	1.4274(21)
ASB2M3	48×16^3	6.7	-1.03	120	0.582272(30)	1.6251(40)
ASB2M4	48×18^3	6.7	-1.04	100	0.585693(30)	1.924(8)
ASB2M5	48×24^3	6.7	-1.045	120	0.587367(22)	2.122(5)
ASB2M6	48×24^3	6.7	-1.05	110	0.588953(21)	2.342(8)
ASB2M7	48×24^3	6.7	-1.055	180	0.590599(15)	2.650(9)
ASB2M8	48×24^3	6.7	-1.06	180	0.592155(13)	2.928(12)
ASB2M9	54×28^3	6.7	-1.063	110	0.593154(13)	3.435(17)
ASB2M10	54×32^3	6.7	-1.065	150	0.593758(9)	3.626(14)
ASB2M11	54×32^3	6.7	-1.067	180	0.594392(8)	3.704(8)
ASB2M12	54×36^3	6.7	-1.069	120	0.595060(9)	4.320(12)
ASB3M1	54×18^3	6.75	-1.03	180	0.590431(21)	2.205(7)
ASB3M2	54×24^3	6.75	-1.041	120	0.593531(15)	2.642(9)
ASB3M3	54×24^3	6.75	-1.046	180	0.595008(12)	3.100(12)
ASB3M4	54×28^3	6.75	-1.051	196	0.596339(10)	3.607(15)
ASB3M5	54×32^3	6.75	-1.055	225	0.597567(8)	4.066(13)
ASB4M1	48×16^3	6.8	-1.0	170	0.589860(24)	1.889(6)
ASB4M2	54×16^3	6.8	-1.02	165	0.597306(19)	2.456(14)
ASB4M3	54×24^3	6.8	-1.03	180	0.597270(13)	2.947(10)
ASB4M4	56×24^3	6.8	-1.035	275	0.598552(10)	3.367(11)
ASB4M5	54×32^3	6.8	-1.04	100	0.599829(10)	3.711(13)
ASB4M7	54×36^3	6.8	-1.046	72	0.601397(10)	4.520(20)

Numerical results: Gradient flow scale



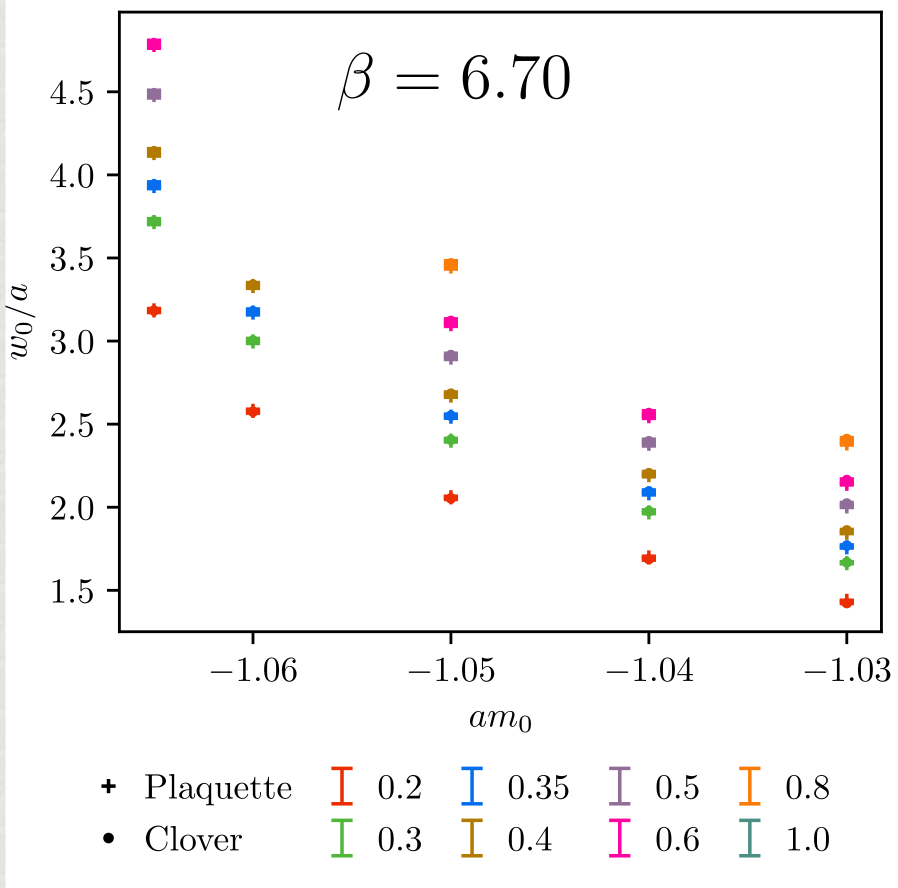
$$E(t) = -\frac{1}{2}\mathrm{Tr}(G_{\mu\nu}G_{\mu\nu})$$

$$k_\alpha t^2 \langle E(t) \rangle \equiv k_\alpha \mathcal{E}(t)$$

$$\mathcal{E}(t)|_{t=t_0} = \mathcal{E}_0$$

$$\mathcal{W}(t)|_{t=w_0^2} = \mathcal{W}_0$$

with $\mathcal{W}(t) \equiv t\dfrac{\mathrm{d}\mathcal{E}(t)}{\mathrm{d}t}$.

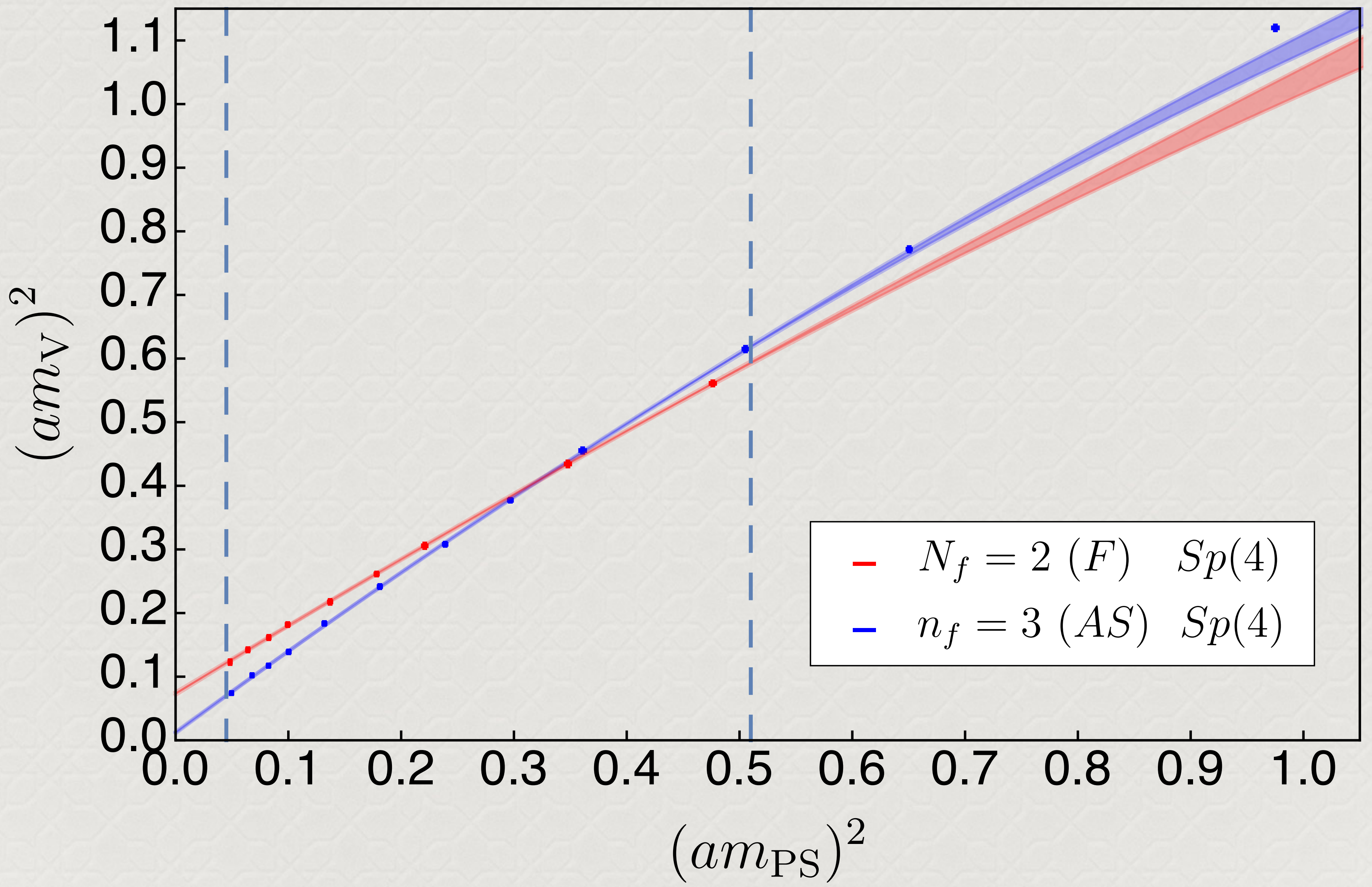


- w_0/a is less affected by the lattice artifacts (UV fluctuation)
- GF scales are showing significant mass dependence
- Reference scale is chosen according to a simple N_c scaling

E. Bennett et al (2022), arXiv:2205.09364

$\mathcal{W}_0(N_c) = c_w C_2(F)\,,\ c_e = 9/40$ *cf.* $\mathcal{W}_0 = 0.3$ for SU(3) $\boxed{\mathcal{W}_0 = 0.28 \text{ for Sp}(4)}$

How far from the massless limit?



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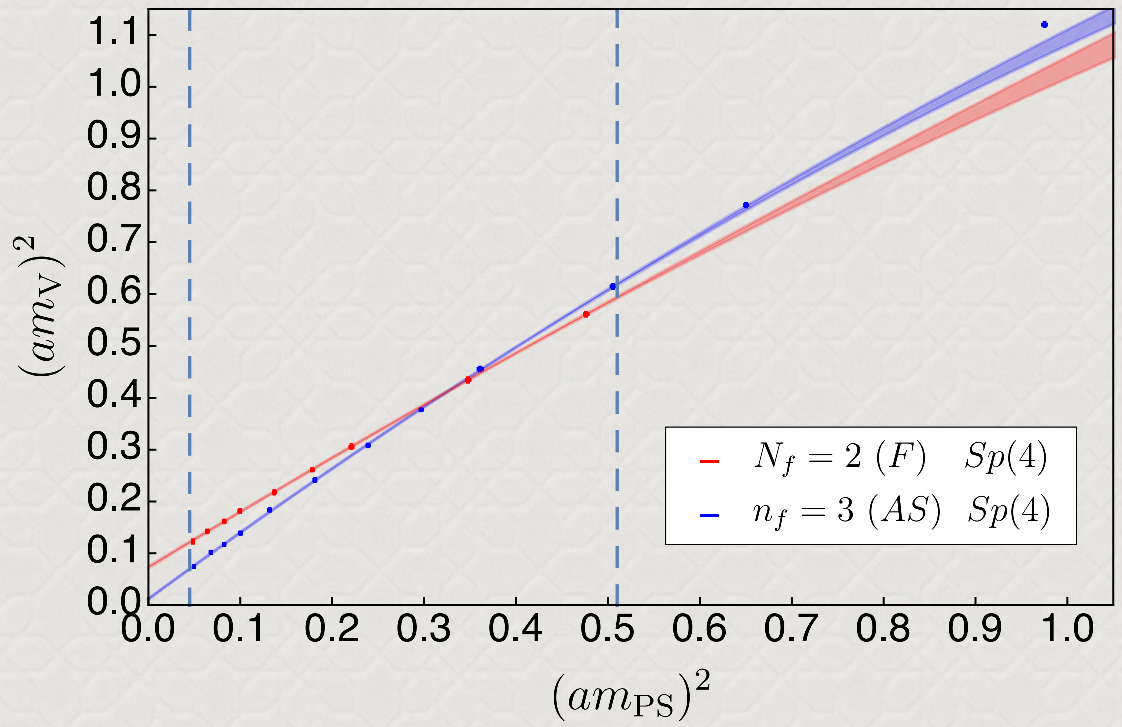
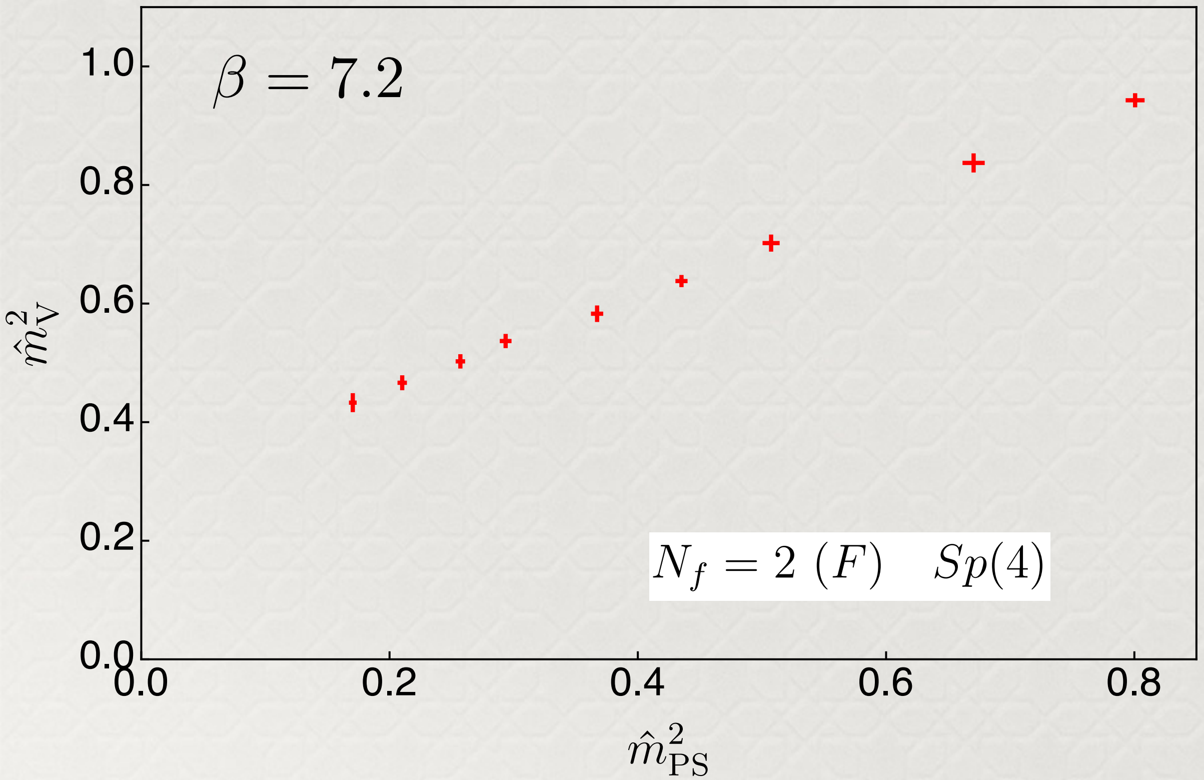
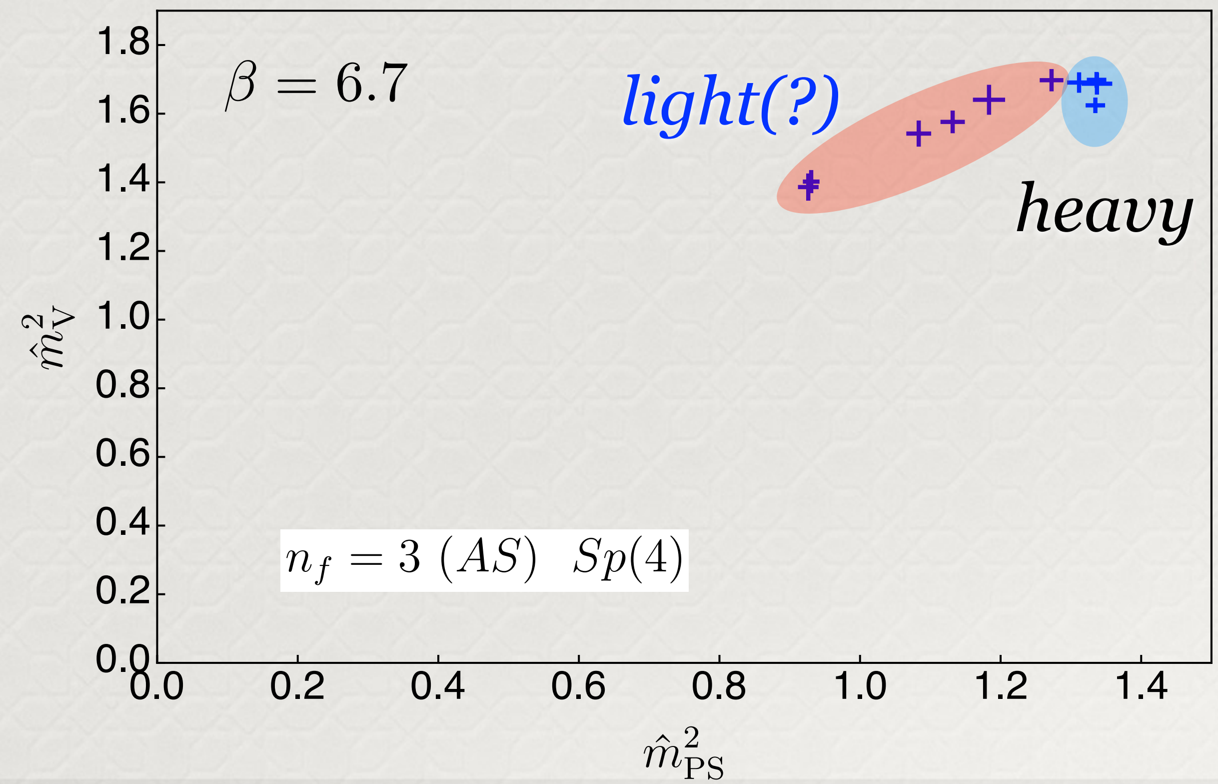
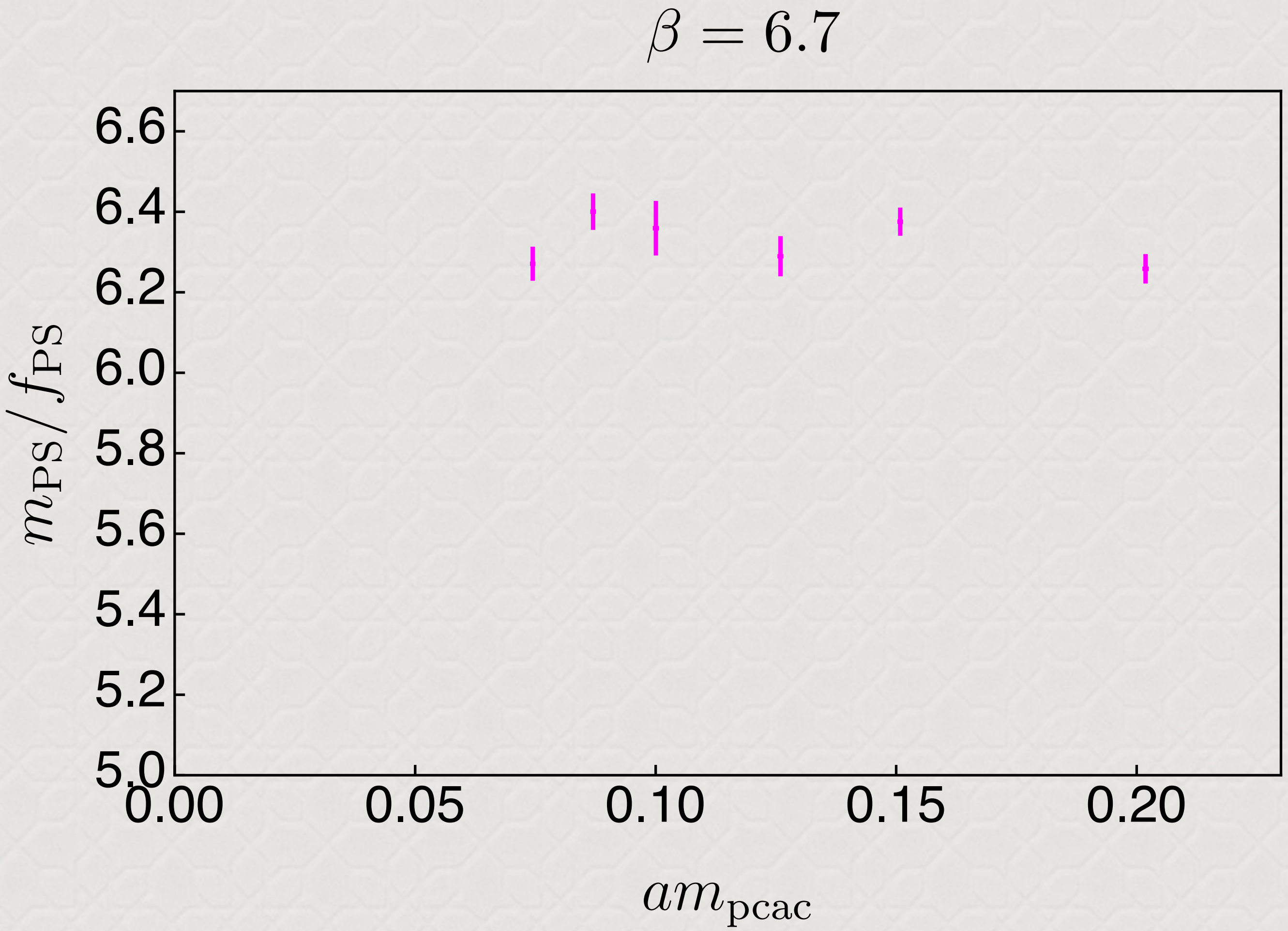




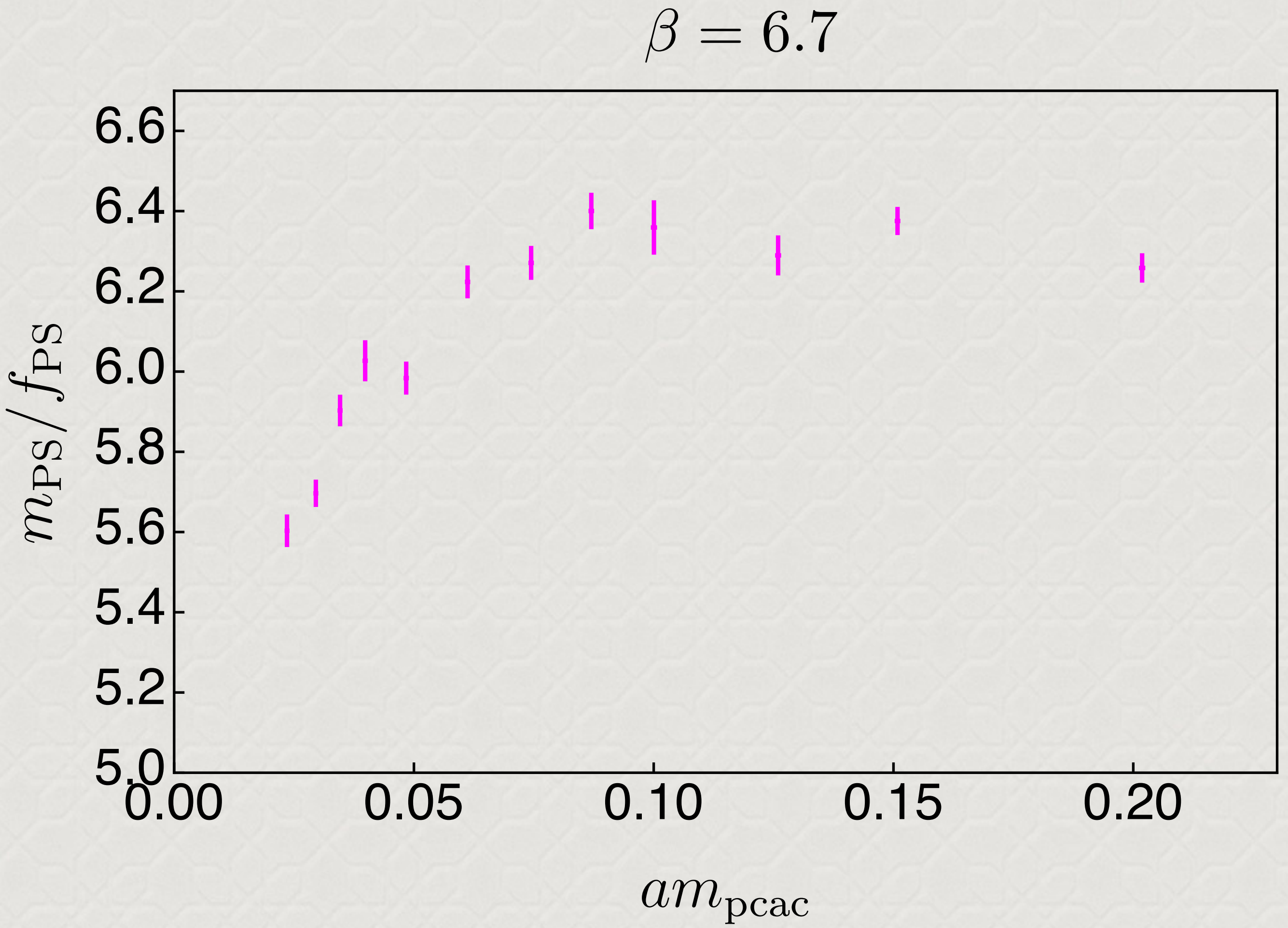


Conformal or chirally broken?



- A sign of conformality?

Conformal or chirally broken?



- A sign of conformality? No!
- indicates that our theory is indeed in the broken phase (as expected)

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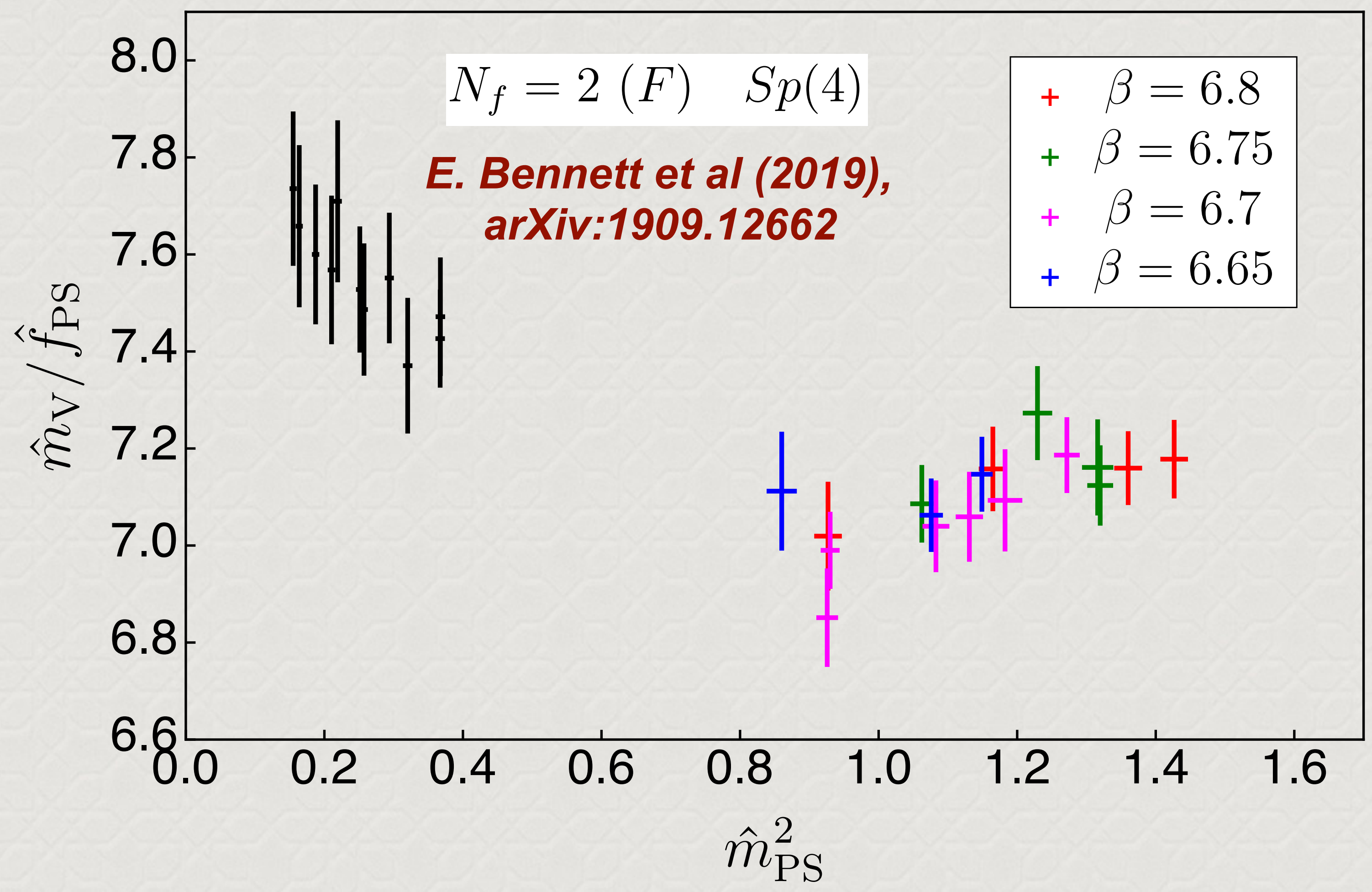
Vector meson mass in units of PS decay constant

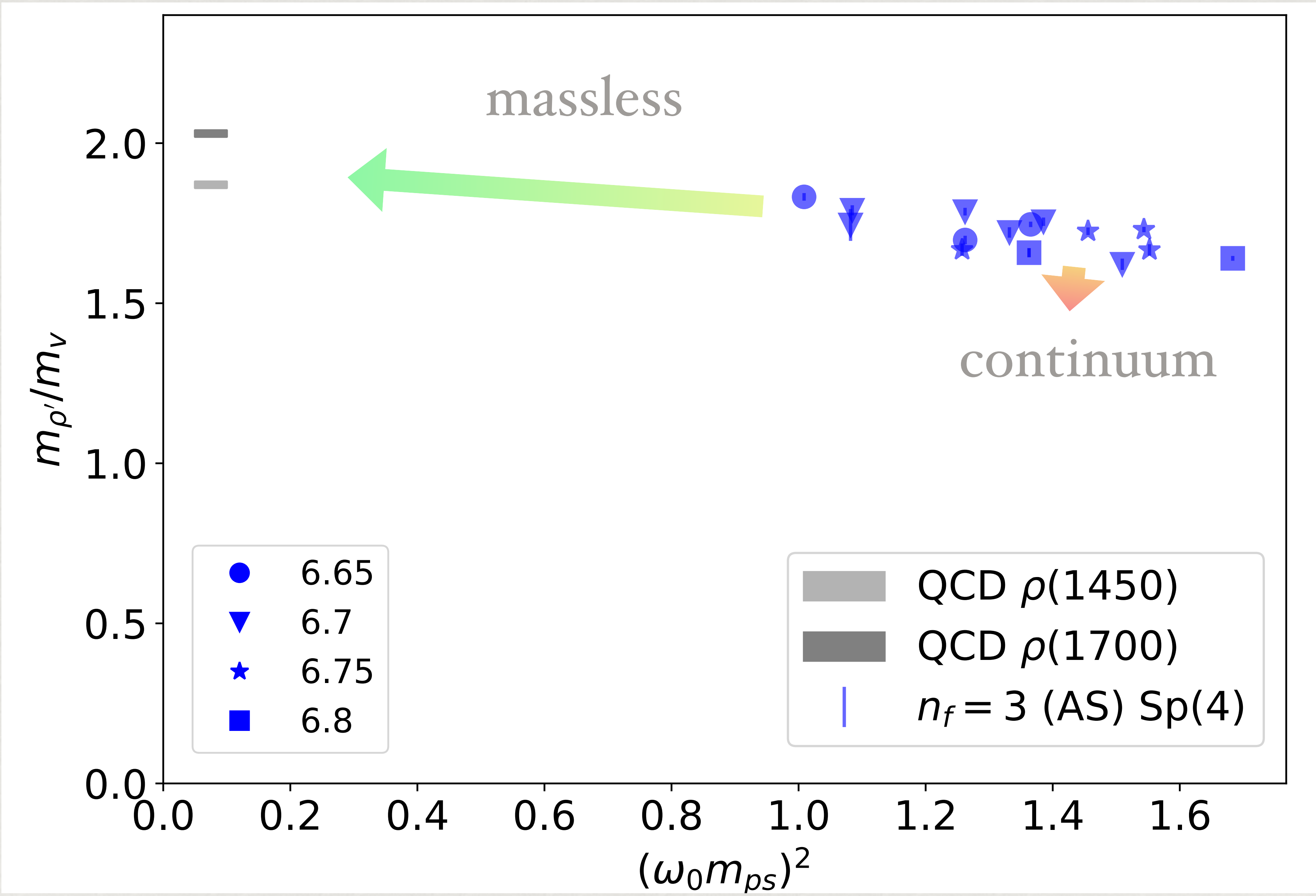


- related to the coupling g_{VPP} through KSRF relation: $$g_{VPP} = \frac{m_{\rm V}}{\sqrt{2} f_{\rm PS}}$$

  Kowarabayashi & Suzuki (1966) *Riazuddin & Fayyazuddin (1966)*

- In the massless limit, it is approaching the value smaller than the one in N_f=2 (F) $Sp(4)$ theory

Results: first excited state of vector meson



Massless and continuum extrapolation

- Despite of a long massless extrapolation, we use the following ansatzs linear in $\hat{m}_{\rm PS}^2$ and $\hat{a}$ to fit the data for ensembles in the linear regime.

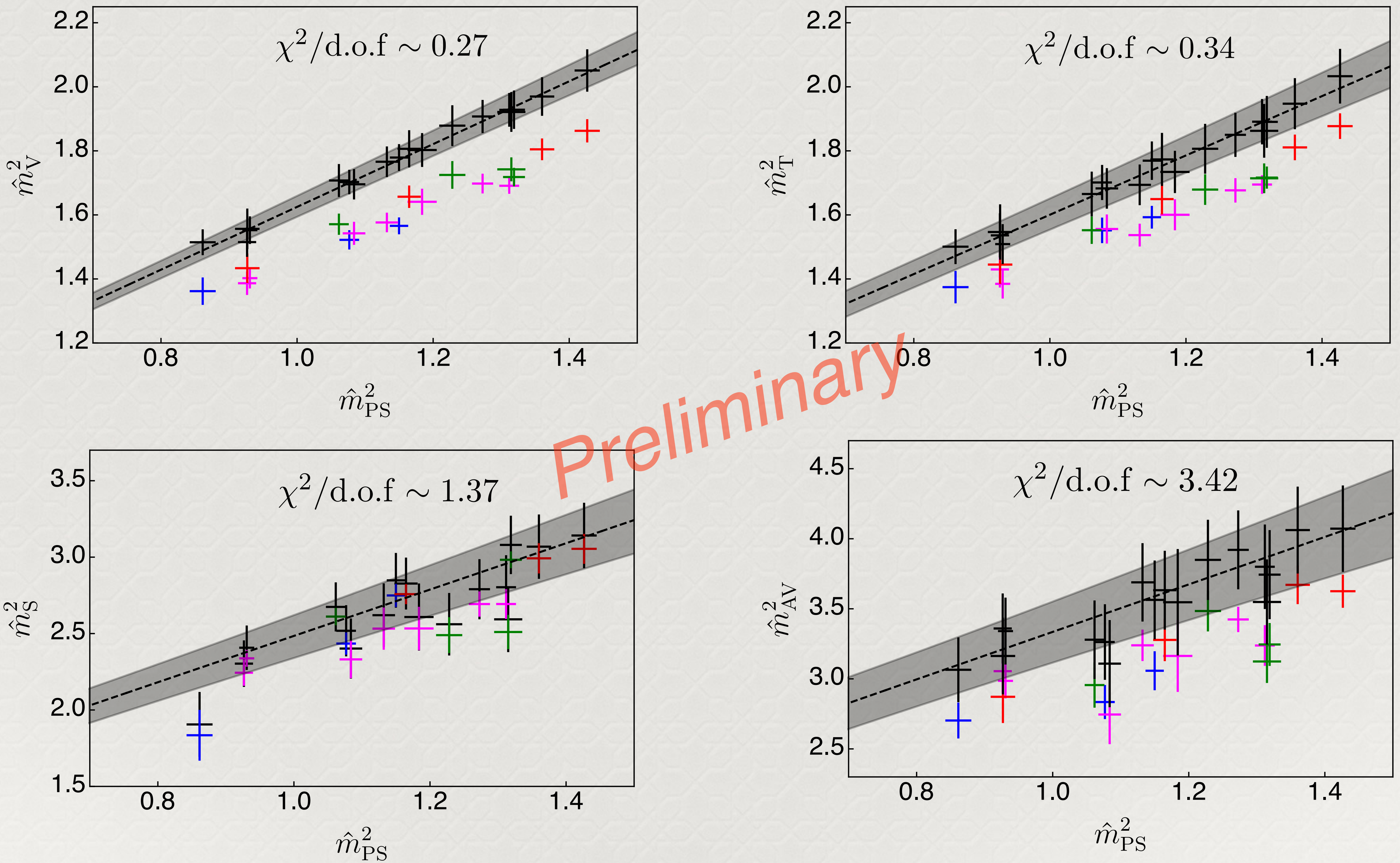
$$\hat{m}_M^{2,\rm NLO} = \hat{m}_M^{2,\chi}\left(1+L^0_{m,M}\hat{m}^2_{\rm PS}\right)+W^0_{m,M}\hat{a}$$

$$\hat{f}_M^{2,\rm NLO} = \hat{f}_M^{2,\chi}\left(1+L^0_{f,M}\hat{m}^2_{\rm PS}\right)+W^0_{f,M}\hat{a}$$

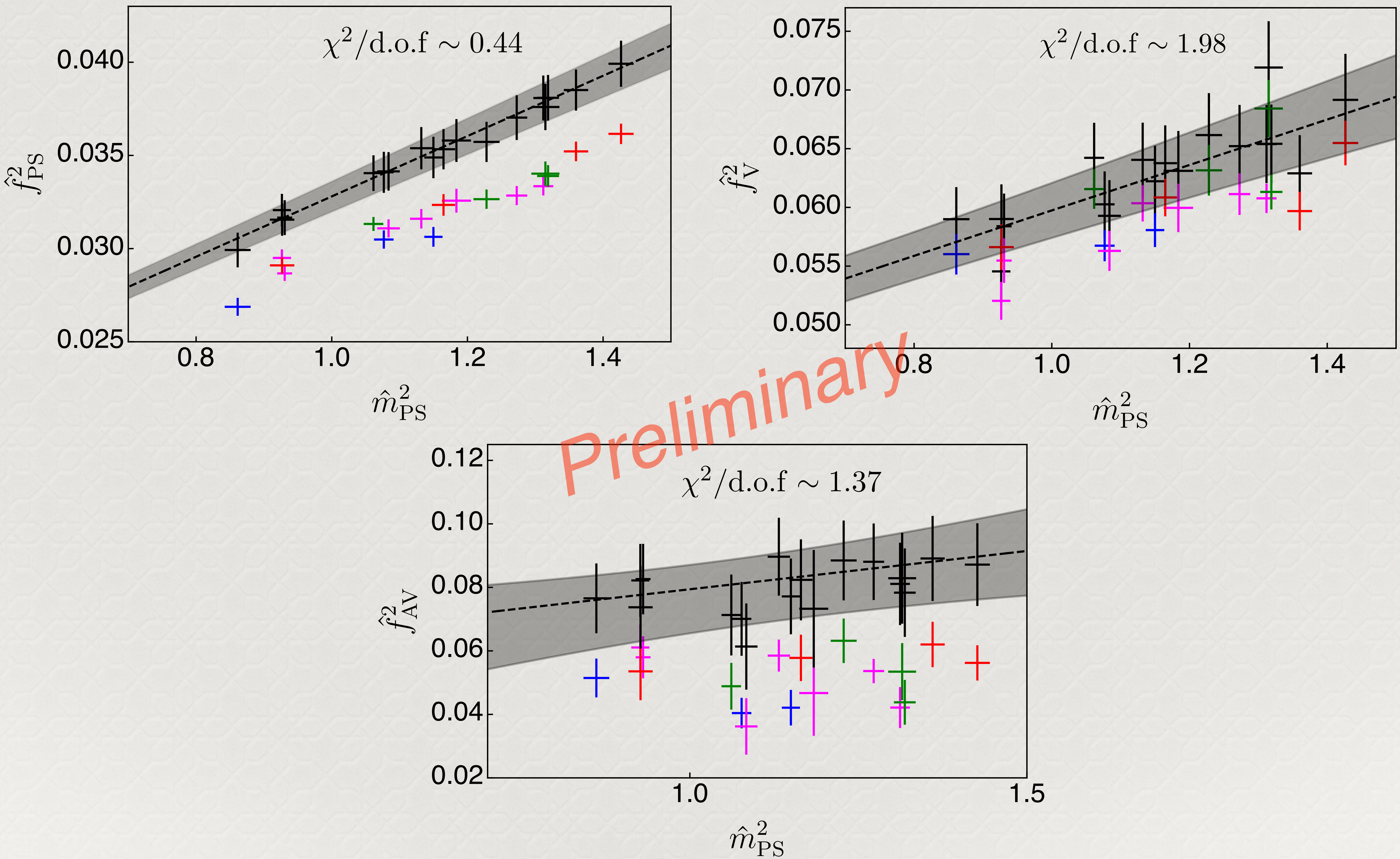
- Restrict to the continuum extrapolated results over the mass range of data available as our final results

- Still useful for the phenomenological model buildings for comp. Higgs, top partial compositeness and dark matter - *massless limit may not be required*

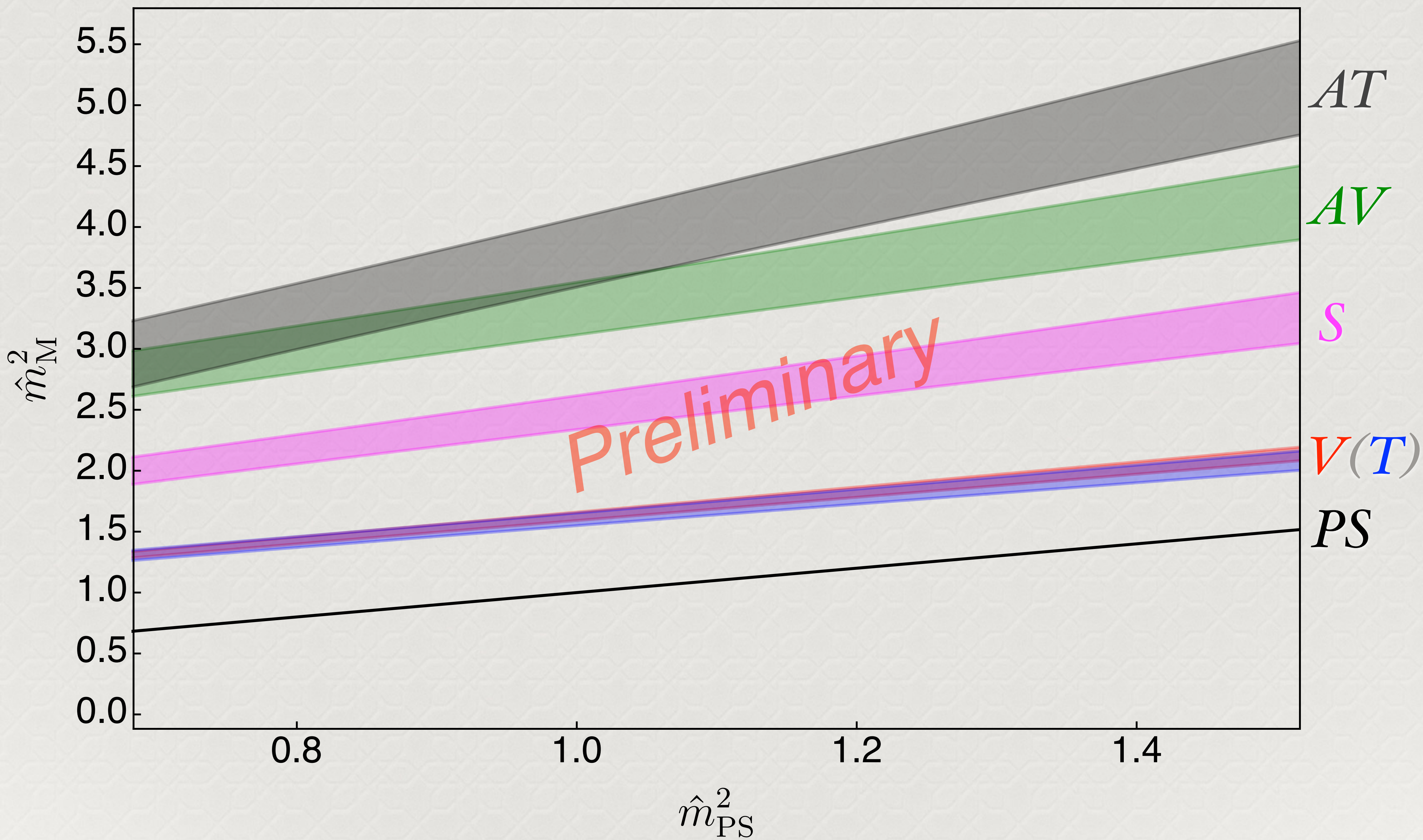
Results: masses in the continuum limit



Results: decay constants in the continuum limit



Meson spectrum of n_f=3 AS $Sp(4)$



Conclusion

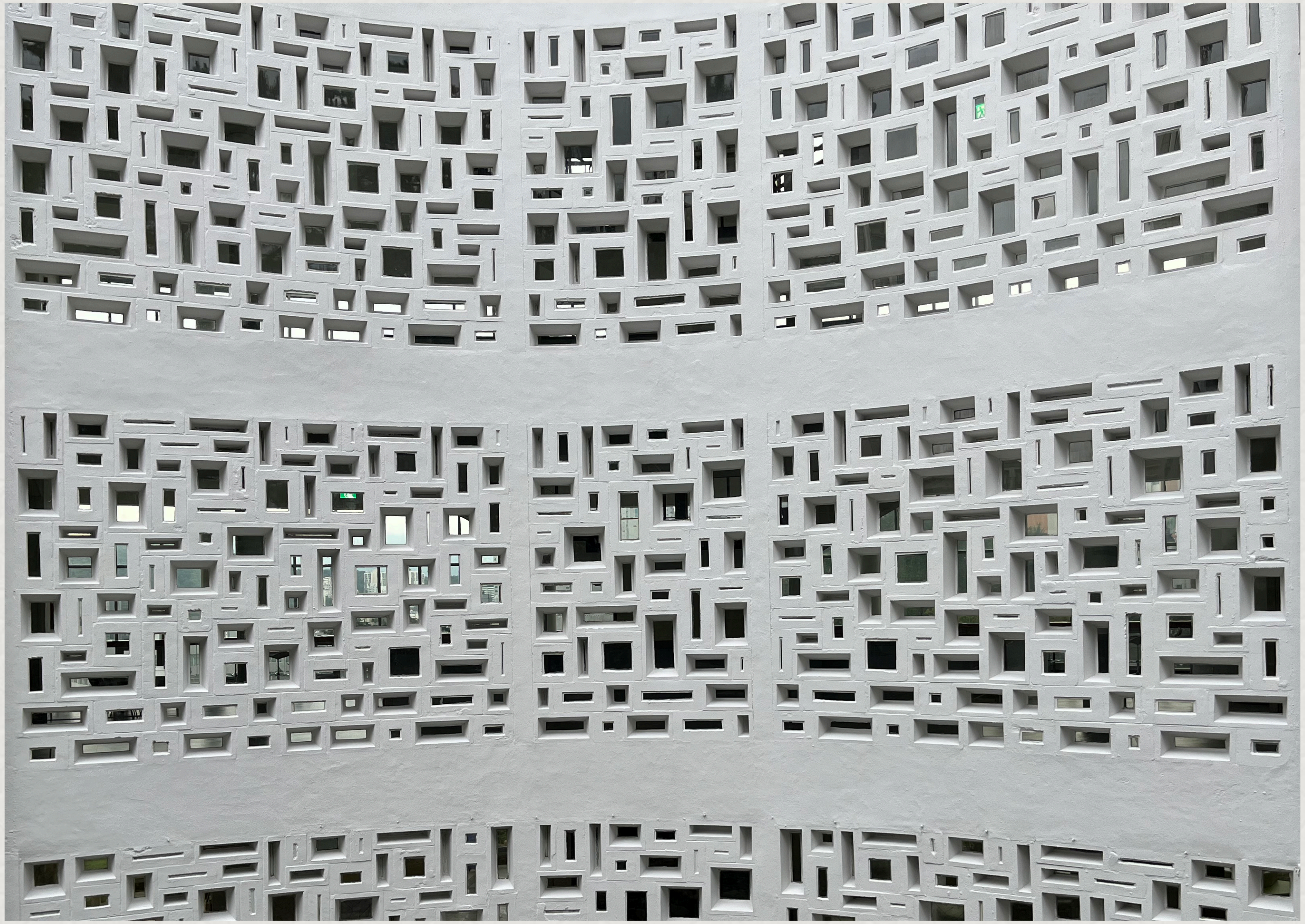
- Sp(4) theory with nf=3 antisymmetric fermions is relevant to top-partial compositeness, composite Higgs and dark matter

- We have studied the spectrum of mesons in spin-0 and 1 channels including the first excited state of the vector meson - no sign of (near) conformality

- Gradient flow scale shows a large mass dependence, which challenges to getting close to the massless limit

- Continuum extrapolation has been carried out only in the large mass regime using a simple linear ansatz for the PS mass squared and the lattice spacing

Could still be phenomenologically interesting for BSM physics based on new strong dynamics!



Thank you for your attention!