Non-perturbative study of Yang-Mills theory with four supercharges in two dimensions

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With Raghav G. Jha, Anosh Joseph, and David Schaich

Quick RECAP

Presented preliminary analysis in **Lattice 2021**.
$\Rightarrow$ Some remarks from the talk:

Scalars behaviour
Existence of bound state at finite temperature for $U(N)$ with $N = 2, 4, 8, 12$.
arXiv:2109.01001 [hep-lat] NSD, Jha, Joseph, Schaich

Comparison with 16 supercharge theory
Theory looks to be in different universality class to maximal theory.

Not discussed
Possible 'Spatial Deconfinement' transition.

This talk

of four supercharge theory on .

with maximal theory in regimes.

Signature of transition and its possible .

.

Two-dimensional $\mathcal{N} = (2, 2)$ SYM

Two-dimensional $N = (2; 2)$ SYM

Constructed from dimensional reduction of four dimensional theory.

$$N = 1, \; d = 4 \quad ! \quad N = (2, 2), \; d = 2$$

Not a "**maximal**" theory.

No holographic dual "**exists**".

Regularised on lattice using "**twisting**".

Phys. Rep. **484** (2009) 71-130

Catterall, Kaplan, Ünsal

Maximal Supersymmetric theories on Lattice talks:

Goksu Toga: Now TD-I

Angel Sherletov: Monday-5:10 pm

David Schaich: Monday-5:30 pm

Arpith Kumar: Wednesday-4:50 pm

Two-dimensional $N = (2;2)$ SYM

Continuum Action

$$S = \frac{N}{4} Q \int d^2x \,\mathrm{Tr} \quad {}_{ab}F_{ab} + \quad \overline{D}_a; D_a \quad \frac{1}{2} \; d$$

After integrating out auxiliary field

$$S = \frac{N}{4} \int d^2x \,\mathrm{Tr} \quad \overline{F}_{ab}F_{ab} + \frac{1}{2} \; \overline{D}_a; D_a \;{}^{2} \quad {}_{ab}D_{[a} \quad {}_{b]} \quad \overline{D}_a \quad {}_a$$

Two-dimensional $\mathcal{N} = (2,2)$ SYM

$$S = \frac{N}{4} \int d^2x \, \mathrm{Tr} \; \overline{F}_{ab} F_{ab} + \frac{1}{2} \overline{D}_a, D_a \;^2 \quad {}_{ab} D_{[a} \quad {}_{b]} \quad \overline{D}_a \quad {}_a$$

Using geometrical discretization $\rightarrow$ theory lives on 2d lattice.
JHEP **11** (2004) 006
Catterall

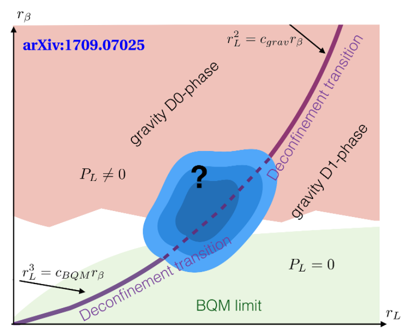
To control flat directions, scalar potential term added to discretized action.
JHEP **11** (2012) 072
Catterall, Damgaard, DeGrand, Galvez, Mehta

Discretization used and all the observables studied can be accessed via publicly available software github.com/daschaich/susy.

Comparison with Two-dimensional $N = (8, 8)$ SYM

Two-dimensional $N = (8; 8)$ SYM

At low temperature and large N) dual to type IIB supergravity.



PRD **97**, 086020 (2018)
Catterall, Jha, Schaich, Wiseman

Maximal theory prediction from gravity dual

Scalars behave as: $\mathrm{Tr}(X^2)$ / t.
JHEP **07** (2013) 101 *Wiseman*

Energy density / t^2 (for $t > 1$),
/ t^3 (for $t < 1$).
JHEP **07** (2013) 101 *Wiseman*

First order GL Phase transition.
PRL **70**, 2837 (1993) *Gregory, Laflamme*

Back to Target Theory

) Lattice simulations for four supercharge theory (

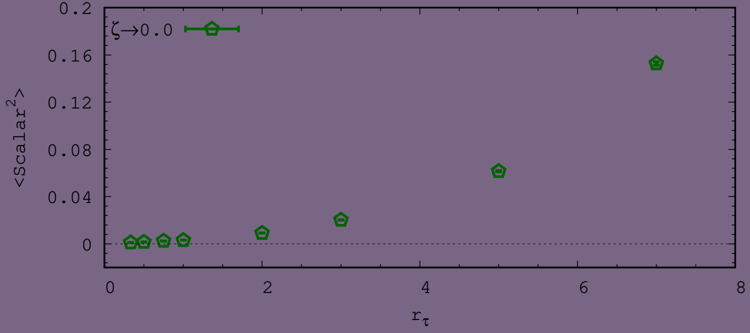
Worked with finite mass deformation parameter ,
$= \frac{r}{N_t} = \sqrt{} a, r = 1{=}t, \quad 0{:}33 \quad r \quad 7{:}0, \quad 2\ (0{:}2; 0{:}3; 0{:}4; 0{:}5)$.

Different Lattice aspect ratios ,
$= \frac{N_x}{N} = \frac{r_x}{r} \quad 2\ (0{:}5; 1{:}0; 1{:}5; 2{:}0)$.

Different gauge groups, $2 \quad N \quad 20$

Anti-periodic boundary conditions for fermions along temporal direction.

Scalar behaviour



Scalar^2 $ Tr(X)^2

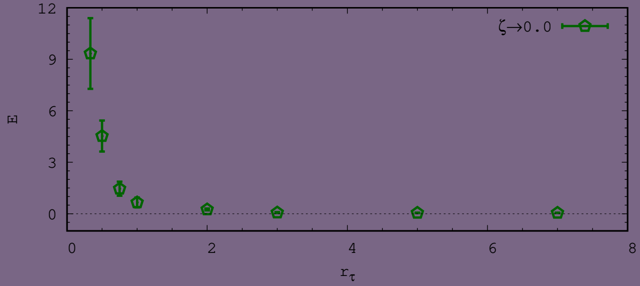
24 24 lattice,

N = 12.

r > 1 ! r^3 behaviour, Maximal case ! 1=r .

r < 1 ! r behaviour, Maximal case ??

Energy density



$E = \frac{3}{lat} \;\; 1 \qquad \frac{2}{3N^2} S_B$

24 24 lattice, $N = 12$.

$r \;\; > 1$! r^0 behaviour, Maximal case ! $1{=}r^3$.

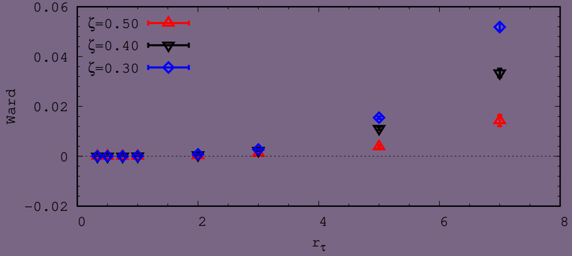
$r \;\; < 1$! $1{=}r^2$ behaviour, Maximal case ! $1{=}r^2$.

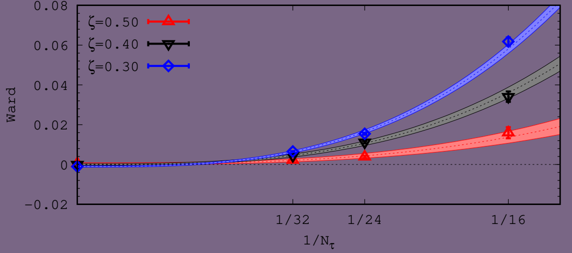
Vanishing energy density at zero temperature ! Preserved SUSY.

PRD **80**, 065014 (2009) *Hanada, Kanamori*

PRD **97**, 054504 (2018) *Catterall, Jha, Joseph*

Ward Identity





Ward Identity:

$Q \; P_a \; U_a U_a$

At larger temperatures (r < 1), ward identity satisfied.

At smaller temperatures (r > 1), satisfied at larger volume.

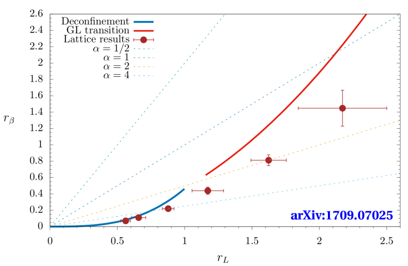
24 24 lattice, N = 12
Bottom left plot with
r = 5:0.

‘Spatial deconfinement’ in two-dimensional $\mathcal{N} = (2,2)$ SYM

Spatial deconfinement

Four supercharge theory:
so far .....

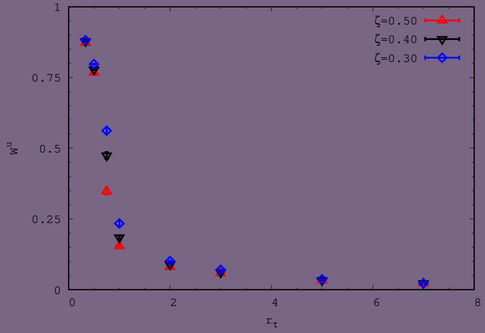
- Preserved SUSY.
- Different behaviour compared with maximal case.
- What about deconfinement transition? which exists in sixteen supercharge theory.

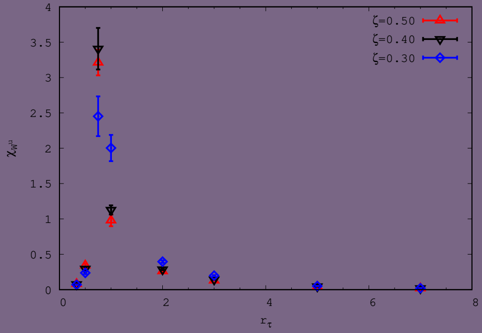


PRD **97**, 086020 (2018)

Catterall, Jha, Schaich, Wiseman

Spatial deconfinement - Signal



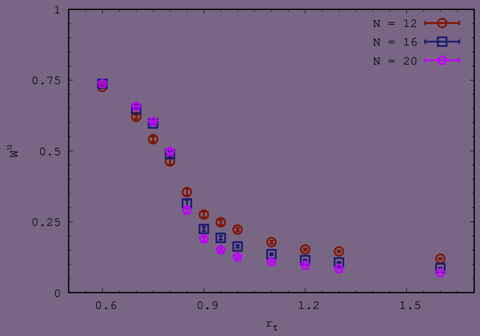


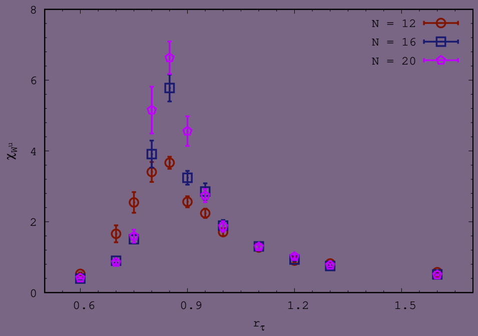
Spatial wilson lines and its susceptibilty as order parameter for deconfinement transition.

24 24 lattice, N = 12.

Transition around
r = 1:0.
Slight dependence.

Spatial deconfinement - Order





Spatial wilson lines and its susceptibilty for different *N* values.

12 12 lattice, = 0:30.

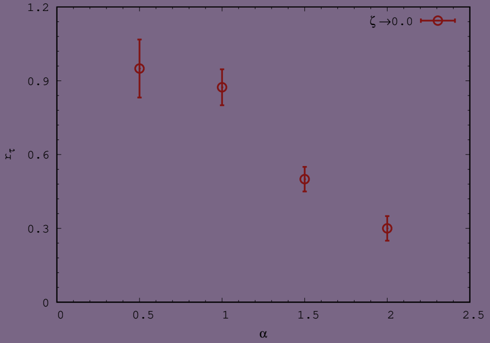
Critical *r* independent of *N*.

Hints of second-order transition.

PRL **113**, 091603 (2014)

Azuma, Morita, Takeuchi

Phase transition vs Lattice aspect ratio



N = 12, Lattices used
12 24; 12 12; 24 16; 24 12.
r (critical) has dependence for 1.

Spatial deconfinement transition similar to maximal theory but restricted only in weaker coupling regime.

Conclusions

Spatial deconfinement

Spatial deconfinement phase transition observed in this theory with different lattice volumes.

Weak coupling behaviour

Similar to maximal theory,
with different normalizations

- Phase transition observed.
- Energy density behaviour same.

Strong coupling behaviour

Different from maximal theory,

- No Phase transition.
- Scalars behaviour different.
- Energy density behaviour different.

Open question

Holographic dual to two-dimensional $\mathcal{N} = (2,2)$ SYM - - - - ???